To: AISI Committee Members  
Subject: Progress Report No. 2

Direct Strength Design for Cold-Formed Steel Members with Perforations

Please find enclosed the second progress report summarizing our continuing research work on expanding the capabilities of the Direct Strength Method to cold-formed steel members with perforations. The valuable comments received at the AISI meeting in February 2006 have focused our elastic buckling studies in this report on key areas of interest for the committee. In addition to the elastic buckling work, we have made significant strides in our ability to predict the ultimate strength prediction of cold-formed steel members with holes using nonlinear finite element techniques. The preliminary ultimate strength results presented here are exciting and reinforce the importance of considering holes in the design of cold-formed steel members.

We look forward to your comments regarding this ongoing research.

Sincerely,

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Summary of Progress

The primary goal of this AISI-funded research is to extend the Direct Strength Method to cold-formed steel members with holes.

Research begins September 2006

Progress Report #1 February 2006

Accomplishments:

- Evaluated the ABAQUS S9R5, S4, and S4R thin shell elements for accuracy and versatility in thin-walled modeling problems
- Studied the influence of element aspect ratio and element quantity when modeling rounded corners in ABAQUS
- Developed custom MATLAB tools for meshing holes, plates, and cold-formed steel members in ABAQUS
- Determined the influence of a slotted hole on the elastic buckling of a structural stud channel and classified local, distortional, and global buckling modes
- Investigated the influence of hole size on the elastic buckling of a structural stud channel
- Performed a preliminary comparison of existing experimental data on cold-formed steel columns with holes to DSM predictions
- Conducted a study on the influence of the hole width to plate width ratio on the elastic buckling behavior of a simply supported rectangular plate

Papers from this research:

Moen, C., Schafer, B.W. (2006) “Impact of Holes on the Elastic Buckling of Cold-Formed Steel Columns with Application to the Direct Strength Method”, Eighteenth International Specialty Conference on Cold-Formed Steel Structures, Orlando, FL.

Summary of Progress (continued)

Progress Report #2 August 2006

Accomplishments:

• Evaluated the influence of slotted hole spacing on the elastic buckling of plates (with implications for structural studs)
• Determined the impact of flange holes on the elastic buckling of an SSMA structural stud
• Conducted a preliminary investigation into the nonlinear solution algorithms available in ABAQUS
• Compared the ultimate strength and load-displacement response of a rectangular plate and an SSMA structural stud column with and without a slotted hole using nonlinear finite element models in ABAQUS
• Calculated the effective width of a rectangular plate with and without a slotted hole using nonlinear finite element models in ABAQUS
1 Introduction

The research work presented in this progress report represents the continuing effort to develop a general design philosophy that relates elastic buckling behavior to the ultimate strength of cold-formed steel members with perforations. The general framework for this philosophy is being developed around the Direct Strength Method (DSM), which uses the local, distortional, and global elastic buckling modes to predict the ultimate strength of cold-formed steel members (NAS 2004, Appendix 1).

The final objective of this research project is to extend DSM to cold-formed steel columns and beams with holes, which will be met through research goals defined in three phases:

Phase I

1. Study the influence of holes on the elastic buckling of cold-formed steel members.
2. Formalize the identification of buckling modes for members with holes.
3. Compare existing experimental data on members with holes to the current DSM specification.
Phase II

1. Increase our understanding of post-buckling mechanisms for members with holes through non-linear finite element models and laboratory testing

2. Formalize the relationship between elastic buckling and ultimate strength for members with holes

Phase III

1. Modify the current DSM specification to account for members with holes

2. Experimentally validate DSM as a rational analysis method for any cold-formed member with holes

3. Develop open-source tools that engineers may use for easy application of DSM to members with holes

Research summarized in Progress Report #1 addressed the Phase I goals for cold-formed steel compression members with elastic buckling studies that evaluated the influence of holes on thin plates and cold-formed steel channel studs. This report, Progress Report #2, continues the elastic buckling research guided by the helpful discussion and comments from the February 2006 AISI meeting in Denver, Colorado. Two studies with direct implications to AISI specifications, (1) the influence of flange holes in SSMA structural studs and (2) the impact of slotted web hole spacing on the performance of an SSMA structural...
stud, have now been completed with the results presented in Section 2 of this report.

In addition to the new elastic buckling research, this report presents preliminary nonlinear finite element model results of thin plates and cold-formed steel compression members with holes. Commercial nonlinear finite element programs (in our case ABAQUS) are extremely powerful research tools, with the ability to accurately predict the load-displacement response and ultimate strength of cold-formed steel members. These programs offer much greater flexibility than a laboratory when exploring the boundary and loading conditions, and provide direct access to the stress distributions, yield zones, and failure modes of the specimen.

The difficulty with nonlinear finite element modeling is the large quantity of solution controls and methods available to the user when solving the problem, which can easily be misused and misinterpreted. This is why the first nonlinear finite element study presented in this report is a parameter study which focuses on evaluating result sensitivity to solution controls of two ABAQUS nonlinear solution methods, (1) a modified Riks arc-length solution algorithm and (2) an artificial damping solution algorithm. Both of these methods have been shown to be robust solvers of inherently unstable problems (ABAQUS 2004).

Once a set of viable solution controls is determined, two nonlinear finite element studies are conducted to compare the load-displacement response, ultimate strength, and failure modes of a simply supported rectangular plate and
a structural stud column with and without a slotted hole. The influence of a slotted hole on the AISI code-based “effective width” is also presented. The report is completed with a preliminary attempt to connect the elastic buckling and failure modes of an intermediate length structural stud column, including a comparison of the FEM column strengths and DSM predictions.

2 Elastic Buckling Studies

This section of the report focuses on the influence of holes on the elastic buckling behavior of thin plates and cold-formed steel structural studs. The influence of hole spacing on the elastic buckling load of a long rectangular plate is evaluated and hole spacing limits are defined in a specification-based format. The elastic buckling behavior of intermediate length structural stud columns with flange holes is also studied, with conclusions drawn regarding the impact of these holes on the local, distortional, and global buckling modes of the member.

2.1 Effect of Slotted Hole Spacing on the Elastic Buckling of Uniaxially Loaded Rectangular Plates

2.1.1 Finite Element Modeling and Dimension Nomenclature

The elastic buckling behavior of the plates in this study are obtained with an eigenbuckling analysis in ABAQUS (ABAQUS 2004). All members are modeled with S9R5 reduced integration nine-node thin shell elements. Plate boundary conditions are modeled as simply supported. The plates are loaded from each end with a uniform compressive stress applied as consistent nodal loads in
ABAQUS. Cold-formed steel material properties are assumed as $E=203$ GPa and $\nu=0.3$. Plate boundary and loading conditions are summarized in Figure 2.1(a).

![Figure 2.1](image)

**Figure 2.1** (a) Plate loading and boundary conditions, (b) dimension nomenclature

Nomenclature for the dimensions is summarized in Figure 2.1(b). The slotted hole has dimensions of $h_{\text{hole}}=38.1$ mm, $L_{\text{hole}}=101.6$ mm, and $R_{\text{hole}}=19.1$ mm. Holes are always centered transversely between the unloaded edges of the plate. Plate width $h$ and length $L$ are varied in this study, and the plate thickness $t$ is 0.88 mm.

### 2.1.2 Influence of a single slotted hole on the elastic buckling stress of rectangular plates with varying lengths and widths

This study explores the influence of a single slotted hole on the elastic buckling stress of simply supported rectangular plates. The plate length is varied from three to twenty-four times the slotted hole length, and the width of the plates are chosen to equal the flat web widths of four common Steel Stud Manufacturer Association (SSMA) structural studs listed in Table 2.1.
Table 2.1 Plate widths considered in this study and corresponding SSMA structural stud designations

<table>
<thead>
<tr>
<th>SSMA Designation</th>
<th>h (mm)</th>
<th>$h_{\text{hole}}/h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>250S162-33</td>
<td>57.9</td>
<td>0.66</td>
</tr>
<tr>
<td>362S162-33</td>
<td>86.3</td>
<td>0.44</td>
</tr>
<tr>
<td>600S162-33</td>
<td>146.8</td>
<td>0.26</td>
</tr>
<tr>
<td>800S162-33</td>
<td>197.6</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Figure 2.2 Influence of a slotted hole on the elastic buckling stress of a simply supported rectangular plate with varying length

The results of this study are presented in Figure 2.2 and demonstrate that as the length of a rectangular plate increases, the elastic buckling stress converges to a constant magnitude which is either equal to or lower than the buckling stress of a plate without a hole. The convergence approximately occurs as plate lengths exceed an $L/L_{\text{hole}}$ of 5, which suggests that the influence of the plate end conditions is negligible beyond this length.
When $h_{\text{hole}}/h=0.66$ and $L/L_{\text{hole}}$ is small (see Figure 2.2), the elastic buckling stress of the plate with the hole is as much as seven percent higher than for a plate without a hole. This increase in stress is explained by the buckled mode shapes in Figure 2.3. The plate with the hole in Figure 2.3(a) has a higher elastic buckling stress than the plate without the hole in Figure 2.3(b) because the natural pattern of buckled waves is modified by the hole. The buckled cells adjacent to the hole are shorter (and therefore stiffer), and the thin strips at the hole dampen buckling (due to increased stiffness). These wavelength stiffening and “unstiffened strip” effects are discussed in more detail in Moen and Schafer (2006).

Figure 2.3 Comparison of buckled shape and displacement contours for a rectangular plate with $h_{\text{hole}}/h=0.66$ and $L/L_{\text{hole}}=3$, (a) with slotted hole and (b) without hole. Notice the change in length and quantity of buckled cells with the addition of a slotted hole.

As the plate length increases past $L/L_{\text{hole}}=5$ for the smallest plate width ($h_{\text{hole}}/h=0.66$), the buckling stress converges to that of a plate without a hole. Figure 2.4 demonstrates that for these long, slender plates the slotted hole dampens buckling near the hole but does not appreciably change the natural
half-wavelength of the buckled cells as was observed for the shorter plates in Figure 2.3.

![Buckled shape of a simply supported plate with and without a hole.](image)

**Figure 2.4** Buckled shape of a simply supported plate (a) with a slotted hole and (b) without a hole. $L=15L_{\text{hole}}$, $h_{\text{hole}}/h=0.66$. The slotted hole dampens buckling but does not significantly change the natural half-wavelength of the plate.

For plates with $h_{\text{hole}}/h$ less than 0.66, the slotted hole causes a decrease in the elastic buckling stress, but converges to a constant magnitude as the plate length exceeds $L/L_{\text{hole}} = 5$. Figure 2.5 demonstrates that local buckling near the hole controls the elastic buckling stress of wider and longer plates. This mode is suspected to contribute to distortional buckling modes near a hole (DH modes), which were observed in the eigenbuckling analyses of SSMA channel studs with slotted holes (Moen and Schafer 2006).
2.1.3 Influence of slotted hole spacing on a long rectangular plate

This study evaluates the influence of slotted hole spacing on the elastic buckling stress of a long fixed length simply supported rectangular plate, where \( L=24 \, L_{\text{hole}} \). Slotted holes are added one by one such that the center-to-center spacing \( S \) varies as described in Figure 2.6.

![Figure 2.5](image1.png)  
(a) Slotted hole causes local buckling \((h_{\text{hole}}/h=0.26)\), compared to (b) buckled cells at the natural half-wavelength of the plate

![Figure 2.6](image2.png)  
Figure 2.6 Definition of center-to-center dimension for the slotted holes
Figure 2.7 Influence of slotted hole spacing on the elastic buckling load of a long simply supported rectangular plate

Figure 2.7 demonstrates that as hole spacing decreases, the elastic buckling stress can either increase or decrease depending on the ratio of hole width to plate width. When there are many large holes ($h_{hole}/h=0.66$, $S/L_{hole} < 4$), buckling is dampened at the holes and the buckled cells shorten their lengths to form between adjacent holes (see Figure 2.8 for buckled shape). These changes in wavelength cause an increase in elastic buckling stress of the plate. When the holes are smaller relative to the plate width ($h_{hole}/h<0.44$) and are spaced closely together ($S/L_{hole} < 4$), the local buckling influence of adjacent holes combine to sharply decrease the elastic buckling stress. The inset of Figure 2.7 highlights this reduction in elastic buckling stress for $h_{hole}/h=0.19$ and $h_{hole}/h=0.26$, and Figure 2.8 provides a summary of the associated buckled shapes. When hole
spacing increases above \( S/ L_{\text{hole}} = 5 \), the elastic buckling stresses approach constant magnitudes for all plate widths considered, which is consistent with the trends presented in Figure 2.2.

![Comparison of buckled shapes for a long simply supported rectangular plate (\(L=24 L_{\text{hole}}\) with a slotted hole spacing of \(S/L_{\text{hole}}=4\) and \(h_{\text{hole}}/h=0.66, 0.44, \) and 0.26.](image)

2.1.4 Elastic buckling stress of long plates with slotted holes

A relationship between the plate buckling coefficient \(k\) and the hole width to plate width ratio, \(h_{\text{hole}}/h\), will now be determined for the simply-supported plates considered in the two previous studies. The gross width \(h\) of the plate is used when calculating \(k\).

The theoretical buckling stress for a stiffened plate is:
where $E$ is the modulus of elasticity of the plate material, $\nu$ is the Poisson’s ratio, and $t$ is the thickness of the plate. For a stiffened plate without a hole, $k=4$ as the plate aspect ratio $L/h$ approaches infinity. When slotted holes were added to the plates in the previous studies, $f_{cr}$ (and therefore $k$) either increased or decreased depending on $h_{hole}/h$, and converged to a constant value with a hole spacing $S > 5L_{hole}$.

Table 2.2 summarizes the buckling coefficient $k$ for rectangular plates with slotted holes ($S > 5L_{hole}$).

<table>
<thead>
<tr>
<th>SSMA Designation</th>
<th>$h$ (mm)</th>
<th>$h_{hole}/h$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>250S162-33</td>
<td>57.9</td>
<td>0.66</td>
<td>4.00</td>
</tr>
<tr>
<td>362S162-33</td>
<td>86.3</td>
<td>0.44</td>
<td>3.61</td>
</tr>
<tr>
<td>600S162-33</td>
<td>146.8</td>
<td>0.26</td>
<td>3.39</td>
</tr>
<tr>
<td>800S162-33</td>
<td>197.6</td>
<td>0.19</td>
<td>3.47</td>
</tr>
</tbody>
</table>

Figure 2.9 presents a parabolic curve that provides an approximate prediction of $k$ (or elastic buckling stress) for rectangular plates with slotted holes spaced at $S > 5L_{hole}$.
The prediction of the elastic buckling stress for this type of plate can be written in more formal design language as:

For a stiffened rectangular plate with one or more slotted holes spaced at $S>5L_{\text{hole}}$, the elastic buckling stress shall be calculated as:

$$f_{cr} = \frac{k \pi^2 E}{12(1-\nu^2)} \left( \frac{t}{h} \right)^2$$

where:

$$k = 6 \left( \frac{h_{\text{hole}}}{h} \right)^2 - 4 \left( \frac{h_{\text{hole}}}{h} \right) + 4 \leq 4$$
2.1.5 Conclusions

The study of Sections 2.1.2 to 2.1.3 evaluated the influence of multiple slotted holes on the elastic buckling stress of rectangular simply supported plates. The first phase of the study examined the relationship between elastic buckling stress and plate length when considering a single slotted hole at the plate midlength. The second phase considered the influence of adjacent holes on the elastic buckling stress of a long rectangular plate. The following conclusions are made:

- When the slotted hole width is large compared to the plate widths considered in this study, buckling near the hole is dampened. For short plates or for plates with multiple holes, the location, quantity, and length of the buckled cells may be modified resulting in an increase in the elastic buckling stress of the plate.
- When the slotted hole is small compared to the plate width ($\frac{h_{\text{hole}}}{h} < 0.66$), local buckling decreases the elastic buckling stress and controls the buckled shape of the plate. For slotted hole spacings $\frac{S}{L_{\text{hole}}} < 5$, the influence of the holes becomes cumulative and can lead to a sharp decrease in elastic buckling stress.
- For slotted hole spacings $S > 5 L_{\text{hole}}$ and all plate widths considered, the effect of a hole on the elastic buckling stress of the plate is not cumulative,
but instead corresponds to that of a plate with a single hole at the midlength of the plate.

### 2.2 Effect of Circular Flange Holes on the Elastic Buckling of an Intermediate Length SSMA Structural Stud

The research focus up until now has been on the elastic buckling modes of isolated web plates and cold-formed steel compression members with web holes (Moen and Schafer 2006). We now turn our attention to flange holes and their impact on the elastic buckling of cold-formed steel compression members. An example of a standard connection detail that requires a flange hole is shown in Figure 2.10 (Western States Clay Products Association 2004).

![Connection detail for structural stud to exterior wall](image)

**Figure 2.10** Connection detail for structural stud to exterior wall requires a screw or bolt hole placed in the stud flange (Western States Clay Products Association *Design Guide for Anchored Brick Veneer over Steel Studs*)
2.2.1 Problem Statement

This study evaluates the influence of circular flange holes on the elastic buckling behavior of an intermediate length SSMA 362S162-33 structural stud. A single hole is placed at the midlength of both the top and bottom flanges and centered between the web and lip stiffeners. Five hole diameters consistent with standard bolt holes are considered: \( b_{\text{hole}} / b = 0.178, 0.356, 0.534, 0.713, \) and 0.892 (\( \frac{3}{8}'' , \frac{1}{2}'' , \frac{3}{4}'' , 1'', 1\frac{1}{4}'' \) holes in a \( 1\frac{5}{8}'' \) flange) where the flat flange width \( b = 35.6 \text{ mm}. \) The length \( L \) is 1220 mm for all members which corresponds to half the standard length of an SSMA structural stud ((half of 8'-0''). Nomenclature of the dimensions used to describe the structural studs is summarized Figure 2.11. The cross-section dimensions of the SSMA 362S162-33 are summarized in Table 2.3.

\[ \text{Figure 2.11 SSMA structural stud and hole dimension nomenclature} \]

\[ \text{Table 2.3 SSMA 362S162-33 Cross-section dimensions} \]

<table>
<thead>
<tr>
<th>SSMA Designation</th>
<th>( H ) (mm)</th>
<th>( B ) (mm)</th>
<th>( D ) (mm)</th>
<th>( R ) (mm)</th>
<th>( t ) (mm)</th>
<th>( r ) (mm)</th>
<th>( b ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>362S162-33</td>
<td>91.9</td>
<td>41.3</td>
<td>12.7</td>
<td>1.9</td>
<td>0.88</td>
<td>2.38</td>
<td>35.6</td>
</tr>
</tbody>
</table>
2.2.2 Finite Element Modeling

The elastic buckling behavior of the structural studs in this study are obtained with an eigenbuckling analysis in ABAQUS (ABAQUS 2004). Member boundary and loading conditions are summarized in Figure 2.12 (shown without the holes). The studs are loaded from each end with a uniform compressive stress applied as consistent nodal loads in ABAQUS. Boundary conditions are modeled as warping free at the member ends and warping fixed at the midlength of the member, which mimics the semi-analytical finite strip method. All members are modeled with S9R5 reduced integration nine-node thin shell elements. Cold-formed steel material properties are assumed as \( E = 203 \text{ GPa} \) and \( \nu = 0.3 \).

Figure 2.12 Loading and boundary conditions for the SSMA intermediate length stud. Boundary conditions are modeled to mimic those assumed in the finite strip method and uniform compressive stresses are applied at each end with consistent nodal loads.
2.2.3 Results

Figure 2.13 summarizes the elastic buckling loads for local, distortional, and global modes as the diameter of the flange holes increase. The local buckling load, $P_{cr}$, is not sensitive to small holes although for large holes ($b_{hole}/b=0.713$, $0.891$) a new local buckling mode becomes important. It is noted that the $P_{cr, no hole}$ magnitudes labeled in Figure 2.13 are determined using ABAQUS finite element models (not finite strip models) with boundary and loading conditions consistent with Figure 2.12. A comparison study of finite strip (CUFSM) and finite element results (ABAQUS) can be found in Progress Report #1.

The local hole mode (LH), presented in Figure 2.14, is dominated by local web and flange buckling near the holes and has an elastic buckling load ten percent less than $P_{cr}$ for a stud without a hole. This result suggests that large flange holes adversely affect the beneficial web-flange interaction inherent in structural studs.
Figure 2.13  Influence of flange hole diameter on the local (L), distortional (D), and global (GFT) elastic buckling loads of an SSMA 362S162-33 structural stud

Figure 2.14  The LH mode is dominated by flange and web buckling near the holes

The pure distortional mode (D) and the global flexural-torsional mode (GFT) demonstrate a slight decreasing trend as flange hole size increases, with a maximum decrease in critical load of four percent when compared to a member without a hole. The holes do not initiate local buckling of the flanges or web for these modes.
2.2.4 Conclusions

This study evaluates the influence of a single circular hole in the top and bottom flanges on the elastic buckling behavior of an intermediate length SSMA structural stud. The following conclusions are made:

• The elastic buckling properties of the SSMA stud considered in this study are not affected by flange holes when $b_{hole} / b < 0.50$.

• A new local hole buckling mode (LH) is created by large flange holes which reduces the local buckling load $P_{cr}$ of the member by as much as ten percent. This reduction is caused by the loss in stiffness in the flanges near the holes.

• The distortional and global elastic buckling loads are not significantly influenced by isolated flanges holes, even when the hole diameter is large relative to the flange width.

It should be noted that these conclusions are founded on elastic buckling analyses and do not necessarily correlate to strength predictions. Future work is required to evaluate the influence of flange holes on ultimate member strength. Material and geometrical nonlinearities in the post-buckling regime and the net section loss from the holes are identified as important parameters in this future work.
3 Nonlinear Modeling and Ultimate Strength

It is challenging to obtain physically realistic post-buckling behavior of cold-formed steel members using finite element methods. Consider an intermediate length cold-formed steel structural channel stud in compression. Buckling of the individual elements (webs, flanges, stiffening lips) may occur before material yielding, causing the compressive forces to flow to areas of higher stiffness (intersection of corners and web for instance) resulting in a highly nonlinear stress distribution in the cross section. Initial conditions such as residual stresses from roll-forming, geometric imperfections from fabrication, and pre-punched perforations for utilities combine to produce several potential failure modes with valid equilibrium paths. The challenge with nonlinear finite element modeling is then to navigate these path-dependent variations and find a solution that has real physical meaning.

3.1 Evaluation of ABAQUS Nonlinear Solution Methods

3.1.1 Introduction

The goal of this study is to become familiar with two nonlinear solution tools available in ABAQUS that are known to produce reasonable solutions to difficult nonlinear problems such as determining the ultimate strength of thin-walled structures. The first tool, *STATIC, RIKS, is based on the modified Riks method introduced in the early 1980’s. This algorithm employs an arc length constraint on the Newton-Raphson incremental solution to ensure equilibrium at
highly nonlinear points along the load-deflection curve such as when the peak load of a member has been reached (ABAQUS 2004, Crisfield 1981, Ramm 1981, Powell and Simon 1981). The second tool, *STATIC, STABILIZE, couples the Newton-Raphson solution technique with artificial mass proportional damping. As local instabilities develop (that is, when changes in nodal displacements increase rapidly over a solution increment), damping is added to help the solution algorithm maintain equilibrium (ABAQUS 2004). Local instabilities near the peak load are common in cold-formed steel members, such as when a thin plate develops a fold line under a compressive load.

Both of these solution methods will be evaluated in this study to (1) gain experience with implementing the methods, (2) evaluate their ability to predict the ultimate strength and failure modes of thin plates, and (3) determine solution control sensitivities that will impact upcoming nonlinear modeling work on cold-formed steel columns and beams.

3.1.2 Problem Statement

As stated previously, the goal of this study is to learn about the nonlinear capabilities and modeling requirements of ABAQUS. A thin rectangular simply supported plate is chosen for this study because it is straightforward to implement and is not computationally intensive. Initial geometric imperfections and material nonlinearities are included in the model, and the ultimate strength
and failure modes of the plate are determined. The sensitivity of the solutions to key ABAQUS controls is also evaluated.

![Diagram of load conditions](image)

**Figure 3.1** Loading conditions for (a) modified Riks method and (b) artificial damping solutions

The plate loading conditions are summarized in Figure 3.1. When using the modified Riks solution method, the plate is loaded at both ends with a uniform compressive stress using consistent nodal loads. The magnitude of the load is represented by the parameter $\lambda$, which is an accumulation of load steps $\Delta\lambda$ determined by ABAQUS. This load step selection process uses large $\Delta\lambda$ increments when the Newton-Raphson algorithm converges quickly (along the linear branch of the load-displacement curve) and adjusts to smaller increments as equilibrium becomes more difficult to achieve (near the peak of the load-displacement curve). For the artificial damping solution method, displacement control is used to compress the plate. The total displacement $\delta$ is applied over 100 steps (chosen by the user), where the maximum displacement increment at each step is set at $\Delta\delta=0.0145t$. It should be noted that the modified Riks and
artificial damping methods are capable of solving problems with applied loads or displacements.

Figure 3.2 Simply supported boundary conditions with coupling at loaded edges

The boundary conditions for the plate are summarized in Figure 3.2. The plate is simply supported around the perimeter, and transverse displacements (Poisson effect) are allowed. The nodes at the loaded edges of the plate are coupled to displace together longitudinally (in the 1 direction), which prevents local failure modes of the plate at the loaded edges. The nodal coupling is provided with an equation constraint in ABAQUS.

A plate width $h$ of 86.4 mm, aspect ratio of 8:1, and a thickness $t$ of 0.88 mm are chosen to be consistent with the flat web width and thickness of an intermediate length SSMA 362S162-33 structural stud. Cold-formed steel material properties are assumed as $E=203$ GPa and $\nu=0.3$. The true stress-strain curve presented in Figure 3.3 is input into ABAQUS to model the material
nonlinearity of the steel. This curve was generated from an actual tensile coupon test of a cold-formed steel structural stud, where the yield strength of the steel was determined to be 227.5 MPa (Yu 2005).

![True stress-strain curve used in ABAQUS, derived from a tensile coupon test (Yu 2005)](image)

<table>
<thead>
<tr>
<th>True Stress (MPa)</th>
<th>True Strain</th>
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<td>419.6</td>
<td>0.1407</td>
</tr>
<tr>
<td>451.6</td>
<td>0.1988</td>
</tr>
</tbody>
</table>

Figure 3.3  True stress-strain curve used in ABAQUS, derived from a tensile coupon test (Yu 2005)

Initial geometric imperfections in the plate are assumed to be related to the fundamental elastic buckling mode of the plate. The magnitude of the imperfections is chosen based on a probabilistic treatment developed for cold-formed steel members (Schafer and Peköz 1998). Since the plate considered here has been chosen to be consistent with the web of a structural stud, a Type 1 imperfection is assumed as shown in Figure 3.4. The maximum magnitude of the imperfection field is selected such that there is a fifty percent chance that a
randomly occurring imperfection in the plate, $\Delta$, will have a magnitude less than $d_1$. A $d_1/t=0.34$ corresponds to this probability of occurrence, $P(\Delta<d_1)=0.50$, and is used to scale the initial imperfection field of the plate.

![Cross section with initial geometric imperfections](image)

Figure 3.4 Type 1 imperfection (Schafer and Peköz 1998)

### 3.1.3 Modified Riks Method Solution Results

A range of load-displacement curves for the simply supported rectangular plate problem are presented in Figure 3.5. Several different equilibrium paths were found in ABAQUS by varying $\Delta \lambda_{\text{max}}$, the maximum load increment allowed in the automatic step selection algorithm used by ABAQUS.
The existence of multiple solutions is consistent with a plate containing periodic geometric imperfections, since each half-wavelength has an equal chance of deforming locally into a plastic failure zone. Although there were several different post-peak trends observed depending upon the choice of $\Delta \lambda_{\text{max}}$, the primary failure mechanism for the plate was a sharp yield-line fold occurring transversely across the plate. Figure 3.6 demonstrates that this folding occurs at the crest of the buckled half-wave of the initial geometric imperfection field, suggesting that the failure mechanism of the plate is linked to the initial imperfections. The quantity and location of the folds influence the ductility or energy-absorbing capacity of the plate (the area under the load-displacement curve). As the number of folds increase, the post-peak strength and ductility of
the plate increase. It is important to note that the peak compressive load of the plate, $P_u$, did not exhibit sensitivity to changes in $\Delta \lambda_{max}$. For the range of $\Delta \lambda_{max}$ considered, $P_u$ was consistently determined to be between 0.62 and 0.64 of $P_y$ for the plate.

![Figure 3.6 Correlation between initial imperfections and fold line locations at failure](image)

#### 3.1.4 Artificial Damping Solution Results

In this solution method, artificial mass proportional damping is employed to alleviate local instabilities. The global equilibrium equations at each displacement step can be written as:

$$ P - F - D = 0 $$

where $P$ is the vector of applied external forces, $F$ is the vector of calculated internal forces, and $D$ is the vector of viscous damping forces. The damping force vector $D$ is calculated at each step based on the following relationship:

$$ D = (cM)v $$

where $c$ is the damping ratio, $M$ is an artificial mass matrix calculated with a unit material density, and $v$ represents the change in nodal displacements divided by the size of the “time” step selected by ABAQUS. $v$ is called the “nodal velocity”
in ABAQUS since the dimensions are length/”time” units, which makes $v$ sensitive to the definition of “time”. In this study, the total “time” is selected as one unit and the maximum “time” step allowed is 0.01 units. If the total “time” was chosen as 100 units and the maximum “time” step as 1 unit, it seems that the magnitude of the damping forces $D$ would change. Following the same argument, the magnitude of $v$ will be impacted by the choice of units for the problem (feet, inches, meters, mm) since $v$ has dimensions of length/”time” units. Future work is required to evaluate the influence of “time” and length units on the calculation of the “nodal velocity” $v$. An evaluation of the artificial damping solution sensitivity to the magnitude and distribution of mass in a member will also be pursued in future research.

When the solution is stable, changes in nodal displacements are small and viscous damping is negligible. When large changes in displacements occur between two consecutive load steps (as is the case for a local instability), damping forces are applied to help make up the difference between $P$ and $F$. $v$ may only be high for certain locations in the member, and therefore damping will only be applied in these areas.

The damping factor $c$ is typically chosen as a small number since large damping forces may add too much artificial stiffness to the system, producing an incorrect solution. ABAQUS provides both automatic and manual options for selection the damping factor $c$. When the automatic option is selected, ABAQUS finds $c$ such that the dissipated energy to total strain energy ratio after the first
load step is equal to $2.0 \times 10^{-4}$. This ratio provides a unit and “time” independent selection method for $c$.

The artificial damping solution results for both manually and ABAQUS-selected damping factors are presented in Figure 3.7. One important observation is that the peak load is not sensitive to the damping factor $c$. For the range of $c$ considered, the peak load varied from 0.62 to 0.63 of $P_y$ for the plate which is consistent with the modified Riks results in Section 3.1.3. Also consistent with the Riks solutions are the presence of several types of post-peak responses and failure modes. This is caused by the presence of many post-peak equilibrium paths as discussed previously in Section 3.1.3 of this report.

![Figure 3.7 Artificial damping load displacement solutions and failure modes](image)

**Figure 3.7** Artificial damping load displacement solutions and failure modes
It is interesting to note that the load-displacement curve calculated without damping ($c=0.000$) and with ABAQUS automatic damping ($c=0.0162$) are very similar, both having failure modes with four plate folds as shown in Figure 3.8. There are no clear trends in this study that establish a relationship between the damping factor $c$ and the peak load response and failure modes of the plate.

![Artificial damping solution showing four fold lines at ultimate limit state](image)

**Figure 3.8 Failure mode for artificial damping solution without damping and with $c=0.0162$**

### 3.1.5 Conclusions

ABAQUS provides two nonlinear solution methods, *STATIC, RIKS* and *STATIC, STABILIZE*, that have the ability to solve potentially unstable structural problems. *STATIC, RIKS* is an arc length method which uses a Newton-Raphson convergence algorithm in conjunction with an automatic step selection procedure. *STATIC, STABILIZE* couples Newton-Raphson with artificial mass proportional damping to stabilize areas of the model experiencing large changes in deformation between load steps. The following conclusions are drawn from this study:
• Both artificial damping and modified Riks methods were able to successfully converge on valid equilibrium paths, including the determination of peak load and the post-peak characteristics of the plate.

• The peak load $P_u$ was consistently predicted by both artificial damping and the modified Riks methods, suggesting that a global maximum was correctly determined.

• The typical failure mechanism of the plate was at least two sharp transverse fold lines occurring at the crests of the buckled half-wave imperfections along the plate.

• The post-peak response and failure modes predicted using the modified Riks method were sensitive to the maximum load increment step size $\Delta \lambda_{\text{max}}$. This sensitivity is attributed to the periodic geometric imperfections applied to the plate which produces many possible equilibrium paths. Discrete non-periodic imperfections and larger magnitude periodic imperfections are expected to reduce this sensitivity. Evaluating the sensitivity of a particular problem to $\Delta \lambda_{\text{max}}$ is recommended when the post-peak response is deemed important.

• The artificial damping post-peak responses and failure modes were sensitive to the magnitude of the damping factor $c$. There were no clear trends relating $c$ and the failure modes, although it is noted that the solution with $c=0.0162$ produced a similar load-displacement curve to the solutions without damping. Evaluating the sensitivity of a particular
problem to $c$ is recommended when the post-peak response is important to the solution.

3.2 Ultimate strength of a rectangular plate with and without a slotted hole

3.2.1 Introduction

Section 3.1 presents the preliminary experiences gained using ABAQUS nonlinear solution methods to determine the ultimate strength of rectangular plates. It was observed that equilibrium paths and failure modes can be sensitive to solution controls, although the peak resisting load of the plate was consistently predicted. With this information in mind, the nonlinear solution of a rectangular plate with and without a slotted hole is attempted with the *STATIC, RIKS and *STATIC, STABILIZE solution methods available in ABAQUS. The first step in this study is to determine a set of solutions controls (step size, damping factor, convergence limits) that is capable of capturing the full load-displacement response of the plate. In this study, the algorithms employed in ABAQUS struggled to predict a post-peak response for the plate with the hole, and both successes and failures are documented. Once an ABAQUS solution control set is discovered that produces a complete load-displacement curve, the predicted ultimate strength and post-peak response of the plate with and without a hole are compared. The sensitivity of the load-displacement response and ultimate strength to imperfection magnitude is also evaluated.
### 3.2.2 Problem Description

The selected problem is a simply supported rectangular plate with dimensions, material properties, and loading conditions identical to the plate described in Section 3.1.2, although in this study a plate with a single slotted hole placed at the midlength of the plate and centered between the unloaded edges is also considered. The slotted hole has dimensions of $h_{\text{hole}}=38.1$ mm, $L_{\text{hole}}=101.6$ mm, and $R_{\text{hole}}=19.1$ mm, where the hole dimension nomenclature is summarized in Figure 2.1(b).

The boundary conditions for the plate were initially assumed to be simply supported with both the transverse and longitudinal plate midlines restrained and the loaded edge nodes coupled with constraint equations as described in Figure 3.2. It was later determined that the constraints used to enforce uniform displacements in 1 ($u$) at the loaded edges and the transverse midline restraint in 2 ($w=0$) were potentially causing solution convergence problems, and therefore the boundary conditions were revised to those shown in Figure 3.9.
Since it is known that peak load and failure mode of the plate are sensitive to the initial geometric imperfections, it was important to have the same imperfection shape when comparing the load-displacement responses of the plate with and without a hole. The deformation field for the plate without the hole is the fundamental buckling mode of the plate. This same mode shape is imposed on the plate with the slotted hole by mapping the deformation field to the plate’s nodal coordinates using custom MATLAB code (Mathworks 2006). A comparison of the geometric imperfections assumed for the plate with and without the hole is provided in Figure 3.10. $d_1/t=0.34$ is used to scale the initial imperfection field of the plate. This magnitude corresponds to a probability of occurrence of $P(\Delta<d_1)=0.50$, as discussed in Section 3.1.2 of this report.
3.2.3 Determination of ABAQUS Nonlinear Solution Controls

Local plastic instabilities such as those expected in the plate with the slotted hole have been known to cause convergence problems for the modified Riks method and the standard Newton Raphson algorithms (ABAQUS 2004). ABAQUS provides several solution and convergence controls that can be modified by the user, although it is difficult to determine which combination of parameters are best suited for the problem at hand. Six exploratory ABAQUS models were performed with various combinations of solution controls and boundary conditions. The combinations are intended to determine what controls can help achieve a reasonable solution to this cumbersome nonlinear problem.
Models RIKS1 and RIKS2 employ the modified Riks method to solve the plate problem. The plate is loaded with a uniformly distributed load at both ends as described in Figure 3.1(a). The boundary conditions for the plate are summarized in Figure 3.2. The initial load step and the maximum load step magnitudes are defined for RIKS1 based on experience from the rectangular plate study in Section 3.1.3. RIKS2 allows ABAQUS to select all load stepping parameters automatically.
The load-displacement response from the RIKS1 and RIKS2 models is plotted in Figure 3.11. It is observed that for both models, ABAQUS is not able to capture the peak load and reverts back along the equilibrium path until the plate is loaded to failure in tension! The ABAQUS Theory Manual states that this type of direction switch is possible when the equilibrium path exhibits very high curvature (ABAQUS 2004). The ABAQUS message files (.msg) for these models report that the moment residuals are too high at the loaded edge nodes and at nodes along the transverse midline of the plate, suggesting that these boundary conditions are contributing to the convergence difficulties of the solution.

The next two models, STATIC1 and STATIC2, employ the Newton Raphson algorithm in displacement control. The boundary conditions for these models are described in Figure 3.2. The time stepping parameters are chosen to ensure at least 100 increments are achieved before failure. Also the number of convergence criteria iterations is modified by doubling the ABAQUS parameters \( I_o, I_r, \) and \( I_c \) over their default values. \( I_o \) represents the number of equilibrium iterations before a check is performed to ensure that moment and force residuals are decreasing. After \( I_o \) iterations, if residuals are not decreasing between two consecutive equilibrium iterations then the length of the increment time step is reduced and the equilibrium search is restarted. \( I_r \) represents the number of equilibrium iterations after which the logarithmic rate of convergence check
begins. $I_c$ represents the maximum number of equilibrium iterations within a
time increment step.

A line search algorithm is available in ABAQUS to improve the
convergence of the Newton Raphson algorithm when nodal force and moment
residuals are large. The line search finds the optimum length of the solution
correction vector calculated by the pure Newton Raphson method that
minimizes the residual vector (ABAQUS 2004). This line search tool is turned on
for the STATIC2 run, while the STATIC1 uses the pure Newton Raphson method
to find the load-displacement response.

![Graph showing STATIC1 and STATIC2 load-displacement curves.]

Figure 3.12  STATIC1 and STATIC2 load-displacement curves demonstrate convergence difficulties
near the peak load. The line search algorithm in STATIC2 is able to work past the peak
load before terminating.
The STATIC1 and STATIC2 load-displacement curves are plotted in Figure 3.12. The STATIC1 model finds the peak load but then terminates due to moment residual convergence issues as it attempts to predict the first step of the post-peak response. The ABAQUS message file states that the moment residuals at nodes along the loaded edges, along the transverse midline of the plate, and some nodes at the hole are increasing and convergence is judged unlikely. The solution is terminated after the automatic time stepping procedure requires a smaller time step than the minimum set in this model (1x10^-20). The STATIC2 model with the line search algorithm also finds the peak load of the plate and is able to track onto the vertical portion of the post-peak equilibrium for one iteration before terminating from the same convergence problems experienced by the STATIC1 model. The fact that the STATIC2 model was able to determine equilibrium in the highly nonlinear post-peak range suggests that the line search algorithm is a solution control with potential for solving these types of problems.

The final two models, STAB1 and STAB2, attempt to solve the plate problem using a displacement control Newton Raphson algorithm coupled with automatic artificial damping discussed in Section 3.1.4. The boundary conditions are modified to those summarized in Figure 3.9 because of the convergence issues observed with the constraint equations and transverse midline restraints. As in the case of STATIC1 and STATIC2, the convergence iteration limits \( I_o, I_r, \) and \( I_c \) are doubled from their default values. In an attempt to alleviate the moment residual convergence issues from previous runs, the Newton Raphson
parameter $R_{\alpha}^n$ is modified to loosen the residual requirements when the solution approaches the peak load. $R_{\alpha}^n$ is the allowable limit on the ratio of the largest residual force or moment at a node ($r_{\alpha,max}^\alpha$) to the largest change in force or moment at a node averaged over each time step increment that has been completed ($q^\alpha$). The $\alpha$ superscript indicates that $R_{\alpha}^n$ can be defined for either a displacement field $u$ or a rotation field $\Phi$. The convergence limit can be written mathematically as:

$$r_{\alpha, max}^\alpha \leq R_{\alpha}^\alpha q^\alpha$$

The default for $R_{\alpha}^n$ of 0.005 is used in STAB1 for both $u$ and $\Phi$ fields, whereas in STAB2 $R_u^n=0.005$ and $R_{\Phi}^n=0.100$ (significantly higher than the default).

![Graph](image)

Figure 3.13 STAB1 and STAB2 load-displacement curves demonstrate a highly nonlinear post-peak equilibrium path
The ABAQUS solutions from models STAB1 and STAB2 in Figure 3.13 demonstrate a highly nonlinear post-peak equilibrium path. Both models are able to successfully predict the peak load and then move vertically down until a secondary load path with its own peak is determined. The solution terminates because the maximum number of Newton Raphson iterations is reached. The prediction of a sudden loss in capacity (vertical drop) and the second peak load is a major success, although future work is required to better understand the influence of the artificial damping on the accuracy of the solution. It is also noted that the modification of the moment residual limit $R^\Phi_n$ from 0.005 to 0.100 did not significantly influence the solution.

Figure 3.14  Summary of nonlinear finite element results and a comparison of load paths near the peak load
A summary comparison of the six ABAQUS models, including a close up of the peak load region of the load-displacement curve, is presented in Figure 3.14. All of the solutions predict the same equilibrium path in the elastic regime of the plate. The STATIC2 model with the line search algorithm is able to follow the path for one iteration after the peak load, while the STAB1 and STAB2 models move past the peak and onto a secondary load path with its own defined peak.

3.2.4 Conclusions

This study evaluated the modified Riks and artificial damping methods used for calculating the ultimate strength of a rectangular plate with a slotted hole. Six exploratory finite element models were assembled, with different solution controls for each. Based on the results obtained in this study, nonlinear finite element modeling of the plates and members considered from this point forward in the report will be conducted using the solution controls of the STAB2 model. There are still many questions, though, regarding the nonlinear solution methods considered here. Conclusions, observations, and future work for each method are summarized below.

Modified Riks Method

The modified Riks method models (RIKS1 and RIKS2) struggled to converge as they approached the peak load of the plate. A possible cause of the convergence problems was the constraint equation employed to couple the
loaded edge nodes for longitudinal displacements. Future modeling will be performed to observe if modifying the boundary conditions to a rigid body formulation in ABAQUS (Figure 3.9) improves the performance of the Riks method. Also, an imposed displacement will be implemented instead of a uniform load to evaluate if convergence issues can be improved.

**Newton Raphson**

The models employing the Newton Raphson algorithm (STATIC1 and STATIC2) in displacement control experienced convergence problems near the peak load of the plate, possibly due to the same constraint equation boundary condition issues observed for RIKS1 and RIKS2. STATIC2, which employed a line search algorithm to minimize the force and moment residuals, was able to move past the peak load before terminating due to convergence limits. Future work will be conducted to explore the abilities of this line search algorithm with the more favorable boundary condition formulations used in the STAB1 and STAB2.

**Newton Raphson with Artificial Mass Proportional Damping**

The STAB1 and STAB2 models successfully obtained the peak load of the plate and were able to track along a stable post-peak equilibrium path. Uncertainties regarding the choice of damping factor discussed in Section 3.1.4 must still be resolved. Future work is planned to compare the displacement control results presented in this study with a model employing load control and artificial damping.
3.2.5 Influence of a slotted hole on the ultimate strength of a rectangular simply supported plate

This study attempts to establish some preliminary connections between the elastic buckling and ultimate strength of a simply supported rectangular plate with a slotted hole. Quantifying this connection between elastic buckling and ultimate strength is a key requirement to extending the Direct Strength Method to members with holes. This study will be conducted using the cumulative knowledge of ABAQUS eigenbuckling analysis gained from Progress Report #1 and of ABAQUS nonlinear solution methods from Section 3.1 and Section 3.2.3 of this report.

The ultimate strength and failure mode of a simply supported rectangular plate with and without a slotted hole will be predicted with a nonlinear finite element model in ABAQUS. An elastic eigenbuckling analysis of the plate with and without a slotted hole is also performed. The loading and boundary conditions, dimensions, material properties, and solution controls are the same as those used for the STAB2 model discussed in Section 3.2.3 and Table 3.1. Initial geometric imperfections are not considered.
Figure 3.15 presents the elastic buckling and ultimate strength results for the plate with and without the slotted hole. The slotted hole significantly reduces the strength of the plate, from 1.0 P_{y,g} to 0.58 P_{y,g}. It is interesting to note that the strength of the plate with the hole correlates well with the load associated with yield of the net section, 0.56 P_{y,g}. 

Figure 3.15 Comparison of ultimate limit state and elastic buckling plate behavior, initial imperfections are not considered in these results
Figure 3.16 The failure mode shape suggests that the elastic buckling Mode 2 is influential here.

The elastic buckling modes and failure modes of the plates are shown together for comparison in Figure 3.16. The failure mode of the plate with the slotted hole includes characteristics of the second elastic buckling mode predicted by ABAQUS. This elastic buckling mode occurs in eight half-waves and is consistent with the fundamental mode of the plate without the hole. The failure mode of the plate occurs without any initial geometric imperfections imposed, suggesting that in this case the second elastic buckling mode shape is related to the failure of the plate.
3.2.6 Influence of geometric imperfection magnitudes on the ultimate strength of a simply supported rectangular plate with and without a slotted hole

The ultimate strength of cold-formed steel members has been shown to be sensitive to initial geometric imperfections in the shape of elastic buckling mode shapes. In this study, the influence of imperfection magnitude on the ultimate strength of a simply supported rectangular plate with and without a slotted hole is evaluated. The loading and boundary conditions, dimensions, material properties, and solution controls are the same as those used for the STAB2 model discussed in Section 3.2.3 and Table 3.1. The imperfection field is assumed as the fundamental elastic buckling mode of the plate discussed in Section 3.2.2 and pictured in Figure 3.10. The Type 1 imperfection magnitudes considered in this study are summarized in Table 3.2 and are based on a probabilistic treatment proposed by Schafer and Peköz (1998) summarized in Section 3.1.2 of this report.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Imperfection Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P(\Delta&lt;d_i)$</td>
</tr>
<tr>
<td>0.25</td>
<td>0.14</td>
</tr>
<tr>
<td>0.50</td>
<td>0.34</td>
</tr>
<tr>
<td>0.75</td>
<td>0.66</td>
</tr>
<tr>
<td>0.95</td>
<td>1.35</td>
</tr>
<tr>
<td>0.99</td>
<td>3.87</td>
</tr>
</tbody>
</table>

Figure 3.17 and Figure 3.18 present the load-displacement results for the plate without and with the hole and demonstrate that the Type 1 imperfection magnitude has a large influence on both the peak load and the post-peak
response of the plates. The ABAQUS strength results for the plate with and without the hole can be compared to DSM specification predictions if local buckling is assumed to control the failure mode:

for $\lambda_i \leq 0.776$

$$P_{nf} = P_{ne}$$

for $\lambda_i > 0.776$

$$P_{nf} = \left[1 - 0.15 \left(\frac{P_{crf}}{P_{ne}}\right)^{0.4} \left(\frac{P_{crf}}{P_{ne}}\right)^{0.4}\right] P_{ne}$$

Assuming $P_{ne} = P_{y,g}$ and using $P_{crf} = 0.30 P_{y,g}$ from the fundamental buckling mode of the plate with the slotted hole, the ultimate capacity of the plate with the hole is calculated as $P_{nf} = 0.56 P_{y,g}$. If the net section is assumed ($P_{ne} = P_{y,net}$), then $P_{nf} = 0.38 P_{y,g}$. For the plate without the hole, DSM predicts the capacity as $P_{nf} = 0.58 P_{y,g}$ where $P_{cr} = 0.33 P_{y,g}$. 
Figure 3.17  Load-displacement sensitivity to imperfection magnitude for a plate without a hole

Figure 3.18  Load-displacement sensitivity to imperfection magnitude for a plate with a slotted hole
The results for both plates demonstrate a softening of the elastic stiffness with increasing imperfection magnitude which can be observed by comparing the imperfection results to the linear slope of the load-displacement curve without imperfections. This softening of the plate is a result of the initial out-of-plane deformations which engage more of the bending stiffness of the plate and less of the axial stiffness as the plate is compressed. Out-of-plane deformations (such as initial imperfections) increase the magnitude of the geometric stiffness matrix which negates the initial stiffness of the undeformed system. The load-displacement results of the plate also reveal that the magnitude of the in-plane displacement at failure, $\delta_u$, is significantly smaller for plates with holes than without holes and also for smaller imperfections versus larger imperfections. This trend suggests that for this plate and hole size that the hole produces a less ductile failure mode than without a hole. It can also be concluded that imperfections decrease the strength but increases the ductility of the plate with and without a hole.

Figure 3.19 summarizes the influence of imperfections on the ultimate strength of the plate with and without a hole and provides a comparison between ABAQUS results and DSM strength predictions. The DSM strength predictions for the plate with the hole are more consistent with the ABAQUS strength results when the net area of the plate is assumed ($P_{ne} = P_{y, net}$) instead of the gross area ($P_{ne} = P_{y,g}$). While this observation is only for one plate, and the stress distribution at failure is more complicated than either $P_{y,g}$ or $P_{y,net}$ would
predict, the agreement with $P_{y,\text{net}}$ is somewhat different than in Progress Report #1 and is a topic of future study.

![Graph showing influence of imperfection magnitude on the peak load of a plate with and without a slotted hole.]

Figure 3.19 Influence of imperfection magnitude on the peak load of a plate with and without a slotted hole

3.3 **Determination of plate “effective width” using nonlinear finite element modeling**

3.3.1 **Introduction**

The “effective width” method provides an easily implemented approximation to the complex non-uniform stress distribution in a thin buckled plate under compression. Initially presented in the 1930’s by von Karman and extended to cold-formed steel members by Winter in the 1940’s, the method accounts for the reduction in load-carrying capacity at the center of a simply supported plate (von Karman 1932, Winter 1947). The inability of the center of
the plate to carry compressive load is caused by the out-of-plane deformations in the shape of the fundamental elastic buckling mode. These deformations cause a loss in axial stiffness that concentrates the compressive force at the edges of a plate. The ultimate load of the plate is reached when these edge stresses, carried by the “effective width” of the plate, exceed the yield stress of the plate material. The “effective width” concept is the basis for most cold-formed steel design codes around the world today.

3.3.2 Problem Statement

In this study, a nonlinear finite element model is used to calculate the longitudinal stress distribution at failure for a plate with and without a slotted hole. The distribution of stresses for both cases is compared, and the variation in effective width along the plates is determined. The plate with and without the slotted hole are modeled with the same loading and boundary conditions, dimensions, material properties, and solution controls as those used for the STAB2 model discussed in Section 3.2.3 and Table 3.1. The initial imperfection geometry for the plates correspond to the fundamental elastic buckling mode of the plate without the hole as described in Figure 3.10. \( d_l/t = 0.34 \) is used to scale the initial imperfection field of the plate, which corresponds to a probability of occurrence of \( P(\Delta < d_l) = 0.50 \) as discussed in Section 3.1.2 of this report. The effective width for each plate is calculated by first integrating the longitudinal (S11) membrane stresses over several cross sections along the length of the plate.
and then dividing the resulting areas by the yield stress of the steel as shown in Figure 3.20.

![Diagram](image)

**Figure 3.20** Calculation of “effective width” at a cross section along a thin simply supported plate

The membrane stresses are the longitudinal (S11) stresses that occur at the midplane of the plate as defined in Figure 3.21.

![Diagram](image)

**Figure 3.21** Definition of longitudinal (S11) membrane stress
3.3.3 Results

Figure 3.22 (a) highlights the variation in membrane longitudinal stress (S11) occurring at the failure load of a plate. The highest stresses accumulate along the edges of the plate and decrease toward the center of the plate. The largest edge stresses occur at the crests of the half-waves where the grey stress contours indicate yielding of the plate. The corresponding effective width of the plate is presented in Figure 3.22(b). The maximum effective width of 0.51 \( h_e/h \) occurs at the inflection point between half-waves, while the minimum effective width of 0.48 \( h_e/h \) occurs at the wave crests. The predicted effective width for this plate using Section B2.1 of the AISI specifications is 0.50 \( h_e/h \).
The failure mode of the plate with the slotted hole is significantly different than the plate without the hole. The stresses in Figure 3.23(a) demonstrate that yielding occurs only at the location of the hole when the peak load of the plate is reached. Compressive stresses are highest at the edge of the plate and then transition to tensile stresses at the face of the hole. The effective width of the yielded portion of the plate in Figure 3.23(b) is less than that for the plate without the hole, even with the beneficial tensile stresses at the face. The average effective width for this plate is 0.38 \( \frac{h_e}{h} \), which is 25 percent less than that of the plate without the hole. The predicted effective width using Section B2.2 of the
AISI Specifications is $0.30 \frac{h_e}{h}$. The effective widths of the plate with and without a slotted hole are compared together in Figure 3.24.

![Figure 3.24 Effective width comparison for a plate with and without a slotted hole](image)

3.3.4 Variation in Effective Width through the Thickness of a Plate

The longitudinal stresses ($S_{11}$) in the top and bottom of the plate at failure are different from the membrane stresses at the midplane of the plate, suggesting that the effective width of a plate actually varies through its thickness. Figure 3.25 and Figure 3.26 provide a comparison of this variation for a plate with and without a slotted hole. It is observed that a plate is much more effective on the surface of the plate where the out of plane deformation causes compression. The effective width is significantly reduced when tensile and compressive stresses negate each other, as shown in the 2D plot of extreme fiber and membrane stresses at a representative cross section of the plate in Figure 3.27.
Figure 3.25  Through the thickness variation of effective width of a plate without a hole

Figure 3.26  Through the thickness variation of effective width of a plate with a slotted hole
3.3.5 Conclusions

The longitudinal membrane stresses of a plate with and without a slotted hole are calculated at the peak failure load using nonlinear finite element analysis. Using these stresses, the effective width is calculated at transverse sections along the length of the plate and through the thickness. The following conclusions are drawn from this study:

- The failure modes of a plate with and without a slotted hole are significantly different. For the plate without the hole, the failure of the plate is dominated by yielding at the crests of the buckled half-waves
resulting from the concentration of compressive forces at the plate edges. For the plate with the slotted hole, compressive stresses are again highest at the plate edges, although yielding occurs only at the hole.

- The variation in the effective width along the length of the plate without the hole and is influenced by the magnitudes of the out-of-plane deformations. The maximum effective width occurs at the inflection point between buckled half-waves and the minimum width occurs at the crests of the half-waves.

- For the plate with the slotted hole, the effective width at failure is located at the hole and is 25 percent less than that of the plate without the hole.

- The AISI effective width prediction for the plate without the hole is four percent higher than the minimum calculated with the longitudinal membrane stresses in ABAQUS.

- The AISI effective width prediction for the plate with the slotted hole results is 13 percent lower than the minimum calculated with the longitudinal membrane stresses in ABAQUS.

- The effective width of a plate varies through its thickness. Where the out-of-plane deformations cause tension at the extreme fiber of the plate, the effective width of the plate is appreciably reduced. When out-of-plane deformations cause compression, the extreme fiber of the plate is fully effective.
3.4 Preliminary nonlinear finite element modeling of an intermediate length cold-formed steel column

3.4.1 Introduction

The fundamental goal of this research is to develop a general method for predicting the strength of cold-formed steel members with holes. The basis for this approach will be the Direct Strength Method (DSM), which assumes that the elastic buckling behavior of the member is a predictor of its strength. In this study, the behavior of an intermediate length SSMA 362S162-33 with a slotted hole is evaluated as it is loaded to its ultimate limit state using a nonlinear finite element model in ABAQUS. The stress distribution and deformation of the members at failure are evaluated for both fixed and pinned end conditions, and the ultimate strengths of the columns is compared to current AISI DSM predictions. The results and experiences gained from this study will be the foundation for future work that will quantify the relationship between elastic buckling behavior and the ultimate strength of cold-formed steel members with holes.

3.4.2 Problem Statement

Intermediate length SSMA 362S162-33 structural stud columns with and without a slotted hole are considered in this study. The material properties, solution controls, and slotted hole size assumed in the ABAQUS nonlinear finite element model are the same as the STAB2 plate model discussed in Section 3.2.3 and Table 3.1. The length of the member, L=691.5 mm, is chosen to match that of
the rectangular plate considered throughout this report to facilitate comparison of the plate and column results. Also, local, distortional, and global buckling modes are relevant at the chosen length, which allows for a complete evaluation of the elastic buckling influence on the ultimate limit state behavior of the column. Initial geometric imperfections and residual stresses are not considered, although this study’s results will be used as a baseline for future work in evaluating ultimate strength sensitivities to initial conditions. The predicted strengths in this study are not expected to be similar to an average tested strength, but rather represent the baseline response for the numerical model and an upper bound on the member response.

The boundary and loading conditions are modeled to be consistent with future planned experiments on structural stud columns. The nodes at the ends of the column are rigidly connected to a node at the centroid of the cross section. Warping of the column ends is restrained. The columns are loaded in displacement control, with the displacement applied at the centroid node. The centroid node degrees of freedom are defined to mimic two types of boundary conditions, weak-axis pinned and fixed-fixed. The loading and boundary conditions for the two types of models are summarized in Figure 3.28 and Figure 3.29. The elastic buckling modes of the columns are determined with an eigenbuckling analysis in ABAQUS assuming these same boundary conditions, except that a uniform compressive stress is applied to the member end as a
perturbation load. For more details on the application of this perturbation load, see Progress Report #1 or Moen and Schafer (2006).

Figure 3.28 Pinned-pinned end conditions for structural stud column loaded in displacement control

Figure 3.29 Fixed-fixed end conditions for structural stud column loaded in displacement control
3.4.3 Results

3.4.3.1 Elastic Buckling

The local and distortional elastic buckling modes for the pinned column with and without the slotted hole are presented in Figure 3.30. As was observed in Progress Report #1, the addition of a hole creates additional elastic distortional modes L+DH and DH2. The DH+L mode is a mixed mode where distortional buckling near the hole combines with local web and flange buckling. The DH2 mode is an isolated hole mode where antisymmetric buckling of the flanges occurs at the hole.

![Diagram showing elastic buckling modes](image)

**Figure 3.30** Elastic buckling modes calculated for the pinned-pinned structural stud (the fixed-fixed structural stud elastic buckling results are similar)
3.4.3.2 Ultimate Limit State

The load-displacement curves for the pinned and fixed columns in Figure 3.31 demonstrate the column’s sensitivity to the assumed end conditions and to the addition of a slotted hole. The addition of the hole causes an appreciable reduction in the strength for both fixed and pinned columns, 21 percent and 17 percent respectively. The reduction in ultimate strength changing from fixed to pinned end conditions is also considerable, with a 13 percent reduction for the column with the hole and a 17 percent reduction for the column without the hole.

![Figure 3.31 Load-displacement response of SSMA 362S162-33 with fixed-fixed and pinned-pinned end conditions and with a slotted hole, L=691.5 mm](image)

The mechanics behind the strength reductions observed from the addition of a hole become clearer by observing the von Mises membrane stresses at the ultimate load of the column in Figure 3.32. For the columns without the hole, the
yielding pattern is consistent along the length of the member, occurring near the web-flange intersection for the pinned column and across the entire flange, lip stiffener, and a portion of the web in the fixed column. The yield pattern in the web is consistent with the observed local buckling deformation of the column and the isolated web plate results in Figure 3.23.

For the columns with the hole, yielding occurs only in an isolated portion of the web and flange directly adjacent to the hole, while stresses away from the

**Figure 3.32 von Mises membrane stress distribution at the ultimate load of an SSMA 362S162-33 structural stud with and without a slotted hole**
hole are relatively low. This result is also consistent with the isolated web plate study in Figure 3.22. Also note that the column deformations at failure are consistent with the L+DH elastic buckling mode in Figure 3.30, where local buckling of the web combines with distortion of the flanges at the hole. The observed localized yielding associated with the failure of this column with a hole is more similar to yielding associated with distortional buckling, as observed in the flanges of C- or Z-sections, than local buckling. Sensitivity of the failure modes to imperfection distribution and magnitude (as well as residual stresses) remains to be studied and is anticipated to be influential.

3.4.3.3 Comparison of ABAQUS Ultimate Strengths to DSM Predictions

The column ultimate strengths (calculated with ABAQUS without imperfections) are compared to the Direct Strength Method (DSM) predictions in Table 3.3. It should be noted that future work is required to make definitive conclusions regarding DSM and members with holes, and therefore this comparison is made only to identify general trends in the data.

<table>
<thead>
<tr>
<th>End Conditions</th>
<th>Gross or Net Area</th>
<th>Squash</th>
<th>Elastic Buckling</th>
<th>DSM</th>
<th>FEM Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$P_y$</td>
<td>$P_{cre}$</td>
<td>$P_{crf}$</td>
<td>$P_{crf1}$</td>
</tr>
<tr>
<td>pinned</td>
<td>$A_{g}$ no hole</td>
<td>38.3</td>
<td>172.2</td>
<td>16.0</td>
<td>---</td>
</tr>
<tr>
<td>pinned</td>
<td>$A_{g}$ slotted hole</td>
<td>38.3</td>
<td>172.2</td>
<td>16.1</td>
<td>19.9</td>
</tr>
<tr>
<td>pinned</td>
<td>$A_{net}$ slotted hole</td>
<td>30.7</td>
<td>172.2</td>
<td>16.1</td>
<td>19.9</td>
</tr>
<tr>
<td>fixed</td>
<td>$A_{g}$ no hole</td>
<td>38.3</td>
<td>688.9</td>
<td>16.0</td>
<td>---</td>
</tr>
<tr>
<td>fixed</td>
<td>$A_{g}$ slotted hole</td>
<td>38.3</td>
<td>688.9</td>
<td>16.9</td>
<td>17.9</td>
</tr>
<tr>
<td>fixed</td>
<td>$A_{net}$ slotted hole</td>
<td>30.7</td>
<td>688.9</td>
<td>16.9</td>
<td>17.9</td>
</tr>
</tbody>
</table>

All loads in kN
Yield stress=227.5 MPa
The basic premise of the Direct Strength Method is expressed for a column as \( P_n = f(P_{cr}, P_{crd}, P_{cre}, P_y) \) where \( P_n \) is the nominal column strength, \( P_y \) is the axial load at yielding, and \( P_{cr}, P_{crd}, P_{cre} \) are the axial loads at which elastic local, distortional, and global (flexural, torsional, flexural-torsional) buckling occur (NAS 2004 Appendix 1). \( P_{cr}, P_{crd}, P_{cre} \) are obtained from the results described in Section 3.4.3.1 of this report. \( P_{crd} \) is conservatively chosen as the minimum buckling load from the L+DH, DH2, and D+L modes. \( P_y \) is calculated using the gross and net cross sectional area, \( A_g \) and \( A_{net} \) respectively.

Both DSM and ABAQUS predict a reduction in capacity with the addition of a slotted hole to the column, although the strength loss predicted by ABAQUS is much greater than the small reductions observed in the local (L) and distortional (D+L) elastic buckling loads. The DSM strengths for the fixed columns are less than the pinned columns when compared to the ABAQUS ultimate strengths. The use of \( P_{y,g} \) in the DSM equations produces predictions more consistent with the ABAQUS ultimate strengths in this case, although future consideration of residual stresses and geometric imperfections will most likely influence these current trends.

3.4.3.4 Conclusions

The ultimate behavior of an intermediate length SSMA 362S162-33 structural stud with and without a slotted hole is calculated using nonlinear finite element analysis in ABAQUS. The elastic buckling modes of the columns are also
determined in ABAQUS with an eigenbuckling analysis. The influence of column end conditions and elastic buckling modes on the column strength is evaluated and a preliminary comparison of DSM strength predictions to ABAQUS results is performed. The following conclusions are drawn from this study:

- A slotted hole reduces the strength of an intermediate length SSMA 362S162-33 structural stud column when considering both fixed and pinned end conditions.

- The failure mode of a structural stud column with a slotted hole is different than a stud column without a hole. For the columns without the hole, the yielding pattern is consistent along the length of the member, occurring near the web-flange intersection for the pinned column and across the entire flange, lip stiffener, and a portion of the web in the fixed column. For the columns with the hole, yielding occurs only in an isolated portion of the web and flange directly adjacent to the hole, while stresses away from the hole are relatively low.

- Deformations consistent with both local and distortional modes exist at the failure load of the structural stud with a slotted hole.

- The strength loss from the addition of a slotted hole, as predicted by ABAQUS, is much greater than the small reductions observed in the local (L) and distortional (D+L) elastic buckling loads.
• The use of $P_{y,g}$ in the DSM equations produces predictions more consistent with the ABAQUS ultimate strengths in this case, although definite conclusions cannot be drawn until initial conditions such as residual stresses and geometric imperfections have been accounted for in the nonlinear finite element models.
4 Future Work

The experience and knowledge gained from research completed to date suggests that the primary challenge for extending the Direct Strength Method to members with holes is still to establish a general connection between the elastic buckling modes and the failure mode of a member with holes. The elastic distortional modes unique to compression members with holes (DH and DH2) are presenting themselves as key contributors to this connection. Research efforts will continue through nonlinear FEM, laboratory testing, and the use of modal decomposition methods (Generalized Beam Theory). The details of this planned future work are described below.

Elastic Buckling
- Evaluate the influence of holes on the elastic buckling behavior of cold-formed steel beams
- Extend the plate hole spacing study (Section 2.1) to cold-formed steel columns and beams

Nonlinear Finite Element Modeling\Ultimate Strength
- Continue the evaluation of the modified Riks method and artificial damping solution methods in ABAQUS
- Continue to study the impact of holes on the stress distribution, yielding locations, and failure modes of cold-formed steel members.
- Evaluate the influence of different initial geometric imperfection fields and magnitudes on the strength of cold-formed steel members with holes.
- Determine how residual stresses affect the strength of members with holes. Is the effect different on a member without holes?
- Assess the effect of holes on the residual stresses in cold-formed steel members.
- Use modal decomposition methods to isolate the buckling failure modes and relate them back to the elastic buckling modes of the member

Laboratory Testing
- Conduct laboratory testing of intermediate length SSMA structural studs with holes to compare actual specimen failures to elastic buckling modes and failure mechanisms predicted with ABAQUS
References


Moen, C., Schafer, B.W. (2006). “Impact of Holes on the Elastic Buckling of Cold-Formed Steel Columns with Application to the Direct Strength Method”, *Eighteenth International Specialty Conference on Cold-Formed Steel Structures*, Orlando, FL.


Winter, G. (1947) “Strength of thin steel compression flanges.” *Cornell University Engineering Experiment Station Reprint No. 32*.