FINITE STRIP ANALYSIS IN THE ASSESSMENT OF LOCAL BUCKLING CAPACITY OF THE FEMORAL NECK

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Introduction: Realistic computer simulations of loaded trabeculae in osteoporotic bones show that individual trabeculae fail by Euler buckling. Euler buckling is an instability mode that becomes likely in end-loaded structures when they become too slender and lose lateral support. Increasing slenderness and loss of transverse elements are hallmarks of trabecular degradation in osteoporosis. Since applied loads are mainly borne by the cortex, its stability may be of greater importance. Aging and osteoporosis cause thinning cortices, expansion of outer diameter and loss of internal trabecular support which may lead to local buckling, a different form of instability. In this preliminary study we performed a series of stability analyses on tubular structures simulating femoral neck geometries using observed dimensions from elderly fracture cases and controls.

Method: We analyzed eight cross-section models of the femoral neck cortex using dimensions from studies of hip fracture cases and non-fractured controls. Sections were selected to span the a range of cortical slenderness based on the buckling ratio (BR); i.e., maximum outer radius from the center of mass divided by the mean wall thickness. Three different models were considered: non concentric circles; non-concentric ellipses; and more realistic sections from CT scans of the femoral neck taken from a study comparing femoral neck biopsies in fracture cases and cadaver controls(1).To examine potential for local instability we developed finite strip models of each cross-section and determined the elastic local buckling stress ($f_{cr\ell}$). The finite strip method (2) is a specialized variant of the finite element method; we employed the implementation in CUFSM (3). The model considers the cross-section as an extrusion (no variation along the length). Local buckling occurs at lengths as short as ½ the section diameter, so the assumption of a straight extrusion is reasonable. Material properties assumed E = 18,800 MPa, G= 13620 MPa, ν= 0.31(4).

Table 1 Cross-section	Buckling Ratio	$f_{cr\ell}$ (MPA)	$\sqrt{\frac{f_{yc}}{f_{cr\ell}}}$	f_n (MPA)
#1 Circle Control	8.6	1579	0.36	210
#2 CT Control	9.9	674	0.56	210
#3 CT Case	10	431	0.7	206
#4 Ellipse Control	11.9	948	0.47	210
#5 Ellipse Case #5	15.4	681	0.56	210
#6 CT Case	18.2	209	1.00	163
#7 Circle Case	25.7	562	0.61	210
#8 Circle Case	56.2	135	1.25	139

Results: Analyses generated in pure compression for $f_{cr\ell}$, $\sqrt{(f_{yc}/f_{cr\ell})}$ and f_n are listed in Table 1. BR and $f_{cr\ell}$ do correlate but relatively poorly in the more realistic CT based cross-sections where irregular cortices dominate load behavior. Overall, the predicted elastic local buckling stresses ($f_{cr\ell}$) are quite high for all but the most slender of cross-sections. To put results in context we used a variation on the effective width method, first developed for elastic-plastic metals by von Kármán (5), and widely used for viscoelastic plastics. We employed the expression attributed to Winter(6) to estimate the reduction in material compressive capacity (f_n) due to local buckling as a function of the local slenderness $\sqrt{(f_{yc}/f_{cr\ell})}$. Figure 1 provides a sense of the inter-play between the two compressive limit states: material yielding, and local instability. For cross-sections with the lowest buckling ratio a strong reduction in compressive capacity can still occur due to local instability. The importance of more realistic cortical dimensions is underscored by the CT results, for example #s 2 and 3 have similar BR values but control #2 should experience no reduction in strength; while fracture case #3 shows a small reduction. Further, this more realistically modeled case is expected to experience some reduction while #7 modeled as a circle experiences none. These results suggest that simple BR values and simple regular models may inadequately describe instability at the femoral neck.

Examining local stability of the femoral neck under pure compression provides insights, but in vivo failure demands will include bending. Figure 2 provides the predicted local instabilities for three sections under a combined action of compression plus bending about the horizontal axis: thus increasing compression on the lateral side and relieving it on the medial side. The importance of local geometry is again apparent, where local instabilities can arise in the cross-section well away from the point of maximum stress (left margin), as shown in Figure 2c.

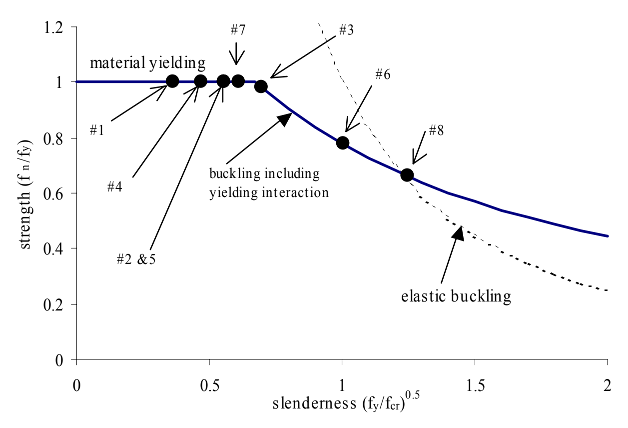
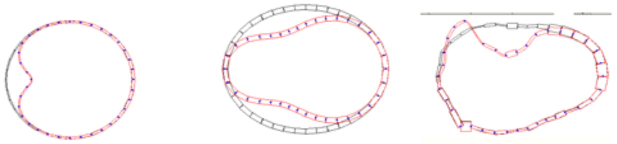


Figure 1: Predicted reduction of compressive limits for the femoral neck due to effect of local buckling (in pure compression).



(a) #8 circle model (b) #5 ellipse model (c) #6 CT based model

Figure 2: Predicted local buckling mode for compression and bending about the vertical axis applied to three models of femoral neck cross-section

Discussion: There is relatively little prior work in modeling instability behavior of whole human bones although the phenomena may be critical in osteoporotic fragility. We have begun a series of proof-of-concept analyses to determine whether cortical geometries seen at the femoral neck are susceptible to buckling failure. Analyses generated in pure compression suggest that geometries seen in fracture cases are less stable than those in controls. The level of model realism appears to be important. Relatively smooth cortices of circle and ellipse models are more stable than more realistic models with irregular cortices. Simple constructs to estimate cortical instability such as the buckling ratio may have some value but may not be sufficiently accurate for predictive use. More realistic analyses generated in a combination of bending and compression should better simulate the fall conditions producing hip fracture. These conditions produce peak compressive stresses on the thinner lateral cortices and further demonstrate the importance of realistic cortical models. Instability behavior is by definition chaotic, difficult to predict, and to observe experimentally especially in objects with the 3D complexity of the human proximal femur. Nevertheless this preliminary work using the finite strip analysis is encouraging and the method may prove to be a useful tool for the investigation of osteoporotic instability. Current limitations such as the assumption of a straight extrusion, can be accommodated. Future analyses will include varying cross-sectional dimensions along the length of the femoral neck, the inclusion of internal trabecular support and finer element sizes to better model irregular cortices. A comparison of failure predictions generated on high resolution CT scans prior to experimentally generated fractures is in the planning stages.

References:

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