

# Tutorial 2: Cross-section stability of a W36x150

Exploring higher modes, and the  
interaction of buckling modes

*prepared by Ben Schafer, Johns Hopkins University, version 1.0*

# Acknowledgments

- Preparation of this tutorial was funded in part through the AISC faculty fellowship program.
- Views and opinions expressed herein are those of the author, not AISC.

# Target audience

- This tutorial is targeted at the advanced undergraduate/beginning graduate level. Some familiarity with structural stability is assumed in the provided discussion.
- It is also assumed that Tutorial #1 has been completed and thus some familiarity with the use of CUFSM is assumed.

# Learning objectives

- Understand the role of “higher” buckling modes in the analysis of a W-section, including
  - how higher buckling modes relate to strong-axis, weak-axis, and torsional buckling in columns
  - what higher buckling modes mean for local buckling
  - when knowledge of higher buckling modes may be useful in design
- Understand how interaction of modes may be identified and quantified using CUFSM for a W-section

# Summary of Tutorial #1

- A W36x150 beam was analyzed using the finite strip method available in CUFSM for pure compression and major axis bending.
- For pure compression local buckling and flexural buckling were identified as the critical buckling modes.
- For major axis bending local buckling and lateral-torsional buckling were identified as the critical buckling modes.

*W36x150 column – review of Tutorial 1*

Load Save

Input Properties Analyze Post

Z R Print Copy Reset ? X

**Plot Shape** ?

separate window in-plane mode

2D  3D  Undef. Scale 1

**half-wavelength = 27.2**

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

**Plot Curve** ?

dump to text

minima  log scale  classify

modes 1

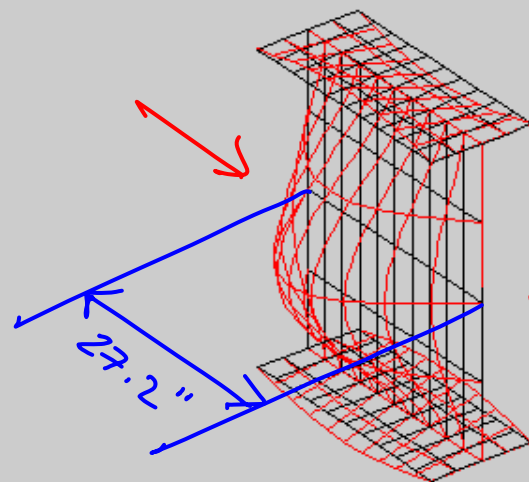
files to be plotted 1

**cFSM Modal Classification**

Classify work norm

cFSM analysis is off

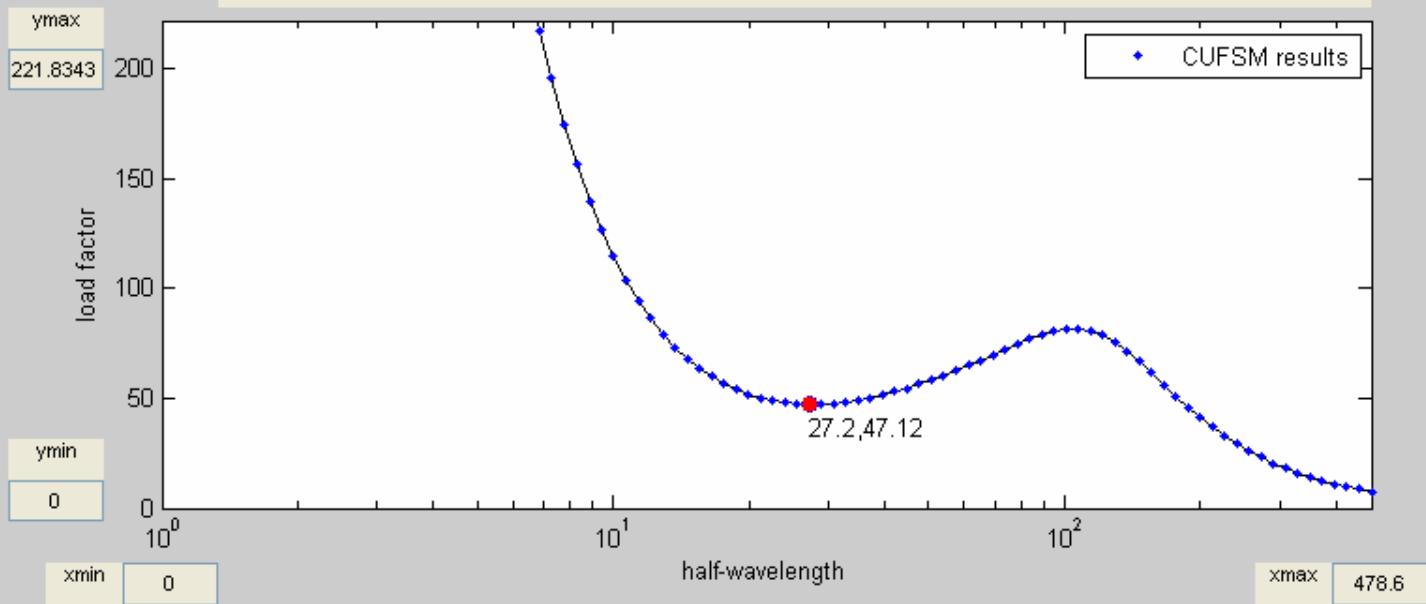
supplemental participation plot



web and flange local buckling is shown

remember, applied load is a uniform compressive stress of 1.0 ksi

Buckled shape for CUF5M results  
 half-wavelength = 27.2 load factor = 47.1237 mode = 1  
 cFSM classification results: off



Load Save

Input Properties Analyze Post

Z R Print Copy Reset ? X

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 27.2

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

files to be plotted 1

cFSM Modal Classification

Classify work norm

cFSM analysis is off

supplemental participation plot

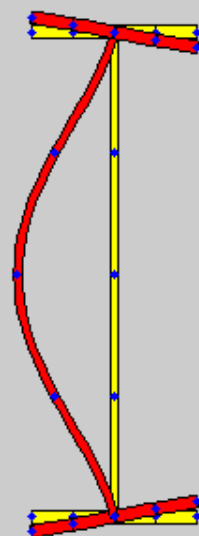
$P_{ref} = 42.6 \text{ k}$   
or  
 $f_{ref} = 1.0 \text{ ksi}$

load factor for local buckling = 47.12

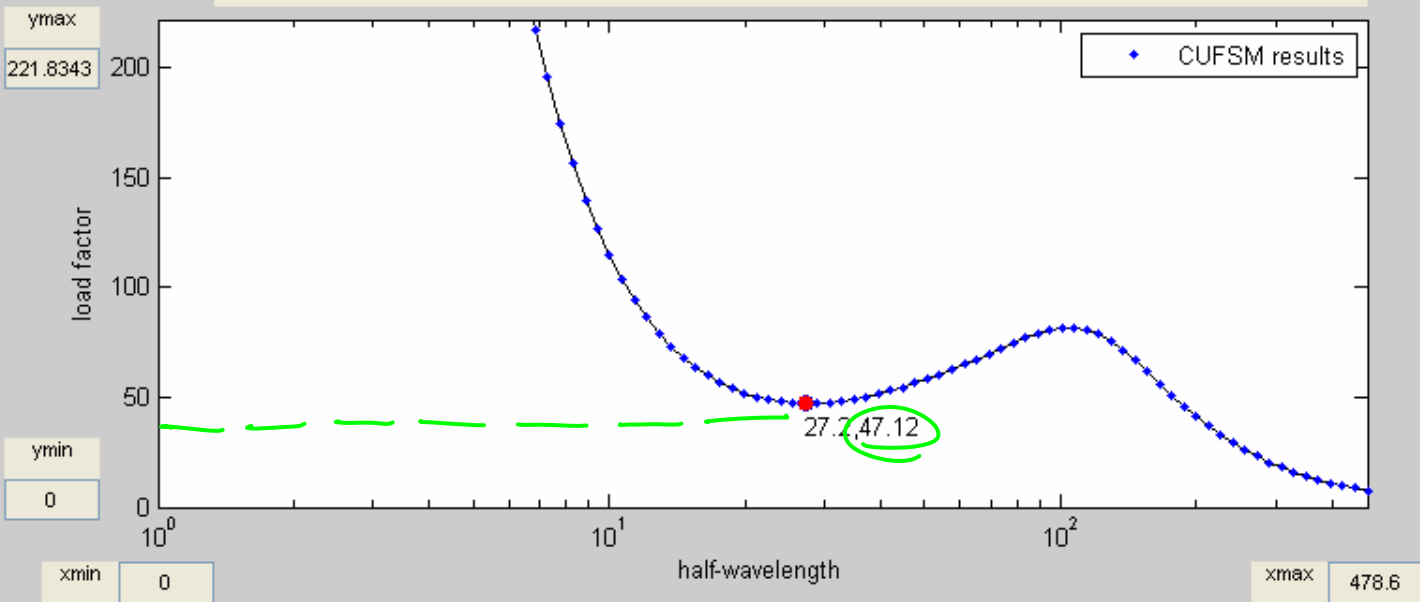
$$P_{cr,local} = 47.12 \times 42.6 = 2007 \text{ k}$$

or

$$f_{cr,local} = 47.12 \times 1.0 \text{ ksi} = 47.12 \text{ ksi}$$



Buckled shape for CUF5M results  
half-wavelength = 27.2 load factor = 47.1237 mode = 1  
cFSM classification results: off





Load Save

Input Properties Analyze Post

Z R Print Copy Reset ? X

Plot Shape ?

separate window in-plane mode

2D  3D  Undef. Scale 1

half-wavelength = 478.6

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

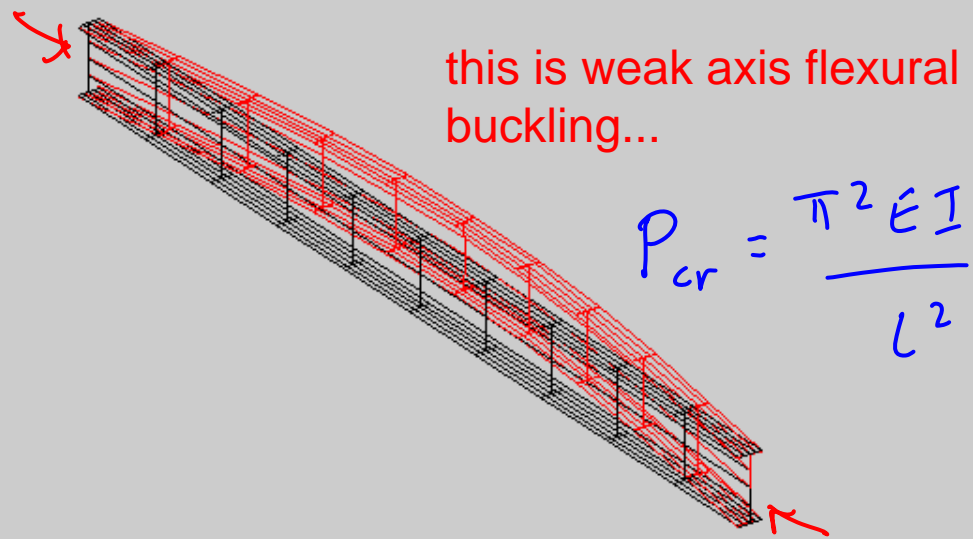
files to be plotted 1

cFSM Modal Classification

Classify work norm

cFSM analysis is off

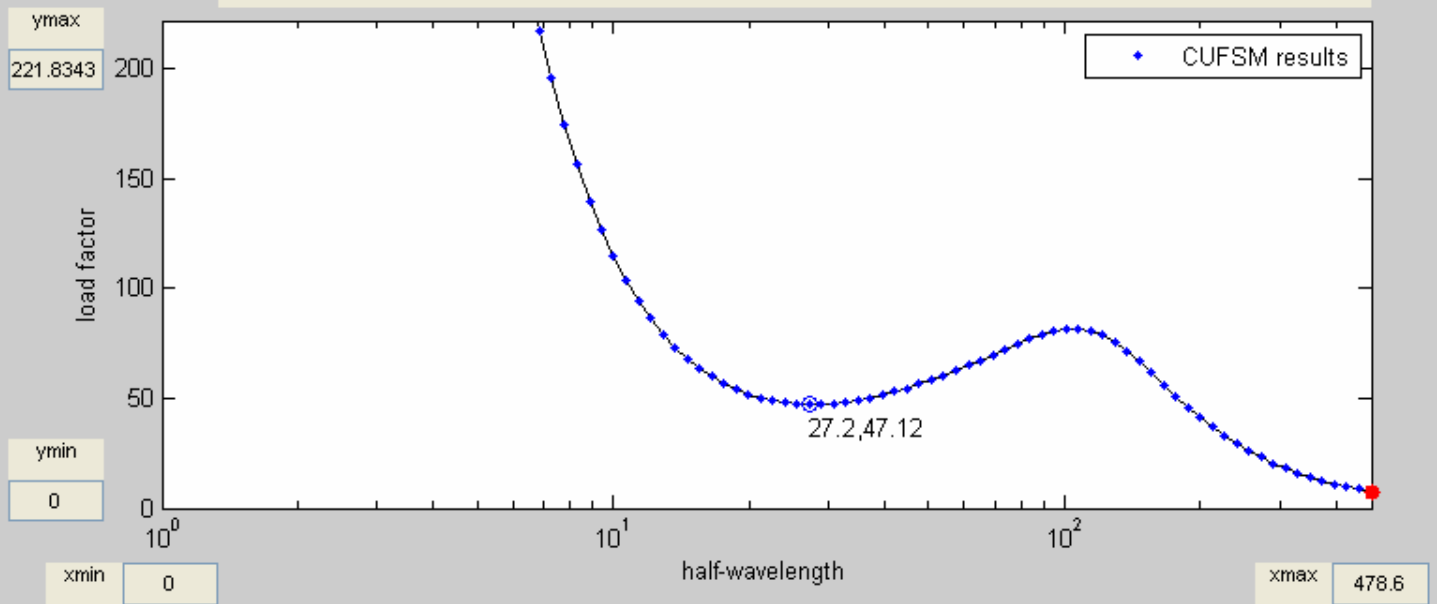
supplemental participation plot



Buckled shape for CUF5M results

half-wavelength = 478.6 load factor = 7.6483 mode = 1

cFSM classification results: off



Load Save

Input Properties Analyze Post

Z R Print Copy Reset ? X

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 478.6

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

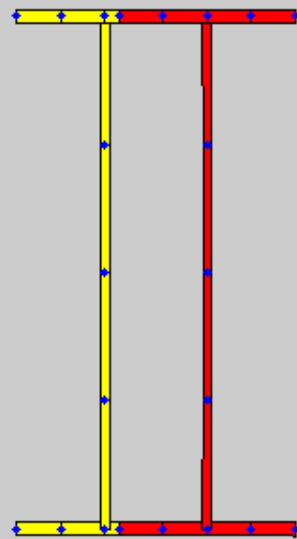
files to be plotted 1

cFSM Modal Classification

Classify work norm

cFSM analysis is off

supplemental participation plot

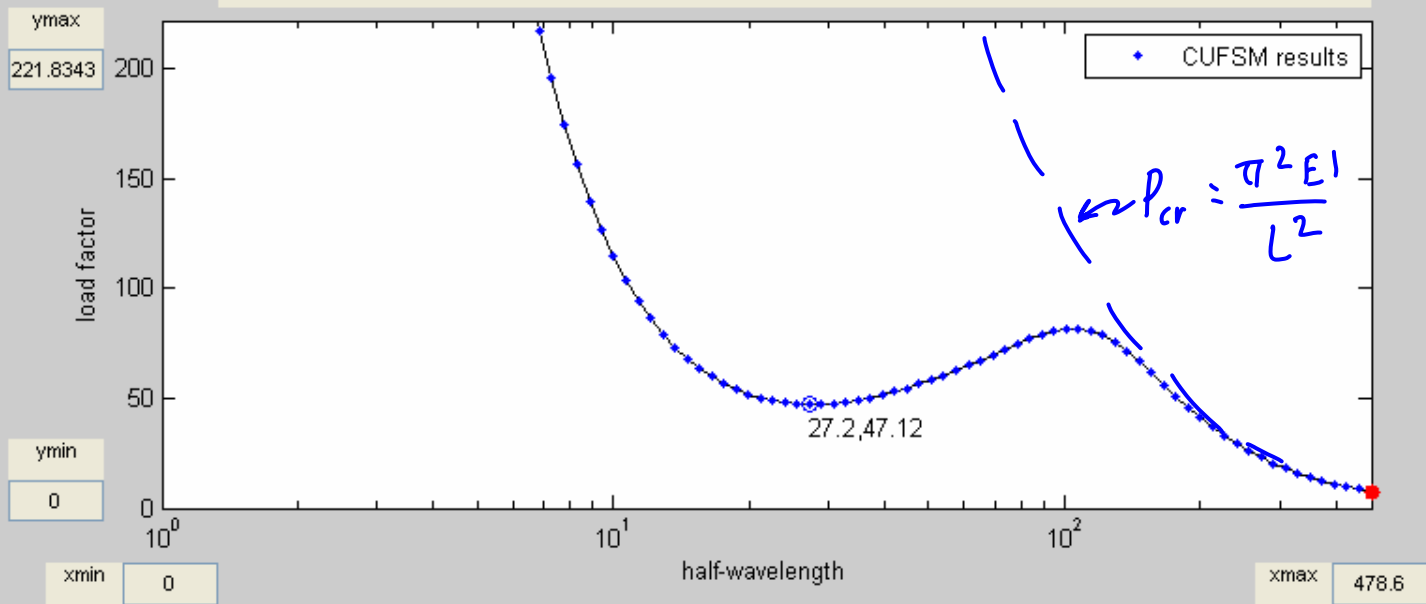


note that for flexural buckling the cross-section elements do not distort/bend, the full cross-section translates/rotates rigidly in-plane.

Buckled shape for CUF5M results

half-wavelength = 478.6 load factor = 7.6483 mode = 1

cFSM classification results: off



Load Save

Input Properties Analyze Post

Z R Print Copy Reset ? X

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 478.6

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

files to be plotted 1

cFSM Modal Classification

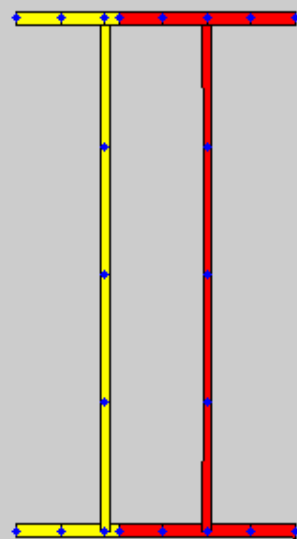
Classify work norm

cFSM analysis is off

supplemental participation plot

$P_{ref} = 42.6 \text{ k}$   
 or  
 $f_{ref} = 1.0 \text{ ksi}$

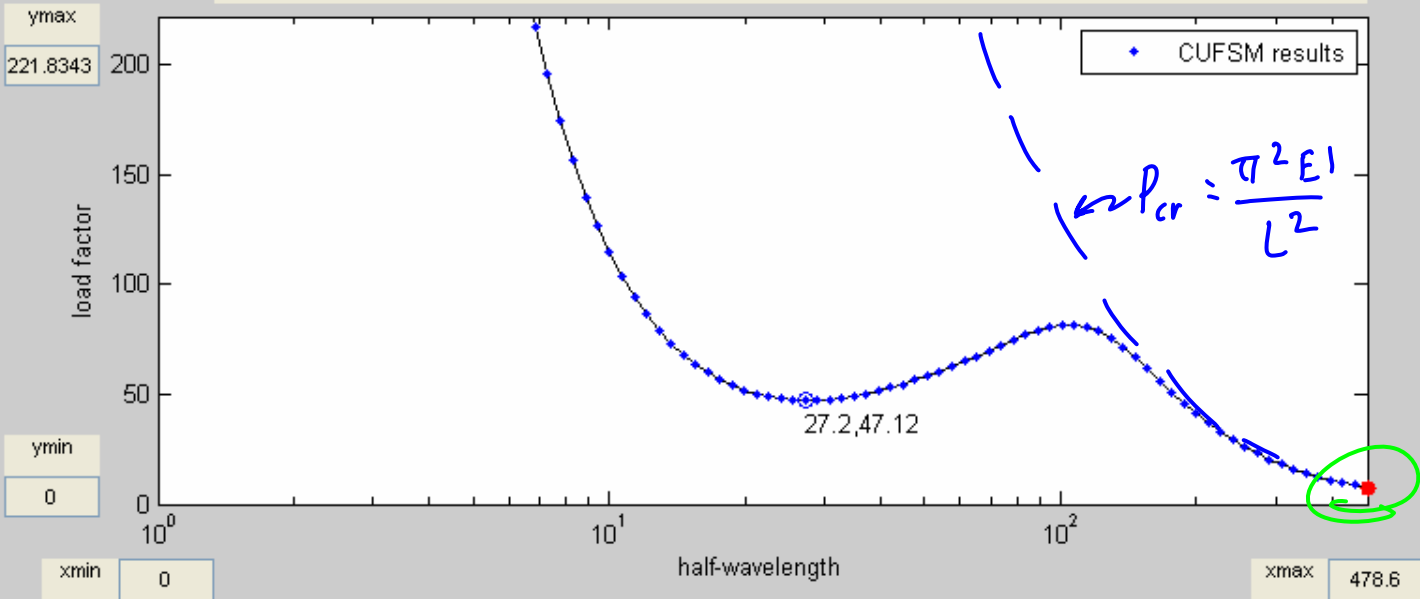
load factor for global flexural buckling = 7.6 at 40 ft. length



$P_{cr} = 7.6 \times 42.6 \text{ k}$   
 $= 324 \text{ k}$

or  
 $f_{cr} = 7.6 \times 1.0 \text{ ksi}$   
 $= 7.6 \text{ ksi}$

Buckled shape for CUF5M results  
 half-wavelength = 478.6 load factor = 7.6483 mode = 1  
 cFSM classification results: off



# Tutorial #1: Column summary

- A W36x150 under pure compression (a column) has two important cross-section stability elastic buckling modes
- (1) Local buckling which occurs at a stress of 47 ksi and may repeat along the length of a member every 27 in. (it's half-wavelength)
- (2) Global flexural buckling, which for a 40 ft. long member occurs at a stress of 7.6 ksi (other member lengths may be selected from the curve provided from the analysis results)

# Higher modes

- We know global buckling of a column has more than one mode.. for instance the buckling can occur about the strong or weak axis:

$$P_{cr1} = \frac{\pi^2 E I_y}{L^2} \quad P_{cr2} = \frac{\pi^2 E I_z}{L^2} \quad \dots$$

- How is this reflected in CUFSM?
- Let's investigate higher modes...

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 478.6

mode 2

file

CUFSM results

select

torsional buckling, mode 2 at 40 ft. is torsional buckling of the W36x150 at 21ksi.



a mid-height brace would remove weak-axis flexural buckling, but may still allow this torsional buckling mode, so in some very specific situations the higher mode results could be quite useful.

loaded files: Load another file

1 = CUFSM results

Buckled shape for CUFSM results

half-wavelength = 478.6 40'

load factor = 21.2875

cFSM classification results: off

mode = 2

Plot Curve ?

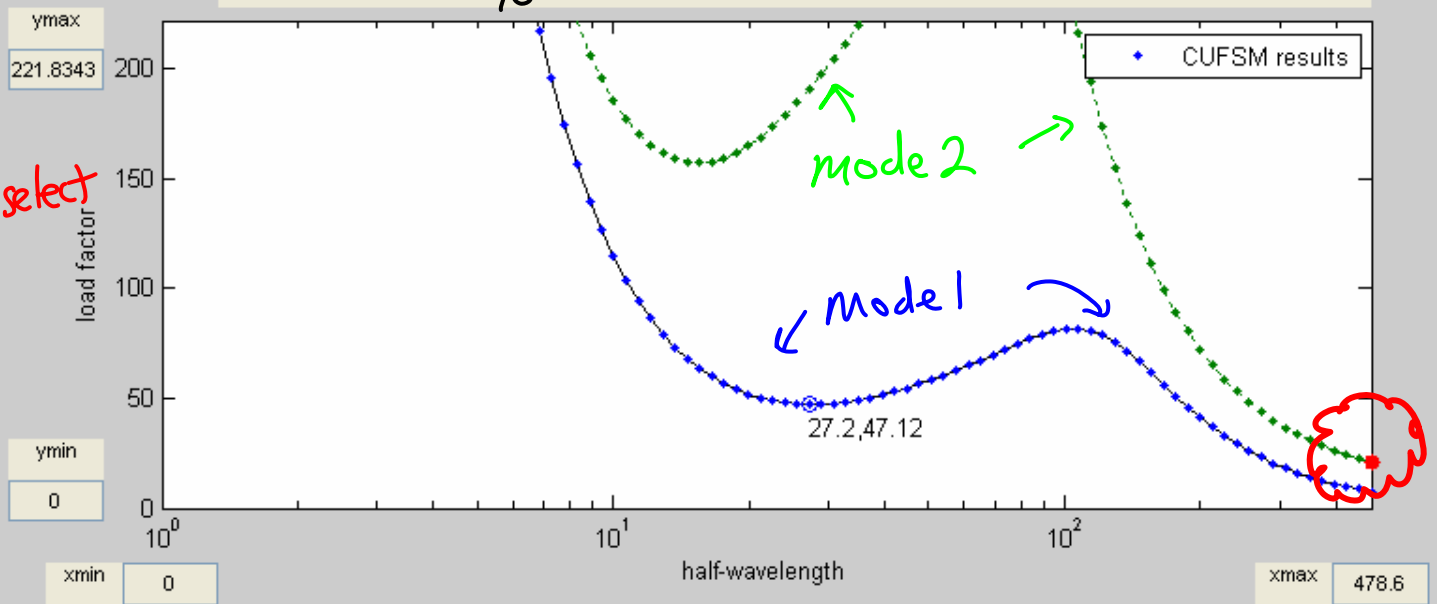
dump to text

minima  log scale  classify

modes 2

files to be plotted 1

select



cFSM Modal Classification

Classify work norm

cFSM analysis is off

supplemental participation plot

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 478.6

mode 3

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 3

files to be plotted 1

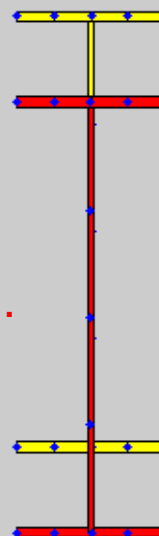
cFSM Modal Classification

Classify work norm

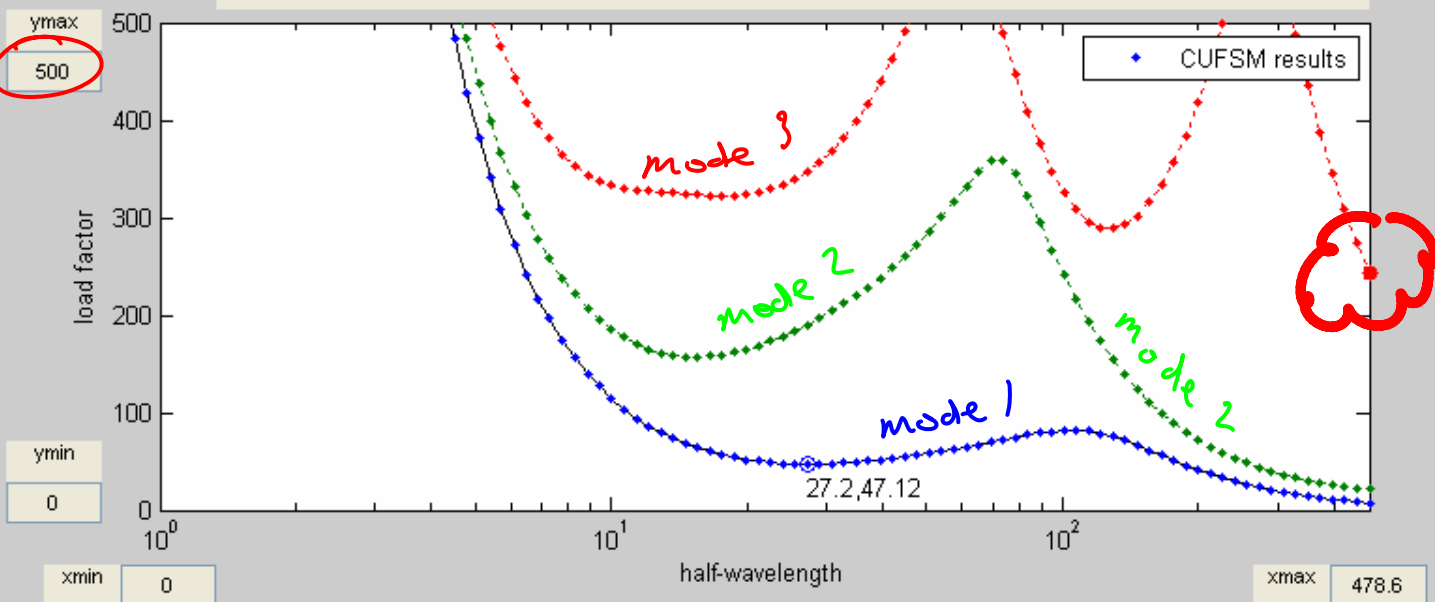
cFSM analysis is off

supplemental participation plot

strong axis flexural buckling, mode 3 at 40 ft. is strong axis flexural buckling of the W36x150 at 244ksi.



Buckled shape for CUF5M results  
 half-wavelength = 478.6 load factor = 243.7799 mode = 3  
 cFSM classification results: off



Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 478.6

mode 3

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 3

files to be plotted 1

cFSM Modal Classification

Classify work norm

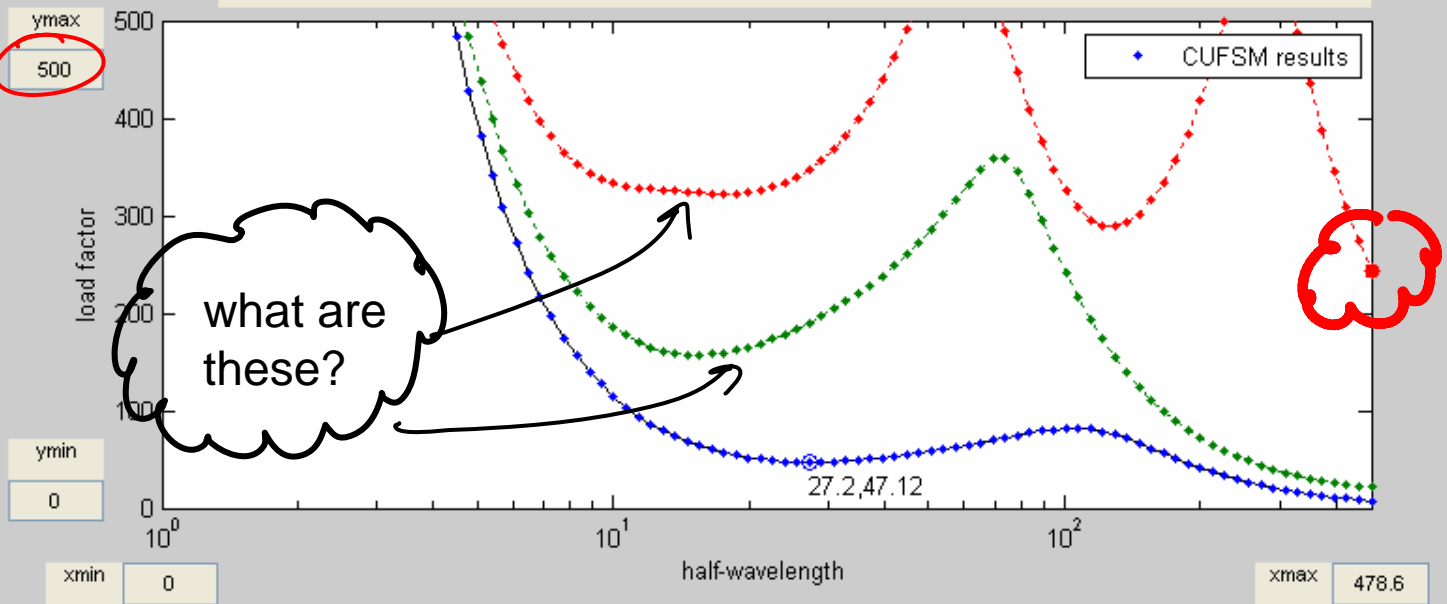
cFSM analysis is off

supplemental participation plot

strong axis flexural buckling, mode 3 at 40 ft. is strong axis flexural buckling of the W36x150 at 244ksi.



Buckled shape for CUF5M results  
 half-wavelength = 478.6 load factor = 243.7799 mode = 3  
 cFSM classification results: off





Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 15.5

mode 2

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 3

files to be plotted 1

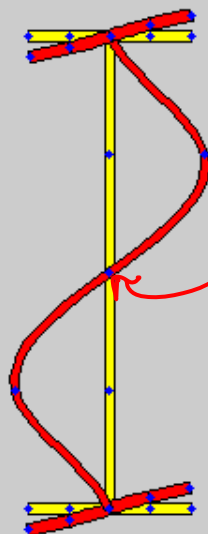
cFSM Modal Classification

Classify work norm

cFSM analysis is off

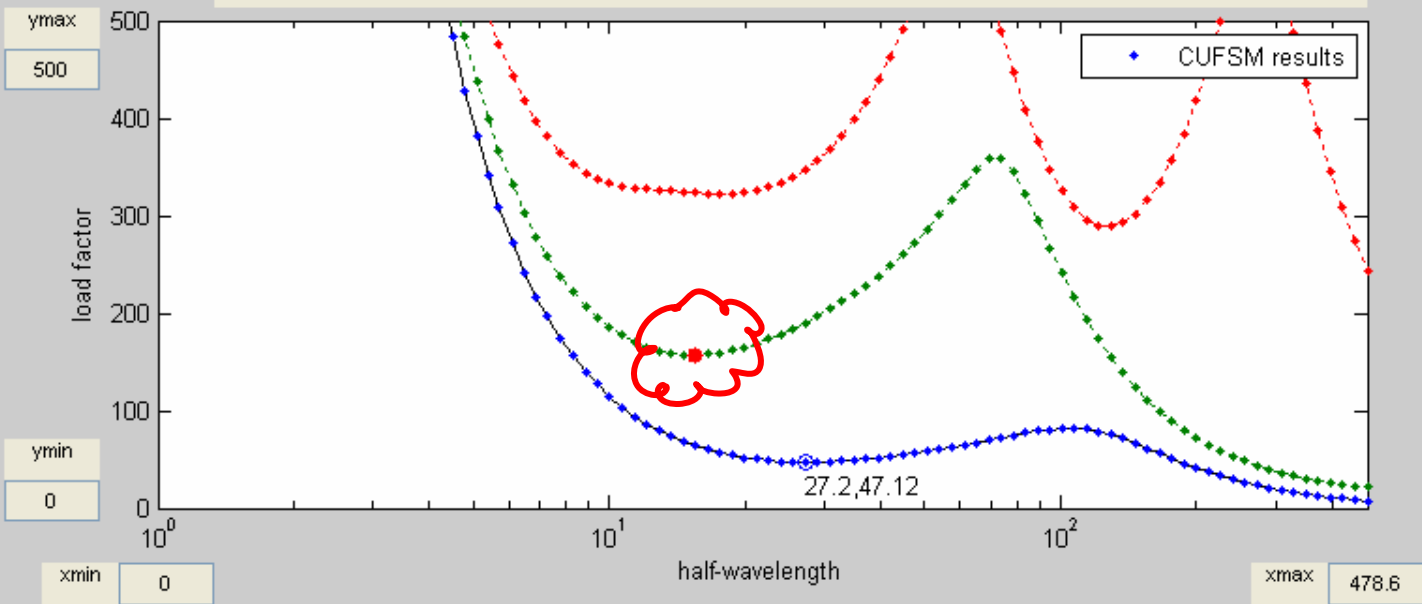
supplemental participation plot

local buckling, mode 2 has a half-wavelength of 15.5 in. and a buckling stress of 157 ksi.



a mid-height brace would remove mode 1 local buckling, but may still allow this mode 2 local mode, so in some very specific situations the higher mode results could be quite useful.

Buckled shape for CUF5M results  
 half-wavelength = 15.5 load factor = 157.0853 mode = 2  
 cFSM classification results: off



# Higher modes summary

- At every half-wavelength investigated many buckling modes are revealed – the lowest of which is the 1<sup>st</sup> mode.
- Higher modes are those buckling modes at a given half-wavelength that have higher buckling stresses (load or moment) than the 1<sup>st</sup> mode.
- Higher buckling modes become more important as bracing and different boundary conditions are considered.
- In this W36x150 example it may be of surprise to some that torsional buckling may be the limiting global buckling mode when weak-axis flexural buckling is restricted

*W36x150 beam – review of Tutorial 1*

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 25.6

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

files to be plotted 1

cFSM Modal Classification

Classify work norm

cFSM analysis is off

supplemental participation plot

Local buckling..

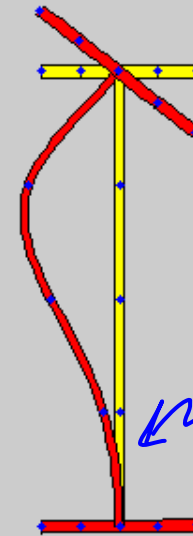
$$M_{cr,local} = 231 \times 500 \text{ kip-in.}$$

$$= 115,500 \text{ kip-in.}$$

$$= 9,625 \text{ kip-ft}$$

$$f_{cr,local} = 231 \times 1.0 \text{ ksi}$$

$$= 231 \text{ ksi}$$



compression

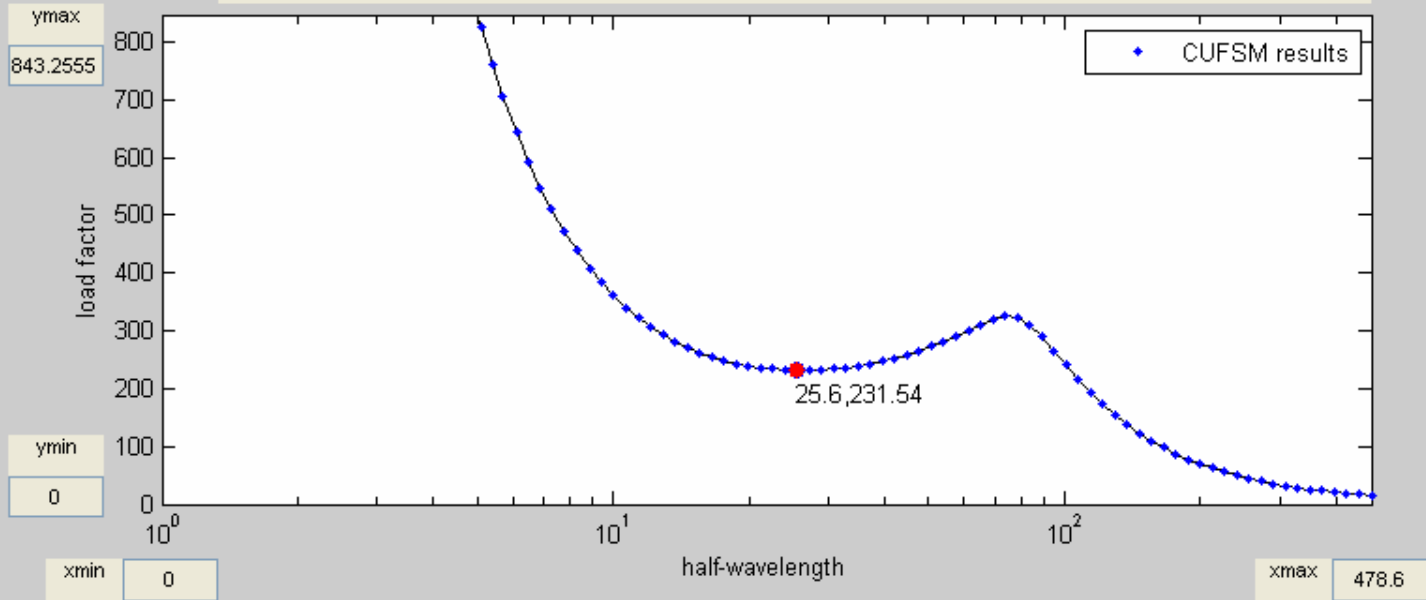
tension helps stiffen the bottom of the web and elevates local buckling a great deal.

tension

Buckled shape for CUF5M results

half-wavelength = 25.6 load factor = 231.5403 mode = 1

cFSM classification results: off



Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 478.6

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

files to be plotted 1

cFSM Modal Classification

Classify work norm

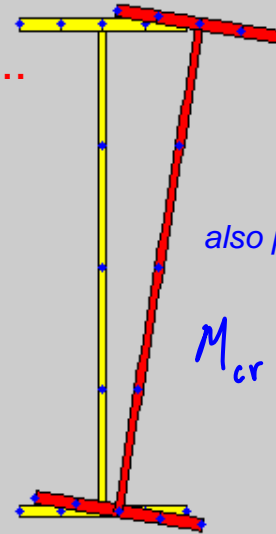
cFSM analysis is off

supplemental participation plot

Lateral-torsional buckling..

$M_{cr} = 15.8 \times 500 \text{ kip-in.}$   
 $= 7,900 \text{ kip-in.}$   
 $= 660 \text{ kip-ft}$

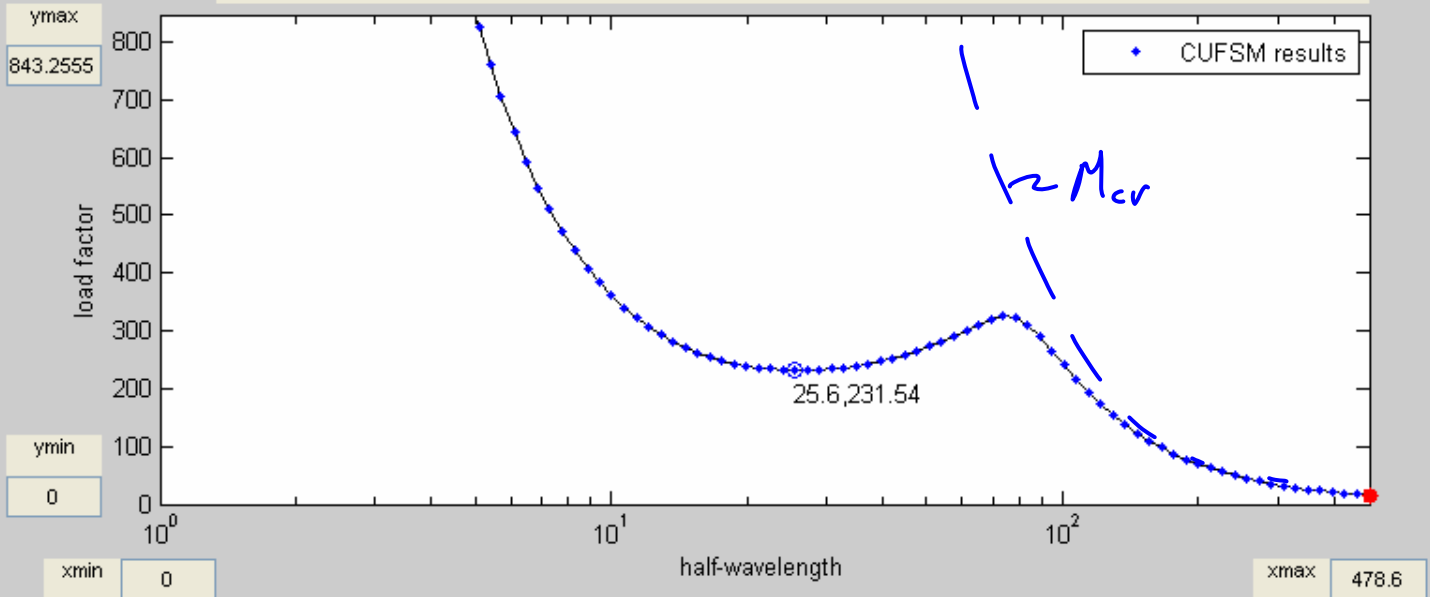
$f_{cr} = 15.8 \times 1.0 \text{ ksi}$   
 $= 15.8 \text{ ksi}$



also predicted by this classical formula:

$$M_{cr} = \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi}{L}\right)^2 EI_y EC_w}$$

Buckled shape for CUF5M results  
 half-wavelength = 478.6 load factor = 15.801 mode = 1  
 cFSM classification results: off



# Tutorial #1: Beam summary

- A W36x150 under major-axis bending (a beam) has two important cross-section stability elastic buckling modes
- (1) Local buckling which occurs at a stress of 231 ksi and may repeat along the length of a member every 26 in. (it's half-wavelength)
- (2) Global lateral-torsional buckling, which for a 40 ft. long member occurs at a stress of 15.8 ksi (other member lengths may be selected from the curve provided from the analysis results)

# Interaction of modes

- In the analysis so far we have looked at two half-wavelengths:
  - 26 in.: which is the first minimum in the curve and exhibits local buckling
  - 480in. or 40 ft.: which is the longest length investigated & exhibits lateral-torsional buckling
- What happens at other half-wavelengths?  
*How about at 100in.?*

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 100.7

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

files to be plotted 1

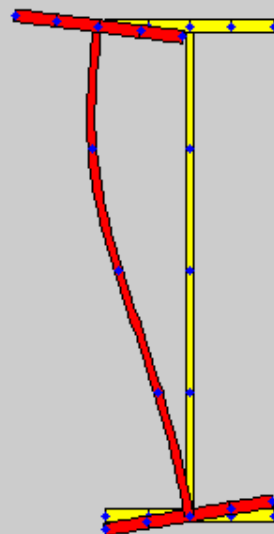
cFSM Modal Classification

Classify work norm

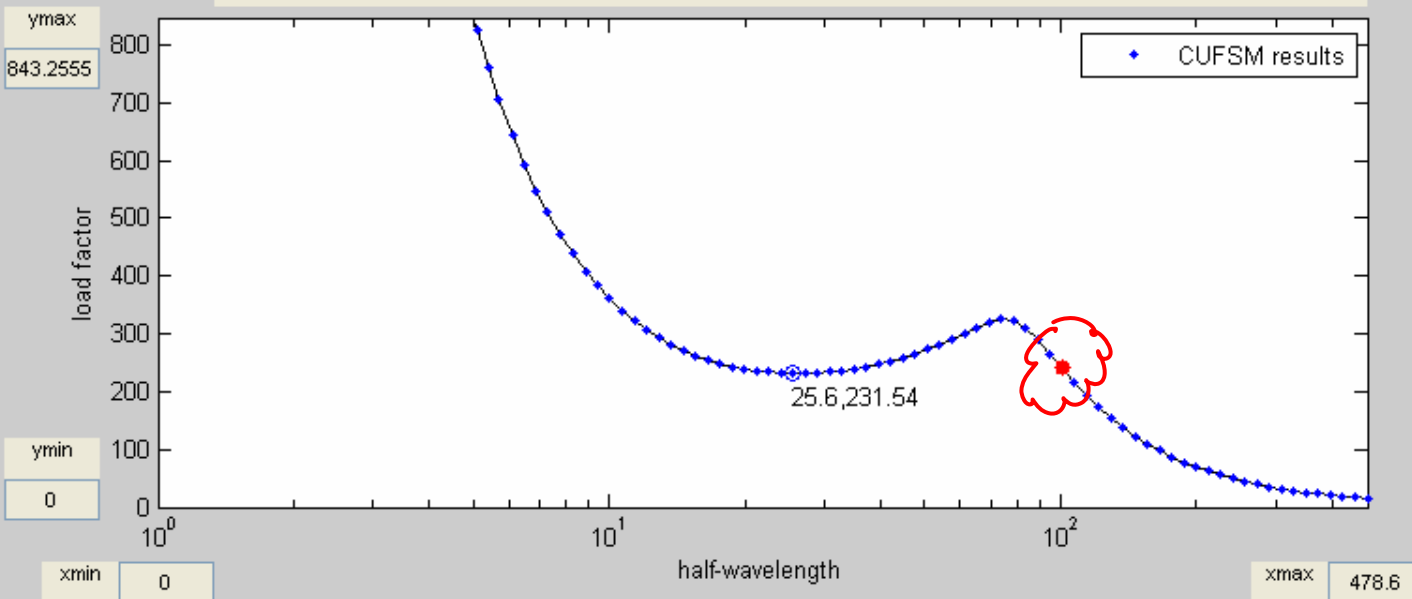
cFSM analysis is off

supplemental participation plot

local and lateral-torsional buckling interacting... (this result is at a lower stress/moment than just lateral-torsional buckling)



Buckled shape for CUF5M results  
 half-wavelength = 100.7 load factor = 240.2291 mode = 1  
 cFSM classification results: off





Load Save

Input Properties Analyze Post

Z R Print Copy Reset ? X

Plot Shape ?

separate window in-plane mode

2D  3D  Undef. Scale 1

**half-wavelength = 100.7**

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

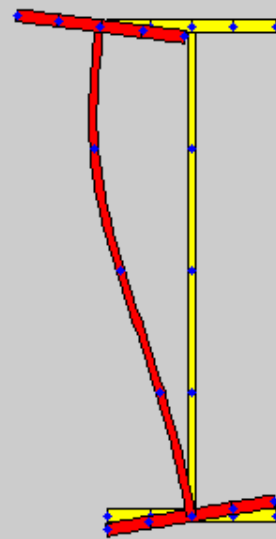
files to be plotted 1

cFSM Modal Classification

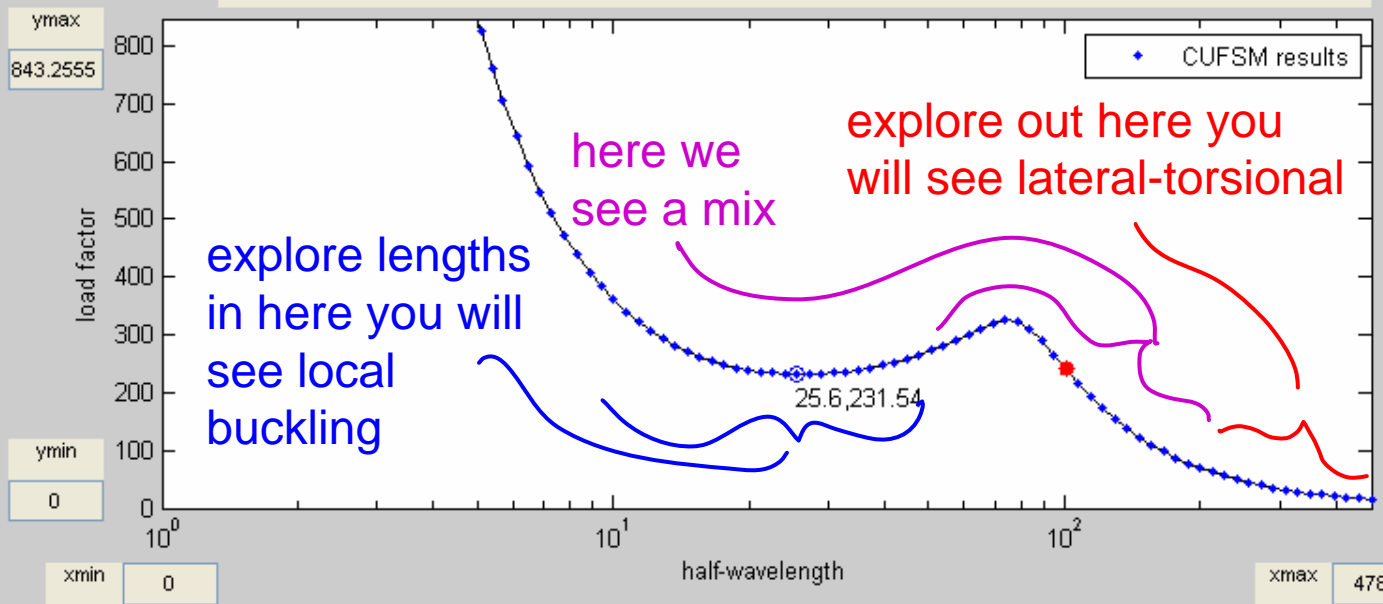
Classify work norm

cFSM analysis is off

supplemental participation plot



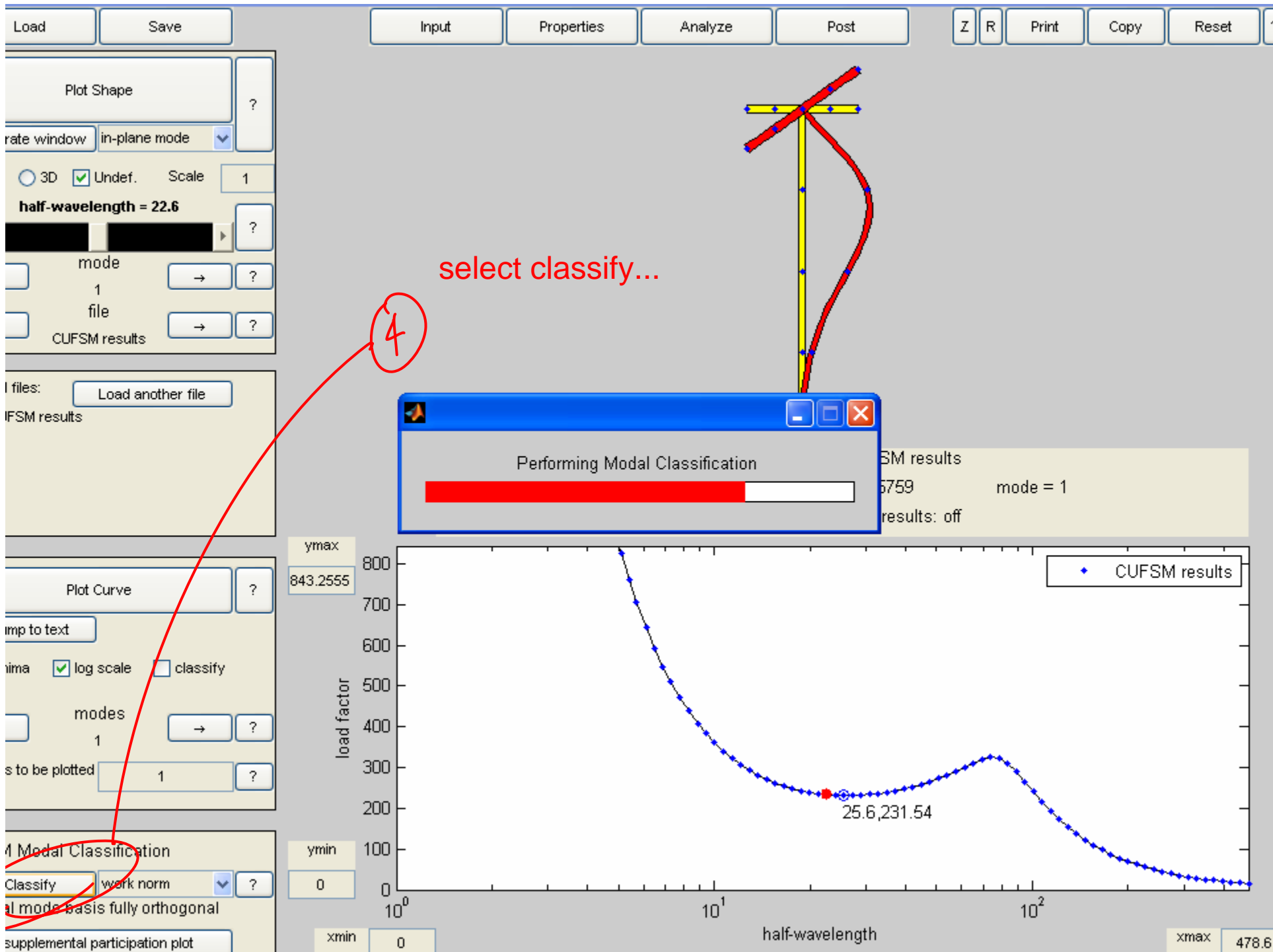
Buckled shape for CUF5M results  
 half-wavelength = 100.7 load factor = 240.2291 mode = 1  
 cFSM classification results: off



# constrained Finite Strip Method

- Using the constrained Finite Strip Method (cFSM) we can formalize our predictions of modal interactions.
- The cFSM was developed and implemented by Schafer and Adany and can be explored in CUFSM
- Let's run an analysis with cFSM on and examine the modal interactions..





Plot Shape ?

separate window in-plane mode

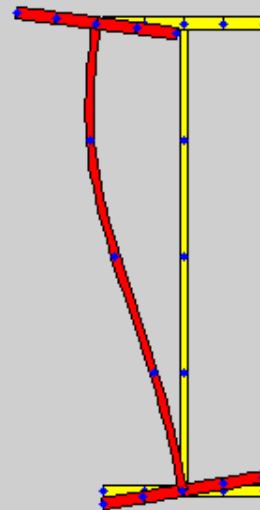
2D 3D  Undef. Scale 1

half-wavelength = 100.7

mode 1

file CUF5M results

cFSM modal classification results at half-wavelength of 100 in. As given, 70% lateral-torsional 21% local 9% other (other buckling modes primarily include shear effects)



loaded files: Load another file

1 = CUF5M results

Buckled shape for CUF5M results  
 half-wavelength = 100.7 load factor = 240.2291 mode = 1  
 cFSM classification results: work norm G=70.3% D=0.0% L=20.7% O=8.9%

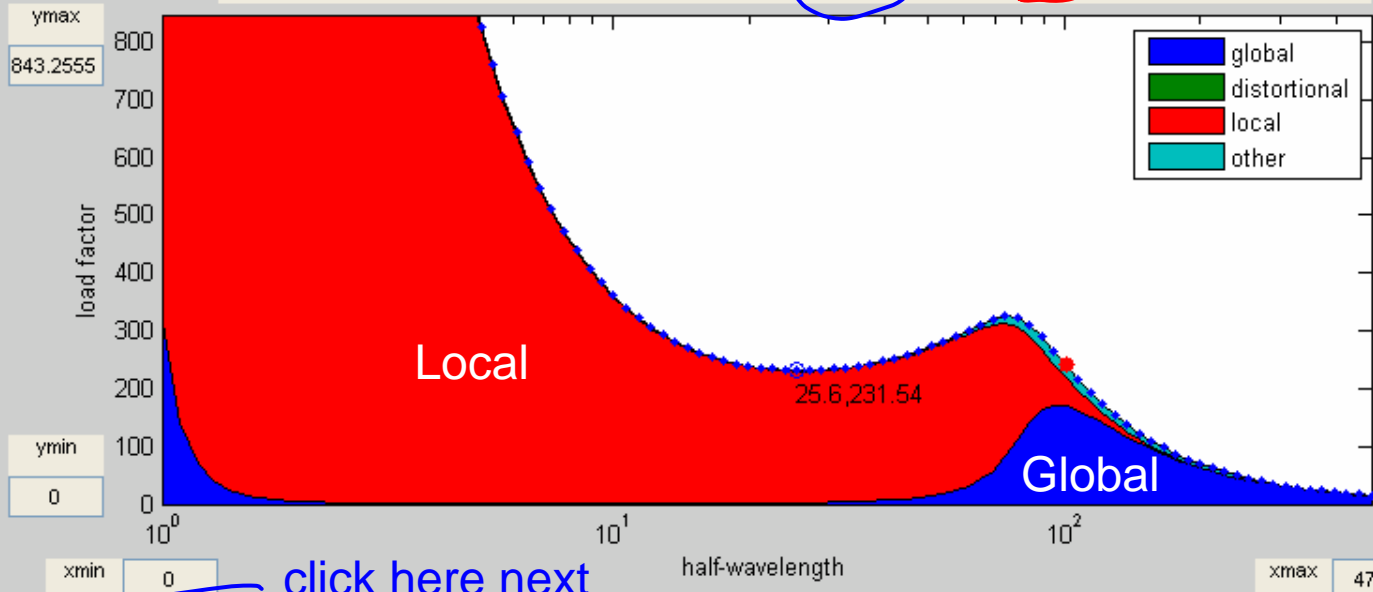
Plot Curve ?

dump to text

minima  log scale  classify

modes 1

files to be plotted 1



cFSM Modal Classification

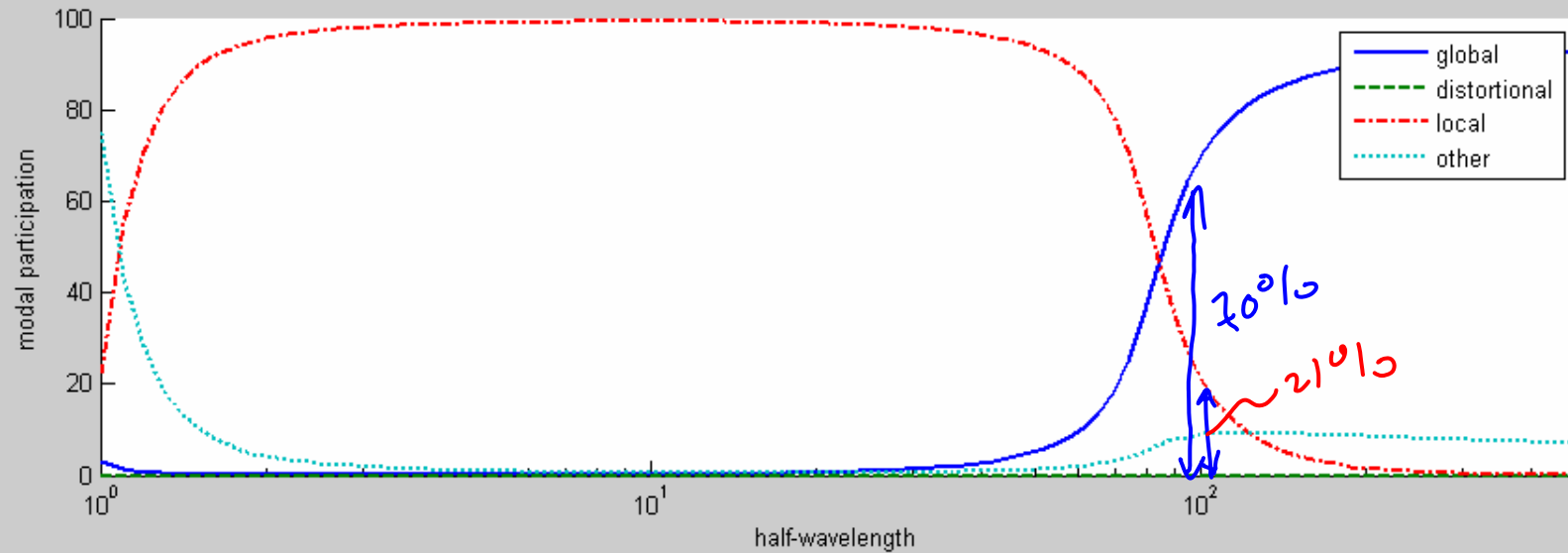
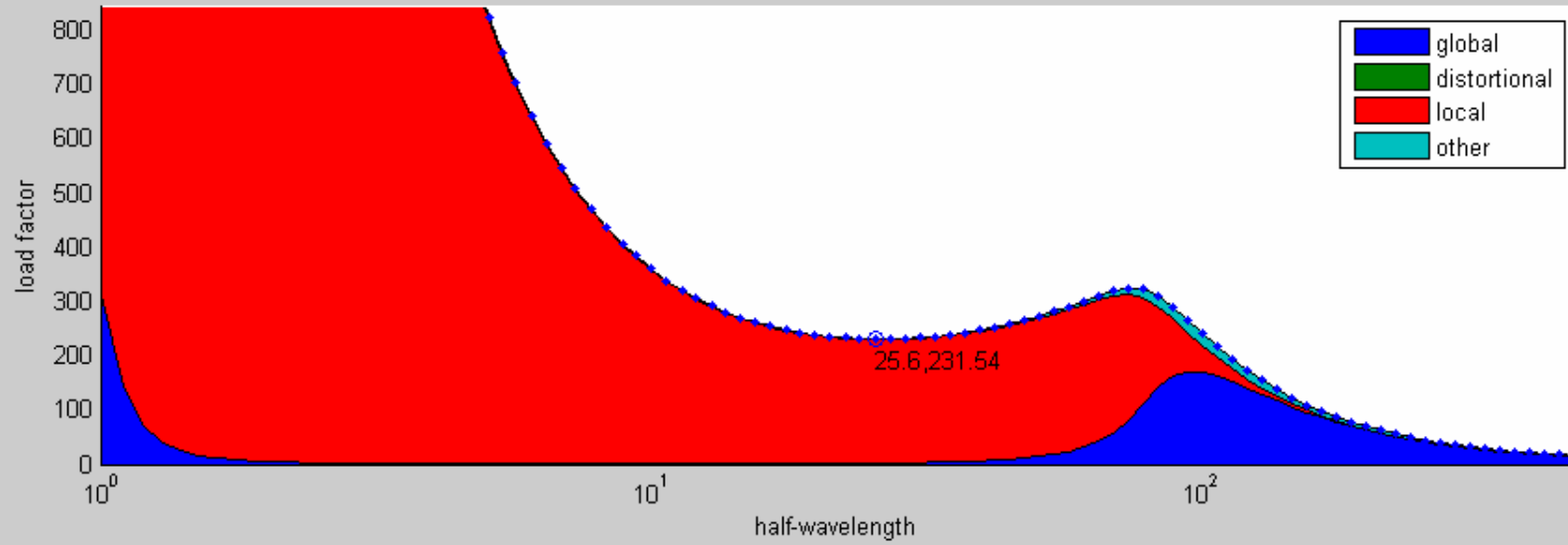
Classify work norm

Axial mode basis fully orthogonal

supplemental participation plot

click here next

CUFSM results - BASIS: Axial mode basis fully orthogonal NORM: work norm



# cFSM decomposition

- One of the useful features of cFSM is the ability to focus on only one type of buckling mode at a time, for example, local buckling..

**Material Properties** ?

mat# | Ex | Ey | vx | vy | Gxy

1	29000.00	29000.00	0.30	0.30	11153.85
---	----------	----------	------	------	----------

**Nodes** ?

node# | x | z | xdof | zdof | ydof | qdof | stress

4	9.00	0.00	1	1	1	-1.00
5	12.00	0.00	1	1	1	-1.00
6	6.00	8.75	1	1	1	-0.50
7	6.00	17.50	1	1	1	0.00
8	6.00	26.25	1	1	1	0.50
9	0.00	35.00	1	1	1	1.00
10	3.00	35.00	1	1	1	1.00
11	6.00	35.00	1	1	1	1.00
12	9.00	35.00	1	1	1	1.00
13	12.00	35.00	1	1	1	1.00

**Elements** ?

elem# | nodei | nodej | thickness | mat#

4	4	5	0.900000	1
5	3	6	0.600000	1
6	6	7	0.600000	1
7	7	8	0.600000	1
8	8	11	0.600000	1
9	9	10	0.900000	1
10	10	11	0.900000	1
11	11	12	0.900000	1
12	12	13	0.900000	1

Update Plot

Plot Options:

node #

element #

material #

stress mag.

stress dist.

coordinates

constraints

springs

origin

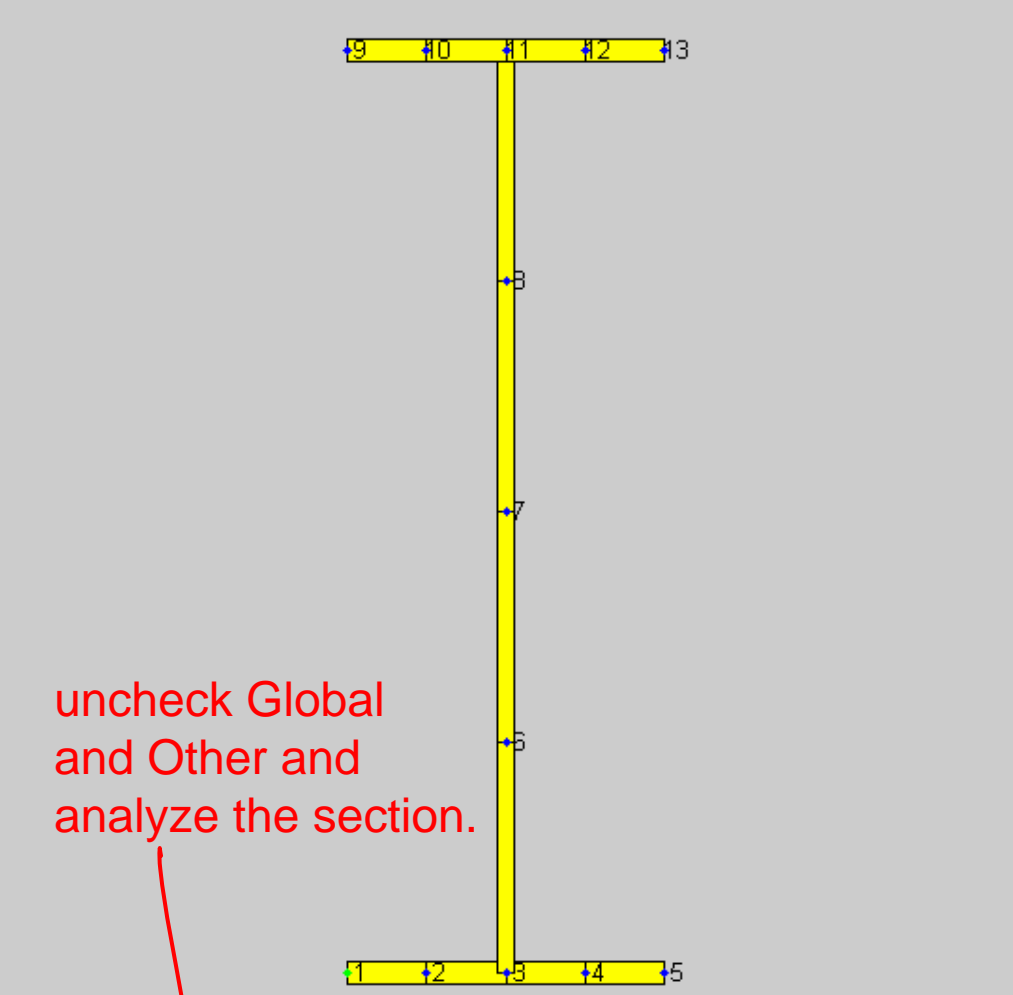
C/Z Template

Double Elem.

Divide Elem.

Delete Elem.

Trans. Node



uncheck Global and Other and analyze the section.

**Lengths** ?

8 78.5 83.6 88.9 94.7 100.7 107.2 114.1 121.5 129.3 137.6 146.4 155.9 165.9 176.6 187.9 200.0 212.9 226.5 241.1 256.6 273.1 290.7 309.4 329.3 350.5 373.0 397.0 422.5 449.7 478.6

**Springs** ?

node# | DOF(x=1,z=2,y=3,theta=4) | kspring | kflag

0			
---	--	--	--

**General Constraints** Master-Slave ?

node#e | DOFe | coeff. | node#k | DOFk

0			
---	--	--	--

**cFSM**

On/Off

Global

Dist.

Local

Other

Basis for cFSM

Natural modes

Axial modes fully orthogonal 0 modes

0	0	0	0
1	1	1	1
0	0	0	0



Load Save

Input Properties Analyze Post

Z R Print Copy Reset ? X

Plot Shape ?

separate window in-plane mode

2D 3D  Undef. Scale 1

half-wavelength = 25.6

mode 1

file CUF5M results

loaded files: Load another file

1 = CUF5M results

Plot Curve ?

dump to text

minima  log scale  classify

modes 1

files to be plotted 1

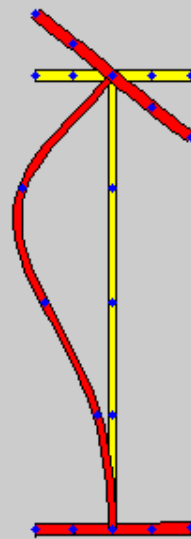
cFSM Modal Classification

Classify work norm

Axial mode basis fully orthogonal

supplemental participation plot

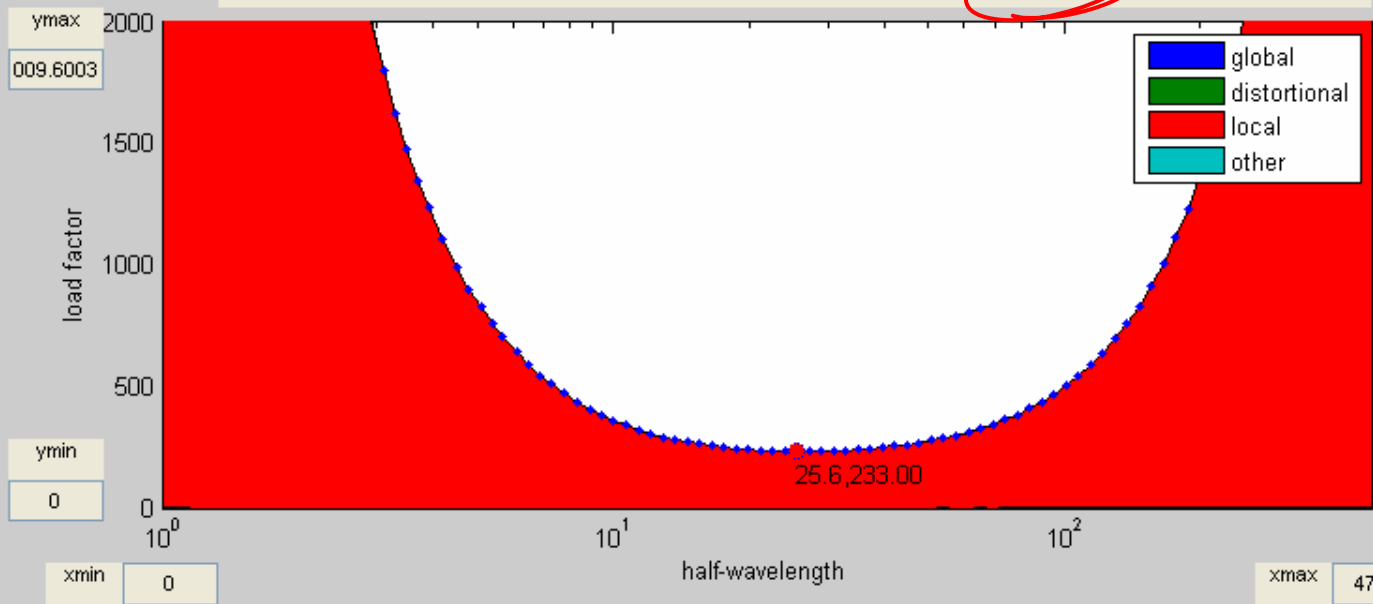
Local only,  
result after  
cFSM analysis  
and classification



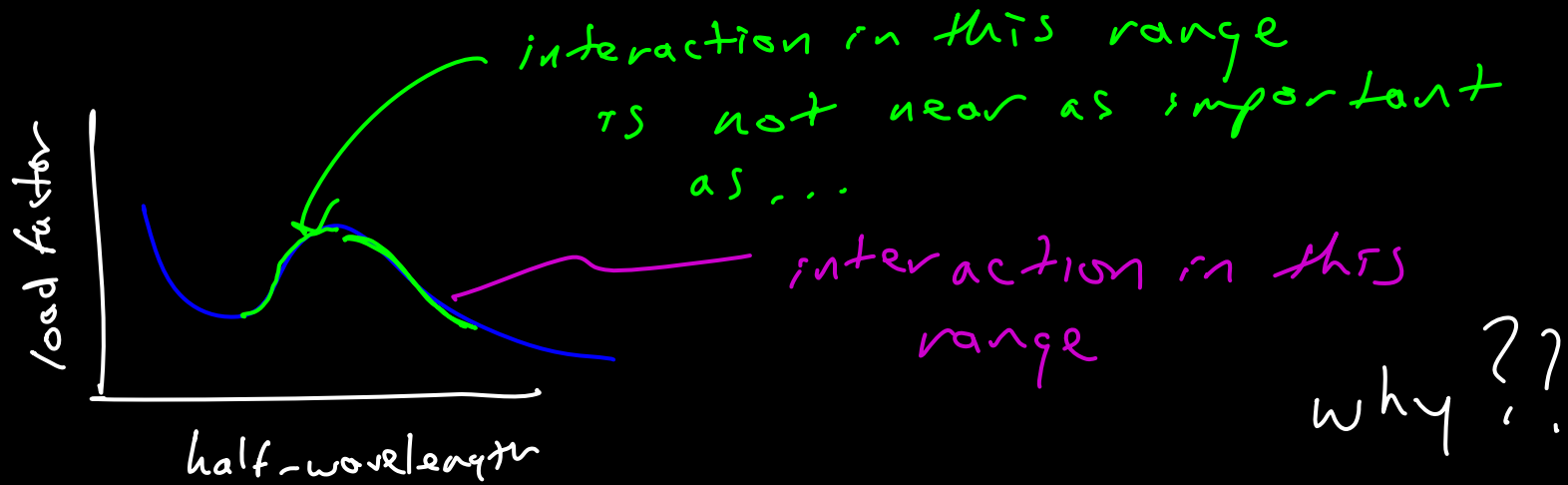
Buckled shape for CUF5M results

half-wavelength = 25.6 load factor = 232.9994 mode = 1

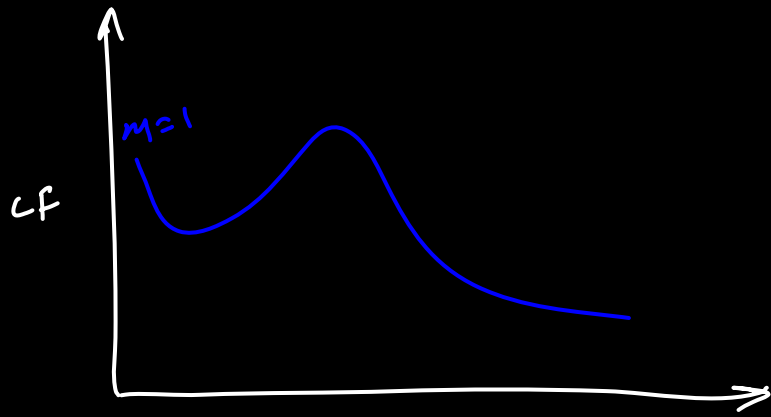
cFSM classification results: work norm G=0.0% D=0.0% L=100.0% O=0.0%



*some more detailed thoughts about when these interactions matter*

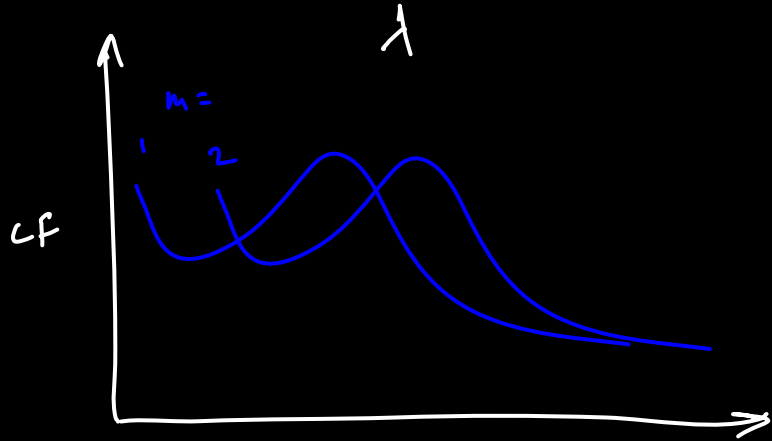


The "up" part of an FSM curve (as shown in green) is typically not the lowest buckling mode because all buckling modes may repeat along the length.

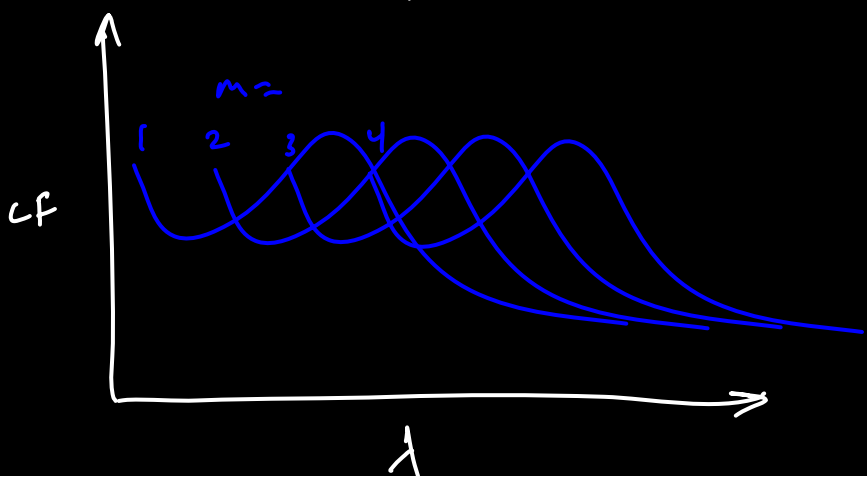


$$\text{shape} \equiv \sin \frac{m\pi x}{\lambda}$$

← traditional FSM



here is what we get if we look at two half-waves along the length (compared to 1)



m=1



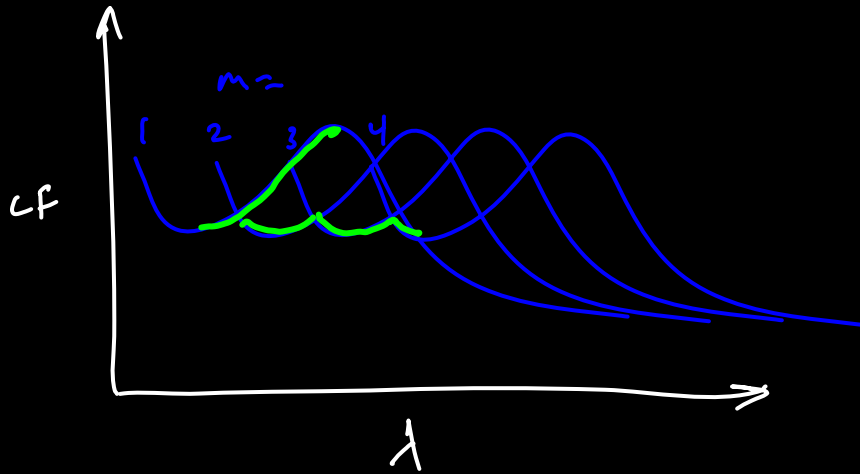
m=2



m=3



m=4



"Green" or up part of the traditional FSM curve does not control, instead

the  $m=2, 3, 4, \text{etc.}$

solutions control (i.e. provide the lowest elastic buckling results.)

conclusion? interaction of the buckling modes is only of serious concern from an FSM analysis when it is identified on the downward slope of a traditional ( $m=1$ , single half-wave) finite strip result.

It should also be noted that interactions such as those identified here in these elastic buckling results are not considered in the AISC Spec.