Improved Ponding Criteria for Cantilever Framing Systems

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Abstract

Current AISC ponding provisions [AISC, 1989b, 1999] are derived from simply-supported primary framing systems. When applied to framing systems where the primary framing is cantilevered over the supports (cantilever framing), the existing equations are overly conservative and take away much of the potential economy presented by cantilever framing. This paper provides a proposed modification to the current provisions, by deriving a modification to the flexibility constant, C_p, of the primary framing. This proposed modification is equally applicable to Load and Resistance Factor Design and Allowable Stress Design methods.

Introduction

Ponding of rainwater on flat or nearly flat roofs is a condition that must be considered by structural engineers when designing buildings. When rain falls for long periods of time, drainage systems may be unable to dispose of the water quickly enough, especially if roof drains become plugged or sufficient overflow scuppers are not provided. Ponding of rainwater occurs in areas on a roof that have initial deflection due to gravity loads or negative camber.

The current ponding provisions [AISC, 1989b, 1999], which are located in section K2 of the specifications, are based on provisions derived by Marino [1966] for ponding as a two-dimensional phenomenon. In the derivation, Marino considered both the primary and secondary framing to be simply-supported. Marino’s work is an extension of Chinn [1965] who considered ponding as a one-dimensional phenomenon. The specifications state that,

\[ C_p + 0.9 C_s \geq 0.25 \]  

(1)

The roof system shall be considered stable and no further investigation is needed if:
where,

$$C_p = \frac{\gamma L S L_p^4}{\pi^4 E I_p} \tag{2}$$

and

$$C_s = \frac{\gamma S L_s^4}{\pi^4 E I_s} \tag{3}$$

However, since the primary framing flexibility constant, $C_p$, is derived for simply supported framing, it may be excessively conservative for primary framing which is continuous over supports, such as cantilever framing.

**Cantilever Framing Systems**

In a cantilever beam framing system, the primary framing member is cantilevered a distance past a support into the next span. The cantilevered end of one beam supports the end of the suspended span using a simple shear connection. This type of framing system is efficient, both in terms of strength and stiffness. The positive moment in the cantilevered beam is reduced by the negative moment created at the support, while shortening the length of the suspended member reduces the positive moment in the suspended beam. Additionally, deflections of cantilever-framed systems are less than those of similar simply-supported systems. Hemstad [1999] places cantilever framing into two categories, which are illustrated in Figure 1. The cantilever/suspended span/cantilever (CSC) arrangement includes a cantilever span at each end with a suspended span in the center. The suspended span/cantilever/suspended span (SCS) arrangement has a simply-supported span at each end with a center span that is cantilevered over each of its supports.

![Figure 1. Cantilever framing systems](image)

**Derivation of Modified Method**

Currently, no provisions exist for explicitly considering ponding in cantilever beam framing systems. It is conservative to use the standard ponding equation derived by Marino [1966] because deflections of a cantilever beam framing system are less than those in a simply-supported system using the same beam sizes. A cantilever framing system that might be considered to be unserviceable using the current specification
criteria may be acceptable for ponding if the actual stiffness of the cantilever system is considered.

The maximum deflection of a simply-supported beam that carries a uniformly distributed load is:

$$\Delta_{ss} = \frac{5WL^3}{384EI}$$

(4)

For the general case of any rotational end restraint, this equation may be re-written as:

$$\Delta_{A} = \frac{C_{A}WL^3}{EI}$$

(5)

where:

$$C_{A} = \frac{5}{384} \text{ for pinned ends; and } \frac{1}{384} \text{ for fixed ends}$$

For a beam, the flexural stiffness is defined as:

$$K = \frac{EI}{L^3}$$

(6)

Substituting Equation (6) into Equation (5), the deflection may be written as:

$$\Delta_{A} = \frac{C_{A}W}{K}$$

(7)

It is evident from Equation (7) that the deflection of a beam is inversely proportional to its flexural stiffness and that the flexural stiffness is directly proportional to the moment of inertia of the beam. Simply stated, for a given span the deflection of a beam under any loading may be reduced by one-half by doubling the stiffness of that beam. The stiffness will be increased by a factor of two by doubling the moment of inertia. In general, the relative stiffness of any elastic beam can be related to the stiffness of a simply-supported beam of equal length as the ratio of the deflections under the same loading conditions. Therefore, it may be written that:

$$\frac{\Delta_{ss}}{\Delta_{A}} = \frac{K_{A}}{K_{ss}} = \frac{I_{A}}{I_{ss}}$$

(8)

The deflection of a cantilever framing system beam is within the envelope of deflections defined by pinned ends and fixed ends. Substituting the stiffness of a cantilever span beam ($K_{c}$) for the general stiffness ($K_{A}$) and solving for the stiffness:

$$K_{c} = K_{ss}\left(\frac{I_{c}}{I_{ss}}\right) = K_{ss}\left(\frac{\Delta_{ss}}{\Delta_{c}}\right)$$

(9)

Therefore, the ratio of the stiffnesses may be represented as a ratio of the deflections, where $n$ is the ratio of the deflection of a beam in a simply-supported condition relative to that same beam cross-section in a cantilever condition.

$$n = \frac{\Delta_{ss}}{\Delta_{c}}$$

(10)
The AISC ponding provisions are based on evaluating the flexibility of the framing. The flexibility of a beam is the reciprocal of its stiffness.

\[
C = l/K \tag{11}
\]

Therefore, the equivalent stiffness of the cantilever beam portion of the cantilever framing system may be represented as:

\[
K_C = \frac{E(nl)}{L^3} \tag{12}
\]

Equation 2 from the AISC specifications (AISC, 1989a, 1999) can now be modified to reflect this equivalent stiffness by substituting nIp for Ip giving:

\[
C_{PC} = \frac{32L_sL_p^4}{10^nlp} \tag{13}
\]

**CALCULATION OF EQUIVALENT STIFFNESS FACTOR (n)**

The calculation of the equivalent stiffness factor n requires calculation of the deflections of the cantilever beam and an identical beam in a simply-supported condition. In a cantilever beam framing system, the maximum deflection is not necessarily located at the center of the span as it is in a simply-supported beam. Sputo et al. (2004) illustrates two methods of calculating the value of the equivalent stiffness factor: an approximate method and an ‘exact’ method. The derivation of the approximate method is given below (refer to Figure 2 for all dimensions). Both methods replace the moment at the support created by a unit load placed at the end of the cantilever span with an equivalent moment located at the support.

**Approximate Calculation of Equivalent Stiffness Factor**

A beam with a cantilever on one end only can be modeled as a simply-supported beam with a concentrated moment at one end, as shown in Figure 3. The maximum deflection and the deflection at the midspan of the beam may be written as:

\[
\Delta_{MAX} = \frac{ML^2}{9\sqrt{3} EI} \quad \text{at} \quad x = \frac{L}{\sqrt{3}} = 0.577L \tag{14}
\]
\[
\Delta_{\text{MID}} = \frac{ML^2}{16EI} \quad \text{at} \quad x = \frac{L}{2} = 0.500L \tag{15}
\]
\[
\frac{\Delta_{\text{MAX}}}{\Delta_{\text{MID}}} = \frac{16}{9\sqrt{3}} = 1.03 \tag{16}
\]

Because the ratio of maximum deflection to the midspan deflection is approximately one, it is acceptable to assume that the midspan deflection is functionally equivalent to the maximum deflection for reasonable design purposes. Equation 17 gives the maximum deflection for the case of a simply-supported beam with a uniformly distributed load over the entire length as shown in Figure 4.

\[
\Delta_{\text{MAX}} = \Delta_{\text{MID}} = \frac{5WL^4}{384EI} \tag{17}
\]

![Figure 3. Simplified cantilever beam model](image)

![Figure 4. Simply-supported beam with uniform load](image)

It is assumed that instead of \( I_p \), we substitute \( nI_p \) as an increased effective stiffness for a cantilever framing system over an equivalent simply-supported framing system. Then, the flexibility constant for a simply-supported beam:

\[
C_p = \frac{32L_xL_p^4}{10^2I_p} \tag{18}
\]
may be re-written for a cantilever span beam as:

\[ C_{PC} = \frac{32L_S L_p^4}{10^7 n I_p} \]  

(19)

where \( L_S \) and \( L_p \) are in feet, and \( I_p \) is in in.\(^4\). Now, consider a beam with a cantilever on one end only as shown in Figure 5.

\[ M_i = \frac{P_i B}{2} \]  

(20)

\[ \Delta_{\text{MID assumed}} = \frac{M_i C^2}{9 \sqrt{3} E I} = \frac{wBAC^2}{(2)9 \sqrt{3} E I} \]  

(21)

\[ \Delta_{\text{TOTAL}} = \frac{5wC^4}{384 E I} - \frac{wBAC^2}{18 \sqrt{3} E I} \]  

(22)

\[ \frac{1}{n} = \frac{\Delta_{\text{TOTAL}}}{\Delta_{\text{UNIFORM SS}}} \]  

(23)

\[ \frac{1}{n} = \left[ \frac{5wC^4}{384 E I} - \frac{wBAC^2}{18 \sqrt{3} E I} \right] = \left[ 1 - \frac{wBAC^2 (384 E I)}{18 \sqrt{3} E I (5wC^4)} \right] = \left[ 1 - \frac{2.463BA}{C^2} \right] \]  

(24)

or,

\[ n = \frac{1}{1 - \left( \frac{2.463BA}{C^2} \right)} \]  

(25)

\[ \frac{wA}{2} \]

\[ a \rightarrow b \rightarrow c \]

Figure 5. Cantilever beam with applied loads

**Example 1**

If \( A = 20 \) feet, \( B = 5 \) feet, \( C = 30 \) feet

\[ n = \frac{1}{1 - \left( \frac{2.463(5)(20)}{(30)^2} \right)} = 1.37 \]

\[ n I_p = 1.37 I_p \]
Therefore, this cantilever framing system is 37% stiffer than an equivalent length simply-supported system of three equal spans.

Now consider a beam with unequal cantilevers at both ends as shown in Figure 6.

\[ P_1 = \frac{wA}{2}, \quad M_1 = P_1 B = \frac{wBA}{2} \]  
(26)

\[ P_2 = \frac{wF}{2}, \quad M_2 = P_2 D = \frac{wFD}{2} \]  
(27)

\[ \Delta_{\text{mid assumed}} = \frac{(M_1 + M_2)C^2}{9\sqrt{3EI}} = \frac{w(BA + DF)C^2}{18\sqrt{3EI}} \]  
(28)

\[ \Delta_{\text{total}} = \frac{5wC^4}{384EI} - \frac{w(BA + DF)C^2}{18\sqrt{3EI}} \]  
(29)

\[ \frac{1}{n} = \frac{\Delta_{\text{total}}}{\Delta_{\text{uniform ss}}} \]  
(30)

\[ \frac{1}{n} = \left[ \frac{5wC^4}{384EI} - \frac{W(BA + DF)C^2}{18\sqrt{3EI}} \right] = \left[ 1 - \frac{2.463(BA + DF)}{C^2} \right] \]  
(31)

or,

\[ n = \frac{1}{1 - \frac{2.463(BA + DF)}{C^2}} \]  
(32)

\[ \frac{wA}{2} \quad \frac{wF}{2} \]

\[ b \quad c \quad w \quad d \quad e \]

Figure 6. Cantilever beam with unequal cantilevers

**Example 2**

If \( A = 20 \) feet, \( B = 5 \) feet, \( C = 30 \) feet, \( D = 5 \) feet, \( F = 20 \) feet

\[ n = \frac{1}{1 - \frac{2.463[5(20) + 5(20)]}{(30)^2}} = 2.21 \]

\[ nI_p = 2.21I_p \]
Therefore, this cantilever framing system is 121% stiffer than a similar simply-supported system.

**DESIGN EXAMPLE**

Design primary and secondary framing for the loading condition given below, using the AISC-ASD specification provisions [AISC, 1989a]

- Columns are 30.0 ft o.c.
- Secondary framing spans are 35.0 ft
- DL + LL = 45.0 psf (including all framing weight)
- Framing is ASTM A992, Grade 50

**Simply-Supported Primary Framing**

*Design primary framing*

\[ w = 45.0 \text{psf}(35.0 \text{ft}) = 1580 \text{ plf} \]
\[ M = \frac{wL^2}{8} = \frac{1.58(35.0)^2}{8} = 177 \text{ f-k} \]
\[ \therefore \text{Use W16x40} \quad M_r = 178 \text{ f-k}(\text{AISC-ASD pg 2-12}) \]

*Secondary framing design*

Try beams at 6'- 0” o.c.

\[ w = 45.0 \text{psf}(6.0 \text{ft}) = 270 \text{plf} \]
\[ M = \frac{0.270(35.0)^2}{8} = 41.4 \text{ f-k} \]

Try W12x16 \[ \rightarrow I = 103 \text{ in}^4 \quad M_r = 47 \text{ f-k} \]
\[ \Delta = 3.05 \text{ in} = \ell/138 > \ell/360 \quad \text{(NG, deflection excessive)} \]

Try W16x26 \[ \rightarrow I = 301 \text{ in}^4 \quad M_r = 106 \text{ f-k} \]
\[ \) = 1.04 \text{ inches} = \ell/402 \quad \text{(deflection OK)} \]

**Calculate Flexibility Constants**

*Simply-Supported System* \[ \text{W16x40} \rightarrow I_p = 518 \text{ in}^4 \]
\[ C_p = \frac{32L_S L_p^4}{10^7 I_p} = \frac{32(35)(30)^4}{10^7(518)} = 0.175 \]
\[ \text{W16x26} \rightarrow I_S = 301 \text{ in}^4 \]
\[ C_S = \frac{32SL_S^4}{10^7 I_S} = \frac{32(6)(35)^4}{10^7(301)} = 0.0960 \]

**Check Ponding**

*Simply-Supported System*

A system is considered acceptable if \[ C_p + 0.9C_S \leq 0.25 \quad (K2-1) \quad \text{AISC - ASD} \]
\[ 0.175 + 0.9(0.096) = 0.261 > 0.25 \quad \text{Exceeds Limit, but probably acceptable} \]
Re-design System using a SCS Cantilever Framing System

Design Primary Framing
Suspended span (see Figure 7)
\[
M = \frac{wL^2}{8} = \frac{1.58(20.0)^2}{8} = 78.8 \text{ f-k}
\]
∴ Use W14x22 → \(M_r = 80.0 \text{ f-k}\)

Cantilever beam (see Figure 8)
\[
M = 98.4 \text{ f-k} \text{ (Moment at support controls)}
\]
∴ Use W16x26 → \(M_r = 106 \text{ f-k}\)

\[1.575 \text{ kip/ft}\]

![Figure 7. Suspended span of design example](image)

\[15.75 \text{ K}\]
\[1.575 \text{ kip/ft}\]
\[47.25 \text{ K}\]

![Figure 8. Cantilever span of design example](image)

Design Secondary Framing
Secondary framing remains as previously designed

Calculate Flexibility Constants
SCS Cantilever Framing System using current unmodified AISC-ASD equation.

First, check \(C_p\) for cantilever span,
\[W16x26 \rightarrow I_p = 301 \text{ in}^4\]
\[
C_p = \frac{32L_sL_p^4}{10^7 I_p} = \frac{32(35)(30)^4}{10^7 (301)} = 0.301
\]

Next, check \(C_p\) for suspended span,
\[W14x22 \rightarrow I = 199 \text{ in}^4\]
Secondary Framing
For W16x26 secondary beams
\[
C_s = \frac{32SL_s^4}{10^7 I_s} = \frac{32(6)(35)^4}{10^7(301)} = 0.0960
\]

Check Ponding
A system is considered acceptable if \( C_p + 0.9C_s \leq 0.25 \) (K2 – 1) AISC – ASD

\[
0.301 + 0.9(0.096) = 0.387 > 0.25 \quad \text{Exceeds limit, not acceptable}
\]

Re-evaluate SCS Cantilever Framing System using proposed modified AISC criteria.

A = 20 feet, B = 5 feet, C = 30 feet, D = 5 feet, F = 20 feet

\[
n = 2.209
\]

\[
C_{pc} = \frac{32L_pL_p^4}{10^7 nI_p} = \frac{32(35)(30)^4}{10^7(2.209)(301)} = 0.136
\]

Evaluate Cantilever Span,
\( C_{pc} = 0.136; \quad C_s = 0.096 \)

\[
0.136 + 0.9(0.096) = 0.222 < 0.25 \quad \text{Acceptable, meets AISC criteria}
\]

Evaluate Suspended Span,
\( C_p = 0.090 \)
\( C_s = 0.096 \)

\[
0.0900 + 0.9(0.0960) = 0.176 < 0.25 \quad \text{Acceptable, meets AISC criteria}
\]

As illustrated by this example, the cantilever framing system that would be shown as unacceptable for ponding stability using the current AISC criteria, would be adequate if the actual stiffness of the framing is considered.

**CONCLUSION**

Current AISC ponding provisions were developed for simply-supported primary framing. The provisions have been shown to be conservative when applied to cantilever primary framing. It is possible to simply modify the flexibility constant of the primary framing to take into account the actual stiffness of the cantilever framing, resulting in lighter designs which meet the intent of the AISC ponding criteria.
NOMENCLATURE

CA   Deflection constant
CP   Flexibility constant of primary member
CPc  Modified flexibility constant of primary member in cantilever framing system
CS   Flexibility constant of secondary member
E    Modulus of elasticity (29,000,000 lb/in.²)
Ip   Moment of inertia, primary member (in.⁴)
Is   Moment of inertia, secondary member (in.⁴)
K    Flexural Stiffness (kips/in.)
Lp   Length of primary member (ft.)
ls   Length of secondary member (ft.)
M    Moment (f-k)
S    Spacing of secondary members (ft.)
w    Distributed load on span
W    Total load on the span = wL
γ    Unit weight of water (0.0361 lb/in.³)
Δ    Additional ponding deflection (in.)

REFERENCES