

# COMPARISON OF DESIGN METHODS FOR LOCALLY SLENDER STEEL COLUMNS

*B.W. Schafer<sup>1</sup> and M. Seif<sup>2</sup>*

## ABSTRACT

The objective of this paper is to provide a comparison between three Specification approved methods currently applicable to the design of locally slender steel columns. The AISC W-section is selected as the geometry for the comparisons. The local cross-section stability of all W-sections in the AISC Manual is assessed using finite strip analysis and compared with plate buckling solutions in common use. Significant web-flange interaction in local buckling is observed in the majority of sections. The design strength formulas for a locally slender W-section column performed by the AISC Q-factor approach, AISI Effective Width Method, and AISI Direct Strength Method are all provided in a common set of notation. The role of cross-section stability in the prediction equations is highlighted. The potential to use cross-section stability solutions for local stability instead of plate buckling solutions is investigated. Through parametric studies the Q-factor treatment of unstiffened elements is shown to be more conservative than the Effective Width Method, particularly as the unstiffened element becomes more slender. The Q-factor treatment of stiffened elements is generally found to be similar, but slightly less conservative than the Effective Width Method. Also, the Direct Strength Method is shown to sometimes follow different trends than the other methods, particularly with respect to web-flange interaction. The parameters that lead to significant differences between the design methods are the focus of a nonlinear finite element analysis study currently getting underway.

---

<sup>1</sup> Associate Professor, Johns Hopkins University, [schafer@jhu.edu](mailto:schafer@jhu.edu)

<sup>2</sup> Graduate Research Asst., Johns Hopkins University, [mseif1@jhu.edu](mailto:mseif1@jhu.edu)

## INTRODUCTION

In the design of hot-rolled steel structural members typical practice is to avoid locally slender (noncompact) cross-sections. However, this strategy becomes impossible with standard shapes if high or ultra-high yield strength steels are used, since flange and web slenderness limits are a function of yield stress. The Q-factor approach in the American Institute for Steel Construction (AISC) Specification is currently used for designing such slender cross-sections, and has been in-place for a number of years. However, new open source analysis packages now allow local buckling to be determined with full accounting of web-flange interaction, and new design methods have been developed that take advantage of such analysis. With the potential for higher yield stress steels and the availability of new analysis and design methods now seems a good time to take a fresh look at the design of locally slender steel cross-sections.

## DEFINITION OF VARIABLES

$b$  : Half of the flange width ( $b_f = 2b$ ).

$t_f$  : Flange thickness.

$h$  : centerline web height.

$t_w$  : Web thickness.

$E$  : Young's modulus of elasticity.

$\nu$  : Poisson's ratio.

$L$  : Length of the member.

$r$  : Governing radius of gyration.

$K$  : Effective length factor.

$k_f$  : Flange local plate buckling coefficient.

$k_w$  : Web local plate buckling coefficient.

$P_n$  : Nominal section compressive strength.

$A_g$  : Gross area of the section.

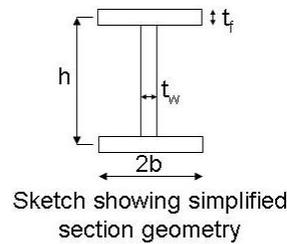
$f_y$  : Yield stress.

$f_e$  : Global elastic buckling stress, e.g.,  $\pi^2 E / (KL/r)^2$ .

$f_{crb}$  : Flange elastic local buckling stress =  $k_f \left( \pi^2 E / (12(1 - \nu^2)) \right) (t_f / b)^2$ .

$f_{crh}$  : Web elastic local buckling stress =  $k_w \left( \pi^2 E / (12(1 - \nu^2)) \right) (t_w / h)^2$ .

$f_{crl}$  : cross-section elastic local buckling stress. e.g. by finite strip

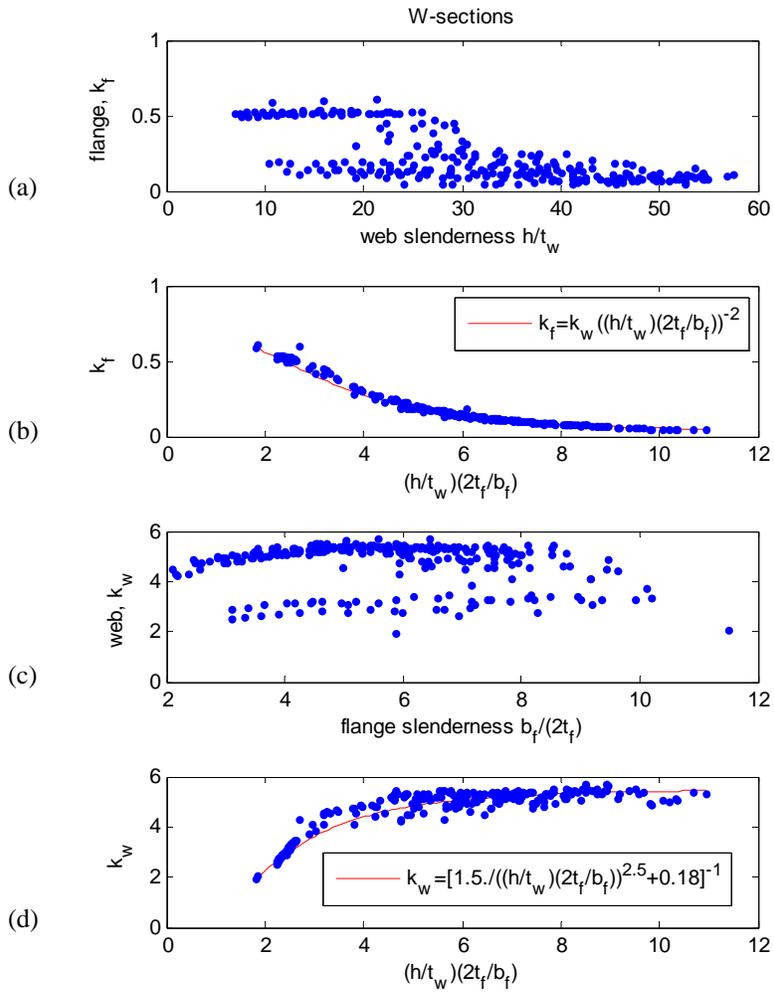


### **CROSS-SECTION BUCKLING BY FINITE STRIP ANALYSIS**

Finite strip analysis was performed for compressive load on the W-sections from the AISC (2005b) Manual of Steel Construction. The analysis was completed using CUFSM version 3.12 (Schafer and Adany 2006). Sections were simplified to their centerline geometry (the increased width in the k-zone was thus ignored). The cross-section local buckling stress ( $f_{cr}$ ) was identified from the buckling half-wavelength vs. load factor curve. Exact (elastic) plate buckling coefficients are found, for example, by setting  $f_{crb}=f_{cr}$  and solving for  $k_f$ . The resulting  $k_f$  and  $k_w$  values are provided in Figure 1. The results of Figure 1 underscore the significant impact of web-flange interaction on local buckling.

The typically cited theoretical limits for the local plate buckling coefficient,  $k_f$ , of an isolated flange (an unstiffened element) vary from 0.43, simply supported on one longitudinal edge free on the other longitudinal edge, to 1.3, fixed on one longitudinal edge free on the other longitudinal edge (Galambos 1998). The AISC Specification assumes a  $k_f$  value of 0.7 (Salmon and Johnson 1996). Figure 1 shows  $k_f$  varies from 0.04 to 0.62 for elastic local buckling. For webs, The theoretical limits for the local plate buckling coefficient,  $k_w$ , of an isolated web (a stiffened element) vary from 4 to 7 (simply supported to fixed edges) in pure compression. The AISC Specification assumes a  $k_w$  of 5. The exact elastic local buckling  $k_w$  values vary from 1.9 to 5.7. As with the flange values, it is clear that web-flange interaction plays a significant role.

For both the web and flange results, not only is there a large difference between the assumed  $k$  values and those calculated, but also the calculated values can be outside expected bounds. For example, for the flange numerous  $k_f$  values are below 0.43. In these cases web local buckling is driving the flange local buckling. The situation for the flange is worse than simply supported, as the flange must provide rotational stiffness to the web for the section to remain stable. Traditionally, it has been assumed that plate buckling coefficients between simply supported and fixed values provide reasonable bounds (e.g., see Salmon and Johnson 1996), but if local buckling of the entire cross-section is considered a much wider range of  $k$  values are possible.



**Figure 1** Flange and web local buckling coefficients for AISC W-sections under axial loading.

Given the functional relationships that are revealed in Figure 1(b) and (d) empirical expressions closely matching the finite strip results are possible. Such expressions for W-sections are provided in Schafer and Seif 2007 and for all other AISC Manual sections (WT, C, L, etc) in compression and bending, in Seif and Schafer 2007.

#### **COMPARING THE AISC, AISI, AND DSM DESIGN METHODS**

A number of different methods exist for the design of steel columns with slender cross-sections, three of which are detailed here: AISC, AISI, and DSM\*. The AISC method, as embodied in the 2005 AISC Specification, uses the  $Q$ -factor approach to adjust the global slenderness in the inelastic regime of the column curve to account for local-global interaction, and further uses a mixture of effective width (for stiffened elements) and average stress (for unstiffened elements) to determine the final reduced strength.

The AISI method, from the main body of the 2007 AISI Specification for cold-formed steel, uses the effective width approach. In the AISI method the global column curve is unmodified but the column area is reduced to account for local buckling in both stiffened and unstiffened elements via the same effective width equation. Finally, the DSM or Direct Strength Method, as given in Appendix 1 of the 2007 AISI Specification for cold-formed steel, uses a new approach where the global column strength is determined and then reduced to account for local buckling based on the local buckling cross-section slenderness.

To provide a more definitive comparison between these three methods the formulas are detailed in the subsequent sections for a centerline model of a W-section in compression. The formulas are presented in a common set of notation. Intermediate derivation steps are shown only for the AISC formulas. In addition, the format of presentation is modified from that used directly in the respective Specifications so that (1) the methods may be most readily compared to one another and (2) the key input parameters are brought to light.

---

\* Strictly, only the AISC method is applicable to the locally slender hot-rolled steel columns studied here. Comparison of  $Q$ -factor and Effective Width methods has been completed by a number of researchers (see, e.g. Galambos 1998 for further discussion). Inclusion of DSM and cross-section local buckling are novel aspects of this paper.

### AISC SLENDER COLUMN DESIGN (Q-FACTOR APPROACH)

The AISC procedure for a column with slender elements is summarized in Section E7 of the 2005 AISC Specification. Specifically, the compressive strength for a centerline model of a W-section is:

$$P_n = A_g \begin{cases} Q(0.658)^{Q(f_e/f_y)} f_y & \text{for: } f_e \geq 0.44Qf_y \\ 0.877f_e & f_e < 0.44Qf_y \end{cases} \quad (1)$$

where:

$$Q = Q_s Q_a \quad (2)$$

and  $Q_s$  is a flange reduction factor for unstiffened elements that depends on the flange slenderness as follows:

$$Q_s = 1.0 \quad \text{if } b/t_f \leq 0.56\sqrt{E/f_y} \quad (3)$$

$$Q_s = 1.415 - 0.74 \left( \frac{b}{t_f} \right) \sqrt{\frac{f_y}{E}} \quad \text{if } 0.56\sqrt{E/f_y} < b/t_f < 1.03\sqrt{E/f_y} \quad (4)$$

$$Q_s = \frac{0.69E}{f_y \left( \frac{b}{t_f} \right)^2} \quad \text{if } b/t_f \geq 1.03\sqrt{E/f_y} \quad (5)$$

$Q_a$  is a web reduction factor, defined as the ratio between the effective area of the cross section and the total cross sectional area:

$$Q_a = A_{eff} / A_g = h_e t_w / A_g, \quad (6)$$

where  $h_e$  is defined as

$$h_e = h \quad \text{if } h/t_w < 1.49\sqrt{E/f} \quad (7)$$

$$h_e = 1.92t_w \sqrt{\frac{E}{f}} \left[ 1 - \frac{0.34}{(h/t_w)} \sqrt{\frac{E}{f}} \right] \leq h \quad \text{if } h/t_w \geq 1.49\sqrt{E/f} \quad (8)$$

and

$$f = P_n / A_{eff} \quad (9)$$

In this form determination of  $f$ , and thus  $h_e$  and  $Q_a$  requires iteration. The AISC Specification notes that  $f$  may be conservatively set to  $f_y$ . More practically, a reasonable estimate of the  $f$  from the iteration may be had without iteration – simply by using the stress from the global buckling column curve with  $Q = 1$ , i.e.,

$$\text{estimated } f = \begin{cases} (0.658)^{(f_e/f_y)} f_y & \text{for: } f_e \geq 0.44 f_y \\ 0.877 f_e & f_e < 0.44 f_y \end{cases} \quad (10)$$

This approximation to  $f$  is conservative since Eq. 10 will always be greater than the  $f$  resulting from Eq. 1 (because  $Q$  is strictly less than 1), but Eq. 10's approximation for  $f$  is also always less than or equal to  $f_y$ . The AISI expressions may be rewritten to better contrast them with their AISI counterparts and highlight the role of cross-section stability:

$$P_n = A_g \hat{f}_n \quad (11)$$

$$\hat{f}_n = \begin{cases} Q_s Q_a (0.658)^{Q_s Q_a (f_e/f_y)} f_y & \text{if } f_e \geq 0.44 Q_s Q_a f_y \\ 0.877 f_e & \text{if } f_e < 0.44 Q_s Q_a f_y \end{cases} \quad (12)$$

The  $Q$  factors may be written directly in terms of the flange and web critical buckling stresses as shown in Eq.'s 13 through 17.  $Q_s$ , the flange reduction factor depends on  $f_{crb}$  as follows:

$$f_{crb} \geq 2f_y : \quad Q_s = 1.0 \quad (13)$$

$$\frac{3}{5} f_y < f_{crb} < 2f_y : \quad Q_s = 1.415 - 0.59 \sqrt{\frac{f_y}{f_{crb}}} \quad (14)$$

$$f_{crb} \leq \frac{3}{5} f_y : \quad Q_s = 1.1 \frac{f_{crb}}{f_y} \quad (15)$$

while  $Q_a$ , the web reduction factor depends on  $f_{crh}$  as follows:

$$f_{crh} > 2f : \quad Q_a = 1.0 \quad (16)$$

$$f_{crh} \leq 2f : \quad Q_a = 1 - \left( 1 - 0.9 \sqrt{\frac{f_{crh}}{f}} \left( 1 - 0.16 \sqrt{\frac{f_{crh}}{f}} \right) \right) \frac{ht_w}{A_g} \quad (17)$$

Note, that the ratio of the web area to the gross area appears in Eq. 17 due to the AISI methodology where only stiffened elements are treated as being reduced to effective width, and hence effective area.

#### **AISI (AISI – EFFECTIVE WIDTH METHOD)**

The AISI Effective Width method is detailed in the 2007 AISI Specification (AISI-S100 2007). The long column (global buckling) design expressions are provided in Section C4.1 of AISI-S100, the effective width reductions follow Section B2.1 for the web (stiffened

element) and B3.1 for the flange (unstiffened element). The expressions provided in Table 1 and Table 2 are not in the same format as AISI-S100 but have been derived here for the purposes of comparison.

### **DSM (AISI – DIRECT STRENGTH METHOD)**

The AISI Direct Strength Method (DSM) is detailed in Appendix 1 of the 2007 AISI Specification. The long column (global buckling) design expression is identical to that in C4.1 of the main AISI Specification. The local buckling strength uses the long column strength as its maximum capacity. The DSM expressions provided in Table 1 and Table 2 have been formulated for comparison to the AISC and AISI Effective Width expressions, and are not in the same form as shown in DSM Appendix 1.

### **DIRECT COMPARISON OF DESIGN EXPRESSIONS**

The design expressions for all three methods, in a common notation system, are provided in Table 1 for the general case of a W-section column and Table 2 for a W-section stub column assuming cross-section local buckling ( $f_{cr}$ ) is used in place of isolated plate buckling solutions ( $f_{crb}$  and  $f_{crh}$ ). Although the expressions appear quite different in the format of their original Specification's – in this format (Table 1) they can be seen to have many similarities.

The number of free parameters in slender column design is actually significantly less than one might typically think. Based on Table 1, and performing a simple non-dimensional analysis, the parameters for determining the column strength of an idealized W-section are:

$$\text{AISC: } P_n/P_y = f(f_e/f_y, f_{crb}/f_y, f_{crh}/f_y, ht_w/A_g)$$

$$\text{AISI: } P_n/P_y = f(f_e/f_y, f_{crb}/f_y, f_{crh}/f_y, ht_w/A_g \text{ or } 2bt_f/A_g)$$

$$\text{DSM: } P_n/P_y = f(f_e/f_y, f_{cr}/f_y)$$

The central role of elastic buckling prediction both globally ( $f_e$ ) and locally ( $f_{crb}$ ,  $f_{crh}$  or  $f_{cr}$ ) in determining the strength of the column is clear. Further, the “direct” nature of the DSM approach is highlighted as DSM only uses ratios of critical buckling values to determine the strength; where AISC and AISI still involve cross-section parameters beyond determination of gross area and critical stress.

**Table 1 Comparison of column design equations for a slender W-section in a common notation\***

<p><b>AISC</b>            inputs to find <math>P_n</math>  <math>A_g</math> = gross area  <math>f_e</math> = global buckling stress  <math>f_y</math> = yield stress  <math>f_{crb}</math> = flange local buckling stress  <math>f_{crh}</math> = web local buckling stress  <math>h_t w / A_g</math> = web/gross area            Comments: shifts the slenderness in the global column curve in the inelastic range only, assumes that unstiffened elements (flange) should be referenced to <math>f_y</math>, only applies an effective width style reduction to stiffened elements (the web), includes an iteration for web stress <math>f</math>.</p>	$P_n = A_g \hat{f}_n$ $\hat{f}_n = \begin{cases} Q_c Q_e (0.658)^{\beta_{0.44} f_e / f_y} f_y & \text{if } f_e \geq 0.44 Q_c Q_e f_y \\ 0.877 f_e & \text{if } f_e < 0.44 Q_c Q_e f_y \end{cases}$ $Q_c = \begin{cases} 1.0 & \text{if } f_{crh} \geq 2f_y \\ 1.415 - 0.59 \sqrt{\frac{f_e}{f_{crb}}} & \text{if } \frac{3}{5} f_y < f_{crh} < 2f_y \\ 1.1 \frac{f_{crh}}{f_y} & \text{if } f_{crh} \leq \frac{3}{5} f_y \end{cases}$ $Q_e = \begin{cases} 1.0 & \text{if } f_{crh} > 2f \\ 1 - \left( 1 - 0.9 \sqrt{\frac{f_{crh}}{f}} \left( 1 - 0.16 \sqrt{\frac{f_{crh}}{f}} \right) \right) \frac{h_t w}{A_g} & \text{if } f_{crh} \leq 2f \end{cases}$ $f = \frac{P_n}{Q_e A_g} - \hat{f}_n \text{ determined with } Q_c = Q_e = 1$
<p><b>AISI - Effective Width</b>            inputs to find <math>P_n</math>  <math>A_g</math> = gross area  <math>f_e</math> = global buckling stress  <math>f_y</math> = yield stress  <math>f_{crb}</math> = flange local buckling  <math>f_{crh}</math> = web local buckling  <math>bt_f</math> = flange area  <math>ht_w</math> = web area            Comments: no shift in global column curve, effective width used for stiffened and unstiffened elements.</p>	$P_n = A_{eff} f_n$ $f_n = \begin{cases} (0.658)^{f_e / f_y} f_y & \text{if } f_e \geq 0.44 f_y \\ 0.877 f_e & \text{if } f_e < 0.44 f_y \end{cases}$ $A_{eff} = 4\rho_b bt_f + \rho_h ht_w$ $b_e = \rho_b b \text{ where } \rho_b = \begin{cases} 1 & \text{if } f_{crb} \geq 2.2 f_n \\ 1 - 0.22 \sqrt{\frac{f_{crb}}{f_n}} \sqrt{\frac{f_{crh}}{f_n}} & \text{if } f_{crb} < 2.2 f_n \end{cases}$ $h_e = \rho_h h \text{ where } \rho_h = \begin{cases} 1 & \text{if } f_{crh} \geq 2.2 f_n \\ 1 - 0.22 \sqrt{\frac{f_{crh}}{f_n}} \sqrt{\frac{f_{crb}}{f_n}} & \text{if } f_{crh} < 2.2 f_n \end{cases}$
<p><b>AISI - DSM</b>            inputs to find <math>P_n</math>  <math>A_g</math> = gross area  <math>f_e</math> = global buckling stress  <math>f_y</math> = yield stress  <math>f_{crf}</math> = local buckling stress            Comments: similar to AISI but reductions on whole section and "effective width" equation modified.</p>	$P_n = A_{eff} f_n$ $f_n = \begin{cases} (0.658)^{f_e / f_y} f_y & \text{if } f_e \geq 0.44 f_y \\ 0.877 f_e & \text{if } f_e < 0.44 f_y \end{cases}$ $A_{eff} = \rho A_g$ $\rho = \begin{cases} 1 & \text{if } f_{crf} \geq 1.66 f_n \\ 1 - 0.15 \left( \frac{f_{crf}}{f_n} \right)^{0.4} \left( \frac{f_{crf}}{f_n} \right)^{0.4} & \text{if } f_{crf} < 1.66 f_n \end{cases}$

\* centerline model of W-section (ignores k-zones) in practice AISI and AISI use slightly different  $k$  values for  $f_{crb}$  and  $f_{crh}$ .

**Table 2 Comparison of stub column design equations for a slender W-section when cross-section elastic local buckling replaces isolated plate buckling solutions, i.e.,  $f_{crL} = f_{crb} = f_{crh}$  and when global buckling is assumed to be fully braced.**

<p><b>AISC</b>  inputs to find <math>P_n</math>  <math>A_g</math> = gross area  <math>f_y</math> = yield stress  <math>f_{crL}</math> = local buckling stress  <math>ht_w/A_g</math> = web/gross area  Comments: adoption of <math>f_{crL}</math> for <math>f_{crb}</math> and <math>f_{crh}</math> does not simplify the AISC methodology significantly. Unstiffened and stiffened elements are treated inherently differently in the AISC methodology.</p>	$P_n = Q_c Q_a A_g f_y$ $Q_c = \begin{cases} 1.0 & \text{if } f_{crL} \geq 2f_y \\ 1.415 - 0.59 \sqrt{\frac{f_y}{f_{crL}}} & \text{if } \frac{3}{5}f_y < f_{crL} < 2f_y \\ 1.1 \frac{f_{crL}}{f_y} & \text{if } f_{crL} \leq \frac{3}{5}f_y \end{cases}$ $Q_a = \begin{cases} 1.0 & \text{if } f_{crL} > 2f_y \\ 1 - \left( 1 - 0.9 \sqrt{\frac{f_{crL}}{f_y}} \left( 1 - 0.16 \sqrt{\frac{f_{crL}}{f_y}} \right) \right) \frac{ht_w}{A_g} & \text{if } f_{crL} \leq 2f_y \end{cases}$
<p><b>AISI – Effective Width</b>  inputs to find <math>P_n</math>  <math>A_g</math> = gross area  <math>f_y</math> = yield stress  <math>f_{crL}</math> = local buckling stress  Comments: when <math>f_{crL}</math> is used for <math>f_{crb}</math> and <math>f_{crh}</math> the methodology becomes the same as DSM, but with a more conservative local buckling predictor equation.</p>	$P_n = A_{eff} f_y$ $A_{eff} = \rho A_g$ $\rho = \begin{cases} 1 & \text{if } f_{crL} \geq 2.2f_y \\ 1 - 0.22 \sqrt{\frac{f_{crL}}{f_y}} \sqrt{\frac{f_{crL}}{f_y}} & \text{if } f_{crL} < 2.2f_y \end{cases}$
<p><b>AISI – DSM</b>  inputs to find <math>P_n</math>  <math>A_g</math> = gross area  <math>f_y</math> = yield stress  <math>f_{crL}</math> = local buckling stress  Comments: no change from general case</p>	$P_n = A_{eff} f_y$ $A_{eff} = \rho A_g$ $\rho = \begin{cases} 1 & \text{if } f_{crL} \geq 1.66f_y \\ 1 - 0.15 \left( \frac{f_{crL}}{f_y} \right)^{0.4} \left( \frac{f_{crL}}{f_y} \right)^{0.4} & \text{if } f_{crL} < 1.66f_y \end{cases}$

### STUB COLUMN COMPARISON

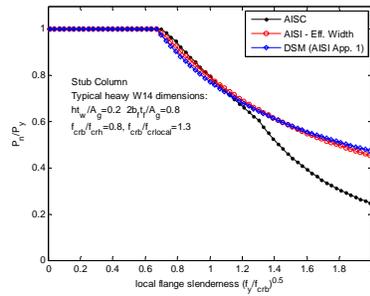
Since all three methods use the same global buckling column curve (though AISC uses the Q-factor approach which adjusts the slenderness used within the curve) the initial focus of the comparison is on a stub column – and thus local buckling only. Predicted stub column

capacities via the three design methods are provided in Figure 2. Since the results are dependent on the cross-section geometry (namely, the  $ht_w/A_g$  ratio) some care must be taken when comparing the methods.

Figure 2(a) provides the stub column comparison for the range of geometry typical of heavier W14 columns. For W14 columns all three methods yield nearly the same strength even for cross-sections reduced as much as 40% from the squash load due to local buckling (i.e.  $P_n/P_y = 0.6$ ). For more slender cross-sections (i.e.,  $(f_y/f_{crb})^{0.5} > 1.2$ ) the AISC method becomes more conservative than AISI and DSM; which essentially provide the same solution for this column.

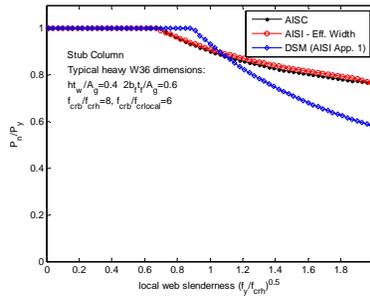
For a W36 column  $f_{crb}$  and  $f_{crh}$  are very different (as opposed to a W14 when they are nearly the same), with the web local buckling stress,  $f_{crh}$ , being significantly lower than the flange local buckling stress,  $f_{crb}$ . In addition, W36 columns have a greater percentage of total material in the web (higher  $ht_w/A_g$  than a W14). For the W36's AISC and AISI provide essentially the same solution over the anticipated flange slenderness range. However, DSM which accounts for the web-flange interaction in a very different manner from the other two methods, assumes the W36 remains compact up to higher flange slenderness, but provides a more severe reduction as the flange slenderness increases.

Since the W36 provides a definite contrast between DSM, and AISI and AISC, the analysis is extended over a wider slenderness range in Figure 3. (Note, flange slenderness  $(f_y/f_{crb})^{0.5}$  greater than 2 is rare for these sections even at yield stress approaching 100 ksi). For the W36 geometry AISI and AISC provide the same solution even as reductions move from just the web, to include the flange. Only when the flange reduction reaches the final branch of the AISC  $Q_s$  curve ( $f_{crb} < 3/5 f_y$ ) and the design stress is reduced essentially to its elastic value of  $1.1 f_{crb}$  does the AISC method diverge from AISI, and in assuming essentially no post-buckling reserve for the unstiffened element flange, provide a more conservative solution. In contrast, the DSM solution provides a continuous reduction and at high slenderness predicts strength between AISI and AISC.



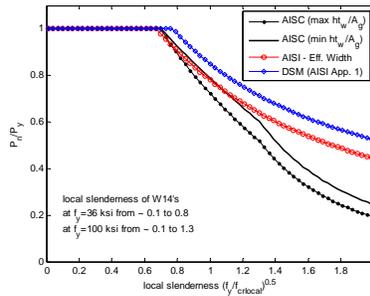
(a) W14 stub column

(flange slenderness varies within W14 series and due to change in  $f_y$ )



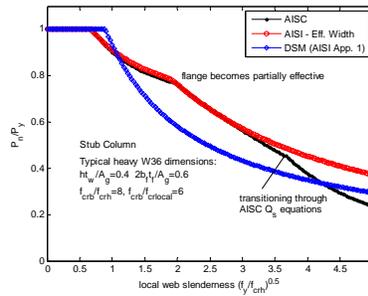
(b) W36 stub column

(web slenderness varies within W36 series and due to change in  $f_y$ )



(c) Any W-section stub column, but cross-section local buckling  $f_{crf}$  has replaced plate buckling  $f_{crb}, f_{crh}$  in the design expressions per Table 2

**Figure 2 Predicted stub column capacities**



**Figure 3 W36 stub column capacity, same as Figure 2(b), but examined over a wider slenderness range to highlight the different predictions**

The stub column strength for the case where cross-section elastic local buckling analysis ( $f_{cr}$ ) is used instead of the isolated plate solutions ( $f_{crb}$  and  $f_{crh}$ ) is provided in Figure 2(c), while the actual design expressions for this case are provided in Table 2. Figure 2(c) provides an interesting contrast to the previous two plots of Figure 2, as it shows that directly introducing  $f_{cr}$  into existing AISC or AISI methods may be overly conservative. The DSM solution provides a strictly greater prediction of a column's strength compared with AISI and AISC for a stub column capacity calculated in this manner. The development of the DSM to an expression different than AISI is exactly because comparisons to cold-formed steel columns showed that when cross-section local buckling was used as the parameter stub column strength follows the DSM curve, not the AISI curve. It is postulated that similar conclusions will be reached for AISC W-sections, though the exact change to a similar DSM curve is not yet known.

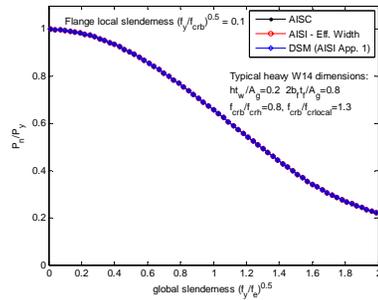
### LONG COLUMN COMPARISONS

The column design expressions for AISC, AISI, and DSM, as summarized in Table 1, are examined for the same three cases as the stub columns in the previous section: W14 columns (Figure 4), W36 columns (Figure 5), and general W-sections where  $f_{cr}$  is substituted for  $f_{crb}$  and  $f_{crh}$  (Figure 6). For each case all three methods are examined as the global slenderness  $((f_y/f_e)^{0.5})$  is varied from 0 to 2, and for four different cross-section slenderness values (subfigures (a) – (d)). The

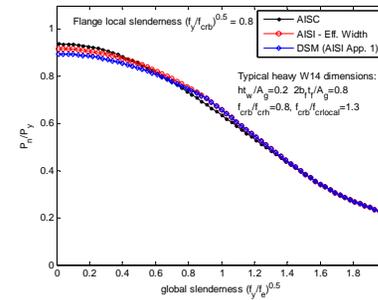
cross-section slenderness is systematically increased in the subfigures: (a) provides the results for a fully compact section, (b) for a local slenderness of 0.8, which corresponds approximately to the most slender W14 at  $f_y=36$  ksi, (c) for a local slenderness of 1.3, a locally slender W14 at  $f_y=100$  ksi, and (d) for a local slenderness of 2 which corresponds to a section with high local slenderness –  $f_{cr}=1/4f_y$ .

The results for the W14 long columns are provided in Figure 5, and the basic conclusions are similar in many respects to the stub column results of Figure 2(a): AISC, AISI, and DSM provide similar capacities except at high local slenderness where AISC provides a much more conservative prediction than AISI or DSM. AISC's Q-factor approach changes the shape of the column curve (i.e.,  $0.658^{Q/(e/f_y)}$  instead of  $0.658^{(e/f_y)}$ ) and the asymptote ( $Qf_y$ ) for a stub column. Figure 3 shows that the change in shape is not significant as neither AISI nor DSM make this change and the basic results are similar as long as the stub column asymptote is similar. Thus, for the AISC curve the stub column asymptote ( $Qf_y$ ) is the only change of practical significance. This is not particularly surprising since prior to the adoption of the unified method in AISI, the cold-formed steel specification also used the Q-factor approach. Part of the justification for moving to a unified effective width approach was that the most significant change to the column curve results was the asymptote (stub column value) not the global slenderness change.

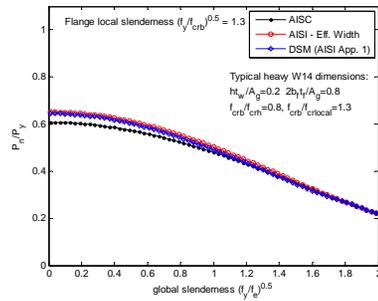
Comparison of the W36 columns is also provided in Figure 5. The most interesting results occur for the most slender cross-section, Figure 5(d), which shows that AISC provides the most liberal prediction of the column strength (though still similar to AISI), which is the opposite of the W14's where AISC provided the most conservative prediction. In practice this implies that AISC penalizes slender unstiffened elements (the flange) more than AISI and rewards slender stiffened elements (the web) more than AISI, thus the ratio of the area of stiffened elements to the area of unstiffened elements or the web-to-flange area ratios influence the AISC predictions relative to AISI or DSM a great deal.



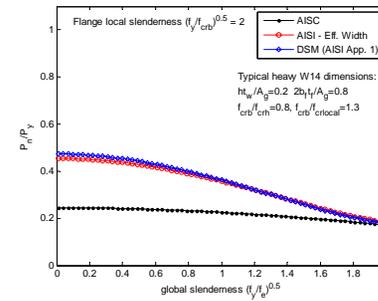
(a) compact: flange slenderness of 0.1, i.e.  $f_{crb} = 100f_y$



(b) local flange slenderness of 0.8, i.e.  $f_{crb} = 1.56f_y$

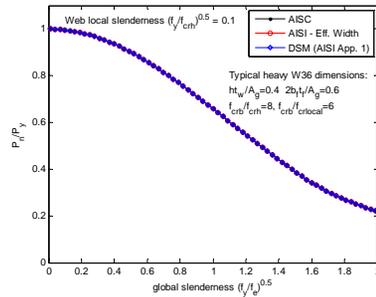


(c) local flange slenderness of 1.3, i.e.  $f_{crb} = 0.59f_y$

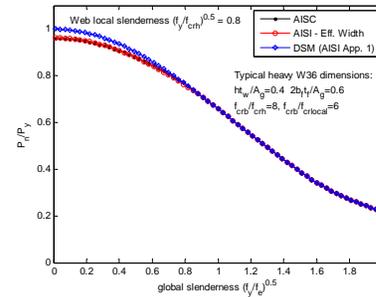


(d) local flange slenderness of 2, i.e.  $f_{crb} = 0.25f_y$

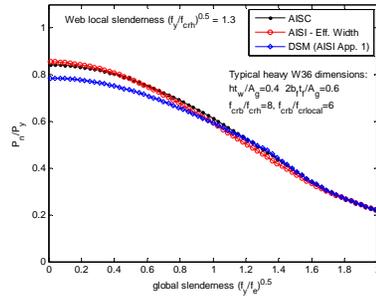
**Figure 4 Predicted long column capacities of typical W14 columns by the AISC, AISI, and DSM methods for varying flange local slenderness following the formulas of Table 1**



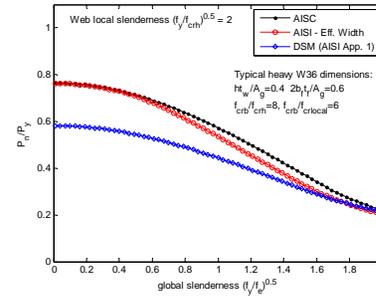
(a) compact: web slenderness of 0.1, i.e.  $f_{crh} = 100f_y$



(b) local web slenderness of 0.8, i.e.  $f_{crh} = 1.56f_y$



(c) local web slenderness of 1.3, i.e.  $f_{crh} = 0.59f_y$



(b) local web slenderness of 2, i.e.  $f_{crh} = 0.25f_y$

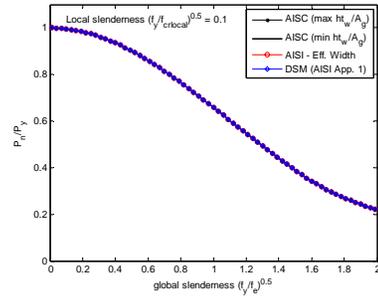
**Figure 5 Predicted long column capacities of typical W36 (as a column) by the AISC, AISI, and DSM methods for varying web local slenderness following the formulas of Table 1**

The long column prediction behavior of DSM is similar to what was observed in the stub column predictions of Figure 2(b): DSM provides a higher capacity than AISC or AISI at low web (local) slenderness, but as the web slenderness increases the predicted overall decrease in the capacity is greater than AISC or AISI. Thus, DSM assumes a greater reduction in the slender column strength due to local buckling driven by the web than AISC or AISI. Finally, as is true for all of the long column methods, since the same global buckling column curve is used, at high global slenderness all of the methods eventually converge.

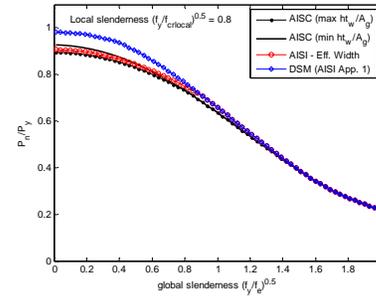
A completely general comparison of the AISC, AISI, and DSM design methods for W-sections is possible if the local cross-section stability solution ( $f_{cr,d}$ ) is used in place of the isolated plate buckling solutions ( $f_{cr,b}$  and  $f_{cr,h}$ ) - such a comparison is provided in Figure 6. Comparisons between the design methods remain similar to the stub column comparisons of Figure 2(c): DSM predicts a consistently greater strength than AISC or AISI, and AISC is most conservative when the flange (unstiffened element) contributes more to the strength. The DSM column curve is known to fit available cold-formed steel column data better than the AISI Effective Width method, when the plate buckling solutions ( $f_{cr,b}$  and  $f_{cr,h}$ ) are replaced by the cross-section local buckling ( $f_{cr,d}$ ) solution. The difference in strength predictions at high local slenderness is large - and suggests that the AISC design philosophy may be overly conservative if cross-section stability solutions are adopted with no other change. Further, this conservatism is increasing as higher yield stress cross-sections are considered.

#### **FUTURE WORK**

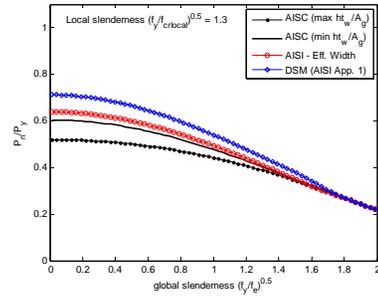
Current efforts include gathering available test data for hot-rolled steel and cold-formed steel to make a statistical comparison of the predictive methods. In addition, we have initiated a parametric study, using ABAQUS, to extend the test database. The role of cross-section details (k-zone, etc.) imperfections, residual stresses, and material yield stress and parameters (strain hardening, etc.) on the results and comparisons will be examined. Particular attention will be placed on understanding the regimes where the AISC and DSM methods give divergent results. The goal of this research is to propose improvements to DSM for its application to locally slender hot-rolled structural steel.



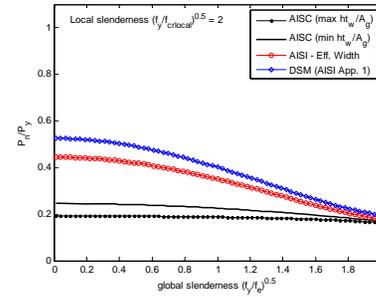
(a) compact: local slenderness of 0.1, i.e.  $f_{cr\ell} = 100f_y$



(b) local slenderness of 0.8, i.e.  $f_{cr\ell} = 1.56f_y$



(c) local slenderness of 1.3, i.e.  $f_{cr\ell} = 0.59f_y$



(d) local slenderness of 2, i.e.  $f_{cr\ell} = 0.25f_y$

**Figure 6 Long column capacities for slender cross-sections. following Table 1, but where cross-section elastic local buckling replaces isolated plate buckling, i.e.,  $f_{crb} = f_{crh} = f_{cr\ell}$  and AISC  $ht_w/A_g$  range reflects range of W14's**

## **CONCLUSIONS**

Using the AISC Q-factor and AISI Effective Width approaches it is shown that the design strength of slender steel columns is specified as a function of (a) global buckling stress, (b) web local plate buckling stress, (c) flange local plate buckling stress, (d) yield stress and (e) the ratio of web area to gross area. However, the flange local buckling stress and web local buckling stress are not unique parameters. As shown herein finite strip analysis of W-section columns provides a ready means for incorporating web-flange interaction and replacing the flange and web local plate buckling stress with a single cross-section local buckling stress. The AISI Direct Strength method replaces the five parameters above with only three parameters: (a) global buckling stress, (b) local buckling stress, and (c) yield stress.

Through parametric studies on W-section columns the AISC Q-factor approach and AISI Effective Width Method are shown to provide similar strengths in most practical regimes. However, as the flanges (unstiffened elements) become more slender the AISC  $Q_s$  term becomes systematically more conservative than the AISI Effective Width expressions (as it effectively ignores post-buckling reserve). The Direct Strength Method provides similar strength predictions to Q-factor and Effective Width, but predicts a greater influence of web-flange interaction. Replacing the plate local buckling solutions with cross-section local buckling solutions within the existing AISC Q-factor and AISI Effective Width Methods is possible, but is shown to predict capacities systematically lower than the Direct Strength Method. Additional work (primarily nonlinear finite element analysis) to better understand the geometric regimes where the methods provide differing strength capacities and to determine the most efficient and accurate solution for locally slender steel columns is underway.

## **ACKNOWLEDGMENTS**

The authors of this paper gratefully acknowledge the financial support of the AISC, and the AISC Faculty Fellowship program. However, any views or opinions expressed in this paper are those of the authors. The authors would also like to thank Don White for discussions related to material in this paper during the development of the 2005 AISC Specification.

## REFERENCES

- AISC (2005). Specification for Structural Steel Buildings. American Institute of Steel Construction, Chicago, IL. ANSI/AISC 360-05
- AISC (2005b). Manual for Steel Construction. American Institute of Steel Construction, Chicago, IL. 13<sup>th</sup> Ed., AISC 325-05
- AISI (2007). North American Specification for the Design of Cold-Formed Steel Structures. American Iron and Steel Institute, Washington, D.C., ANSI/AISI-S100-07.
- Galambos, T. (1998). Guide to Stability Design Criteria for Metal Structures. 5th ed., Wiley, New York, NY, 815-822.
- Salmon, C.G., Johnson, J.S. (1996). Steel structures design and behavior: emphasizing load and resistance factor design. Harper Collins College Publishers, New York, NY.
- Schafer, B.W., Adány, S. (2006). "Buckling analysis of cold-formed steel members using CUFSM: conventional and constrained finite strip methods." Proceedings of the Eighteenth International Specialty Conference on Cold-Formed Steel Structures, Orlando, FL. 39-54.
- Schafer, B.W., Seif, M. (2007). "Cross-section Stability of Structural Steel." American Institute of Steel Construction, Progress Report No. 1. AISC Faculty Fellowship, 25 June 2007.
- Seif, M., Schafer B.W. (2007). "Cross-section Stability of Structural Steel." American Institute of Steel Construction, Letter Report AISC Faculty Fellowship, 12 November 2007.