Mechanism of Fluid-Mud Interactions under Waves
—— Large-Scale Simulation and Modeling

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OBJECTIVE

To integrate the theoretical studies, direct numerical and large-eddy simulation developments, tank measurements, and field experiments into an effective prediction tool for wavefield evolution over muddy bottom and topography.

• Development of a direct phase-resolved wavefield prediction tool (SNOW) for evolution over a horizontal length scale of $O(100 \lambda)$ (~10km) per dimension integrating through closure modeling wave transformation and dissipation mechanisms obtained at smaller scales ($O(\lambda)$) via asymptotic theory and DNS/LES, cross-calibrated with measurements

• Investigation of statistical characteristics of broad-band wave dissipation and dependence on mud layer properties using direct Monte-Carlo SNOW simulations

• Development of effective wave dissipation models for phase-averaged wavefield predictions using tools such as SWAN on scales of $O(10^{2-3})$km per dimension
**APPROACH**

- Wavefield evolution — phase-resolved simulation of nonlinear ocean waves (SNOW)
  - Nonlinear wave-wave and wave-current interactions
  - Nonlinear wave reflection/refraction/diffraction by bottom topography
  - Wave breaking dissipation
  - Large-scale broadband wavefield
  - Parallelized computations on HPC platforms

- Wave dissipation by muddy bottom — physics-based modeling in SNOW
  - Direct/indirect mechanisms: modifying bottom boundary conditions using equivalent damping
  - Resonant wave-wave interactions: nonlinear multi-layer flow with dissipations on interfaces
  - Turbulent mud-flow interactions: coupling LES/LWS with SNOW
  - Broadband wavefields: large-scale computations on HPC platforms

- Wavefield statistics — Monte Carlo simulations with SNOW computations

- Verification and parameterization of dissipation modeling — direct comparisons of SNOW computations with asymptotic theory, DNS/LES, laboratory experiments, and field measurements

- Understanding of dissipation mechanisms — Large-scale SNOW computations
  - Characteristics of wave dissipation and dependence on mud/wave properties
  - Development of dissipation models for phase-averaged SWAN computations
Sample SNOW Simulation of Large-Scale Nonlinear Ocean Wave-field Evolution

Rogue wave development in a short-crested nonlinear wavefield

Shown is a small portion of the large computational domain of 30km × 30km

Computing platform: IBM SP4 (256 processors)

Computation time: ~100 hours
Nonlinear Evolution of Directional Ocean Wave Spectrum

- Dissipation of short waves
- Broadening of spectrum
- Downshifting of peak

$H_s = 12m$
$T_p = 12sec$

JONSWAP spectrum: $\gamma = 3.3$
$\cos^2$ spreading: $\Theta = 80^\circ$
Nonlinear ocean wavefields exhibit frequency-dependent angular spreading with bi-modal spreading for short wave components. SNOW simulations capture these features in agreement with observations.

\[ H_s = 12m \]
\[ T_p = 12sec \]
Nonlinear ocean wavefields obtained from SNOW simulations exhibit a spectral slope of -2.5 for $k$ between $1.5k_p$ and $5k_p$, which is in agreement with observations.
Kurtosis Variation during Long-time Evolution of Long-crested Nonlinear Ocean Wave

Initial condition: JONSWAP spectrum ($\lambda_p=3.5\text{m}, \alpha=0.0135$)

- Wave statistics varies significantly during long-time nonlinear evolution for certain wave spectra. Nonlinear simulation results agree with experimental observations.
Dissipation Model in SNOW for Direct/Indirect Wave-Mud Interaction Mechanism

- Mud flow is assumed to be a viscoelastic flow
- Muddy bottom is modeled as a mass-spring-damper system over which surface waves propagate
- Surface wave motion is fully coupled with the muddy bottom dynamics
- In SNOW, muddy bottom effect is accounted for by the addition of equivalent damping and restoring terms in free-surface boundary conditions (and with a rigid bottom).
- Equivalent damping and restoring are derived such that the simulated wavefield (in SNOW) possesses the same linear dispersion relation as the original wave-mud system.
Formulation

Governing Equations:

\[ \nabla^2 \phi = 0 \]
\[ \phi_{tt} + g \phi_z = 0 \quad @ z = 0, \]
\[ \phi_z - \eta_{b,t} = 0 \quad @ z = -h, \]
\[ m^* \eta_{b,tt} + b^* \eta_{b,t} + k^* \eta_b + P_b = 0 \quad @ z = -h, \]
\[ \phi_t + g \eta_b + \frac{P_b}{\rho} = 0 \quad @ z = -h. \]

\[ m^* \phi_{zz,tt} + b^* \phi_{zt} + (k^* - \rho g) \phi_z - \rho \phi_{tt} = 0 \quad @ z = -h \]

\[ m^* = \frac{4}{\pi^2} \rho \ h_s \quad k^* = \frac{E}{h_s} \]

For Steel \[ = 3100 \text{ kg/m}^2 \quad =200E9 \text{ Kg/m}^2/\text{s}^2 \]
Dispersion Relation

Assuming a solution in the form
\[ \phi = (A e^{kz} + B e^{-kz}) e^{ikx + \omega t} \]

\[ B = A \frac{1 + \Omega^2}{1 - \Omega^2}, \quad \Omega^2 = \frac{\omega^2}{gk} \]

We get the Dispersion Relation:

\[
(\Omega_r^2 + \kappa \tanh \mu) \Omega^4 + 2\zeta \Omega_r \Omega^3 + (\Omega_r^2 \tanh \mu + 1)\Omega^2 + 2\zeta \Omega_r \tanh \mu \Omega + (1 - \kappa) \tanh \mu = 0
\]

where
\[ \Omega_r = \sqrt{\frac{m^*gk}{k^*}}, \] = frequency of the incident water wave / natural frequency of bottom
\[ \kappa = \frac{\rho g}{k^*}, \] = Restoring force of water / Restoring force of the Bottom \(< 1\)
\[ \zeta = \frac{b^*}{2\sqrt{k^*m^*}}, \] = Damping Coefficient \(<< 1\)
\[ \mu = k^*h. \] = Shallowness \(> 1\)
Surface and Bottom Eigen-Modes

\[ \kappa = 0.8 \quad , \quad \Omega_r = 2 \quad , \quad \zeta = 0.2 < \zeta_{cr} = 0.49, \]
Surface Boundary Condition Modification

Effect of the bottom is expressed as a modification term to the surface dynamic boundary condition.

Equivalent B.V.P.

\[
\nabla^2 \phi = 0 \\
\left[ \phi_{tt} + g \phi_z + \frac{b^*}{m^*} \phi_t + \frac{k^*}{m^*} \phi \right]_{zztt} \\
+ \frac{\rho}{m^*} \phi_{ztttt} + \frac{b^* g}{m^*} \phi_{zzzt} + \frac{g(k^* - \rho g)}{m^*} \phi_{zzz} = 0 \quad @ \ z = 0, \\

\phi_z = 0 \quad @ \ z = -h,
\]
Simplified Equivalent System

\[ \nabla^2 \phi = 0 \]

\[ \phi_{tt} + g(1 - A) \phi_z + \sqrt{\frac{g}{k \Omega}} \frac{BA}{B + 1} \phi_{zt} = 0 \quad \text{@ } z = 0 \]

\[ \phi_z = 0 \quad \text{@ } z = -h, \]

where

\[ A = \kappa \frac{1 - \Omega^4}{1 + \Omega_r^2 \Omega^2}, \quad B = \frac{2 \zeta \Omega_r \Omega}{1 + \Omega_r^2 \Omega^2} \]
The graph shows the rate of decay ($r$) as a function of some parameter ($\zeta$). The data points are marked with red circles, and the theoretical curve is represented by a blue line. Two specific values are highlighted as $\varepsilon_s = 0.1$ and $\varepsilon_s = 0.3$. The graph indicates a general increasing trend with $\zeta$.
Initial Spectrum

Normalized Spectral Density $S/f_0^2$

Normalized Wave Number ($kL_0$)

$\zeta=0.02$

$\zeta=0.1$

$\zeta=0.5$
Extension of SNOW to Multi-Layer Wave Motions and Interactions with Variable Bottom Topography

- Arbitrary high-order interactions of surface waves, interfacial waves, and bottom topography
- Broad-band surface/interfacial waves and bottom variations
- Multiple triad, quartet, and quintet resonant interactions of surface/interfacial/bottom waves
- High scalability in HPC platforms

Comparison of SNOW computation with theoretical predictions:

Generation of a reflected interfacial wave by triad resonant interactions between surface waves and bottom ripples:
• Development of direct and indirect wave dissipation models and integrate them into SNOW computations for phase-resolved wavefield evolution prediction

• Extending multi-layer wave simulations and modeling capabilities for generalized three-dimensional surface-wave and mud-layer wave resonant interactions