Comparison of design methods for locally slender steel columns

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acknowledgments

- AISC Faculty Fellowship Program
- Don White, particularly during the development of the 2005 AISC Spec.

overview

- motivation
- cross-section stability
  - AISC w/t limits
  - plate stability
  - local buckling
- design methods
  - stub columns
  - long columns
- conclusions

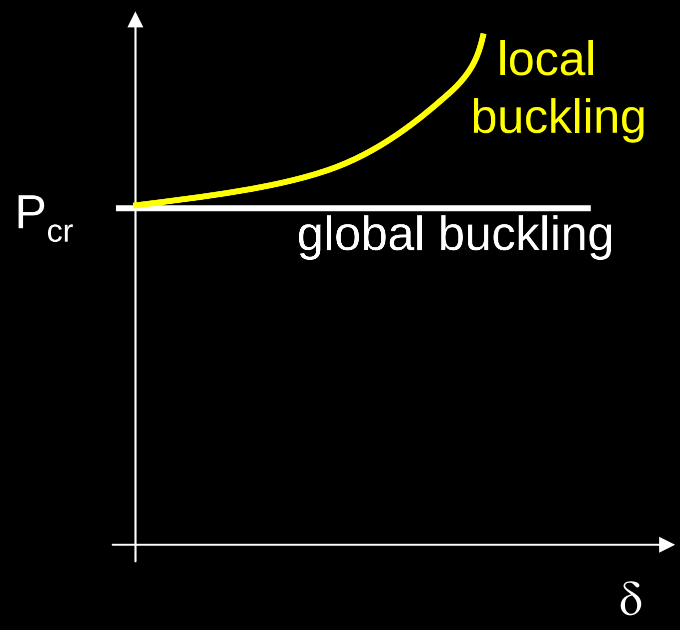
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling.
Why consider cross-section stability?

motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

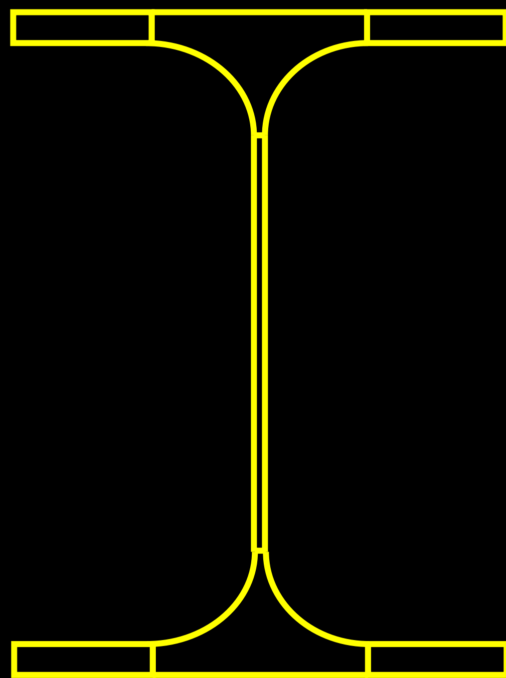
- post-buckling reserve



motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

- post-buckling reserve
- minimizing material



motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling.
Why consider cross-section stability?

- post-buckling reserve
- minimizing material
- increasing yield stress

Based on flange slenderness
at 36 ksi, only 1 of the 267 standard W-sections is noncompact, but
at 50 ksi 11 W-sections,
at 65 ksi 27 W-sections,
at 70 ksi 39, W-sections
at 100 ksi 94, W-sections
at 120 ksi 119 W-sections...

motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

- post-buckling reserve
- minimizing material
- increasing yield stress
- extreme loads

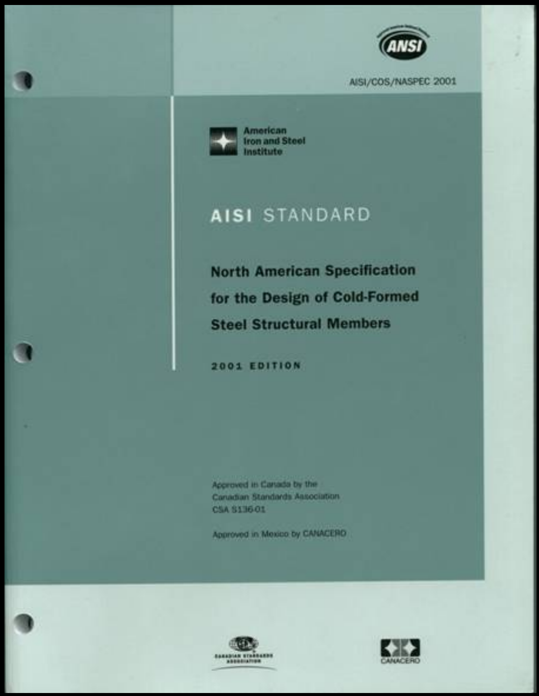
buckle

or **frac ture** ?

motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why reconsider cross-section stability?

- post-buckling reserve
- minimizing material
- increasing yield stress
- extreme loads
- new design methods



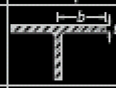
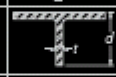
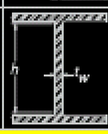
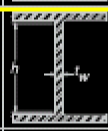
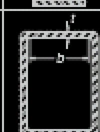
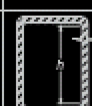
motivation

Common structural steel design practice is to keep sections below w/t limits and avoid local buckling. Why consider cross-section stability?

- post-buckling reserve
- minimizing material
- increasing yield stress
- extreme loads
- new design methods
- accessible mechanics

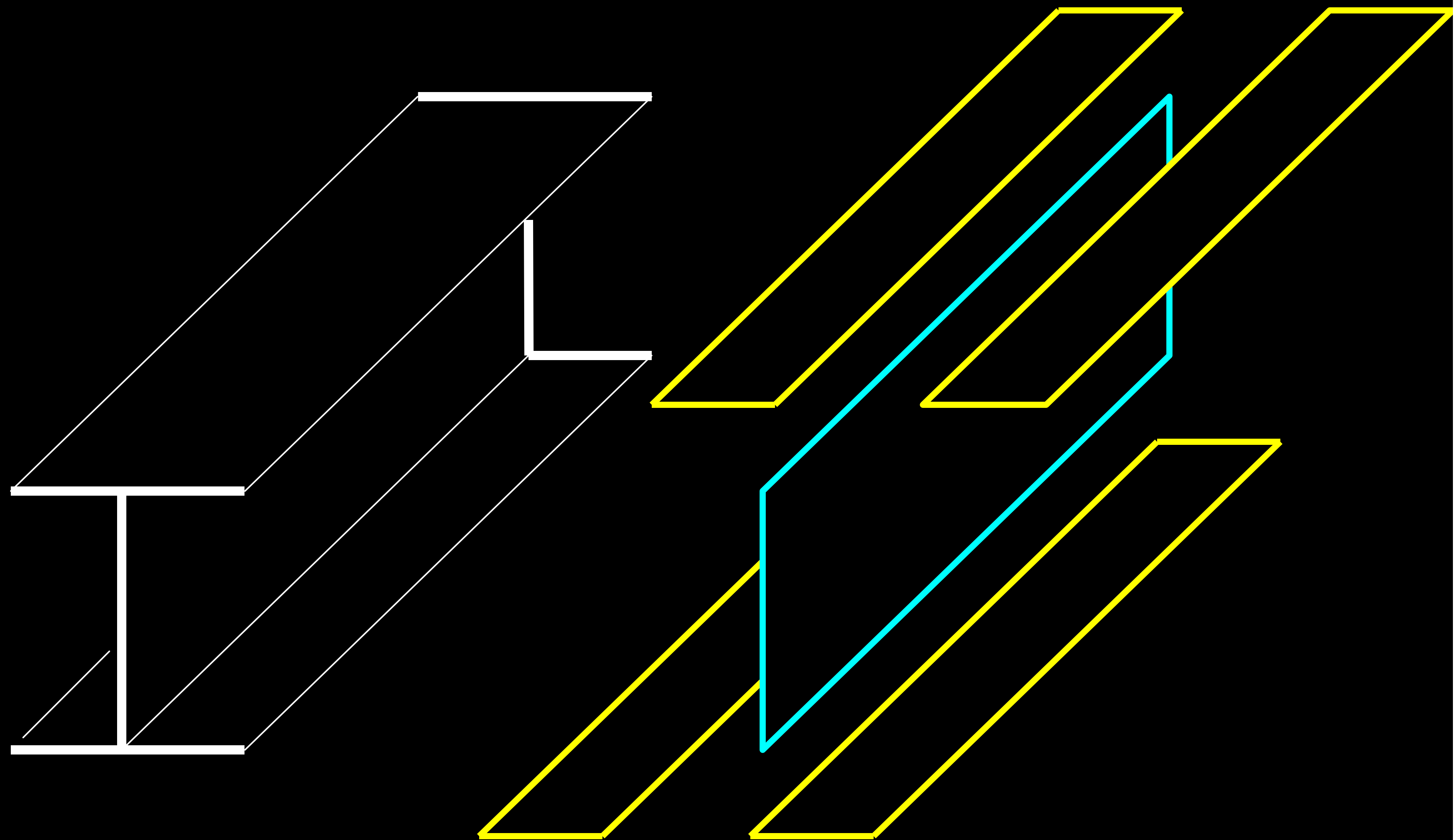
AISC definition of locally slender

TABLE B4.1 (cont.)
Limiting Width-Thickness Ratios for Compression Elements

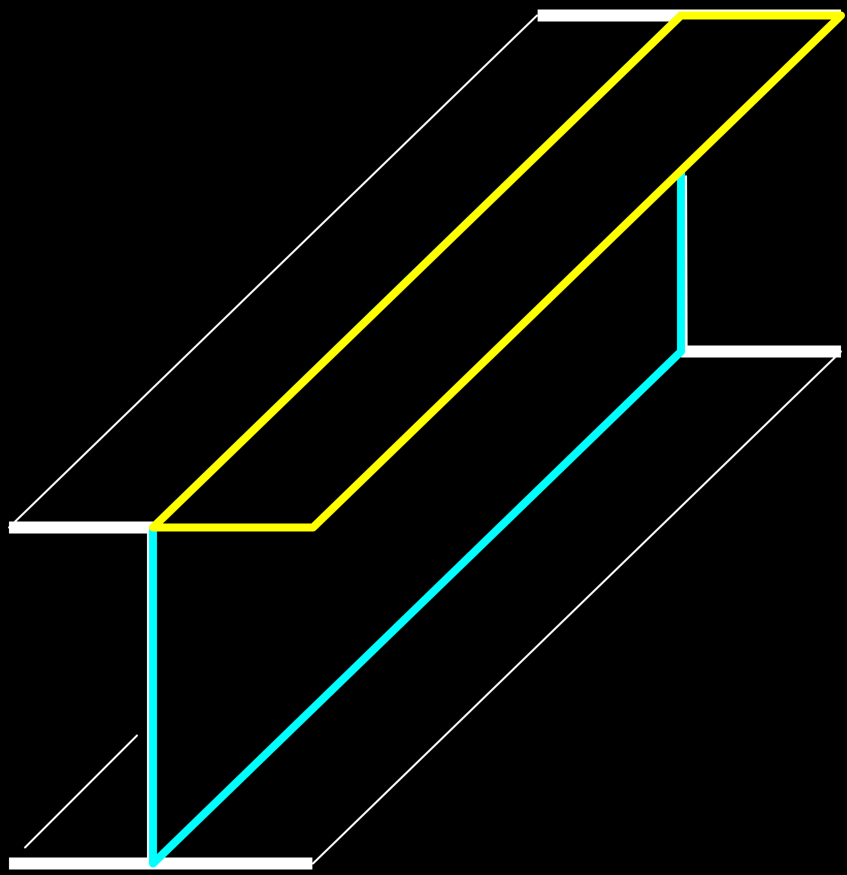
	Case	Description of Element	Width Thick-ness Ratio	Limiting Width-Thickness Ratios: λ_p (compact)	Limiting Width-Thickness Ratios: λ_r (noncompact)	Example
	7	Flexure in flanges of tees	b/t	$0.38\sqrt{E/F_y}$	$1.0\sqrt{E/F_y}$	
	8	Uniform compression in stems of tees	d/t	NA	$0.75\sqrt{E/F_y}$	
	9	Flexure in webs of doubly symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$	
Stiffened Elements	10	Uniform compression in webs of doubly symmetric I-shaped sections	h/t_w	NA	$1.49\sqrt{E/F_y}$	
	11	Flexure in webs of singly-symmetric I-shaped sections	h_c/t_w	$\dfrac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y}-0.09\right)^2} \le \lambda_r$	$5.70\sqrt{E/F_y}$	
	12	Uniform compression in flanges of rectangular box and hollow structural sections of uniform thickness subject to bending or compression; flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$	
	13	Flexure in webs of rectangular HSS	h/t	$2.42\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$	

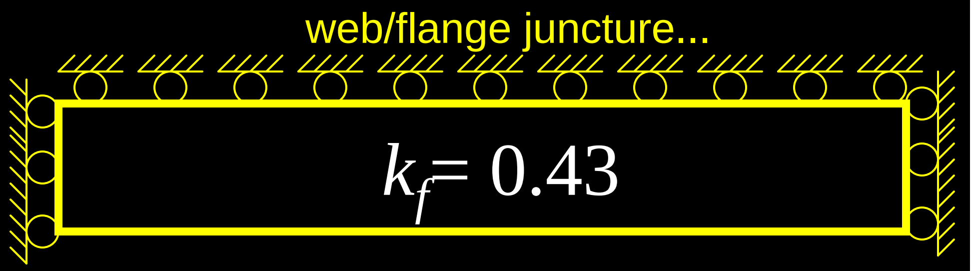
$$\text{AISC limit}: \left(\frac{h}{t_w}\right)_{lim} = 1.49\sqrt{\frac{E}{F_y}}$$

model behind w/t limits

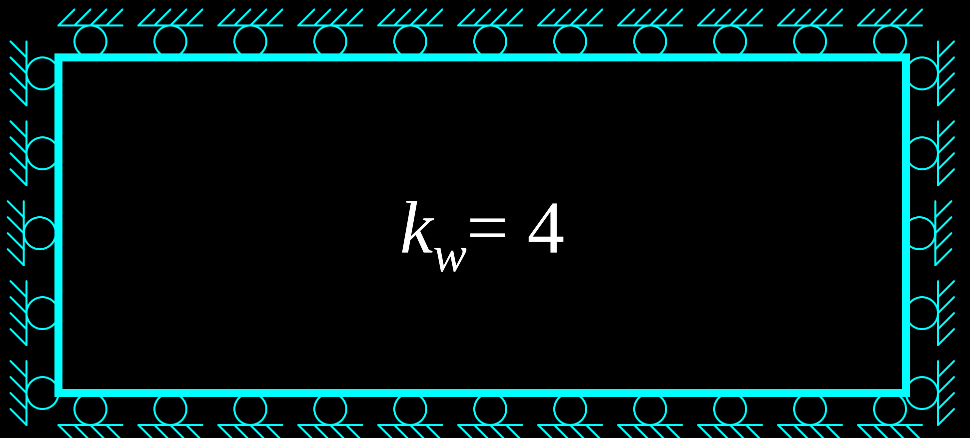


no restraint between web and flange

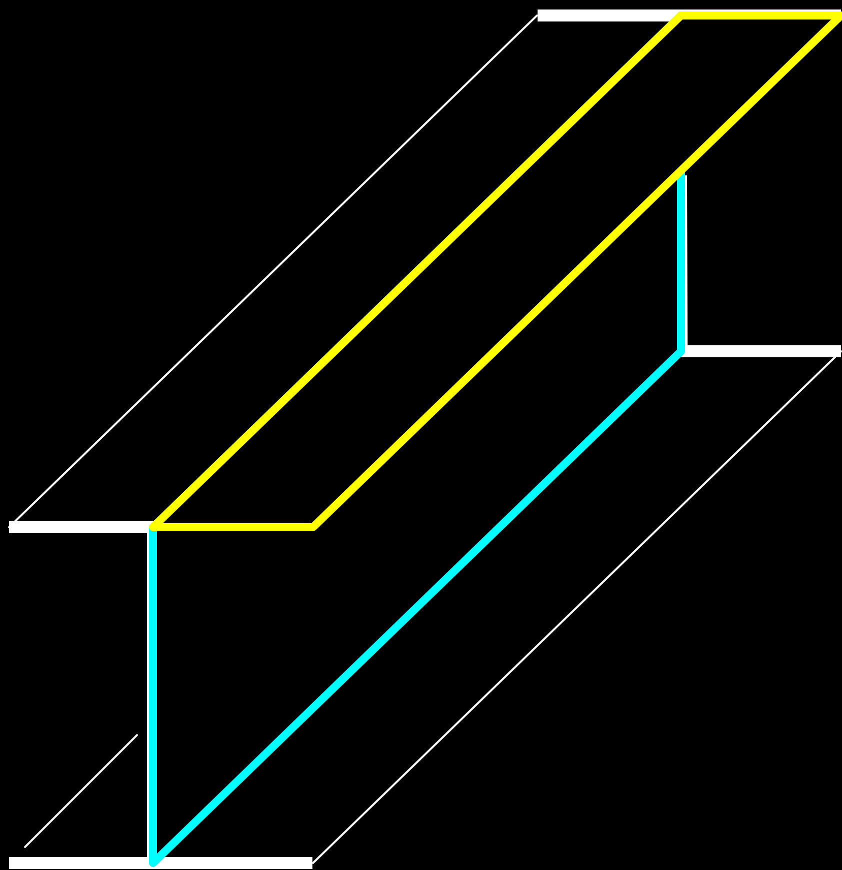




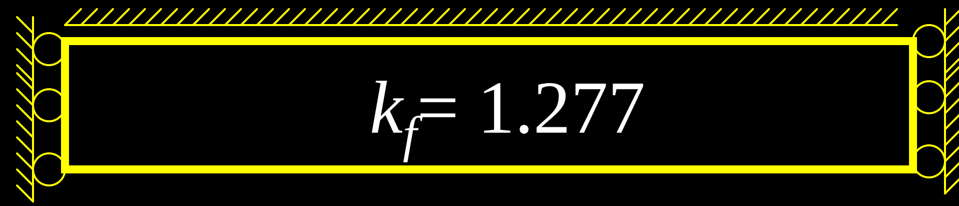
$$f_{cr} = k \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{w}\right)^2$$



full restraint between web and flange



web/flange juncture...



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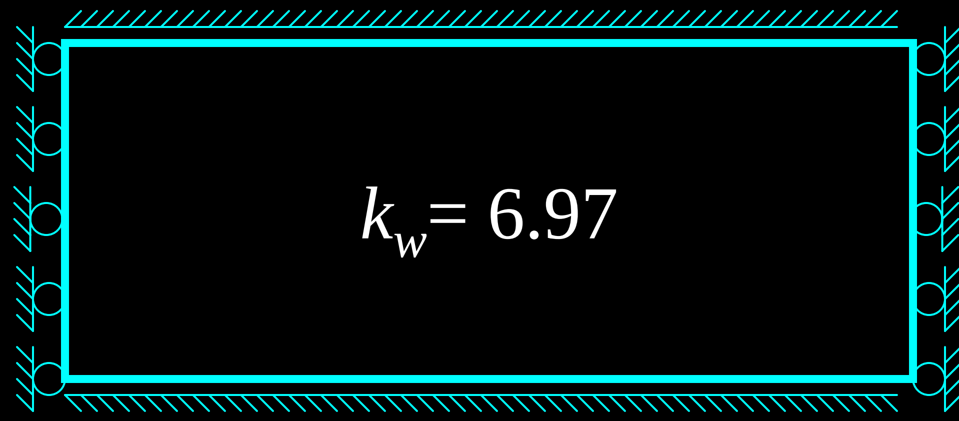
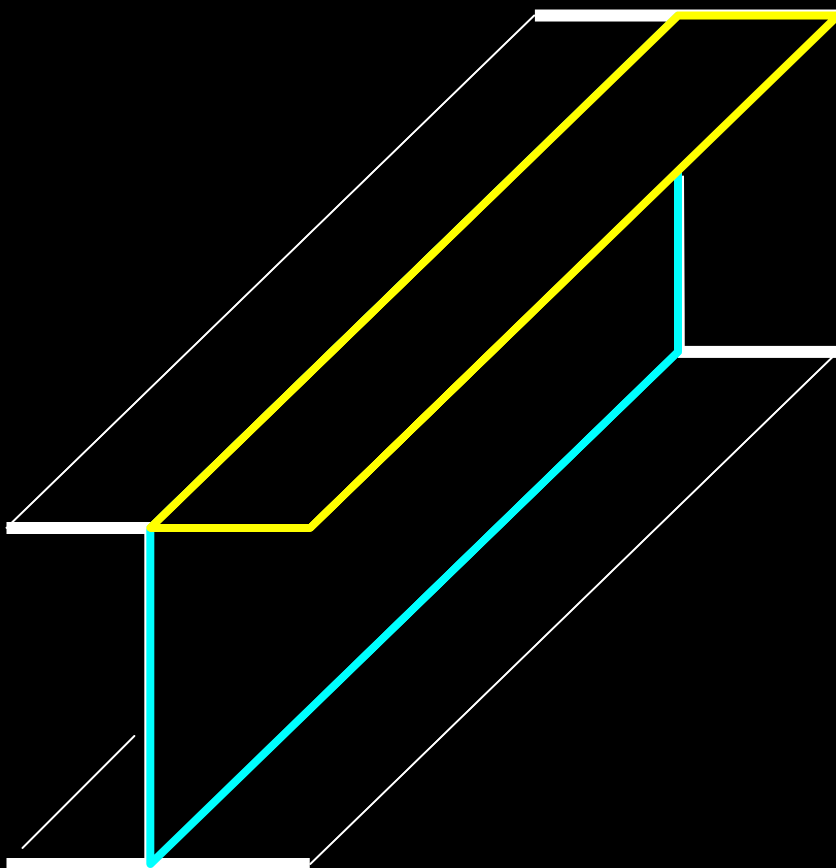
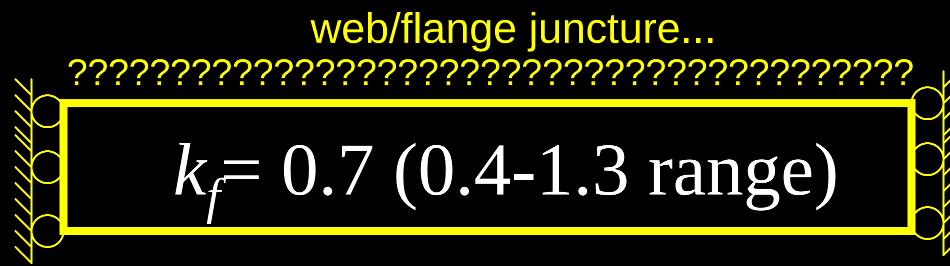


assumed restraint between web and flange





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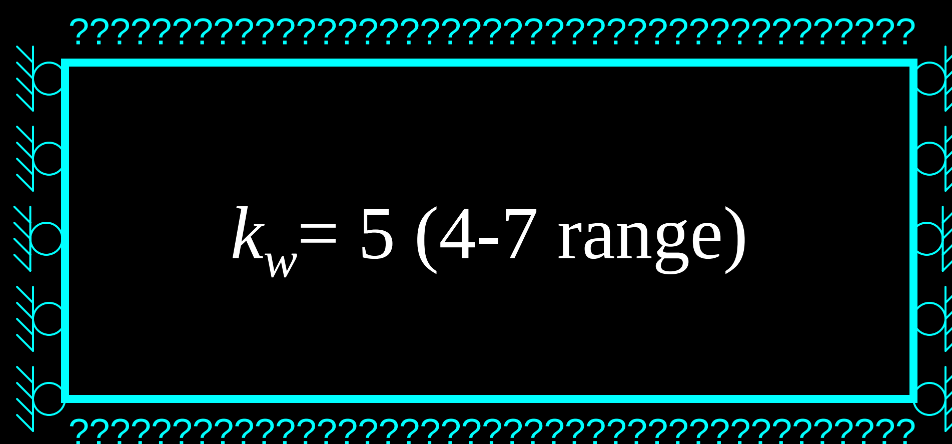


limitations

- Equilibrium

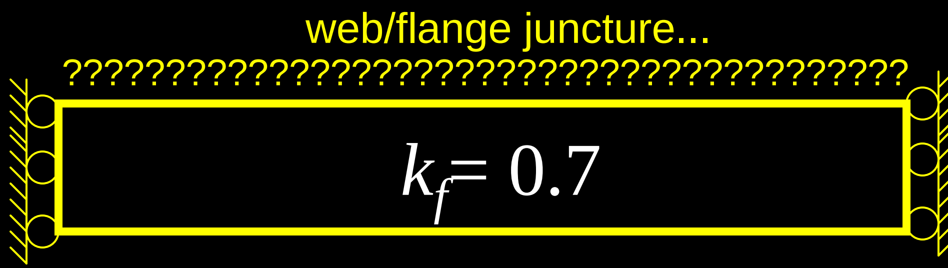
$$f_{cr} = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{w}\right)^2$$

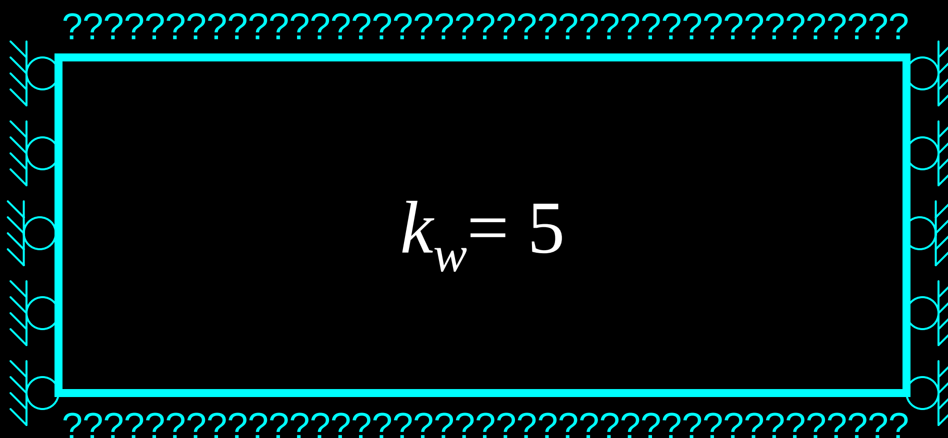
$$f_{cr} = 5.0\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_w}{h}\right)^2$$

$$= 0.7\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_f}{b}\right)^2$$

$$\text{only true for } \frac{h}{t_w} = \sqrt{\frac{5.0}{0.7}}\left(\frac{b}{t_f}\right)!$$

cannot be just one *k* value..





limitations

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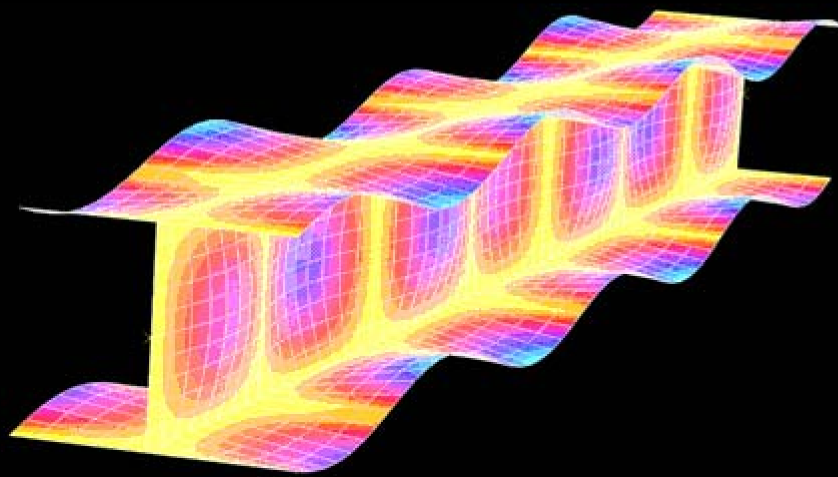
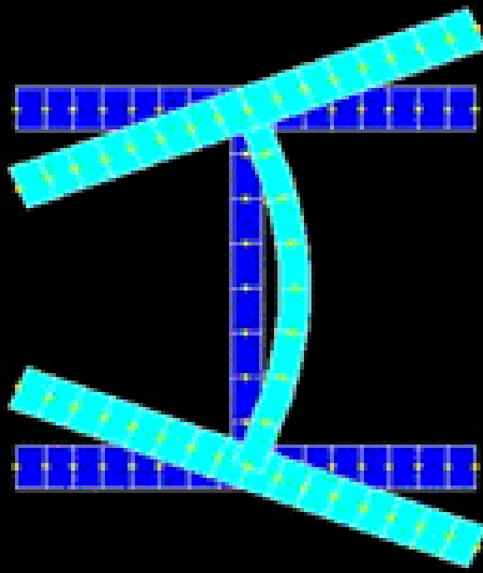
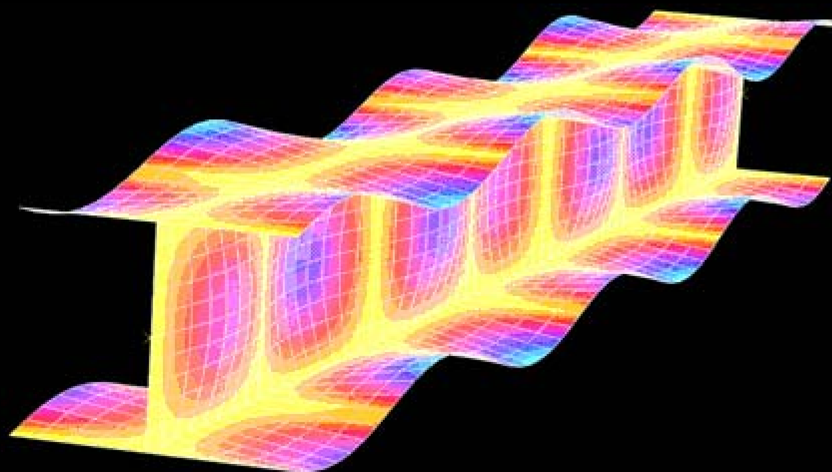
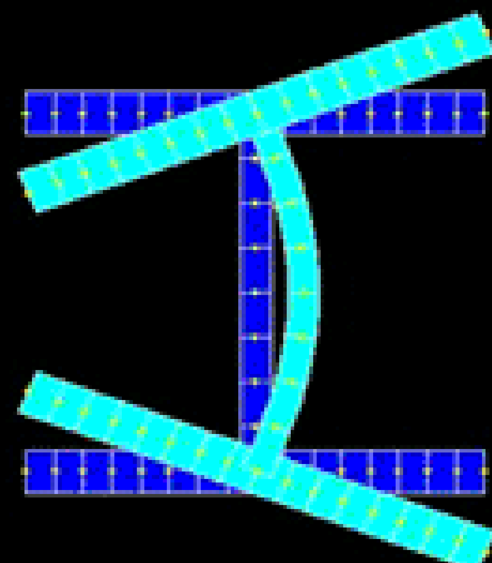




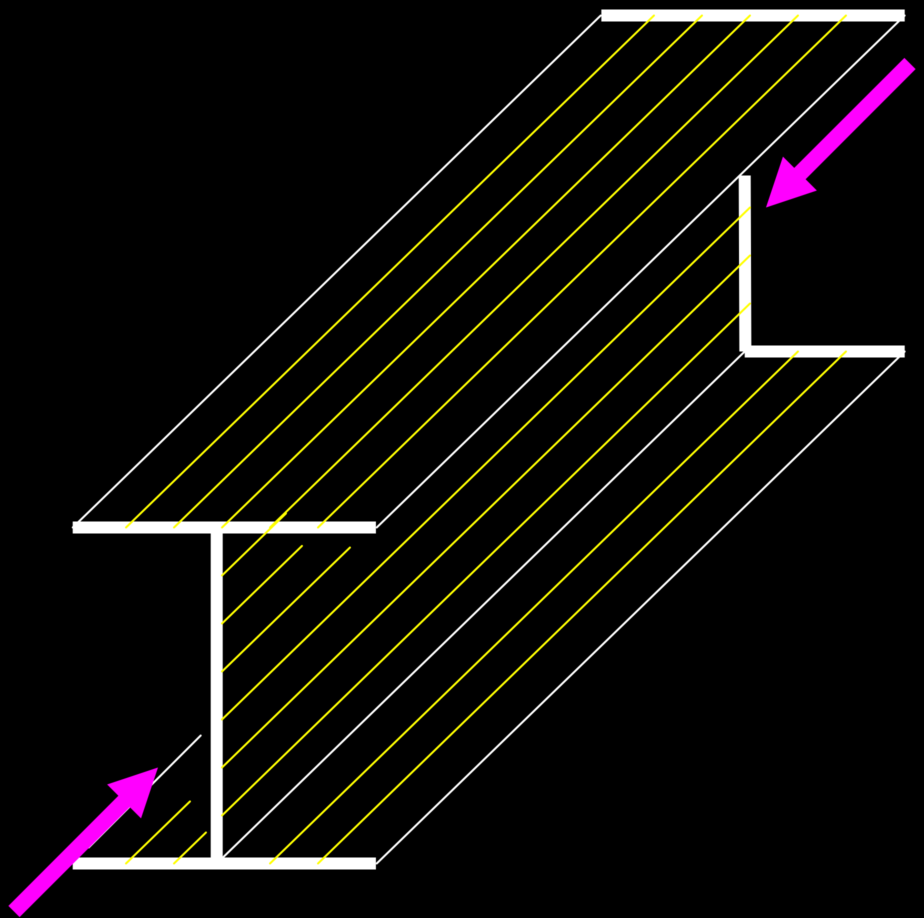
limitations

- Compatibility
  - rotation at web/flange juncture
- Bounds are false
  - if one element buckles before another it demands rotational restraint from the neighbor. simple support condition is not a lower bound!
- cross-section local buckling is needed

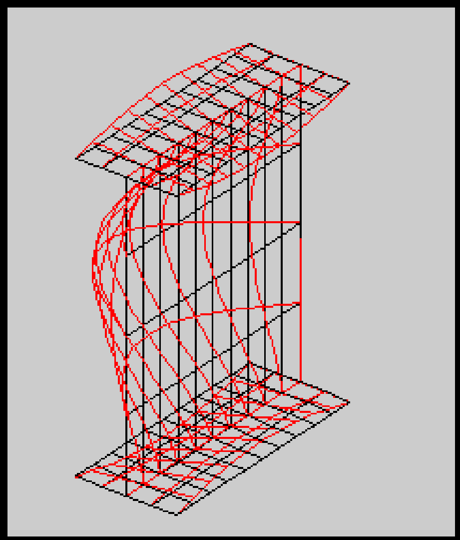


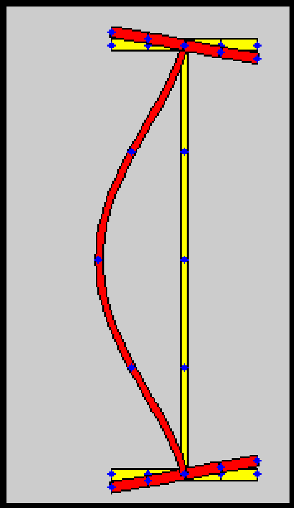


cross-section stability by finite strip method



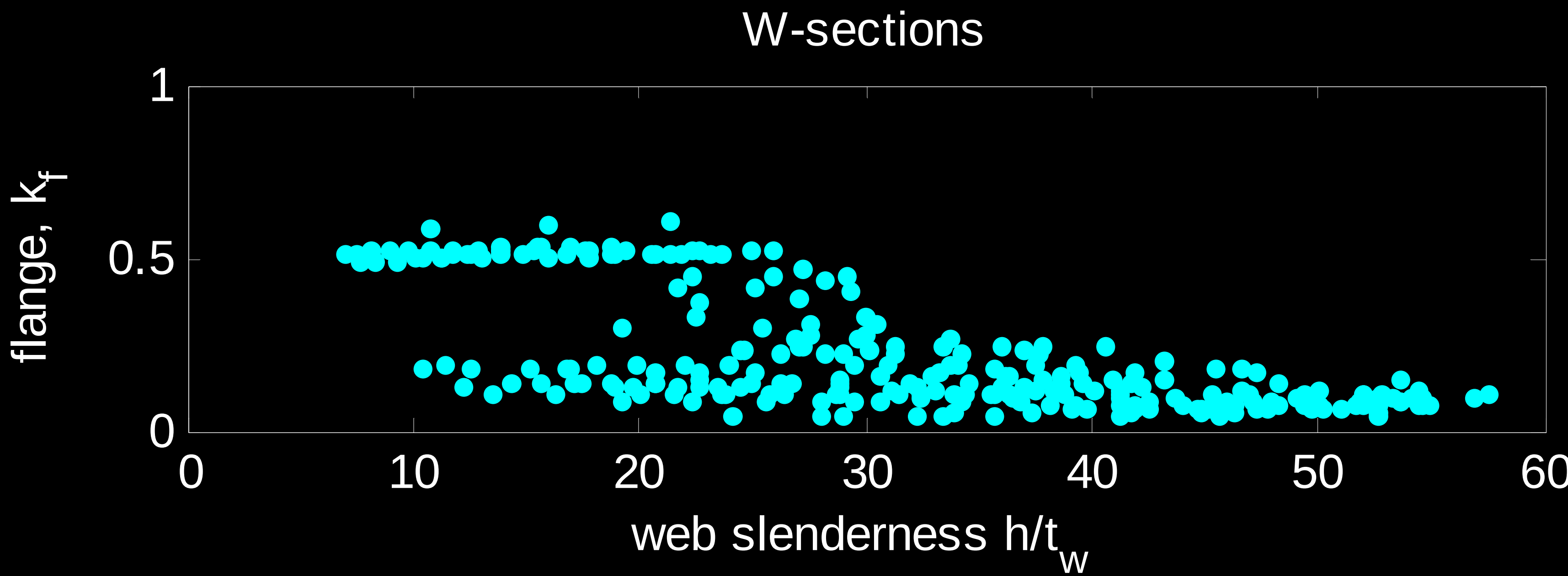
- mesh cross-section with element strips
- each element follows classical plate bending theory
- result of buckling analysis is the local buckling mode and stress of the full cross-section





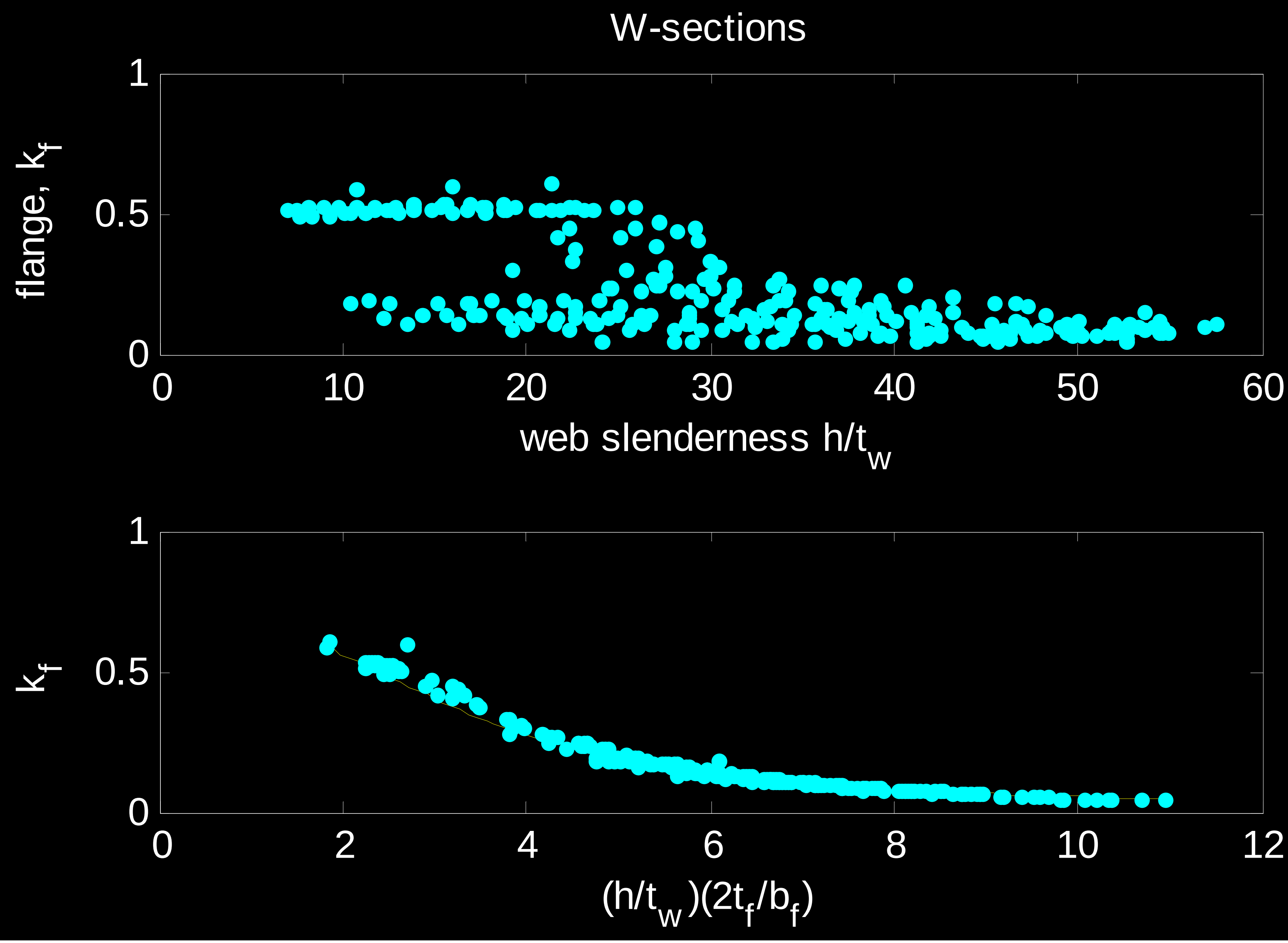
$f_{cr\ell}$=xx ksi

finite strip results for W-sections



$$f_{crb} = k_f \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t_f}{b}\right)^2$$

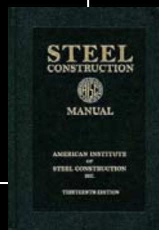
simple relations for cross-section stability



overview

- motivation
- cross-section stability
  - AISC w/t limits
  - plate stability
  - local buckling
- design methods
  - stub columns
  - long columns
- conclusions

AISC
Q-factor

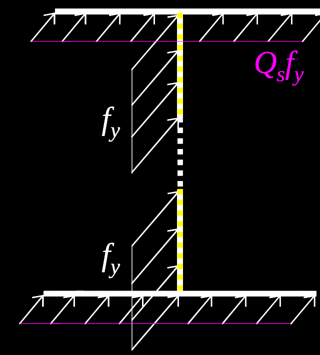


AISI
Effective
Width

AISI
Direct
Strength

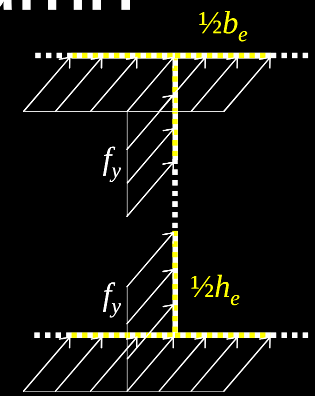
- Locally slender stub column
  - design expressions
  - comparison of predictions
- Locally slender long column
  - design expressions
  - comparison of predictions

AISC – Locally slender stub column



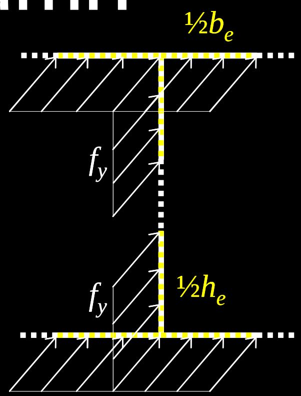
$$P_n = Q_s Q_a A_g f_y$$

AISI – Locally slender stub column

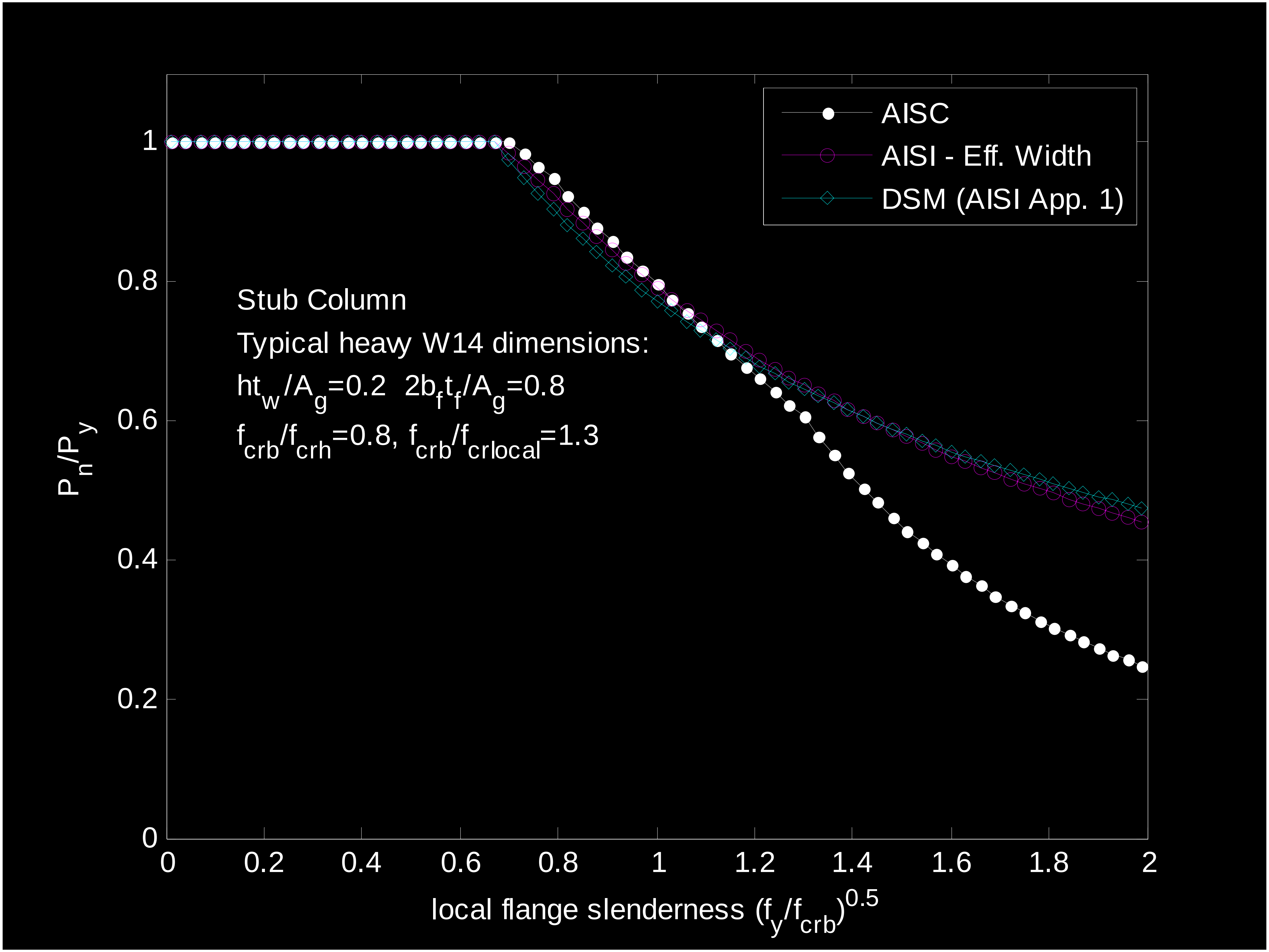


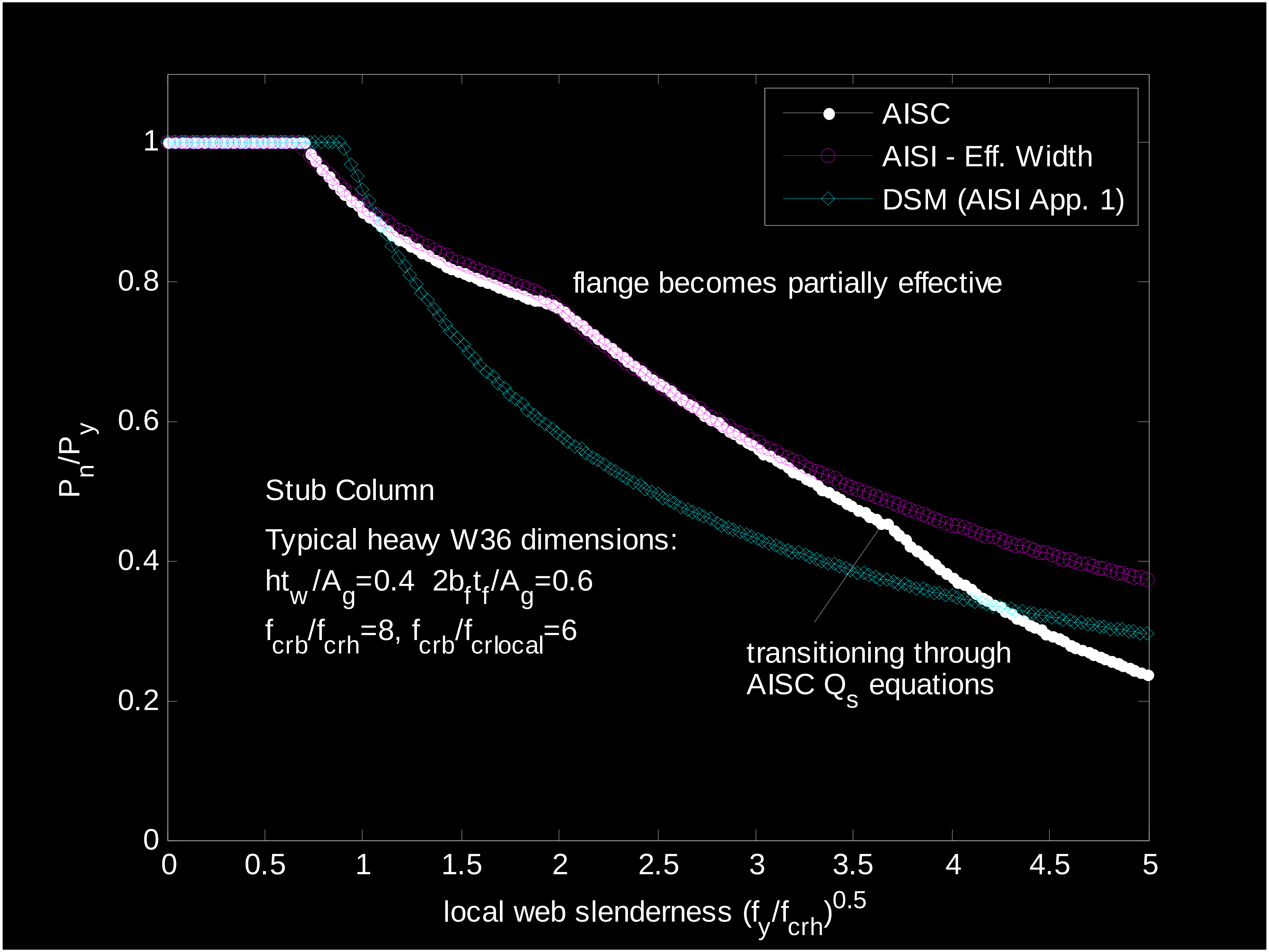
$$P_n = A_{eff}\, f_y$$

DSM – Locally slender stub column



$$P_n = A_{eff}\, f_y$$


AISC
AISI - Eff. Width
DSM (AISI App. 1)
Stub Column
Typical heavy W14 dimensions:
ht_w/A_g=0.2 $2b_f t_f/A_g$=0.8
f_{crb}/f_{crh}=0.8, $f_{crb}/f_{crlocal}$=1.3
P_n/P_y
local flange slenderness $(f_y/f_{crb})^{0.5}$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



AISC
AISI - Eff. Width
DSM (AISI App. 1)
flange becomes partially effective
Stub Column
Typical heavy W36 dimensions:
ht_w/A_g=0.4 $2b_ft_f/A_g$=0.6
f_{crb}/f_{crh}=8, $f_{crb}/f_{crlocal}$=6
transitioning through
AISC Q_s equations
P_n/P_y
local web slenderness $(f_y/f_{crh})^{0.5}$
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

consider replacing f_{crb} and f_{crh} with $f_{cr\ell}$ ...

AISI

$$P_n = A_{eff} f_y$$

$$A_{eff} = 4\rho_b b t_f + \rho_h h t_w$$

$$b_e = \rho_b b \ \text{ where } \ \rho_b = \begin{cases} 1 & \text{if} \quad f_{crb} \geq 2.2 f_y \\ \left(1 - 0.22\sqrt{\dfrac{f_{crb}}{f_y}}\right)\sqrt{\dfrac{f_{crb}}{f_y}} & \text{if} \quad f_{crb} < 2.2 f_y \end{cases}$$

$$h_e = \rho_h h \ \text{ where } \ \rho_h = \begin{cases} 1 & \text{if} \quad f_{crh} \geq 2.2 f_y \\ \left(1 - 0.22\sqrt{\dfrac{f_{crh}}{f_y}}\right)\sqrt{\dfrac{f_{crh}}{f_y}} & \text{if} \quad f_{crh} < 2.2 f_y \end{cases}$$

introduce $f_{cr\ell}$

$$P_n = A_{eff}\, f_y$$

$$A_{eff} = 4\rho_b b t_f + \rho_h h t_w$$

$$b_e = \rho_b b \ \text{ where } \ \rho_b = \begin{cases} 1 & \text{if} \quad f_{cr\ell} \geq 2.2 f_y \\ \left(1 - 0.22\sqrt{\dfrac{f_{cr\ell}}{f_y}}\right)\sqrt{\dfrac{f_{cr\ell}}{f_y}} & \text{if} \quad f_{cr\ell} < 2.2 f_y \end{cases}$$

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ρ is now for the section

$$P_n = A_{eff} f_y$$

$$A_{eff} = 4\rho\, bt_f + \rho\, ht_w$$

$$b_e = \rho\, b \;\text{ where }\; \rho = \begin{cases} 1 & \text{if} \quad f_{cr\ell} \geq 2.2 f_y \\ \left(1 - 0.22\sqrt{\dfrac{f_{cr\ell}}{f_y}}\right)\sqrt{\dfrac{f_{cr\ell}}{f_y}} & \text{if} \quad f_{cr\ell} < 2.2 f_y \end{cases}$$

$$h_e = \rho\, h \;\text{ where }\; \rho = \begin{cases} 1 & \text{if} \quad f_{cr\ell} \geq 2.2 f_y \\ \left(1 - 0.22\sqrt{\dfrac{f_{cr\ell}}{f_y}}\right)\sqrt{\dfrac{f_{cr\ell}}{f_y}} & \text{if} \quad f_{cr\ell} < 2.2 f_y \end{cases}$$

implying a simpler procedure

$$P_n = A_{eff}\, f_y$$

$$A_{eff} = 4\rho\, bt_f + \rho\, ht_w = \rho\, A_g$$

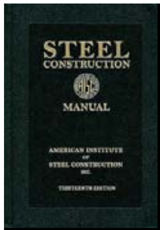
$$\text{where }\ \rho = \begin{cases} 1 & \text{if } \ f_{cr\ell} \geq 2.2 f_y \\ \left(1 - 0.22\sqrt{\dfrac{f_{cr\ell}}{f_y}}\right)\sqrt{\dfrac{f_{cr\ell}}{f_y}} & \text{if } \ f_{cr\ell} < 2.2 f_y \end{cases}$$

similarity to DSM very strong

$$P_n = A_{eff} f_y$$

$$A_{eff} = \rho A_g$$

$$\rho = \begin{cases} 1 & \text{if} \quad f_{cr\ell} \geq 1.66 f_y \\ \left(1 - 0.15\left(\dfrac{f_{cr\ell}}{f_y}\right)^{0.4}\right)\left(\dfrac{f_{cr\ell}}{f_y}\right)^{0.4} & \text{if} \quad f_{cr\ell} < 1.66 f_y \end{cases}$$

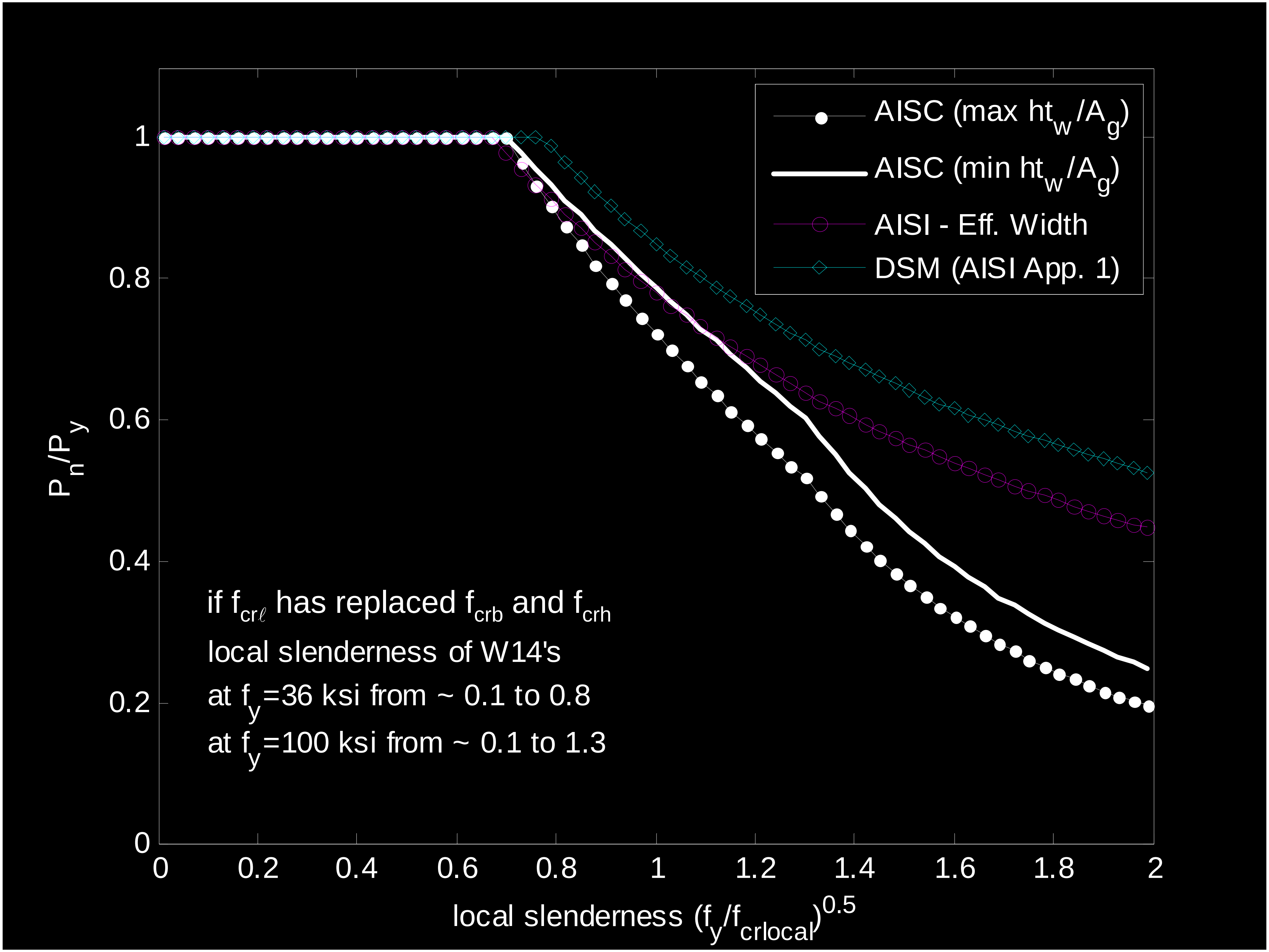


AISC approach does not simplify similarly

$$P_n = Q_s Q_a A_g f_y$$

$$Q_s = \begin{cases} 1.0 & \text{if} \quad f_{cr\ell} \geq 2 f_y \\ 1.415 - 0.59\sqrt{\dfrac{f_y}{f_{cr\ell}}} & \text{if} \quad \dfrac{3}{5} f_y < f_{cr\ell} < 2 f_y \\ 1.1\dfrac{f_{cr\ell}}{f_y} & \text{if} \quad f_{cr\ell} \leq \dfrac{3}{5} f_y \end{cases}$$

$$Q_a = \begin{cases} 1.0 & \text{if} \quad f_{cr\ell} > 2 f_y \\ 1 - \left(1 - 0.9\sqrt{\dfrac{f_{cr\ell}}{f_y}}\left(1 - 0.16\sqrt{\dfrac{f_{cr\ell}}{f_y}}\right)\right)\dfrac{h t_w}{A_g} & \text{if} \quad f_{cr\ell} \leq 2 f_y \end{cases}$$



AISC (max ht_w/A_g)
AISC (min ht_w/A_g)
AISI - Eff. Width
DSM (AISI App. 1)
if $f_{cr\ell}$ has replaced f_{crb} and f_{crh}
local slenderness of W14's
at f_y=36 ksi from ~ 0.1 to 0.8
at f_y=100 ksi from ~ 0.1 to 1.3
P_n/P_y
local slenderness $(f_y/f_{crlocal})^{0.5}$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

overview

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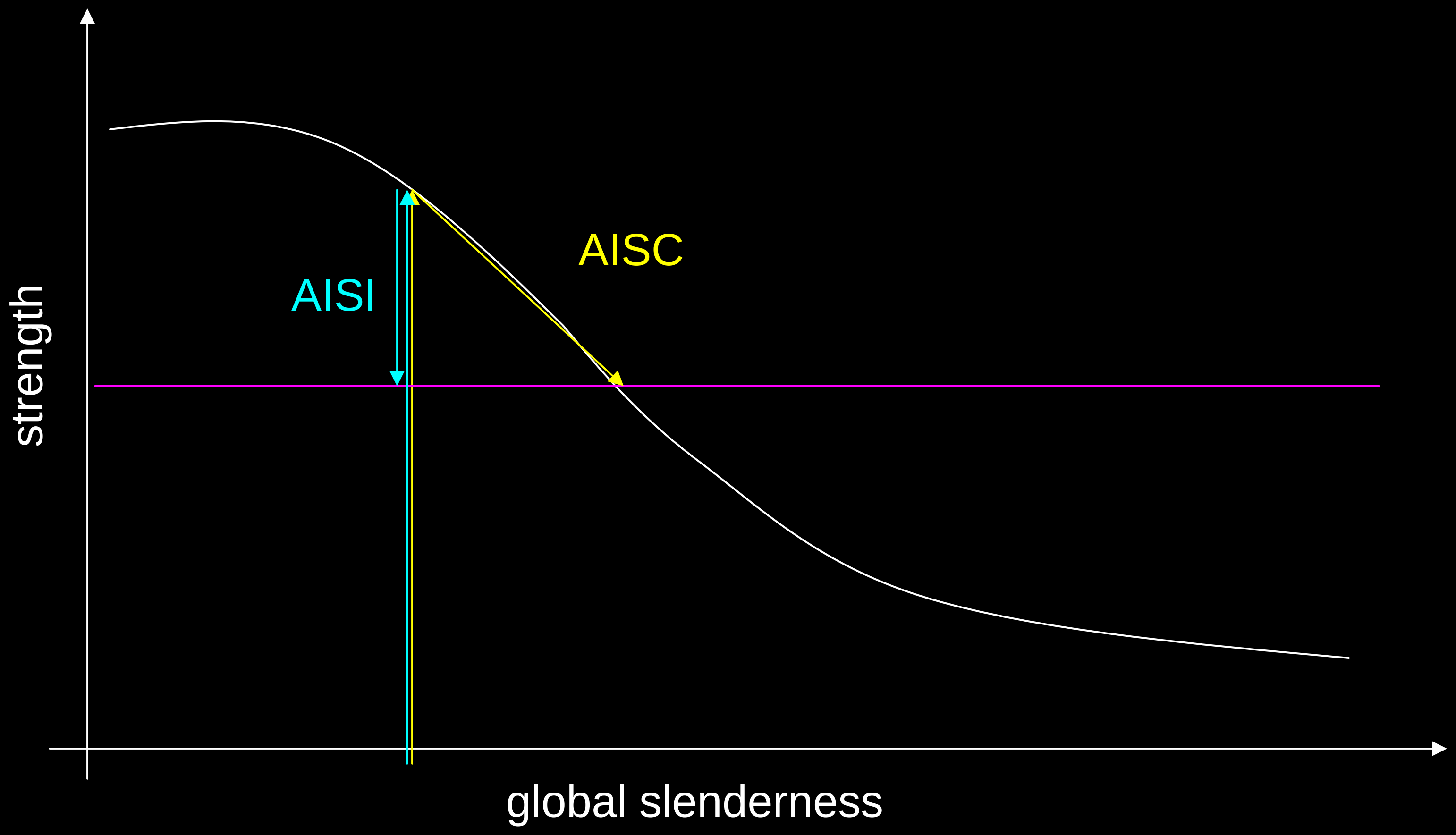
modifying the global column curve



AISC

$$P_n = A_g \hat{f}_n$$

$$\hat{f}_n = \begin{cases} Q_s Q_a (0.658)^{Q_s Q_a (f_e / f_y)} f_y & \text{if } f_e \geq 0.44 Q_s Q_a f_y \\ 0.877 f_e & \text{if } f_e < 0.44 Q_s Q_a f_y \end{cases}$$

AISI

$$P_n = A_{eff} f_n$$

$$f_n = \begin{cases} (0.658)^{(f_e / f_y)} f_y & \text{if } \; f_e \geq 0.44 f_y \\ 0.877 f_e & \text{if } \; f_e < 0.44 f_y \end{cases}$$

$$A_{eff} = 4\rho_b b t_f + \rho_h h t_w$$

$$b_e = \rho_b b \;\text{ where }\; \rho_b = \begin{cases} 1 & \text{if } \; f_{crb} \geq 2.2 f_n \\ \left(1 - 0.22\sqrt{\dfrac{f_{crb}}{f_n}}\right)\sqrt{\dfrac{f_{crb}}{f_n}} & \text{if } \; f_{crb} < 2.2 f_n \end{cases}$$

$$h_e = \rho_h h \;\text{ where }\; \rho_h = \begin{cases} 1 & \text{if } \; f_{crh} \geq 2.2 f_n \\ \left(1 - 0.22\sqrt{\dfrac{f_{crh}}{f_n}}\right)\sqrt{\dfrac{f_{crh}}{f_n}} & \text{if } \; f_{crh} < 2.2 f_n \end{cases}$$

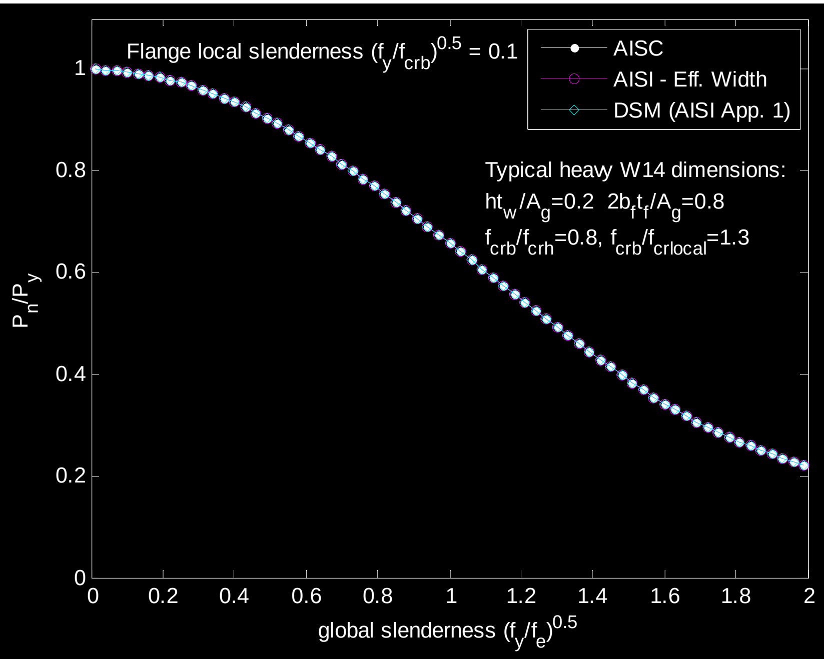
DSM

$$P_n = A_{eff}\, f_n$$

$$f_n = \begin{cases} (0.658)^{(f_e / f_y)}\, f_y & \text{if} \quad f_e \geq 0.44\, f_y \\ 0.877\, f_e & \text{if} \quad f_e < 0.44\, f_y \end{cases}$$

$$A_{eff} = \rho A_g$$

$$\rho = \begin{cases} 1 & \text{if} \quad f_{cr\ell} \geq 1.66\, f_n \\ \left(1 - 0.15\left(\dfrac{f_{cr\ell}}{f_n}\right)^{0.4}\right)\left(\dfrac{f_{cr\ell}}{f_n}\right)^{0.4} & \text{if} \quad f_{cr\ell} < 1.66\, f_n \end{cases}$$



(a) compact: flange slenderness of 0.1, i.e. $f_{crb} = 100f_y$

Concluding thoughts

- Cross-section stability is worthy of continued study and current approaches employed in structural steel design have inherent limitations
- Strength models for locally slender columns all essentially use the same parameters

AISC: $P_n/P_y = f(f_{cr\text{-}global}, f_y, f_{crb}, f_{crh}, ht_w/A_g)$

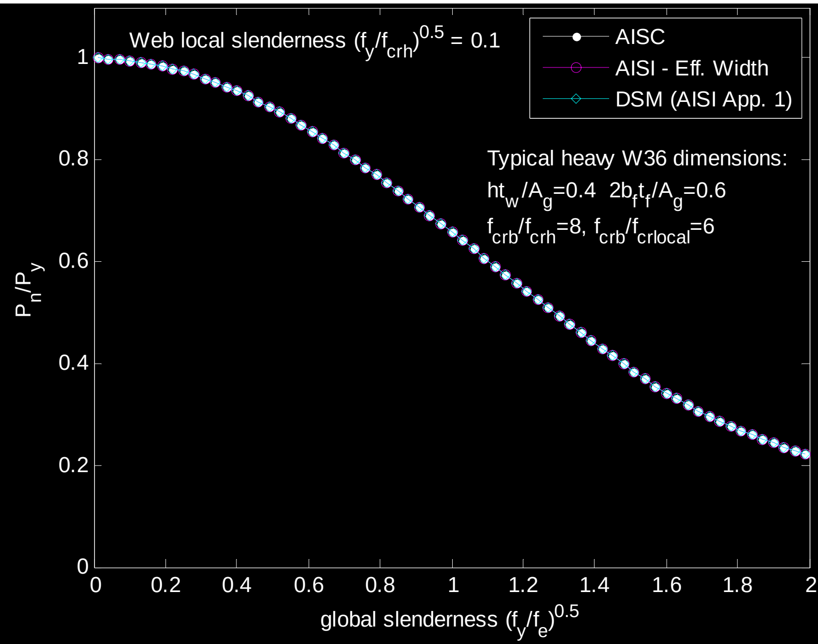
AISI: $P_n/P_y = f(f_{cr\text{-}global}, f_y, f_{crb}, f_{crh}, ht_w/A_g)$

DSM: $P_n/P_y = f(f_{cr\text{-}global}, f_y, f_{cr\ell})$

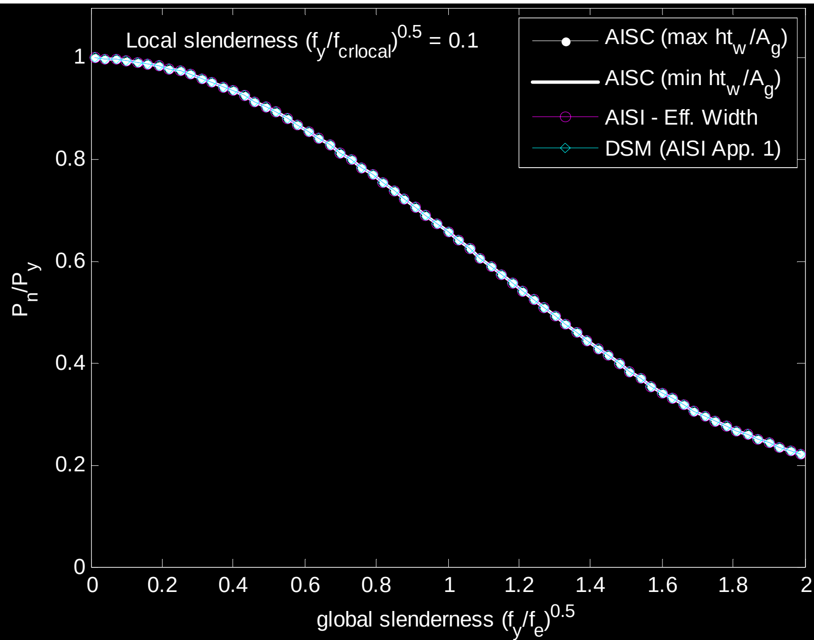
- When local flange slenderness controls predicted strength AISC Q-factor predictions are lower than AISI or DSM
- Work continues

more at www.ce.jhu.edu/bschafer/aisc

[redacted]



(a) compact: web slenderness of 0.1, i.e. $f_{crh} = 100 f_y$



(a) compact: local slenderness of 0.1, i.e. $f_{cr\ell} = 100 f_y$

AISC Faculty Fellowship

Cross-section Stability of Structural Steel

Research

Objective

Investigate the application of the Direct Strength Method (DSM) to structural steel shapes, and to provide the necessary research advances to make this a viable option for the design of noncompact and slender structural steel shapes. A goal of DSM is to provide a design method which is robust enough to allow engineers to realistically explore novel cross-sections, yet make this exploration simple. Investigate the potential of DSM to take a fresh look at hot-rolled steel structural shapes.

Work Products

Proposal (pdf - February 2005)
Progress Report #1 (pdf - June 2007)
Progress Report #2 (pdf - April 2008)

Related Links

CUFSM
constrained Finite Strip Method
Direct Strength Method for Cold-Formed Steel
Reliability and Advanced Analysis of Steel Frames
Frame design/robustness under unforeseen events

Education

Objective

Provide tools, tutorials, and educational aids related to cross-section stability of structural steel shapes so that educators, students, and engineers may explore these concepts more readily. Provide educational aids appropriate for courses in steel design using structural steel at the undergraduate and graduate levels.

Work Products

CUFSM, software for cross-section stability analysis

Example files for structural steel shapes W36x150, W14x120, C5x9, L4x4x1/2, WT 18x150, HSS 4x4x1/2 (all example files in a zip folder)

Tutorial 1: **Cross-section stability of a W36x150 using the finite strip method (ppt) (pdf)**
Learning objectives: (1) Identify all the buckling modes in a W-section, for columns explore flexural (Euler) buckling and local buckling, for beams explore lateral-torsional buckling and local buckling; (2) Predict the buckling stress (load or moment) for identified buckling modes, (3) Learn the interface of a simple program for exploring cross-section stability of

www.ce.jhu.edu/bschafer/aisc