THIN-WALLED COLUMN DESIGN CONSIDERING LOCAL, DISTORTIONAL AND EULER BUCKLING

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ABSTRACT

Open cross-section, thin-walled, cold-formed steel columns have at least three competing buckling modes: local, distortional, and Euler (i.e., flexural or flexural-torsional) buckling. Current North American design specifications for cold-formed steel columns ignore local buckling interaction and do not provide an explicit check for distortional buckling. Closed-form prediction of the buckling stress in the local mode, including interaction of the connected elements, and the distortional mode, including consideration of the elastic and geometric stiffness at the web/flange juncture, are provided and shown to agree well with numerical methods. Numerical analyses and experiments indicate post-buckling capacity in the distortional mode is lower than in the local mode. Existing experiments on cold-formed channel, zed, and rack columns indicate inconsistency and systematic error in current design methods and provide validation for alternative methods. A new method is proposed for design that explicitly incorporates local, distortional and Euler buckling, does not require calculations of effective width and/or effective properties, gives reliable predictions devoid of systematic error, and provides a means to introduce rational analysis for elastic buckling prediction into the design of thin-walled columns.

INTRODUCTION

Compared with conventional structural columns, the pronounced role of instabilities complicates behavior and design of thin-walled columns. Elastic buckling analysis of open cross-section, thin-walled columns typically reveal at least three buckling modes: local, distortional, and Euler (global). Finite strip analysis of a cold-formed steel lipped channel in pure compression (Figure 1) shows that for practical

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member lengths all modes occur at stresses low enough that they must be considered in understanding and predicting behavior. Therefore, in addition to usual considerations for columns: material non-linearity (e.g., yielding), imperfections, residual stresses, etc., the individual role and potential for interaction of buckling modes must also be considered.

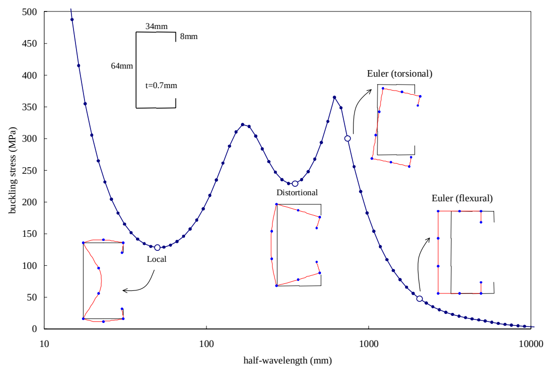

Figure 1 Finite strip analysis of a lipped channel in compression

Work in the last two decades (see Schafer 2000 for full details) has added much to our understanding of thin-walled columns, but a consistent design method that incorporates current knowledge is lacking. The combination of more refined methods for local and distortional buckling prediction, improved understanding of the post-buckling strength and imperfection sensitivity in distortional failures, and the relatively large amount of available experimental data allow for a re-assessment of existing design methods and development of new procedures. Consistent integration of local, distortional and Euler buckling into the design of thin-walled columns is needed.

ELASTIC BUCKLING

Local Buckling Prediction

Closed-form prediction of local buckling is examined using two methods: the element approach, and a semi-empirical interaction approach. The element approach is the classic solution for buckling of an isolated plate. For lipped channel and zed columns with web depth, h, flange width, b, and lip length, d, the local buckling stress ($f_{cr\ell}$) is:

$$\left(f_{cr\ell}\right)_{web} = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{h}\right)^2 \text{ and } k = 4 \quad (1)$$

$$\left(f_{cr\ell}\right)_{flange} = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 \text{ and } k = 4 \quad (2)$$

$$\left(f_{cr\ell}\right)_{lip} = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{d}\right)^2 \text{ and } k = 0.43 \quad (3)$$

For the element approach local buckling of a member may be approximated by taking the minimum of Eq.'s 1-3, or weighted averages. Alternatively, local interaction may be ignored; current design largely follows this approach (AISI 1996).

The semi-empirical interaction approach accounts for local buckling interaction in a single connected element. Expressions for k are determined for both flange/lip local buckling and flange/web local buckling by empirical close-fit solutions to finite strip analysis results. The solutions for k (of Eq. 2) including interactions are:

$$k_{flange/lip} = -11.07\left(\frac{d}{b}\right)^2 + 3.95\left(\frac{d}{b}\right) + 4 \quad (d/b < 0.6) \quad (4)$$

$$\text{if } \frac{h}{b} \geq 1 \; k_{flange/web} = 4\left(\frac{b}{h}\right)^2\left(2-\left(\frac{b}{h}\right)^{0.4}\right) \quad (5)$$

$$\text{if } \frac{h}{b} < 1 \; k_{flange/web} = 4\left(2-\left(\frac{h}{b}\right)^{0.2}\right) \quad (6)$$

Member local buckling may be predicted by the minimum of Eq. 4 and the appropriate expression from Eq. 5 or 6 and substituting into Eq. 2.

Distortional Buckling Prediction

Methods for closed-form prediction in the distortional mode include Desmond et al. (1981) as employed in North American design specifications (AISI 1996), and Lau and Hancock (1987) as employed in Australian/New Zealand design practice. Closed-form prediction of distortional buckling of beams is derived in Schafer and Peköz (1999); extension of this method to columns is completed here. The rotational stiffness (k_ϕ) at the juncture of the flange and the web may be expressed as the summation of the **e**lastic and stress dependent **g**eometric stiffness terms with contributions from both the **f**lange and the **w**eb

$$k_\phi = \left(k_{\phi f} + k_{\phi w}\right)_e - \left(k_{\phi f} + k_{\phi w}\right)_g \tag{7}$$

Buckling ensues when the elastic stiffness at the web/flange juncture is eroded by the geometric stiffness, i.e., when

$$k_\phi = 0 \tag{8}$$

If the stress dependent portion of the geometric stiffness is linearized and written explicitly then the critical buckling stress for distortional buckling (f_{crd}) may be found as

$$k_\phi = k_{\phi fe} + k_{\phi we} - f_{crd}\left(\tilde{k}_{\phi fg} + \tilde{k}_{\phi wg}\right) = 0 \tag{9}$$

$$f_{crd} = \frac{k_{\phi fe} + k_{\phi we}}{\tilde{k}_{\phi fg} + \tilde{k}_{\phi wg}} \tag{10}$$

Expressions for the flange remain unchanged from that of the previously derived work on beams, therefore:

$$k_{\phi fe} = \left(\frac{\pi}{L}\right)^4 \left(I_{xf}\left(x_o - h_x\right)^2 + C_{wf} - \frac{I_{xyf}^2}{I_{yf}}\left(x_o - h_x\right)^2 \right) E + \left(\frac{\pi}{L}\right)^2 GJ_f \tag{11}$$

$$\tilde{k}_{\phi fg} = \left(\frac{\pi}{L}\right)^2 \left[A_f \left(\begin{array}{l} \left(x_o - h_x\right)^2 \left(\dfrac{I_{xyf}}{I_{yf}}\right)^2 + h_x^2 \\ -2y_o\left(x_o - h_x\right)\left(\dfrac{I_{xyf}}{I_{yf}}\right) + y_o^2 \end{array} \right) + I_{xf} + I_{yf} \right] \tag{12}$$

All expressions with subscript f (I_{xf}, I_{yf}, J_f etc.) refer to section properties for the flange alone. Explicit expressions for all quantities in Eq.'s 11 and 12 are provided in Schafer and Peköz (1999).

The mechanical model employed for the web is a single, simply supported, lower order, plate bending, finite strip (Cheung and Tham 1998). The rotational stiffness contribution of the web at the web/flange juncture is assumed equal at each end. When the lateral translation of the web at the web/flange juncture is restrained, expressions simplify greatly (see Schafer 2000 for complete details):

$$k_{\phi we} = \frac{Et^3}{6h(1-\nu^2)} \tag{13}$$

$$\tilde{k}_{\phi wg} = \left(\frac{\pi}{L}\right)^2 \frac{th^3}{60} \tag{14}$$

Considering the lateral translation of the web at the web/flange juncture free leads to overly conservative predictions of the distortional buckling stress and is thus not detailed here. Determination of the critical length for distortional buckling of columns follows that of the previous derivation for beams, with appropriate substitutions reflecting Eq.'s 13 and 14. The resulting critical length is:

$$L_{cr} = \left(\frac{6\pi^4 h(1-\nu^2)}{t^3}\left(I_{xf}(x_o - h_x)^2 + C_{wf} - \frac{I_{xyf}^2}{I_{yf}}(x_o - h_x)^2\right)\right)^{1/4} \tag{15}$$

The elastic distortional buckling stress (f_{crd}) is found by determining the critical length, L_{cr}, using Eq. 15, substituting $L = L_{cr}$ into Eq.'s 11 – 14 to determine the appropriate rotational stiffness terms and then using Eq. 10 to calculate the buckling stress.

Euler Buckling Prediction

Closed-form predictions of the Euler buckling modes for x-axis and y-axis flexural buckling as well as flexural-torsional buckling are given in current design specifications (e.g., AISI 1996).

Accuracy of Elastic Buckling Models

Accuracy of the closed-form methods are summarized in Table 2. For local buckling prediction the semi-empirical interaction model (Eq.'s 4-6) is more accurate than the element model (Eq.'s 1-3). Local buckling prediction by the element model is only appropriate when the web depth (h) and flange width (b) are approximately equal ($h \cong b$). For distortional buckling prediction current design specifications (AISI 1996) are flawed or inapplicable, while both Lau and Hancock (1987) and the proposed method (Eq.'s 7-15) work reasonably well. For members with slender webs and small flanges the Lau and Hancock (1987) approach conservatively converges to a buckling stress of zero (these members are ignored in the summary statistics of Table 2). For the same members, the proposed method converges to the correct solution: the web local buckling stress.

Table 1 Member geometry for elastic buckling study

	h/b		h/t		b/t		d/t		
	max	min	max	min	max	min	max	min	count
Schafer (1997) Members	3.0	1.0	90	30	90	30	15.0	2.5	32
Commercial Drywall Studs	4.6	1.2	318	48	70	39	16.9	9.5	15
AISI Manual C's	7.8	0.9	232	20	66	15	13.8	3.2	73
AISI Manual Z's	4.2	1.7	199	32	55	18	20.3	5.1	50
	7.8	0.9	318	20	90	15	20.3	2.5	170

Table 2 Performance of prediction methods for elastic buckling

		Local		Distortional		
		(fcr)true / (fcr)element	(fcr)true / (fcr)interact	(fcr)true / (fcr)Schafer	(fcr)true / (fcr)Hancock	(fcr)true / (fcr)AISI
All Data	avg.	**1.34**	**1.03**	**0.93**	**0.96**	**0.79**
	st.dev.	0.13	0.06	0.05	0.06	0.33
	max	1.49	1.15	1.07	1.08	1.45
	min	0.96	0.78	0.81	0.83	0.18
	count	149	149	89	89	89
Schafer (1997) Members	avg.	**1.16**	**1.02**	**0.92**	**0.96**	**1.09**
	st.dev.	0.15	0.08	0.07	0.06	0.16
Commercial Drywall Studs	avg.	**1.38**	**1.07**	**0.93**	**1.00**	**0.81**
	st.dev.	0.09	0.05	0.02	0.07	0.26
AISI Manual C's	avg.	**1.33**	**1.01**	**0.93**	**0.99**	**0.81**
	st.dev.	0.13	0.07	0.05	0.03	0.26
AISI Manual Z's	avg.	**1.39**	**1.04**	**0.92**	**0.92**	**0.41**
	st.dev.	0.03	0.04	0.03	0.06	0.18

(fcr)true = local or distoritonal buckling stress from finite strip analysis
(fcr)element = minimum local buckling stress of the web, flange and lip via Eq.'s 1-3
(fcr)interact = minimum local buckling stress using the semi-empirical equations (Eq.'s 4-6)
(fcr)Schafer = distortional buckling stress via Eq.'s 7-15
(fcr)Hancock = distortional buckling stress via Lau and Hancock (1987)
(fcr)AISI = buckling stress for an edge stiffened element via AISI (1996) from Desmond et al. (1981)

ULTIMATE STRENGTH

Numerical Studies on Distortional Failures

Nonlinear finite element analysis of an isolated flange and lip, compressed to failure, is reported in Schafer and Peköz (1999). The

ABAQUS model uses nine-node reduced integration shell elements, elastic-plastic material with strain hardening, and imperfections and residual stresses as suggested in the modeling guidelines for cold-formed steel by Schafer and Peköz (1998). From this analysis it is concluded that: distortional failures have lower post-buckling capacity than local failures, distortional buckling may control the failure mechanism even when the elastic distortional buckling stress (f_{crd}) is higher than the elastic local buckling stress ($f_{cr\ell}$), and distortional failures have higher imperfection sensitivity. Further analysis on complete lipped channel columns (geometry summarized as Schafer 1997 Members in Table 1) supports these conclusions; except that reduction of the post-buckling capacity for distortional failures is less in the members than observed in models of the isolated flange and lip.

Distortional Failures of Rack Columns

The most extensive experimental work on the strength of cold-formed steel columns failing in the distortional mode is from the University of Sydney: Lau and Hancock (1987), Kwon and Hancock (1992), Hancock et al. (1994). Compression tests were conducted on: (a) lipped channels, (b) rack column uprights, (c) rack column uprights with additional outward edge stiffeners, (d) hats, and (e) lipped channels with a web stiffener as shown in Figure 2. The column curve fit to the distortional buckling failures may be expressed as:

$$\frac{P_n}{P} = \left(1 - 0.25\left(\frac{P_{crd}}{P}\right)^{.6}\right)\left(\frac{P_{crd}}{P}\right)^{.6} \text{ for } \sqrt{\frac{P}{P_{crd}}} > 0.561 \text{, else } P_n = P. \quad (16)$$

where: P_n is the nominal capacity in distortional buckling

P is the squash load ($P = P_y = A_g f_y$) when interaction with other modes is not considered, otherwise $P = A_g f$, where f is the limiting stress of a mode that may interact

P_{crd} is the critical elastic distortional buckling load ($A_g f_{crd}$)

Figure 2 provides strong evidence that if failure is known to occur in the distortional mode, then the elastic distortional buckling load (stress) may be used to directly predict the ultimate strength.

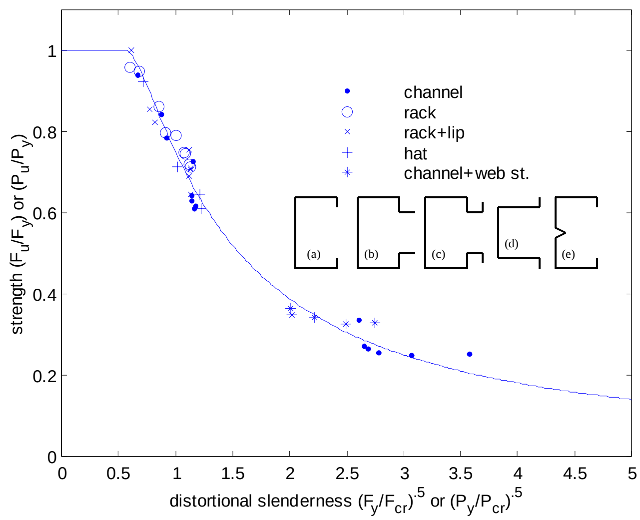

Figure 2 Columns failing in distortional mode (Sydney tests)

Experiments on lipped channel and zed Columns

To evaluate existing and proposed methods for the design of cold-formed steel columns experimental data on lipped channel and zed columns are gathered (Thomasson 1978, Loughlan 1979, Mulligan 1983, Peköz 1987, Polyzois and Charnvarnichborikarn 1993, Miller and Peköz 1994). Only unperforated lipped channel and zed sections, with 90 degree edge stiffeners, tested in a pin-pin configuration are selected. The geometry of the tested sections is summarized in Table 3. The available experimental data on lipped channels represents a wide variety of sections: slender webs, slender flanges, and relatively long lips are all included. However, in 95 out of 102 members h/b is greater than 1.6. Therefore, in most members the local buckling stress is lower than the distortional buckling stress due to the high slenderness of the web (only members with small lip length are an exception). For the lipped zed columns h/b ratios are similar to those of the lipped channels – thus this data suffers from the same limitations. However, the researchers specifically investigated the case of small, or no edge stiffening lip.

Table 3 Geometry of experimental data on lipped channel columns

	h/b		h/t		b/t		d/t		
	max	min	max	min	max	min	max	min	count
lipped channel columns									
Loughlan (1979)	5.0	1.6	322	91	80	30	33	11	33
Miller and Pekoz (1994)	4.6	2.5	170	46	38	18	8	5	19
Mulligan (1983)	2.9	1.0	207	93	93	64	16	14	13
Mulligan (1983) Stub Columns	3.9	0.7	353	65	100	33	22	7	24
Thomasson (1978)	3.0	3.0	472	207	159	69	32	14	13
summary	5.0	0.7	472	46	159	18	33	5	102
lipped zed columns									
Polyzois et al. (1993)	2.7	1.5	137	76	56	30	36	0	85

Experiments on Lipped Channels with Web Stiffeners

Thomasson (1978) tested cold-formed columns with up to two stiffeners in the web. The members without web stiffeners are recorded in the previous section. The tests included channels with slender webs, flanges, and lips – thickness as low as 0.63 mm (0.025 in.). Initial testing of specimens with an intermediate stiffener buckled in a distortional mode. In subsequent tests a bar was attached to the lips of the channels to restrict symmetric distortional buckling.

COLUMN DESIGN METHODS

Current thin-walled column design requires identification of the failure mode/mechanism of interest (e.g., local buckling), determination of elastic buckling characteristics (e.g., f_{cr}), and calculation of ultimate strength using empirical expressions – that are a function of material (e.g., f_y) and elastic buckling (e.g., f_{cr}). For a given failure mode, the elastic buckling calculations may be organized into two groups: (1) element methods, e.g., Eq.’s 1-3 for local buckling, or (2) member methods, which include closed-form expressions (e.g., Eq’s 4-6 for local buckling, Eq.’s 7-15 for distortional buckling) and numerical methods (e.g., finite strip analysis or finite element analysis). Ultimate strength calculation generally includes one of two basic approaches: effective width or column curve / “direct strength” methods.

Effective width uses empirical expressions to determine the portion of an element which is effective in resisting the load at the full applied stress (an element is a part of a member: i.e., the flange, web, lip, etc.). For example, the effective width method commonly implemented in design specifications:

$$\frac{b_{eff}}{b} = \left(1 - 0.22\sqrt{\frac{f_{cr\ell}}{f}}\right)\left(\sqrt{\frac{f_{cr\ell}}{f}}\right) \text{ for } \sqrt{\frac{f}{f_{cr\ell}}} > 0.673 \text{, else } b_{eff} = b \quad .(17)$$

where: b_{eff} is the effective width of an element with gross width b

f is the yield stress ($f = f_y$) when interaction with other modes is not considered, otherwise f is the limiting stress of a mode interacting with local buckling

$f_{cr\ell}$ is the local buckling stress

Column curve, or "direct strength" methods use gross properties of a member to determine the reduced strength of a column in a given mode due to buckling and/or yielding. Column curves have been typically applied to Euler buckling modes such as:

$$P_n = A_g f_n \text{ for } \lambda_c \leq 1.5 \;\; f_n = \left(0.658^{\lambda_c^2}\right) f_y \;\; \text{for } \lambda_c > 1.5 \;\; f_n = \left(\frac{0.877}{\lambda_c^2}\right) f_y \quad (18)$$

where: P_n is the nominal capacity

$\lambda_c = (f_y/f_e)^{1/2}$, and f_e is Euler buckling stress (min of flexural and flexural-torsional, with appropriate braced lengths).

f_y is the yield stress.

Direct strength methods are the extension of column curves to others modes such as local and distortional buckling, e.g., Eq. 16 for distortional buckling. Based on existing direct strength work for beams (Schafer and Peköz 1998b), the following form is suggested for local buckling of columns:

$$\frac{P_n}{P} = \left(1 - 0.15\left(\frac{P_{cr\ell}}{P}\right)^{.4}\right)\left(\frac{P_{cr\ell}}{P}\right)^{.4} \text{ for } \sqrt{\frac{P}{P_{cr\ell}}} > 0.776 \text{, else } P_n = P \;. \quad (19)$$

where: P_n is the nominal capacity

P is the squash load ($P = P_y = A_g f_y$) except when interaction with other modes is considered, then $P = A_g f$, where f is the limiting stress of the interacting mode.

$P_{cr\ell}$ is the critical elastic local buckling load ($A_g f_{cr\ell}$)

Three different assumptions on the level of interaction between local (L), distortional (D) and Euler (E) modes are considered (see summary below). Simultaneous L+D+E interaction is not considered in any of the methods. Interaction is considered by modifying the limiting stress in the strength equation. For example, interaction with Euler buckling is completed by limiting f used in Eq.'s 16, 17 or 19 to f_n of Eq. 18. Details and complete design examples for the eleven considered methods are given in Schafer (2000) only a summary is given here:

A1: Current Practice (AISI 1996)
A2: Current Practice with a distortional check

B1: Effective width with L+E and D interactions considered
B2: Direct Strength (hand*) with L+E and D interactions considered
B3: Direct Strength (numeric**) with L+E and D interactions

C1: Effective width with L+E and D+E interactions considered
C2: Direct Strength (hand) with L+E and D+E interactions considered
C3: Direct Strength (numeric) with L+E and D+E interactions

D1: Effective width with L+D, L+E and D+E interactions considered
D2: Direct Strength (hand) with L+D, L+E and D+E interactions
D3: Direct Strength (numeric) with L+D, L+E and D+E interactions

* hand methods use the formulas presented herein for buckling
** numeric methods use finite strip for buckling prediction

PERFORMANCE OF DESIGN METHODS

Current Practice (A1)

For common lipped channels and zeds average performance is 6% unconservative for method A1. Lack of an explicit treatment for distortional buckling does not introduce significant error for common members; but, ignoring local web/flange interaction does result in systematic error. Members with high web slenderness (h/t), consistently give unconservative predictions. Members with high h/t and h/b also suffer from systematically unconservative predictions. For local buckling in element approaches, as used in current design (as well as methods B1, C1, and D1) no matter how high the slenderness of the web becomes it has no effect on the solution for the flange.

Experimental data on zed sections further highlights these issues, and also demonstrates additional difficulties with empirical expressions in current use. Consider the A1 (AISI) results in Figure 3, for small lip length (d), when distortional buckling controls, predictions are adequate. However, for intermediate d, as behavior transitions from distortional to local predictions may be significantly unconservative (e.g., d ~ 20 in Figure 3) due to ignoring local web/flange interaction. Finally, for large d predicted strength is overly conservative – and generally follows the wrong trend. While "on average" current methods may be adequate, systematic errors exist and can be rectified.

Current Practice with a Distortional Check (A2)

For common members adding a separate distortional buckling check to current methods does not significantly benefit prediction. Instead, errors relate primarily to local web/flange interaction. Nonetheless, rack sections, intermediately stiffened sections, high strength steel sections, and other shapes prone to distortional buckling do require accurate, reliable design methods, and would benefit from a distortional buckling check, even the simple check used in this method.

Alternative Effective Width Method (B1)

Current design uses an element based effective width method for determining strength. However, buckling predictions are not always based on simple element methods (i.e., Eq.'s 1-3), but rather on empirical corrections that include limited aspects of local buckling interaction and even distortional buckling. If distortional buckling is treated separately from local buckling then many of the empirical corrections can be removed and replaced by simple expressions (e.g., Eq.'s 1-3). This approach is investigated in methods B1, C1 and D1.

The predicted strength for B1, (see Figure 3), is the minimum of the distortional buckling curve and the local buckling curve. Strength in the distortional mode is well predicted, but strength in the local mode suffers from systematic error due to ignoring web/flange interaction. Method B1 provides an upper bound solution, works as well as existing design methods (A1), removes problems with the strength prediction as lip length is increased, and explicitly separates local and distortional modes in a manner more consistent with observed behavior.

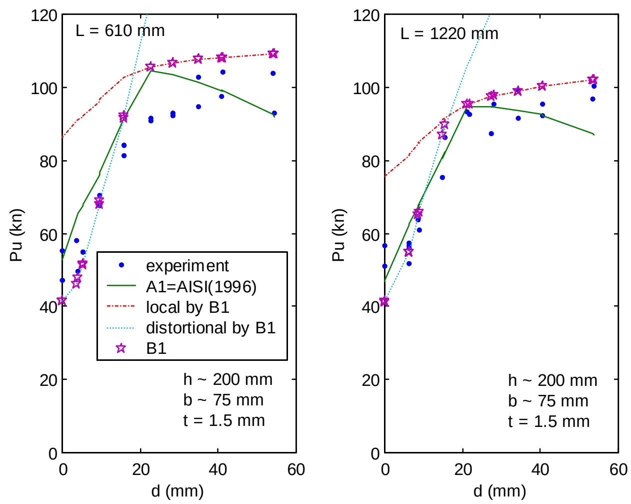

Figure 3 Performance of method A1(AISI) and B1 for sample of Z's

Direct Strength Methods (B2 and B3)

The direct strength methods are based on the use of separate strength (column) curves for local (Eq. 19) and distortional (Eq. 16) buckling. Method B2 relies on hand methods for predicting the local and distortional buckling stress (Eq.'s 4-6 and 7-15), while method B3 uses numerical methods (finite strip analysis). For lipped channel and zed section data the performance of the strength curves is shown as slenderness vs. strength in Figure 4 and summarized in Table 5. The method performs well, and given typical scatter in column data, appears to be a good predictor over a wide range of slenderness. The increased accuracy of the method (over methods A1, A2 and B1) occurs due to improvements in the local buckling prediction. Again using the zed column data as an example, the local buckling curves for direct strength methods B2 or B3 (Figure 5) can be compared with the element based methods B1 or A1 (Figure 3) to demonstrate that local web/flange interaction is the key difference in the methods.

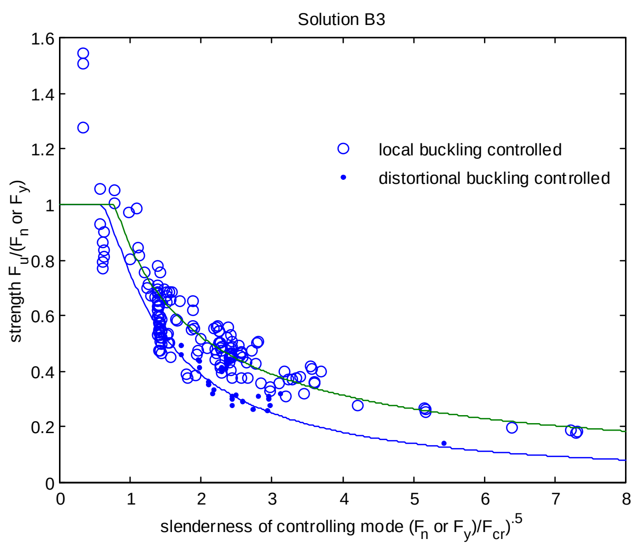

Figure 4 Slenderness vs. strength for C and Z sections, method B3

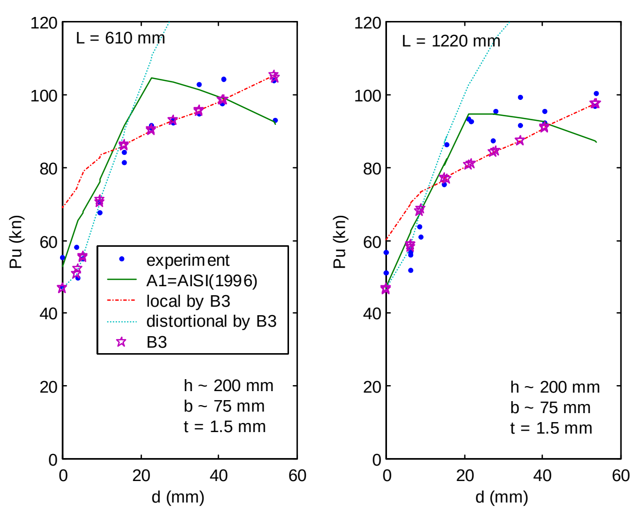

Figure 5 Performance of method B3 for sample of zed columns

Examination of the data with respect to h/t, h/t·h/b, and distortional slenderness, as well as other variables reveals no systematic error. The direct strength method postulates that if elastic critical buckling loads in the local and distortional mode are known this information is enough to determine the member strength – for this data, the notion appears validated.

Distortional and Euler Interaction Methods (C1,C2,C3)

Design methods C1, C2, and C3 are nearly identical to their counterparts; methods B1, B2, and B3 respectively, except that in the strength calculation for distortional buckling interaction with Euler buckling is considered (f limited to f_n). For the lipped channel and zed sections little overall difference occurs when distortional and Euler interaction is considered. Interaction of distortional buckling with Euler buckling cannot be definitively recognized nor rejected on this basis. This is discussed in more detail in Schafer (2000).

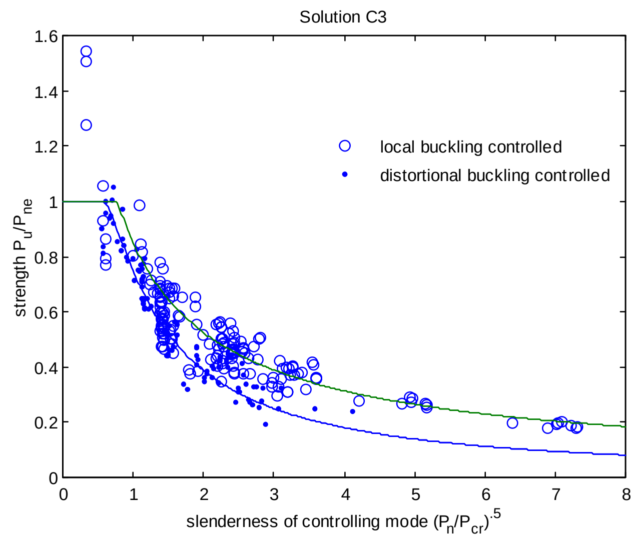

Figure 6 Slenderness vs. strength, all available column data (channels, zeds, C's with int. web stiffeners, racks, racks with compound lips)

Direct Strength - Distortional & Euler Interaction (C3)

Overall performance of the direct strength method (C3) for all experimental data is shown in Figure 6. Individual test to predicted ratios for the channel and zed sections are in Table 5, except for Thomasson's data with web stiffeners (note an attached bar restricting the symmetrical distortional mode is used in those tests, thus antisymmetric distortional buckling is used for f_{crd}) the average test to predicted ratio is 0.94 with a standard deviation of 0.13, and for the University of Sydney data the test to predicted ratio is 1.01 with a standard deviation of 0.07. Though scatter certainly exists, the direct strength approach is viable as a general method for prediction of the strength of cold-formed steel columns in local, or distortional buckling with consideration of interaction with Euler buckling.

Local and Distortional Interaction Methods (D1,D2,D3)

All "D" methods (D1, D2 and D3) allow local and distortional interaction and perform poorly (overly conservative). Interaction may still exist between these two modes (e.g. in perforated rack columns Baldassino and Hancock 1999), but in the available data, local and distortional interaction does not appear significant.

DISCUSSION

Reliability

Table 4 presents the reliability of the examined design methods, assessed by calculating the resistance factor (ϕ) for a reliability (β) of 2.5 via the guidelines of Section F in AISI (1996). Continued use of $\phi \sim 0.85$ appears appropriate for cold-formed steel columns.

Restriction of the Distortional Mode

In many cases attachments to other members (e.g. sheeting), as well as discrete braces may hinder the distortional mode and thus increase the strength. For discrete braces the best current practice is to compare the unbraced length (L_m) with the half-wavelength of the mode (L_{cr} of Eq. 15). If $L_m < L_{cr}$ it may be used in place of L_{cr} in Eq's 11-14. The bracing should restrict rotation of the flange and cause the distortional buckling wave to occur within the unbraced segment.

Table 4 Resistance Factors (ϕ) for the Design Methods by Limit State

Design method	L+E	D or D+E	L+D	All Data[1]
A1: AISI (1996) Specification	0.79			**0.82**
A2: AISI (1996) plus distortional check	0.78	0.94		**0.83**
B1: Effective width with distortional check	0.75	0.84		**0.81**
B2: Direct Strength by hand w/ dist. check	0.82	0.92		**0.86**
B3*: Direct Strength numerical wi/ dist. check	0.79	0.90		**0.84**
C1: same as B1 but consider D+E interaction	0.75	0.84		**0.81**
C2: same as B2 but consider D+E interaction	0.82	0.89		**0.86**
C3*: same as B3 but consider D+E interaction	0.80	0.89		**0.84**
D1: same as C1 but consideer D+L interaction	0.72		0.81	**0.88**
D2: same as C2 but consider D+L interaction	0.73	0.99	0.80	**0.89**
D3: same as C3 but consider D+L interaction	0.66	0.70	0.70	**0.85**

[1] resistance factor calculations for "all data" use a weighted standard deviation, i.e., the standard deviation for all the data is weighted by the number of samples conducted by each researcher.
* resistance factors calculated for methods B3 and C3 include Thomasson's data as well as all the cited University of Sydney data. Other methods only include the lippped channel and Z's used in this report.

Recommendations for Column Design

Two methods are recommended for thin-walled column design: C1 and C2/C3. If current design practice continues with element based effective width procedures then method C1 provides the best alternative to current practice. C1 removes complicated empirical expressions for local buckling and replaces them with simple formula (Eq.'s 1-3) and adds an explicit check on distortional buckling. Compared with current practice (method A1, AISI 1996) adoption of C1 favors members with longer lips (higher d/b) For common lipped channel and zed members the average change in predicted strength is less than 1%.

Whether implemented as a traditional hand method (C2) or one that allows rational analysis for elastic buckling determination (C3) the direct strength method provides a reliable alternative design procedure for thin-walled compression members. The method holds several advantages: calculations do not have to be performed for individual elements, interaction of the elements (e.g., in local buckling) is accounted for, distortional buckling is explicitly treated as a unique limit state, a means for introducing rational analysis through numerical prediction of elastic buckling is provided, and thus a general method for design of members with stiffener configurations or other geometries in which current rules are inapplicable can be completed. Compared to current practice (AISI 1996), narrow members (high h/b) with slender webs (high h/t) and short lips (low d/b) will be specifically discouraged. Members with longer lips (higher d/b) are encouraged. The direct strength method integrates known behavior into a rational design procedure, removes systematic error, and has a mean test to predicted ratio of 1.01.

CONCLUSIONS

Behavior and design of thin-walled, cold-formed steel columns requires consideration of local, distortional and Euler (i.e., flexural, or flexural-torsional) buckling. Accurate closed-form methods are provided for prediction of local buckling, including interaction, and distortional buckling. Current design methods ignore local buckling interaction and do not explicitly consider distortional buckling. Ignoring local buckling interaction leads to systematic error in strength prediction. Further, experimental and numerical studies indicate that post-buckling strength in the distortional mode is less than in the local mode. In pin-ended lipped channel and zed columns local and Euler interaction is well established. Comparisons with experimental data indicate local and distortional interaction is not significant, but are inconclusive regarding distortional and Euler interaction – for now it is proposed to include this interaction in design. A direct strength method (C2 and/or C3) is proposed for column design. The method uses separate column curves for local buckling (Eq 19) and distortional buckling (Eq 16), with the slenderness and maximum capacity in each mode controlled by consideration of Euler buckling (Eq 18). The method considers all the buckling modes in a consistent manner, does not require effective width calculations, and demonstrates that numerical elastic buckling solutions (e.g., finite strip) may be used as the key input to determining the strength of a large variety of thin-walled compression members.

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Table 5 Test to Predicted Ratios for all 11 Solution Methods, Broken Down by Controlling Limit State

design method:	A1: AISI (1996) Specification								
limit state[1]:	L+E			-			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**0.97**	0.04	13						
Miller and Pekoz (1994)	**0.86**	0.04	13						
Mulligan (1983)	**0.86**	0.12	33						
Mulligan (1983) Stub Col.	**1.05**	0.06	24						
Thomasson (1978)	**0.99**	0.23	19						
Polyzois at al. (1993)	**0.93**	0.10	85						
All Data	**0.94**	0.13	187						

design method:	A2: AISI (1996) Specification with Distortional Check								
limit state[1]:	L+E			D			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**0.97**	0.04	13						
Miller and Pekoz (1994)	**0.86**	0.04	13						
Mulligan (1983)	**0.86**	0.12	33						
Mulligan (1983) Stub Col.	**1.06**	0.06	20	**1.10**	0.09	4			
Thomasson (1978)	**1.00**	0.24	18	**0.98**		1			
Polyzois at al. (1993)	**0.92**	0.10	60	**1.12**	0.12	25			
All Data	**0.93**	0.13	157	**1.11**	0.11	30			

design method:	B1: Effective Width Method with L+E and D Check								
limit state[1]:	L+E			D			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**0.97**	0.04	13						
Miller and Pekoz (1994)	**0.86**	0.04	13						
Mulligan (1983)	**0.83**	0.12	33						
Mulligan (1983) Stub Col.	**1.05**	0.06	19	**1.10**	0.07	5			
Thomasson (1978)	**1.00**	0.24	18	**0.98**		1			
Polyzois at al. (1993)	**0.87**	0.08	47	**1.03**	0.16	38			
All Data	**0.91**	0.14	143	**1.04**	0.15	44			

design method:	B2: Hand Based Direct Strength Method with L+E & D								
limit state[1]:	L+E			D			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**1.11**	0.07	13						
Miller and Pekoz (1994)	**1.01**	0.07	13						
Mulligan (1983)	**0.94**	0.12	33						
Mulligan (1983) Stub Col.	**1.15**	0.09	24						
Thomasson (1978)	**1.01**	0.22	19						
Polyzois at al. (1993)	**0.95**	0.12	60	**1.09**	0.11	25			
All Data	**1.00**	0.15	162	**1.09**	0.11	25			

design method:	B3: Numerical Direct Strength Method with L+E & D								
limit state[1]:	L+E			-			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**1.08**	0.07	13						
Miller and Pekoz (1994)	**0.99**	0.06	12	**1.09**		1			
Mulligan (1983)	**0.92**	0.13	33						
Mulligan (1983) Stub Col.	**1.10**	0.09	20	**1.28**	0.04	4			
Thomasson (1978)	**1.00**	0.23	18	**1.02**		1			
Polyzois at al. (1993)	**0.92**	0.12	60	**1.05**	0.10	25			
All Data	**0.97**	0.14	156	**1.08**	0.12	31			

design method:	C1: Effective Width Method with L+E and D+E Check								
limit state[1]:	L+E			D+E			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**0.97**	0.04	12	**0.94**		1			
Miller and Pekoz (1994)	**0.85**	0.04	8	**0.93**	0.03	5			
Mulligan (1983)	**0.84**	0.12	32	**0.74**		1			
Mulligan (1983) Stub Col.	**1.04**	0.06	18	**1.10**	0.07	6			
Thomasson (1978)	**1.04**	0.25	14	**0.97**	0.10	5			
Polyzois at al. (1993)	**0.87**	0.08	47	**1.07**	0.17	38			
All Data	**0.91**	0.14	131	**1.04**	0.16	56			

design method:	C2: Hand Based Direct Strength with L+E & D+E								
limit state[1]:	L+E			D+E			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**1.12**	0.05	12	**0.94**		1			
Miller and Pekoz (1994)	**1.01**	0.07	13						
Mulligan (1983)	**0.94**	0.12	33						
Mulligan (1983) Stub Col.	**1.15**	0.09	24						
Thomasson (1978)	**1.01**	0.25	14	**1.04**	0.09	5			
Polyzois at al. (1993)	**0.96**	0.12	57	**1.12**	0.15	28			
All Data	**1.00**	0.15	153	**1.10**	0.15	34			

design method:	C3: Numerical Direct Strength Method with L+E & D+E								
limit state[1]:	L+E			D+E			-		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)	**1.09**	0.05	12	**1.00**		1			
Miller and Pekoz (1994)	**0.99**	0.06	12	**1.38**		1			
Mulligan (1983)	**0.92**	0.13	33						
Mulligan (1983) Stub Col.	**1.10**	0.10	18	**1.26**	0.06	6			
Thomasson (1978)	**1.07**	0.30	9	**1.00**	0.13	10			
Polyzois at al. (1993)	**0.92**	0.12	60	**1.12**	0.10	25			
All Data	**0.97**	0.15	144	**1.11**	0.14	43			

design method:	D1: Effective Width with L+E, D+E, and L+D Checks								
limit state[1]:	L+E			D+E			L+D		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)							**1.25**	0.15	13
Miller and Pekoz (1994)							**1.46**	0.12	13
Mulligan (1983)	**0.94**	0.16	7				**0.94**	0.13	26
Mulligan (1983) Stub Col							**1.47**	0.21	24
Thomasson (1978)	**1.04**	0.25	14				**1.12**	0.24	5
Polyzois at al. (1993)	**0.86**	0.05	11				**1.14**	0.22	74
All Data	**0.96**	0.20	32				**1.19**	0.26	155

design method:	D2: Hand Based Direct Strength with L+E, D+E, & L+D								
limit state[1]:	L+E			D+E			L+D		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)							**1.43**	0.21	13
Miller and Pekoz (1994)							**1.77**	0.14	13
Mulligan (1983)	**1.01**	0.16	7				**1.07**	0.17	26
Mulligan (1983) Stub Col							**1.64**	0.37	24
Thomasson (1978)	**1.03**	0.27	12	**1.08**	0.04	2	**1.21**	0.27	5
Polyzois at al. (1993)	**0.88**	0.07	11				**1.25**	0.24	74
All Data	**0.97**	0.20	30				**1.34**	0.33	155

design method:	D3: Numerical Direct Strength with L+E, D+E, and L+D								
limit state[1]:	L+E			D+E			L+D		
test to predicted stats[2]:	**mean**	std	count	**mean**	std	count	**mean**	std	count
Loughlan (1979)							**1.41**	0.20	13
Miller and Pekoz (1994)							**1.86**	0.22	13
Mulligan (1983)	**0.99**	0.21	4				**1.09**	0.19	29
Mulligan (1983) Stub Col							**1.76**	0.50	24
Thomasson (1978)	**1.07**	0.30	9	**0.95**	0.14	5	**1.27**	0.28	5
Polyzois at al. (1993)							**1.24**	0.29	85
All Data	**1.05**	0.27	13				**1.35**	0.39	169

[1] L=Local buckling, D=Distortional buckling, E=Euler (overall) buckling, L+E =Limit State that consider Local buckling interaction with Euler (overall) buckling, etc.

[2] test to predicted ratios are broken down by the controlling limit state