Design Examples: Concentrically Loaded Lipped Channel Column

The following 11 examples include the methodology for all methods considered in the report: Distortional Buckling of Columns. In brief, the 11 methods include:

A. AISI (1996) Methods and Simple Modifications
- **A1. Current AISI (1996) Method**
- **A2. AISI (1996) with a Distortional Check**

B. New Methods which include only Local+Euler Check and Distortional Check
- **B1. Effective width "element" based method**
- **B2. Hand Implementation of Direct Strength "member" based method**
- **B3. Numerical Implementation of Direct Strength "member" based method**

C. New methods which include Local+Euler Check and Dist+Euler Check
- **C1 - C3 same as B methods with interactions listed above**

D. New methods which include Local+Euler, Dist+Euler, and Local+Dist Check
- **D1 - D3 same as B and C methods with interactions listed above**

Specimen Dimensions and Properties:

$h := 5.034 \cdot in$ $\qquad K_x := 1$

$b := 1.992 \cdot in$ $\qquad K_y := 1$

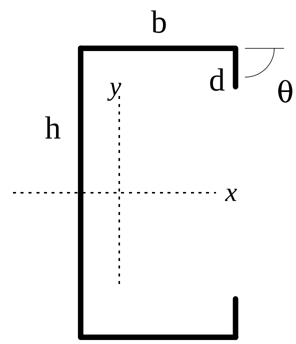
$d := 0.735 \cdot in$ $\qquad K_t := 0.5$

$t := 0.031 \cdot in$ $\qquad L_x := 75 \cdot in$

$E := 29500 \cdot ksi$ $\qquad L_y := 75 \cdot in$

$\nu := 0.3$

$f_y := 35.1 \cdot ksi$ $\qquad L_t := 75 \cdot in$



Dimensions of the above example are based on Loughlan (1979) specimen #L6

Glossary of Variables:

h = web height	K_x = x-axis effective length
b = flange width	K_y = y axis effective length
d = lip length	K_t = torsion effective length
θ = lip angle (radians)	L_x = x-axis unbraced length
t = thickness	L_y = y-axis unbraced length
E = Young's modulus	L_t = torsion unbraced length
ν = Poisson's ratio	
fy = yield stress	

$$\theta := 90 \cdot \frac{\pi}{180}$$

The following solution only applies for θ = 90, due to explicit formulas used in the calculation of C_w for overall (Euler) buckling. Those formulas only apply to lipped C's.

Whole Section Material and Gross Properties Required for Overall Buckling Analysis of Lipped Channel

Material Property: $G := \frac{E}{2\cdot(1+\nu)}$ $\qquad G = 1.135 \cdot 10^{4}\ \text{ksi}$

Gross Section Properties:

This is a series of "canned" formulas for gross property calculations of a lipped channel. They do not apply to other cross-section geometry. The C_w formula is from Yu, Cold-Formed Steel Design.

$$A := t\cdot(h + 2\cdot b + 2\cdot d) \qquad A = 0.325\ \text{in}^2$$

$$J := \frac{1}{3}\cdot h\cdot t^3 + 2\cdot\frac{1}{3}\cdot b\cdot t^3 + 2\cdot\frac{1}{3}\cdot d\cdot t^3 \qquad J = 1.041 \cdot 10^{-4}\ \text{in}^4$$

$$y_{cg} := \frac{1}{2}\cdot h \qquad y_{cg} = 2.517\ \text{in}$$

$$I_x := \frac{1}{12}\cdot\left(h^3\cdot t\right) + \frac{1}{2}\cdot b\cdot t\cdot h^2 + \frac{2}{3}\cdot d^3\cdot t + \frac{1}{2}\cdot d\cdot t\cdot h^2 - d^2\cdot t\cdot h + \frac{1}{6}\cdot b\cdot t^3 \qquad I_x = 1.325\ \text{in}^4$$

$$x_{cg} := \frac{b\cdot(b + 2\cdot d)}{h + 2\cdot b + 2\cdot d} \qquad x_{cg} = 0.658\ \text{in}$$

$$I_y := \frac{1}{12}\cdot h\cdot t^3 + \frac{2}{3}\cdot t\cdot b^3 + \frac{1}{6}\cdot d\cdot t^3 + 2\cdot d\cdot t\cdot b^2 - (h\cdot t + 2\cdot b\cdot t + 2\cdot d\cdot t)\cdot b^2\cdot\frac{(b + 2\cdot d)^2}{(h + 2\cdot b + 2\cdot d)^2} \qquad I_y = 0.204\ \text{in}^4$$

$$x_o := \frac{b\cdot t\cdot(b + 2\cdot d)}{A} + \frac{b\cdot t}{12\cdot I_x}\cdot\left(6\cdot d\cdot h^2 + 3\cdot b\cdot h^2 - 8\cdot d^3\right) \qquad x_o = 1.668\ \text{in}$$

$$m := x_o - x_{cg} \qquad m = 1.01\ \text{in}$$

$$C_{wterm1} := \frac{x_{cg}\cdot A\cdot h^2}{t}\cdot\left(\frac{b^2}{3} + m^2 - m\cdot b\right) \qquad C_{wterm1} = 57.823\ \text{in}^6$$

$$C_{wterm2} := \frac{A}{3\cdot t}\cdot\left[m^2\cdot h^3 + b^2\cdot d^2\cdot(2\cdot d + 3\cdot h)\right] - \frac{I_x\cdot m^2}{t}\cdot(2\cdot h + 4\cdot d) \qquad C_{wterm2} = 12.11\ \text{in}^6$$

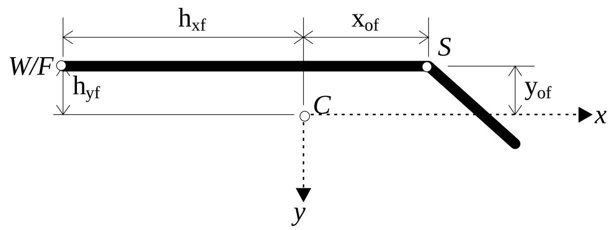
$$C_{wterm3} := \frac{m\cdot d^2}{3}\cdot\left[8\cdot b^2\cdot d + 2\cdot m\cdot(2\cdot d\cdot(d - h) + b\cdot(2\cdot d - 3\cdot h))\right] \qquad C_{wterm3} = -8.058\ \text{in}^6$$

$$C_{wterm4} := \frac{b^2\cdot h^2}{6}\cdot\left[(3\cdot d + b)\cdot(4\cdot d + h) - 6\cdot d^2\right] - \frac{m^2\cdot h^4}{4} \qquad C_{wterm4} = 342.735\ \text{in}^6$$

$$C_w := \frac{t^2}{A}\cdot\left(C_{wterm1} + C_{wterm2} + C_{wterm3} + C_{wterm4}\right) \qquad C_w = 1.196\ \text{in}^6$$

Flange Only Properties Required for Distortional Buckling Calculation

Hand methods for distortional buckling prediction require that section properties of the isolated flange be calculated. The expressions here are only applicable for simple lips. More complicated flanges would follow the same procedures, but new expressions would be required.



Material Properties:

$$G := \frac{E}{2\cdot(1+\nu)}$$

Properties of the Flange Only:

$$A_f := (b+d)\cdot t$$ $A_f = 0.085\,in^2$

$$J_f := \frac{1}{3}\cdot b\cdot t^3 + \frac{1}{3}\cdot d\cdot t^3$$ $J_f = 2.708\cdot 10^{-5}\,in^4$

$$I_{xf} := \frac{t\cdot\left(t^2\cdot b^2 + 4\cdot b\cdot d^3 - 4\cdot b\cdot d^3\cdot\cos(\theta)^2 + t^2\cdot b\cdot d + d^4 - d^4\cdot\cos(\theta)^2\right)}{12\cdot(b+d)}$$ $I_{xf} = 3.279\cdot 10^{-3}\,in^4$

$$I_{yf} := \frac{t\cdot\left(b^4 + 4\cdot d\cdot b^3 + 6\cdot d^2\cdot b^2\cdot\cos(\theta) + 4\cdot d^3\cdot b\cdot\cos(\theta)^2 + d^4\cdot\cos(\theta)^2\right)}{12\cdot(b+d)}$$ $I_{yf} = 0.037\,in^4$

$$I_{xyf} := \frac{t\cdot b\cdot d^2\cdot\sin(\theta)\cdot(b + d\cdot\cos(\theta))}{4\cdot(b+d)}$$ $I_{xyf} = 6.092\cdot 10^{-3}\,in^4$

$$I_{of} := \frac{t\cdot b^3}{3} + \frac{b\cdot t^3}{12} + \frac{t\cdot d^3}{3}$$ $I_{of} = 0.086\,in^4$

$$x_{of} := \frac{b^2 - d^2\cdot\cos(\theta)}{2\cdot(b+d)}$$ x distance from the centroid to the shear center. $x_{of} = 0.728\,in$

$$y_{of} := \frac{-d^2\cdot\sin(\theta)}{2\cdot(b+d)}$$ y distance from the centroid to the shear center. $y_{of} = -0.099\,in$

$$h_{xf} := \frac{-\left(b^2 + 2\cdot d\cdot b + d^2\cdot\cos(\theta)\right)}{2\,(b+d)}$$ x distance from the centroid to the web/flange juncture. $h_{xf} = -1.264\,in$

$$h_{yf} := \frac{-d^2\cdot\sin(\theta)}{2\cdot(b+d)}$$ y distance from the centroid to the web/flange juncture. $h_{yf} = -0.099\,in$

$$C_{wf} := 0\cdot in^6$$ $C_{wf} = 0\,in^6$

All flange properties are given the subscript 'f' to distinguish them from the overall properties of the column

Overall (Global, Long-wavelength) Buckling Modes as per AISI (1996)

Elastic Flexural Buckling about the x-axis:

$$r_x := \sqrt{\frac{I_x}{A}} \qquad r_x = 2.018 \cdot in$$

$$F_{ex} := \frac{\pi^2 \cdot E}{\left(\frac{K_x \cdot L_x}{r_x}\right)^2} \qquad F_{ex} = 210.876 \cdot ksi$$

Slenderness is:

$$\frac{K_x \cdot L_x}{r_x} = 37.158$$

Elastic Flexural Buckling about the y-axis:

$$r_y := \sqrt{\frac{I_y}{A}} \qquad r_y = 0.791 \cdot in$$

$$F_{ey} := \frac{\pi^2 \cdot E}{\left(\frac{K_y \cdot L_y}{r_y}\right)^2} \qquad F_{ey} = 32.417 \cdot ksi$$

Slenderness is:

$$\frac{K_y \cdot L_y}{r_y} = 94.77$$

Elastic Flexural-torsional buckling

$$\sigma_{ex} := F_{ex} \qquad r_o := \sqrt{r_x^2 + r_y^2 + x_o^2} \qquad r_o = 2.735 \cdot in$$

$$\sigma_t := \frac{1}{A \cdot r_o^2} \cdot \left[G \cdot J + \frac{\pi^2 \cdot E \cdot C_w}{\left(K_t \cdot L_t\right)^2} \right] \qquad \beta := 1 - \left(\frac{x_o}{r_o}\right)^2 \qquad \sigma_t = 102.279 \cdot ksi$$

$$\beta = 0.628$$

$$F_{et} := \frac{1}{2 \cdot \beta} \cdot \left[\sigma_{ex} + \sigma_t - \sqrt{\left(\sigma_{ex} + \sigma_t\right)^2 - 4 \cdot \beta \cdot \sigma_{ex} \cdot \sigma_t} \right] \qquad F_{et} = 82.543 \cdot ksi$$

The controlling long-wavelength buckling load is the minimum:

$$F_e := \min\left(\left[F_{ex} \;\; F_{ey} \;\; F_{et}\right]\right) \qquad F_e = 32.417 \cdot ksi$$

mode_is := | "x-axis flexure" if $F_e = F_{ex}$
"y-axis flexure" if $F_e = F_{ey}$
"flexural-torsional" if $F_e = F_{et}$

mode_is = "y-axis flexure"

Note, columns with different effective lengths (K) or braced lengths (Lx, Ly, Lt) are treated in the usual fashion regardless of the design method considered.

A1. Ultimate Strength per Current AISI (1996) Procedure

Determine the long column nominal buckling stress Fn (per AISI section C4):

$\lambda_c := \sqrt{\dfrac{f_y}{F_e}}$ $\qquad \lambda_c = 1.041$

$$F_n := \begin{cases} \left(0.658^{\lambda_c^2} \cdot f_y\right) & \text{if } \lambda_c \le 1.5 \\ \left(\dfrac{0.877}{\lambda_c^2} \cdot f_y\right) & \text{if } \lambda_c > 1.5 \end{cases} \qquad F_n = 22.31 \cdot \text{ksi}$$

Determine the effective area (calculated at the nominal buckling stress, Fn):

Determine the effective width of the web:

The buckling coefficient and stress are:

$k_{web} := 4$

$$f_{cr\_web} := k_{web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{h}\right)^2 \qquad f_{cr\_web} = 4.044 \cdot \text{ksi}$$

The slenderness is:

$$\lambda := \frac{1.052}{\sqrt{k_{web}}} \cdot \left(\frac{h}{t}\right) \cdot \sqrt{\frac{F_n}{E}} \qquad \lambda = 2.349 \qquad \text{or} \qquad \lambda := \sqrt{\frac{F_n}{f_{cr\_web}}} \qquad \lambda = 2.349$$

the two expressions for λ are equivalent.

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.386$$

The effective width of the web:

$h_{eff} := \rho \cdot h$ $\qquad h_{eff} = 1.943 \cdot \text{in}$

A1. Ultimate Strength per Current AISI (1996) Procedure (continued)

Determine the effective width of the flange:

Preliminaries: $S := 1.28 \cdot \sqrt{\frac{E}{F_n}}$ $\quad I_s := \frac{t \cdot d^3 \cdot \sin(\theta)^2}{12}$

$$k_{aisi} := \begin{array}{|l} 4 \quad \text{if } \frac{b}{t} \le \frac{S}{3} \\ \text{if } \frac{S}{3} < \frac{b}{t} \le S \\ \quad \begin{array}{|l} k_u \leftarrow 0.43 \\ I_a \leftarrow t^4 \cdot 399 \cdot \left[\frac{\frac{b}{t}}{S} - \left(\frac{k_u}{4} \right)^{\frac{1}{2}} \right]^3 \\ n \leftarrow \frac{1}{2} \\ C2 \leftarrow \min\left(\begin{bmatrix} \frac{I_s}{I_a} & 1 \end{bmatrix} \right) \\ C1 \leftarrow 2 - C2 \\ k_a \leftarrow \min\left(\begin{bmatrix} 5.25 - 5 \cdot \frac{d}{b} & 4 \end{bmatrix} \right) \\ C2^n \cdot \left(k_a - k_u \right) + k_u \end{array} \\ \text{if } \frac{b}{t} \ge S \\ \quad \begin{array}{|l} k_u \leftarrow 0.43 \\ I_a \leftarrow t^4 \cdot \left(115 \cdot \frac{\frac{b}{t}}{S} + 5 \right) \\ n \leftarrow \frac{1}{3} \\ C2 \leftarrow \min\left(\begin{bmatrix} \frac{I_s}{I_a} & 1 \end{bmatrix} \right) \\ C1 \leftarrow 2 - C2 \\ k_a \leftarrow \min\left(\begin{bmatrix} 5.25 - 5 \cdot \frac{d}{b} & 4 \end{bmatrix} \right) \\ C2^n \cdot \left(k_a - k_u \right) + k_u \end{array} \end{array}$$

Once kaisi is determined from the equations on the left, the effective width of the flange is readily calculated:

The buckling coefficient and stress are:

$k_{aisi} = 3.405$

$$f_{cr\_aisi} := k_{aisi} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b} \right)^2$$

$f_{cr\_aisi} = 21.988 \cdot ksi$

The slenderness is:

$\lambda := \frac{1.052}{\sqrt{k_{aisi}}} \cdot \left(\frac{b}{t} \right) \cdot \sqrt{\frac{F_n}{E}}$ $\qquad \lambda = 1.007$

or $\lambda := \sqrt{\frac{F_n}{f_{cr\_aisi}}}$ $\qquad \lambda = 1.007$

The reduction factor is:

$$\rho := \begin{array}{|l} 1 \quad \text{if } \lambda \le 0.673 \\ \frac{\left(1 - \frac{0.22}{\lambda}\right)}{\lambda} \quad \text{if } \lambda > 0.673 \end{array}$$ $\qquad \rho = 0.776$

The effective width of the flange:

$b_{eff} := \rho \cdot b$ $\qquad b_{eff} = 1.546 \cdot in$

A1. Ultimate Strength per Current AISI (1996) Procedure (continued)

Determine the effective width of the lip

The effective width of the lip is first determined as an unstiffened element in pure compression, labeled as ds_p. Then a second reduction using the C2 term from the flange expressions must also be applied.

First reduction:

$$k_{lip} := 0.43$$

$$f_{cr\_lip} := k_{lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t}{d}\right)^2$$

$$f_{cr\_lip} = 20.395 \cdot ksi$$

$$\lambda := \frac{1.052}{\sqrt{k_{lip}}} \cdot \left(\frac{d}{t}\right) \cdot \sqrt{\frac{F_n}{E}} \quad \text{or} \quad \lambda := \sqrt{\frac{F_n}{f_{cr\_lip}}}$$

$$\lambda = 1.046$$

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases}$$

$$\rho = 0.755$$

$$d_{s\_p} := \rho \cdot d$$

$$d_{s\_p} = 0.555 \cdot in$$

Second reduction

$$C_2 := \begin{cases} 1 & \text{if } \dfrac{b}{t} \le \dfrac{S}{3} \\ \text{if } \dfrac{S}{3} < \dfrac{b}{t} \le S: & I_a \leftarrow t^4 \cdot 399 \cdot \left[\dfrac{\frac{b}{t}}{S} - \left(\dfrac{0.43}{4}\right)^{\frac{1}{2}}\right]^3; \quad \min\left(\left[\dfrac{I_s}{I_a} \quad 1\right]\right) \\ \text{if } \dfrac{b}{t} \ge S: & I_a \leftarrow t^4 \cdot \left(115 \cdot \dfrac{\frac{b}{t}}{S} + 5\right); \quad \min\left(\left[\dfrac{I_s}{I_a} \quad 1\right]\right) \end{cases}$$

$$C_2 = 1$$

$$d_{eff} := C_2 \cdot d_{s\_p}$$

$$d_{eff} = 0.555 \cdot in$$

Determine the effective area and the Ultimate Strength:

The Effective area is:

$$A_e := t \cdot (h_{eff} + 2 \cdot b_{eff} + 2 \cdot d_{eff}) \qquad A_e = 0.19 \cdot in^2 \qquad \text{vs.} \qquad A = 0.325 \cdot in^2$$

The ultimate strength is:

$$P_{n\_A1} := A_e \cdot F_n \qquad P_{n\_A1} = 4.249 \cdot k$$

A2. AISI (1996) with a Distortional Check

Step 1: Complete design method A1.

Step 2: Perform the Distortional Check

Calculate the distortional buckling stress, via hand method of Schafer (1997)

Determine the critical half-wavelength at which distortional buckling occurs:

$$L_{cr} := \left[\frac{6\cdot\pi^4\cdot h\cdot\left(1-\nu^2\right)}{t^3}\cdot\left[I_{xf}\cdot\left(x_{of}-h_{xf}\right)^2+C_{wf}-\frac{I_{xyf}^{\;2}}{I_{yf}}\cdot\left(x_{of}-h_{xf}\right)^2\right]\right]^{\frac{1}{4}} \qquad L_{cr} = 30.007\,\text{in}$$

If bracing is provided that restricts the distortional mode at some length less than Lcr, then the shorter bracing length should be used in place of Lcr in the following calculations.

Determine the elastic and "geometric" rotational spring stiffness of the flange:

$$k_{\phi fe} := \left(\frac{\pi}{L_{cr}}\right)^4\cdot\left[E\cdot I_{xf}\cdot\left(x_{of}-h_{xf}\right)^2+E\cdot C_{wf}-E\cdot\frac{I_{xyf}^{\;2}}{I_{yf}}\cdot\left(x_{of}-h_{xf}\right)^2\right]+\left(\frac{\pi}{L_{cr}}\right)^2\cdot G\cdot J_f$$

$$k_{\phi fe} = 0.035\,\text{k}$$

$$k_{\phi fg} := \left(\frac{\pi}{L_{cr}}\right)^2\cdot\left[A_f\cdot\left[\left(x_{of}-h_{xf}\right)^2\cdot\left(\frac{I_{xyf}}{I_{yf}}\right)^2-2\cdot y_{of}\cdot\left(x_{of}-h_{xf}\right)\cdot\left(\frac{I_{xyf}}{I_{yf}}\right)+h_{xf}^{\;2}+y_{of}^{\;2}\right]+I_{xf}+I_{yf}\right]$$

$$k_{\phi fg} = 2.092\cdot 10^{-3}\,\text{in}^2$$

Determine the elastic and "geometric" rotational spring stiffness from the web:

$$k_{\phi we} := \frac{E\cdot t^3}{6\cdot h\cdot\left(1-\nu^2\right)} \qquad k_{\phi we} = 0.032\,\text{k} \qquad k_{\phi wg} := \left(\frac{\pi}{L_{cr}}\right)^2\cdot\frac{t\cdot h^3}{60} \qquad k_{\phi wg} = 7.224\cdot 10^{-4}\,\text{in}^2$$

$k_{\phi wg}$ is modified due to an error in Schafer (1997) analysis.

Determine the distortional buckling stress:

$$f_{cr\_dist} := \frac{k_{\phi fe}+k_{\phi we}}{k_{\phi fg}+k_{\phi wg}} \qquad f_{cr\_dist} = 23.921\,\text{ksi}$$

A2. AISI (1996) with a Distortional Check (continued)

Calculate the strength reduction factor (ρ) for distortional buckling

Find the reduction factor for the distortional stress

Winter's curve is used to find the strength reduction factor; but the distortional stress is reduced by Rd to account for lower post-buckling capacity in the distortional mode.

$$\lambda_d := \sqrt{\frac{f_y}{f_{cr\_dist}}} \qquad \lambda_d = 1.211$$

$$R_d := \min\left(\begin{bmatrix} 1 & \frac{1.17}{\lambda_d + 1} + 0.3 \end{bmatrix}\right) \qquad R_d = 0.829$$

The increased slenderness is

note that fy, not Fn is used in the strength provisions for distortional buckling in method A2, this is because distortional interaction with long column Euler buckling is ignored in this method.

$$\lambda := \sqrt{\frac{f_y}{R_d \cdot f_{cr\_dist}}} \qquad \lambda = 1.33$$

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \left(1 - 0.22 \cdot \sqrt{\frac{R_d \cdot f_{cr\_dist}}{f_y}}\right) \cdot \sqrt{\frac{R_d \cdot f_{cr\_dist}}{f_y}} & \text{if } \lambda > 0.561 \end{cases} \qquad \rho = 0.627$$

An alternative, but similar method to calculate the strength reduction factor (ρ)

The alternative reduction factor is:

$$\rho_{alt} := \begin{cases} 1 & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{f_{cr\_dist}}{f_y}\right)^{0.6}\right] \cdot \left(\frac{f_{cr\_dist}}{f_y}\right)^{0.6} & \text{if } \lambda > 0.561 \end{cases} \qquad \rho_{alt} = 0.637$$

this method for calculation of ρ is used in methods B2 and B3 and is provided for comparison only.

The Effective area for distortional buckling is:

$$A_e := \rho \cdot A \qquad A_e = 0.204 \cdot in^2 \qquad \text{vs.} \qquad A = 0.325 \cdot in^2$$

The strength prediction for the distortional check is

$$P_{n\_A2dist\_check} := A_e \cdot f_y \qquad P_{n\_A2dist\_check} = 7.16 \cdot k$$

The ultimate strength is the minimum:

$$P_{n\_A2} := \min\left(\begin{bmatrix} P_{n\_A1} & P_{n\_A2dist\_check} \end{bmatrix}\right) \qquad P_{n\_A2} = 4.249 \cdot k$$

$$\text{ultimate\_is} := \begin{cases} \text{"local limited to Fn (L+E)"} & \text{if } P_{n\_A2} = P_{n\_A1} \\ \text{"distortional"} & \text{if } P_{n\_A2} = P_{n\_A2dist\_check} \end{cases}$$

ultimate_is = "local limited to Fn (L+E)"

B1. Effective width "element" based method with L+E and D Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

Determine the effective width of the flange web and lip considering L+E, thus all eff. width calculations are limited to the long column nominal stress Fn

Effective Width of the Web

The plate buckling coefficient is OR the web local buckling stress is

$k_{web} := 4$

$$f_{cr\_w} := k_{web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{h}\right)^2 \qquad f_{cr\_w} = 4.044 \cdot ksi$$

The slenderness is

$$\lambda := \frac{1.052}{\sqrt{k_{web}}} \cdot \left(\frac{h}{t}\right) \cdot \sqrt{\frac{F_n}{E}} \qquad \lambda = 2.349 \qquad \lambda := \sqrt{\frac{F_n}{f_{cr\_w}}} \qquad \lambda = 2.349$$

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.386$$

The effective width of the web:

$$h_{eff} := \rho \cdot h \qquad h_{eff} = 1.943 \cdot in$$

Effective Width of the Flange

The plate buckling coefficient is* OR the flange local buckling stress is

$k_{flange} := 4$

$$f_{cr\_f} := k_{flange} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_f} = 25.829 \cdot ksi$$

The slenderness is

$$\lambda := \frac{1.052}{\sqrt{k_{flange}}} \cdot \left(\frac{b}{t}\right) \cdot \sqrt{\frac{F_n}{E}} \qquad \lambda = 0.929 \qquad \lambda := \sqrt{\frac{F_n}{f_{cr\_f}}} \qquad \lambda = 0.929$$

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.821$$

* note that a k of 4 is used for the flange instead of the AISI rules from B4.2. This implies that only local buckling is considered in this calculation, and distortional buckling will thus be checked separately.

The effective width of the flange:

$$b_{eff} := \rho \cdot b \qquad b_{eff} = 1.636 \cdot in$$

B1. Effective width "element" based method with L+E and D Checks (continued)

Effective Width of the Lip

The plate buckling coefficient is* OR the lip local buckling stress is

$k_{lip} := 0.43$

$f_{cr\_l} := k_{lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t}{d}\right)^2$ $f_{cr\_l} = 20.395 \cdot ksi$

The slenderness is

$\lambda := \frac{1.052}{\sqrt{k_{lip}}} \cdot \left(\frac{d}{t}\right) \cdot \sqrt{\frac{F_n}{E}}$ $\lambda = 1.046$ $\lambda := \sqrt{\frac{F_n}{f_{cr\_l}}}$ $\lambda = 1.046$

The reduction factor is:

$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \frac{\left(1 - \frac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases}$ $\rho = 0.755$

The effective width of the lip:

$d_{eff} := \rho \cdot d$ $d_{eff} = 0.555 \cdot in$

The Effective area is:

$A_e := t \cdot (h_{eff} + 2 \cdot b_{eff} + 2 \cdot d_{eff})$ $A_e = 0.196 \cdot in^2$ $A_{e\_B1local} := A_e$

The strength prediction for local buckling (L) considering long column (E) interaction is

$P_{n\_B1local} := A_e \cdot F_n$ $P_{n\_B1local} = 4.374 \cdot k$

Check distortional buckling (calculations are identical to distortional check in method A2)

The strength prediction for the distortional check from A2 is

$P_{n\_A2dist\_check} = 7.16 \cdot k$

The ultimate strength is the minimum:

$P_{n\_B1} := \min\left(\left[P_{n\_B1local} \;\; P_{n\_A2dist\_check}\right]\right)$ $P_{n\_B1} = 4.374 \cdot k$

$ultimate\_is := \begin{cases} \text{"local (k=4 sol'n) limited to Fn (L+E)"} & \text{if } P_{n\_B1} = P_{n\_B1local} \\ \text{"distortional"} & \text{if } P_{n\_B1} = P_{n\_A2dist\_check} \end{cases}$

ultimate_is = "local (k=4 sol'n) limited to Fn (L+E)"

B2. Hand Based Direct Strength "member" method with L+E and D Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

In the member methods (B2-B3, C2-C3, D2-D3) solutions are written in terms of load, P.

Calculate the **elastic buckling loads** by hand

Long Column Buckling (Euler buckling) see before method A1 for details of hand calculation

$P_{cre} := A \cdot F_e \qquad P_{cre} = 10.54 \cdot k$

Distortional Buckling see method A2 for details of hand calculation

$P_{crd} := A \cdot f_{cr\_dist} \qquad P_{crd} = 7.777 \cdot k$

Local buckling (based on hand expressions for flange/web and flange/lip interaction)

Flange/Web Local Buckling

The plate buckling coefficient for the flange/web interaction expressions are written in terms of the flange:

$$k_{flange\_web} := \begin{cases} \left[\left[2 - \left(\frac{b}{h}\right)^{0.4}\right] \cdot 4 \cdot \left(\frac{b}{h}\right)^2\right] & \text{if } \frac{h}{b} \geq 1 \\ \left[\left[2 - \left(\frac{h}{b}\right)^{0.2}\right] \cdot 4\right] & \text{if } \frac{h}{b} < 1 \end{cases} \qquad k_{flange\_web} = 0.82$$

$$f_{cr\_fw} := k_{flange\_web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_fw} = 5.298 \cdot ksi$$

Flange/Lip Local Buckling

The plate buckling coefficient for the flange/lip interaction expression is also written in terms of the flange:

$$k_{flange\_lip} := -11.07 \cdot \left(\frac{d}{b}\right)^2 + 3.95 \cdot \left(\frac{d}{b}\right) + 4 \qquad k_{flange\_lip} = 3.95$$

$$f_{cr\_fl} := k_{flange\_lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_fl} = 25.508 \cdot ksi$$

Local buckling stress

$$f_{cr\_local} := \min\left(\left[f_{cr\_fw} \;\; f_{cr\_fl}\right]\right) \qquad f_{cr\_local} = 5.298 \cdot ksi$$

Local buckling load

$$P_{crl} := A \cdot f_{cr\_local} \qquad P_{crl} = 1.722 \cdot k$$

Calculate the column squash load

$$P_y := A \cdot f_y \qquad P_y = 11.412 \cdot k$$

B2. Hand Based Direct Strength "member" method with L+E and D Checks (continued)

Calculate the nominal long column (Euler) strength

$$\lambda_c := \sqrt{\frac{P_y}{P_{cre}}} \qquad \lambda_c = 1.041$$

$$P_{ne} := \begin{cases} \left(0.658^{\lambda_c^2} \cdot P_y\right) & \text{if } \lambda_c \le 1.5 \\ \left(\frac{0.877}{\lambda_c^2} \cdot P_y\right) & \text{if } \lambda_c > 1.5 \end{cases} \qquad P_{ne} = 7.253 \cdot k$$

Consider local and long column (Euler) interaction, calculate the strength

$$\lambda := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda = 2.052$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda \le 0.7776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right] \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda > 0.776 \end{cases} \qquad P_{nl} = 3.737 \cdot k$$

Consider distortional, calculate the strength

$$\lambda := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda = 1.211$$

note Py, not Pne is used in the calculation. Thus disotortional and overall interaction is ignored.

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right] \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda > 0.561 \end{cases} \qquad P_{nd} = 7.266 \cdot k$$

The ultimate strength is the minimum:

$$P_{n\_B2} := \min\left(\left[P_{nl} \;\; P_{nd}\right]\right) \qquad P_{n\_B2} = 3.737 \cdot k$$

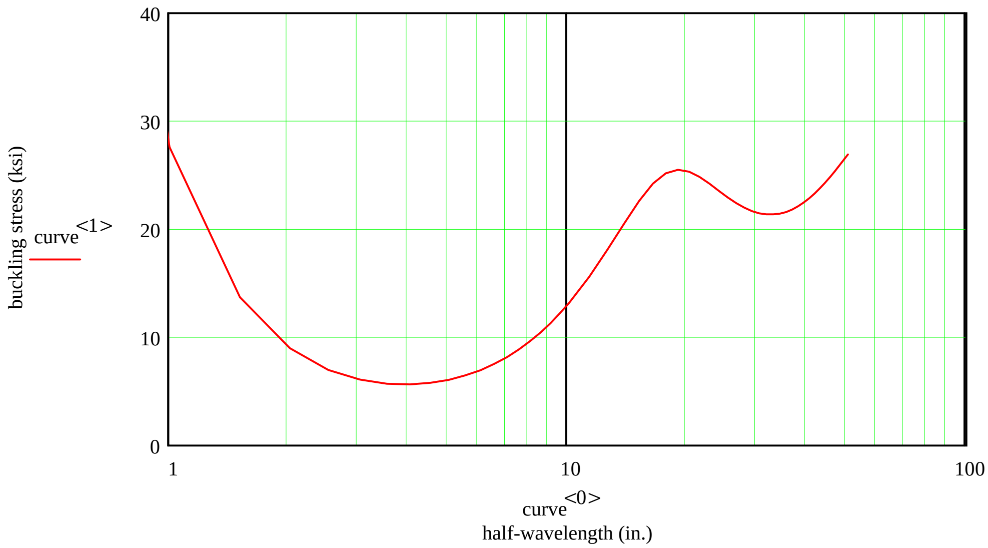
ultimate_is := | "local limited to long column Pne (L+E)" if $P_{n\_B2} = P_{nl}$
| "distortional" if $P_{n\_B2} = P_{nd}$

ultimate_is = "local limited to long column Pne (L+E)"

B3. Numerical Implementation of Direct Strength "member" method with L+E and D Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

Finite Strip Analysis of the member (the raw data is imported from Matlab):



Buckling stresses from the finite strip analysis:

$f_{cr\_locals} := 5.65 \cdot ksi$ These values are manually entered from the curve.

$f_{cr\_dists} := 21.4 \cdot ksi$

$f_{cr\_longs} := F_e$ Since the analysis was stopped at approximately 50 in. the hand solution for overall buckling of the column will be employed.

Elastic Buckling Loads (subscript "s" is added to distinguish from the hand based methods)

Local	$P_{crls} := A \cdot f_{cr\_locals}$	$P_{crls} = 1.837 \cdot k$
Dist.	$P_{crds} := A \cdot f_{cr\_dists}$	$P_{crds} = 6.958 \cdot k$
Long	$P_{cres} := A \cdot f_{cr\_longs}$	$P_{cres} = 10.54 \cdot k$

B3. Numerical Implementation of Direct Strength "member" method with L+E and D Checks (continued)

Calculate the nominal long column strength

$$\lambda_c := \sqrt{\frac{P_y}{P_{cres}}} \qquad \lambda_c = 1.041$$

$$P_{nes} := \begin{cases} \left(0.658^{\lambda_c^2} \cdot P_y\right) & \text{if } \lambda_c \le 1.5 \\ \left(\frac{0.877}{\lambda_c^2} \cdot P_y\right) & \text{if } \lambda_c > 1.5 \end{cases} \qquad P_{nes} = 7.253 \cdot k$$

Consider local and long column (Euler) interaction, calculate the strength

$$\lambda := \sqrt{\frac{P_{nes}}{P_{crls}}} \qquad \lambda = 1.987$$

$$P_{nls} := \begin{cases} P_{nes} & \text{if } \lambda \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crls}}{P_{nes}}\right)^{0.4}\right] \cdot \left(\frac{P_{crls}}{P_{nes}}\right)^{0.4} \cdot P_{nes} & \text{if } \lambda > 0.776 \end{cases} \qquad P_{nls} = 3.825 \cdot k$$

Consider distortional, calculate the strength

note Py, not Pne is used in the calculation. Thus disotortional and overall interaction is ignored.

$$\lambda := \sqrt{\frac{P_y}{P_{crds}}} \qquad \lambda = 1.281$$

$$P_{nds} := \begin{cases} P_y & \text{if } \lambda \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crds}}{P_y}\right)^{0.6}\right] \cdot \left(\frac{P_{crds}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda > 0.561 \end{cases} \qquad P_{nds} = 6.905 \cdot k$$

The ultimate strength is the minimum:

$$P_{n\_B3} := \min\left(\left[P_{nls} \;\; P_{nds}\right]\right) \qquad P_{n\_B3} = 3.825 \cdot k$$

$$\text{ultimate\_is} := \begin{cases} \text{"local limited to long column Pnes (L+E)"} & \text{if } P_{n\_B3} = P_{nls} \\ \text{"distortional"} & \text{if } P_{n\_B3} = P_{nds} \end{cases}$$

ultimate_is = "local limited to long column Pnes (L+E)"

C1. Effective width "element" based method with L+E and D+E Checks

Note, the presented solution for method C1 is in a different format, than that suggested in Appendix F.2. The results are identical, see example C1 (alternate) for solution in the same form as Appendix F.2

Calculations for C1 are nearly identical to B1 except that now the distortional buckling strength check includes the possibility of interaction with the long column (Euler) buckling modes, and thus effective width in both the distortional mode is limited to Fn (instead of Fy).

Strength in local mode from calculation in B1 $P_{n\_B1local} = 4.374 \cdot k$

Strength in distortional mode considering possibility of long column (Euler) interaction

Calculate the strength reduction factor (ρ) for distortional buckling

Find the reduction factor for the distortional stress

Winter's curve is used to find the strength reduction factor; but the distortional stress is reduced by Rd to account for lower post-buckling capacity in the distortional mode.

$\lambda_d := \sqrt{\dfrac{F_n}{f_{cr\_dist}}}$ $\lambda_d = 0.966$

$R_d := \min\left(\begin{bmatrix} 1 & \dfrac{1.17}{\lambda_d + 1} + 0.3 \end{bmatrix}\right)$ $R_d = 0.895$

The increased slenderness is $\lambda := \sqrt{\dfrac{F_n}{R_d \cdot f_{cr\_dist}}}$ $\lambda = 1.021$

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \left(1 - 0.22 \cdot \sqrt{\dfrac{R_d \cdot f_{cr\_dist}}{F_n}}\right) \cdot \sqrt{\dfrac{R_d \cdot f_{cr\_dist}}{F_n}} & \text{if } \lambda > 0.561 \end{cases}$$ $\rho = 0.769$

Alternative method for the strength reduction factor (ρ) used in C2 and C3 and provided here for the purposes of comparison only.

$$\rho_{alt} := \begin{cases} 1 & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\dfrac{f_{cr\_dist}}{F_n}\right)^{0.6}\right] \cdot \left(\dfrac{f_{cr\_dist}}{F_n}\right)^{0.6} & \text{if } \lambda_d > 0.561 \end{cases}$$ $\rho_{alt} = 0.771$

The Effective area for distortional buckling is (same reduction for all elements):

$A_e := \rho \cdot A$ $A_e = 0.25 \cdot in^2$ vs. $A = 0.325 \cdot in^2$

The strength prediction for the distortional check is

$P_{n\_C1dist\_check} := A_e \cdot F_n$ $P_{n\_C1dist\_check} = 5.575 \cdot k$

The ultimate strength is the minimum:

$P_{n\_C1} := \min\left(\begin{bmatrix} P_{n\_B1local} & P_{n\_C1dist\_check} \end{bmatrix}\right)$ $P_{n\_C1} = 4.374 \cdot k$

ultimate_is := | "local limited to long column Pne (L+E)" if $P_{n\_C1} = P_{n\_B1local}$
| "distortional limited to long column Pne (D+E)" if $P_{n\_C1} = P_{n\_C1dist\_check}$

ultimate_is = "local limited to long column Pne (L+E)"

C1 - Alternate. Effective width "element" based method with L+E and D+E Checks

Appendix F.2 provides a proposed method for incorporating method C1 into the AISI Specification. The format is different than that presented in the previous example for C1, but the result is the same. For completeness, this example is provided in the same format as presented in Appendix F.2 and proposed for adoption - however, the final results are identical to method C1 presented above.

Step 1. Determine the effective area for local buckling (consider long column interaction - follow method B1)

$$A_{e\_B1local} = 0.196 \cdot in^2$$

Step 2. Determine the distortional buckling effective area (as described in Appendix F.2)

Determine the long column nominal stress (same as in method A1)

$$F_n = 22.31 \cdot ksi$$

Determine the elastic distortional buckling stress (same as fcr_dist in method A2)

$$f_{ed} := f_{cr\_dist} \qquad f_{ed} = 23.921 \cdot ksi$$

Determine the reduced elastic distortional buckling stress

$$\lambda_d := \sqrt{\frac{F_n}{f_{ed}}} \qquad \lambda_d = 0.966$$

$$R_d := \min\left(\left[1 \quad \frac{1.17}{\lambda_d + 1} + 0.3\right]\right) \qquad R_d = 0.895$$

$$f_d := R_d \cdot f_{ed} \qquad f_d = 21.414 \cdot ksi$$

Determine the effective width of each element, subjected to distortional buckling

Flange

$$k_{d\_flange} := f_d \cdot \frac{12 \cdot \left(1 - \nu^2\right)}{\pi^2 E} \cdot \left(\frac{b}{t}\right)^2 \qquad k_{d\_flange} = 3.316$$

$$\lambda := \frac{1.052}{\sqrt{k_{d\_flange}}} \cdot \left(\frac{b}{t}\right) \cdot \sqrt{\frac{F_n}{E}} \qquad \lambda = 1.021$$

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.768$$

$$b_{eff} := \rho \cdot b \qquad b_{eff} = 1.531 \cdot in$$

C1 - Alternate. Effective width "element" based method with L+E and D+E Checks (continued)

Web

$$k_{d\_web} := f_d \cdot \frac{12 \cdot \left(1 - \nu^2\right)}{\pi^2 E} \cdot \left(\frac{h}{t}\right)^2 \qquad k_{d\_web} = 21.179$$

$$\lambda := \frac{1.052}{\sqrt{k_{d\_web}}} \cdot \left(\frac{h}{t}\right) \cdot \sqrt{\frac{F_n}{E}} \qquad \lambda = 1.021$$

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.768$$

$$h_{eff} := \rho \cdot h \qquad h_{eff} = 3.869 \cdot in$$

Lip

$$k_{d\_lip} := f_d \cdot \frac{12 \cdot \left(1 - \nu^2\right)}{\pi^2 E} \cdot \left(\frac{d}{t}\right)^2 \qquad k_{d\_lip} = 0.451$$

$$\lambda := \frac{1.052}{\sqrt{k_{d\_lip}}} \cdot \left(\frac{d}{t}\right) \cdot \sqrt{\frac{F_n}{E}} \qquad \lambda = 1.021$$

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.768$$

$$d_{eff} := \rho \cdot d \qquad h_{eff} = 3.869 \cdot in$$

The Effective area for distortional buckling is:

$$A_{e\_dist} := t \cdot \left(h_{eff} + 2 \cdot b_{eff} + 2 \cdot d_{eff}\right) \qquad A_{e\_dist} = 0.25 \cdot in^2$$

The governinig effective area is:

$$A_e := \min\left(\left[A_{e\_B1local} \;\; A_{e\_dist}\right]\right) \qquad A_e = 0.196 \cdot in^2$$

$$\text{ultimate\_is} := \begin{cases} \text{"local limited to long column (L+E)"} & \text{if } A_e = A_{e\_B1local} \\ \text{"distortional limited to long column (D+E)"} & \text{if } A_e = A_{e\_dist} \end{cases}$$

ultimate_is = "local limited to long column (L+E)"

Capacity is $\quad P_{n\_C1alt} := A_e \cdot F_n \qquad P_{n\_C1alt} = 4.374 \cdot k$

C2. Hand Based Direct Strength "member" method with L+E and D+E Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

Local Buckling considering long column interaction (same as B2)

$P_{nl} = 3.737 \cdot k$

Distortional Buckling considering long column interaction

$\lambda := \sqrt{\frac{P_{ne}}{P_{crd}}}$ $\lambda = 0.966$

$$P_{nd2} := \begin{cases} P_{ne} & \text{if } \lambda \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6}\right] \cdot \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6} \cdot P_{ne} & \text{if } \lambda > 0.561 \end{cases}$$

$P_{nd2} = 5.592 \cdot k$

note a "2" is added to the subscript of Pnd to distinguish from the calculation method used in example B2, which ignores long column (Euler) interaction, but is otherwise performed in a similar manner.

The ultimate strength is the minimum:

$P_{n\_C2} := \min\left(\left[P_{nl} \ \ P_{nd2}\right]\right)$ $P_{n\_C2} = 3.737 \cdot k$

ultimate_is := | "local limited to long column Pne (L+E)" if $P_{n\_C2} = P_{nl}$
| "distortional limited to long column Pne (D+E)" if $P_{n\_C2} = P_{nd2}$

ultimate_is = "local limited to long column Pne (L+E)"

C3. Hand Based Direct Strength "member" method with L+E and D+E Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

Local Buckling considering long column interaction (same as B3)

$P_{nls} = 3.825 \cdot k$

Distortional Buckling considering long column interaction

$\lambda := \sqrt{\frac{P_{nes}}{P_{crds}}}$ $\lambda = 1.021$

$P_{nd2s} := \begin{cases} P_{nes} & \text{if } \lambda \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crds}}{P_{nes}}\right)^{0.6}\right] \cdot \left(\frac{P_{crds}}{P_{nes}}\right)^{0.6} \cdot P_{nes} & \text{if } \lambda > 0.561 \end{cases}$ $P_{nd2s} = 5.35 \cdot k$

The ultimate strength is the minimum:

$P_{n\_C3} := \min\left(\left[P_{nls} \;\; P_{nd2s}\right]\right)$ $P_{n\_C3} = 3.825 \cdot k$

ultimate_is := | "local limited to long column Pnes (L+E)" if $P_{n\_C3} = P_{nls}$
| "distortional limited to long column Pnes (D+E)" if $P_{n\_C3} = P_{nd2s}$

ultimate_is = "local limited to long column Pnes (L+E)"

D1. Effective width "element" based method with L+E, D+E and L+D Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

This design method is the same as C1 with the addition of a local + distortional check

Check local + distortional interaction

Find the limiting, nominal, distortional stress (Fnd)

Local and long column (Euler) interaction is completed by calculating the effective width for local buckling at the nominal long column stress (Fn). Local and distortional interaction is completed in a similar manner by calculating the effective width for local buckling at the nominal distortional stress (Fnd)

Distortional slenderness is

$$\lambda := \sqrt{\frac{f_y}{f_{cr\_dist}}} \qquad \lambda = 1.211$$

$$F_{nd} := \begin{cases} 1 & \text{if } \lambda \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{f_{cr\_dist}}{f_y}\right)^{0.6}\right] \cdot \left(\frac{f_{cr\_dist}}{f_y}\right)^{0.6} \cdot f_y & \text{if } \lambda > 0.561 \end{cases} \qquad F_{nd} = 22.348 \cdot ksi$$

Determine the effective width of the flange web and lip considering L+D, thus all eff. width calculations are limited to the distortional nominal stress Fnd

Effective Width of the Web

The plate buckling coefficient is

$$k_{web} := 4$$

The slenderness is

$$\lambda := \frac{1.052}{\sqrt{k_{web}}} \cdot \left(\frac{h}{t}\right) \cdot \sqrt{\frac{F_{nd}}{E}} \qquad \lambda = 2.351$$

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \frac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.386$$

The effective width of the web:

$$h_{eff} := \rho \cdot h \qquad h_{eff} = 1.941 \cdot in$$

D1. Effective width "element" based method with L+E, D+E and L+D Checks (continued)

Effective Width of the Flange

The plate buckling coefficient is

$k_{flange} := 4$

The slenderness is

$$\lambda := \frac{1.052}{\sqrt{k_{flange}}} \cdot \left(\frac{b}{t}\right) \cdot \sqrt{\frac{F_{nd}}{E}} \qquad \lambda = 0.93$$

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.821$$

The effective width of the flange:

$b_{eff} := \rho \cdot b \qquad b_{eff} = 1.635 \cdot in$

Effective Width of the Lip

The plate buckling coefficient is $k_{lip} := 0.43$

The slenderness is

$$\lambda := \frac{1.052}{\sqrt{k_{lip}}} \cdot \left(\frac{d}{t}\right) \cdot \sqrt{\frac{F_{nd}}{E}} \qquad \lambda = 1.047$$

The reduction factor is:

$$\rho := \begin{cases} 1 & \text{if } \lambda \le 0.673 \\ \dfrac{\left(1 - \dfrac{0.22}{\lambda}\right)}{\lambda} & \text{if } \lambda > 0.673 \end{cases} \qquad \rho = 0.754$$

The effective width of the lip:

$d_{eff} := \rho \cdot d \qquad d_{eff} = 0.555 \cdot in$

The Effective area is:

$A_e := t \cdot \left(h_{eff} + 2 \cdot b_{eff} + 2 \cdot d_{eff}\right) \qquad A_e = 0.196 \cdot in^2$ vs. $A = 0.325 \cdot in^2$

The strength prediction for local buckling (L) considering long column (E) interaction is

$P_{n\_D1localdist} := A_e \cdot F_{nd} \qquad P_{n\_D1localdist} = 4.378 \cdot k$

The ultimate strength is the minimum:

$P_{n\_D1} := \min\left(\left[P_{n\_B1local} \;\; P_{n\_A2dist\_check} \;\; P_{n\_D1localdist}\right]\right) \qquad P_{n\_D1} = 4.374 \cdot k$

ultimate_is := | "local (k=4 sol'n) limited to Fn (L+E)" if $P_{n\_D1} = P_{n\_B1local}$
| "distortional" if $P_{n\_D1} = P_{n\_A2dist\_check}$
| "local limited to distortional Fnd (L+D)" if $P_{n\_D1} = P_{n\_D1localdist}$

ultimate_is = "local (k=4 sol'n) limited to Fn (L+E)"

D2. Hand Based Direct Strength "member" method with L+E, D+E and L+D Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

Local Buckling considering long column interaction (same as B2)

$P_{nl} = 3.737 \cdot k$

Distortional Buckling considering long column interaction

$P_{nd2} = 5.592 \cdot k$

Local Buckling considering distortional interaction

Consider distortional alone, calculate the strength (done previoulsy in B2)

$P_{nd} = 7.266 \cdot k$

Now consider local limited to nominal distortional load

$\lambda := \sqrt{\frac{P_{nd}}{P_{crl}}}$ $\qquad \lambda = 2.054$

$$P_{nld} := \begin{cases} P_{nd} & \text{if } \lambda \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{nd}}\right)^{0.4}\right] \cdot \left(\frac{P_{crl}}{P_{nd}}\right)^{0.4} \cdot P_{nd} & \text{if } \lambda > 0.776 \end{cases}$$

$P_{nld} = 3.741 \cdot k$

The ultimate strength is the minimum:

$P_{n\_D2} := \min\left(\left[P_{nl} \;\; P_{nd2} \;\; P_{nld}\right]\right)$ $\qquad P_{n\_D2} = 3.737 \cdot k$

ultimate_is := | "local limited to long column Pne (L+E)" if $P_{n\_D2} = P_{nl}$
| "distortional limited to long column Pne (D+E)" if $P_{n\_D2} = P_{nd2}$
| "local limited to distortional Pnd (L+D)" if $P_{n\_D2} = P_{nld}$

ultimate_is = "local limited to long column Pne (L+E)"

D3. Numerical Implementation of Direct Strength "member" method with L+E, D+E and L+D Checks

L=local buckling D=distortional buckling E=Euler (long wavelength) buckling a "+" indicates that interaction in these modes is considered in the design method.

Local Buckling considering long column interaction (same as B3)

$P_{nls} = 3.825 \cdot k$

Distortional Buckling considering long column interaction (same as C3)

$P_{nd2s} = 5.35 \cdot k$

Local Buckling considering distortional interaction

Consider distortional alone, calculate the strength (done previously in B3)

$P_{nds} = 6.905 \cdot k$

Now consider local limited to nominal distortional load

$$\lambda := \sqrt{\frac{P_{nds}}{P_{crls}}} \qquad \lambda = 1.939$$

$$P_{nlds} := \begin{cases} P_{nds} & \text{if } \lambda \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crls}}{P_{nds}}\right)^{0.4}\right] \cdot \left(\frac{P_{crls}}{P_{nds}}\right)^{0.4} \cdot P_{nds} & \text{if } \lambda > 0.776 \end{cases} \qquad P_{nlds} = 3.707 \cdot k$$

The ultimate strength is the minimum:

$$P_{n\_D3} := \min\left(\left[P_{nls} \;\; P_{nd2s} \;\; P_{nlds}\right]\right) \qquad P_{n\_D3} = 3.707 \cdot k$$

ultimate_is := | "local limited to long column Pne (L+E)" if $P_{n\_D3} = P_{nls}$
| "distortional limited to long column Pne (D+E)" if $P_{n\_D3} = P_{nd2s}$
| "local limited to distortional Pnd (L+D)" if $P_{n\_D3} = P_{nlds}$

ultimate_is = "local limited to distortional Pnd (L+D)"

Summary

A. AISI (1996) Methods and Simple Modifications
- **A1. Current AISI (1996) Method**
- **A2. AISI (1996) with a Distortional Check**

B. New Methods which include only Local+Euler Check and Distortional Check
- **B1. Effective width "element" based method**
- **B2. Hand Implementation of Direct Strength "member" based method**
- **B3. Numerical Implementation of Direct Strength "member" based method**

C. New methods which include Local+Euler Check and Dist+Euler Check
- **C1 - C3 same as B methods with interactions listed above**

D. New methods which include Local+Euler, Dist+Euler, and Local+Dist Check
- **D1 - D3 same as B and C methods with interactions listed above**

$P_{n\_A1} = 4.249 \cdot k$

$P_{n\_A2} = 4.249 \cdot k$

$P_{n\_B1} = 4.374 \cdot k$

$P_{n\_B2} = 3.737 \cdot k$

$P_{n\_B3} = 3.825 \cdot k$

$P_{n\_C1} = 4.374 \cdot k$

$P_{n\_C2} = 3.737 \cdot k$

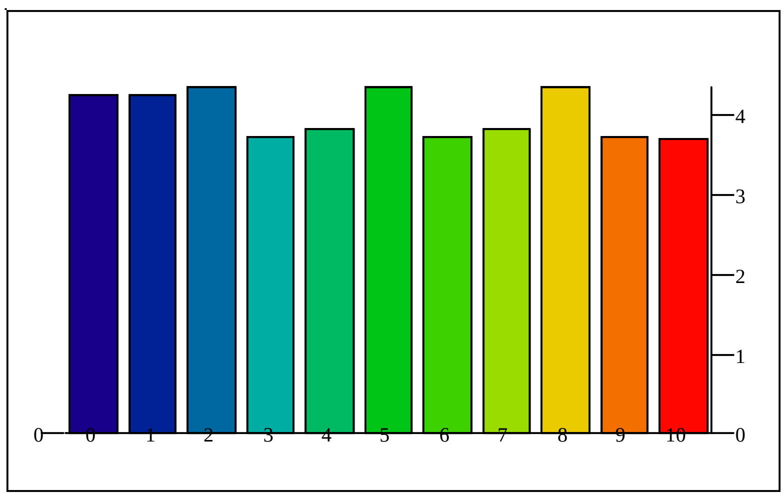
$P_{n\_C3} = 3.825 \cdot k$

$P_{n\_D1} = 4.374 \cdot k$

$P_{n\_D2} = 3.737 \cdot k$

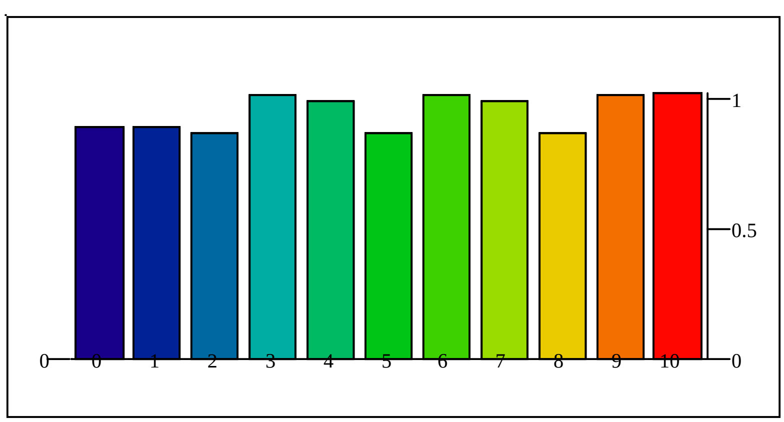
$P_{n\_D3} = 3.707 \cdot k$

Predicted Nominal Capacity $P_{test} := 3.8 \cdot k$



$\frac{P_{n\_all}}{k}$

Test to Predicted Ratio for Loughlan (1979) #L6



R_{tp}

$R_{tp} =$

	0	
0	0.894	A1
1	0.894	A2
2	0.869	B1
3	1.017	B2
4	0.993	B3
5	0.869	C1
6	1.017	C2
7	0.993	C3
8	0.869	D1
9	1.017	D2
10	1.025	D3

see the text for complete discussion of the analyzed design methods.