Comparison of design methods for locally slender steel columns

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acknowledgments

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overview

• motivation

• cross-section stability
  – AISC w/t limits
  – plate stability
  – local buckling

• design methods
  – stub columns
  – long columns

• conclusions
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?
motivation

Common structural steel design practice is to keep sections below yield limits and avoid local buckling. Why consider cross-section stability?

- post-buckling reserve

![Graph](image)
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

- post-buckling reserve
- minimizing material
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

• post-buckling reserve
• minimizing material
• increasing yield stress

Based on flange slenderness at 36 ksi, only 1 of the 267 standard W-sections is noncompact, but at 50 ksi 11 W-sections, at 65 ksi 27 W-sections, at 70 ksi 39 W-sections, at 100 ksi 94 W-sections, at 120 ksi 119 W-sections...
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

• post-buckling reserve
• minimizing material
• increasing yield stress
• extreme loads

buckle or fracture?
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why reconsider cross-section stability?

- post-buckling reserve
- minimizing material
- increasing yield stress
- extreme loads
- new design methods
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

- post-buckling reserve
- minimizing material
- increasing yield stress
- extreme loads
- new design methods
- accessible mechanics
**AISC definition of locally slender**

\[
\text{AISC limit: } \left( \frac{h}{t_w} \right)_{\text{lim}} = 1.49 \sqrt{\frac{E}{F_y}}
\]

<table>
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*Specification for Structural Steel Buildings, March 9, 2005*

AMERICAN INSTITUTE OF STEEL CONSTRUCTION, INC.
model behind w/t limits
no restraint between web and flange

\[ f_{cr} = k \left( \frac{\pi^2 E}{12(1-\nu^2)} \right) \left( \frac{t}{w} \right)^2 \]

\[ k_f = 0.43 \]

\[ k_w = 4 \]
full restraint between web and flange

\[ k_f = 1.277 \]

\[ f_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t}{w} \right)^2 \]

\[ k_w = 6.97 \]
assumed restraint between web and flange

\[ k_f = 0.7 \ (0.4-1.3 \text{ range}) \]

\[ f_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t}{w} \right)^2 \]

\[ k_w = 5 \ (4-7 \text{ range}) \]
• Equilibrium

\[ f_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t}{w} \right)^2 \]

\[ f_{cr} = 5.0 \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t_w}{h} \right)^2 \]

\[ = 0.7 \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t_f}{b} \right)^2 \]

only true for \( \frac{h}{t_w} = \sqrt{\frac{5.0}{0.7}} \left( \frac{b}{t_f} \right) \)!

cannot be just one \( k \) value..
limitations

- Equilibrium

\[ f_{cr} = k \frac{\pi^2 E}{12(1-v^2)} \left( \frac{t}{w} \right)^2 \]

\[ f_{cr} = 5.0 \frac{\pi^2 E}{12(1-v^2)} \left( \frac{t_w}{h} \right)^2 \]

\[ = 0.7 \frac{\pi^2 E}{12(1-v^2)} \left( \frac{t_f}{b} \right)^2 \]

only true for \( \frac{h}{t_w} = \sqrt{\frac{5.0}{0.7} \left( \frac{b}{t_f} \right)} \! \)!

cannot be just one \( k \) value..
limitations

- **Compatibility**
  - rotation at web/flange juncture

- **Bounds are false**
  - if one element buckles before another it demands rotational restraint from the neighbor. simple support condition is not a lower bound!

- **cross-section local buckling is needed**
cross-section stability by finite strip method

- mesh cross-section with element strips
- each element follows classical plate bending theory
- result of buckling analysis is the local buckling mode and stress of the full cross-section

\[ f_{cr\ell} = \text{xx ksi} \]
finite strip results for W-sections

\[
f_{crb} = k_f \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t_f}{b} \right)^2
\]
simple relations for cross-section stability

W-sections

flange, $k_f$

web slenderness $h/t_w$

$k_f = k_w \left(\frac{h}{t_w}\right)^2 \left(\frac{2t_f}{b_f}\right)$
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• design methods
  – stub columns
  – long columns
• conclusions
• Locally slender stub column
  — design expressions
  — comparison of predictions

• Locally slender long column
  — design expressions
  — comparison of predictions
AISC – Locally slender stub column

\[ P_n = Q_s Q_a A g f_y \]
AISI – Locally slender stub column

\[ P_n = A_{\text{eff}} f_y \]
DSM – Locally slender stub column

\[ P_n = A_{\text{eff}} f_y \]
Stub Column

Typical heavy W14 dimensions:

\[ \frac{ht_w}{A_g} = 0.2 \quad 2b_t/A_g = 0.8 \]

\[ \frac{f_{crb}}{f_{crh}} = 0.8, \quad \frac{f_{crb}}{f_{crlocal}} = 1.3 \]
Stub Column

Typical heavy W36 dimensions:

\[ \frac{h_{tw}}{A_g} = 0.4 \quad 2b \frac{t_f}{A_g} = 0.6 \]

\[ \frac{f_{crb}}{f_{crh}} = 8, \quad \frac{f_{crb}}{f_{crlocal}} = 6 \]

transitioning through AISC \(Q_s\) equations

flange becomes partially effective

local web slenderness \(\left(\frac{f_y}{f_{crh}}\right)^{0.5}\)
consider replacing $f_{crb}$ and $f_{crh}$ with $f_{cr\ell}$ ...
\[ P_n = A_{eff} f_y \]

\[ A_{eff} = 4 \rho_b b t_f + \rho_h h t_w \]

\[ b_e = \rho_b b \quad \text{where} \quad \rho_b = \begin{cases} 1 & \text{if} \quad f_{crb} \geq 2.2 f_y \\ 1 - 0.22 \sqrt{\frac{f_{crb}}{f_y}} \sqrt{\frac{f_{crb}}{f_y}} & \text{if} \quad f_{crb} < 2.2 f_y \end{cases} \]

\[ h_e = \rho_h h \quad \text{where} \quad \rho_h = \begin{cases} 1 & \text{if} \quad f_{crh} \geq 2.2 f_y \\ 1 - 0.22 \sqrt{\frac{f_{crh}}{f_y}} \sqrt{\frac{f_{crh}}{f_y}} & \text{if} \quad f_{crh} < 2.2 f_y \end{cases} \]
introduce $f_{cr\ell}$

$$P_n = A_{eff} f_y$$

$$A_{eff} = 4 \rho_b b t_f + \rho_h h t_w$$

$$b_e = \rho_b b \text{ where } \rho_b = \begin{cases} 
1 & \text{if } f_{cr\ell} \geq 2.2 f_y \\
(1 - 0.22 \sqrt{\frac{f_{cr\ell}}{f_y}}) \sqrt{\frac{f_{cr\ell}}{f_y}} & \text{if } f_{cr\ell} < 2.2 f_y
\end{cases}$$

$$h_e = \rho_h h \text{ where } \rho_h = \begin{cases} 
1 & \text{if } f_{cr\ell} \geq 2.2 f_y \\
(1 - 0.22 \sqrt{\frac{f_{cr\ell}}{f_y}}) \sqrt{\frac{f_{cr\ell}}{f_y}} & \text{if } f_{cr\ell} < 2.2 f_y
\end{cases}$$
\( \rho \) is now for the section

\[ P_n = A_{eff} f_y \]

\[ A_{eff} = 4 \rho b t_f + \rho h t_w \]

\[ b_e = \rho b \text{ where } \rho = \begin{cases} 1 & \text{if } f_{cr\ell} \geq 2.2 f_y \\ 1 - 0.22 \frac{f_{cr\ell}}{f_y} & \sqrt{\frac{f_{cr\ell}}{f_y}} \text{if } f_{cr\ell} < 2.2 f_y \end{cases} \]

\[ h_e = \rho h \text{ where } \rho = \begin{cases} 1 & \text{if } f_{cr\ell} \geq 2.2 f_y \\ 1 - 0.22 \frac{f_{cr\ell}}{f_y} & \sqrt{\frac{f_{cr\ell}}{f_y}} \text{if } f_{cr\ell} < 2.2 f_y \end{cases} \]
implying a simpler procedure

\[ P_n = A_{\text{eff}} f_y \]

\[ A_{\text{eff}} = 4 \rho \ b t_f + \rho \ h t_w = \rho \ A_g \]

where \( \rho \) is given by

\[ \rho = \begin{cases} 1 & \text{if } f_{cr} \geq 2.2 f_y \\ 1 - 0.22 \sqrt{\frac{f_{cr}}{f_y}} & \sqrt{\frac{f_{cr}}{f_y}} & \text{if } f_{cr} < 2.2 f_y \end{cases} \]
similarity to DSM very strong

\[ P_n = A_{eff} f_y \]

\[ A_{eff} = \rho A_g \]

\[ \rho = \begin{cases} 
1 & \text{if } f_{cr} \geq 1.66 f_y \\
1 - 0.15 \left( \frac{f_{cr}}{f_y} \right)^{0.4} \left( \frac{f_{cr}}{f_y} \right)^{0.4} & \text{if } f_{cr} < 1.66 f_y 
\end{cases} \]
AISC approach does not simplify similarly

\[ P_n = Q_s Q_a A_g f_y \]

\[ Q_s = \begin{cases} 
1.0 & \text{if } f_{cr\ell} \geq 2f_y \\
1.415 - 0.59 \sqrt{\frac{f_y}{f_{cr\ell}}} & \text{if } \frac{3}{5} f_y < f_{cr\ell} < 2f_y \\
1.1 \frac{f_{cr\ell}}{f_y} & \text{if } f_{cr\ell} \leq \frac{3}{5} f_y 
\end{cases} \]

\[ Q_a = \begin{cases} 
1.0 & \text{if } f_{cr\ell} > 2f_y \\
1 - \left(1 - 0.9 \sqrt{\frac{f_{cr\ell}}{f_y}} \left(1 - 0.16 \sqrt{\frac{f_{cr\ell}}{f_y}} \right) \frac{ht_w}{A_g} \right) & \text{if } f_{cr\ell} \leq 2f_y 
\end{cases} \]
if $f_{cr,l}$ has replaced $f_{cr,b}$ and $f_{cr,h}$

local slenderness of W14's

at $f_y=36$ ksi from ~ 0.1 to 0.8

at $f_y=100$ ksi from ~ 0.1 to 1.3

local slenderness ($f_y/f_{cr,local}^{0.5}$)
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modifying the global column curve

strength

global slenderness

AISI

AISC
\[ P_n = A_g \hat{f}_n \]

\[
\hat{f}_n = \begin{cases} 
Q_s Q_a (0.658) (f_e / f_y) & \text{if } f_e \geq 0.44Q_s Q_a f_y \\
0.877 f_e & \text{if } f_e < 0.44Q_s Q_a f_y 
\end{cases}
\]
\[ P_n = A_{\text{eff}} f_n \]

\[ f_n = \begin{cases} (0.658)^{\left(f_e / f_y\right)} f_y & \text{if } f_e \geq 0.44 f_y \\ 0.877 f_e & \text{if } f_e < 0.44 f_y \end{cases} \]

\[ A_{\text{eff}} = 4 \rho_b b t_f + \rho_h h t_w \]

\[ b_e = \rho_b b \text{ where } \rho_b = \begin{cases} 1 & \text{if } f_{crb} \geq 2.2 f_n \\ 1 - 0.22 \sqrt{\frac{f_{crb}}{f_n}} \sqrt{\frac{f_{crb}}{f_n}} & \text{if } f_{crb} < 2.2 f_n \end{cases} \]

\[ h_e = \rho_h h \text{ where } \rho_h = \begin{cases} 1 & \text{if } f_{crh} \geq 2.2 f_n \\ 1 - 0.22 \sqrt{\frac{f_{crh}}{f_n}} \sqrt{\frac{f_{crh}}{f_n}} & \text{if } f_{crh} < 2.2 f_n \end{cases} \]
\[ P_n = A_{\text{eff}} f_n \]

\[ f_n = \begin{cases} 
(0.658)^{(f_e/f_y)} f_y & \text{if } f_e \geq 0.44 f_y \\
0.877 f_e & \text{if } f_e < 0.44 f_y 
\end{cases} \]

\[ A_{\text{eff}} = \rho A_g \]

\[ \rho = \begin{cases} 
1 & \text{if } f_{cr} \geq 1.66 f_n \\
1 - 0.15 \left( \frac{f_{cr}}{f_n} \right)^{0.4} \left( \frac{f_{cr}}{f_n} \right)^{0.4} & \text{if } f_{cr} < 1.66 f_n 
\end{cases} \]
Typical heavy W14 dimensions: 
\[ \frac{h_{tw}}{A_g} = 0.2, \quad \frac{2b_{tf}}{A_g} = 0.8, \quad \frac{f_{crb}}{f_{crh}} = 0.8, \quad \frac{f_{crb}}{f_{crlocal}} = 1.3 \]

(a) compact: flange slenderness of 0.1, i.e. \( f_{crb} = 100f_y \)
Concluding thoughts

• Cross-section stability is worthy of continued study and current approaches employed in structural steel design have inherent limitations

• Strength models for locally slender columns all essentially use the same parameters

  AISC: \[ \frac{P_n}{P_y} = f(f_{cr-global}, f_y, f_{crb}, f_{crh}, \frac{h t_w}{A_g}) \]

  AISI: \[ \frac{P_n}{P_y} = f(f_{cr-global}, f_y, f_{crb}, f_{crh}, \frac{h t_w}{A_g}) \]

  DSM: \[ \frac{P_n}{P_y} = f(f_{cr-global}, f_y, f_{cr\ell}) \]

• When local flange slenderness controls predicted strength AISC Q-factor predictions are lower than AISI or DSM

• Work continues

more at www.ce.jhu.edu/bschafer/aisc
Web local slenderness \( (f_y/f_{crh})^{0.5} = 0.1 \)

Typical heavy W36 dimensions:
- \( \frac{h_w}{A_g} = 0.4 \)
- \( \frac{2b_f}{A_g} = 0.6 \)
- \( \frac{f_{crb}}{f_{crh}} = 8 \)
- \( \frac{f_{crb}}{f_{crlocal}} = 6 \)

(a) compact: web slenderness of 0.1, i.e. \( f_{crh} = 100f_y \)
(a) compact: local slenderness of 0.1, i.e. $f_{cr} = 100f_y$
AISC Faculty Fellowship

Cross-section Stability of Structural Steel

Research
Objective
Investigate the application of the Direct Strength Method (DSM) to structural steel shapes, and to provide the necessary research advances to make this an advantageous option for the design of noncompact and slender structural steel shapes. A goal of DSM is to provide a design method which is robust enough to allow engineers to realistically explore novel cross-sections, yet make this exploration simple. Investigate the potential of DSM to take a fresh look at hot-rolled steel structural shapes.

Work Products
Proposal (pdf - February 2005)
Progress Report #1 (pdf - June 2007)
Progress Report #2 (pdf - April 2008)

Related Links
CUFSM
constrained Finite Strip Method
Direct Strength Method for Cold-Formed Steel
Reliability and Advanced Analysis of Steel Frames
Frame design/robustness under unforeseen events

Education
Objective
Provide tools, tutorials, and educational aids related to cross-section stability of structural steel shapes so that educators, students, and engineers may explore these concepts more readily. Provide educational aids appropriate for courses in steel design using structural steel at the undergraduate and graduate levels.

Work Products
CUFSM, software for cross-section stability analysis
Example files for structural steel shapes
W36x150, W14x120, C5x9, L4x4x1/2, WT 18x150, HSS 4x4x1/2 (all example files in a zip folder)

Tutorial 1: Cross-section stability of a W36x150 using the finite strip method (ppt) (pdf)
Learning objectives: (1) Identify all the buckling modes in a W-section, for columns explore flexural (Euler) buckling and local buckling, for beams explore lateral-torsional buckling and local buckling; (2) Predict the buckling stress (load or moment) for identified buckling modes, (3) Learn the interface of a simple program for exploring cross-section stability of...