From: Ben Schafer  
To: File  
On: 31 May 2012, 29 July 2012 updated  
Re: Development of DSM Direct Design Formulas for Beam-Columns (Year 1 Proposed)

Abstract:
This memo provides the notes related to the development of a proposed formulation for Direct Strength Method (DSM) design of beam-columns. The formulation developed herein provides, for the first time, integration of inelastic bending, members with holes, and full consideration of the P-M-M space, i.e. integration of axial (P) demands and bi-axial bending (M-M) demands. The formulation presented herein has not been experimentally or computationally verified, and represents the proposed formulation at the end of year 1 of the DSM beam-columns project.

Consider the P-M-M Space defined in z-x-y and in Cartesian or Spherical coordinates

\[
\begin{align*}
M_1, M_2, P &\rightarrow \\
x &= \frac{M_1}{M_y} \\
y &= \frac{M_2}{M_y} \\
z &= \frac{P}{P_y} \\
\beta &= \sqrt{x^2 + y^2 + z^2} \\
\theta_{MM} &= \tan^{-1}(y / x) \\
\phi_{PM} &= \cos^{-1}(z / \beta) \\
\end{align*}
\]

REQUIRED (DEMAND)
Consider the case of design to illustrate (r is a required action, should include P-\delta and P-\Delta)

\[
M_{r1}, M_{r2}, P_r \rightarrow \\
x_r = \frac{M_{r1}}{M_y}, y_r = \frac{M_{r2}}{M_y}, z_r = \frac{P_r}{P_y} \\
\beta_r = \sqrt{x_r^2 + y_r^2 + z_r^2}, \theta_{MM} = \tan^{-1}(y_r / x_r), \phi_{PM} = \cos^{-1}(z_r / \beta_r)
\]

or in long/explicit form

\[
\beta_r = \sqrt{\left(\frac{M_{r1}}{M_y}\right)^2 + \left(\frac{M_{r2}}{M_y}\right)^2 + \left(\frac{P_r}{P_y}\right)^2}, \theta_{MM} = \tan^{-1}\left(\frac{M_{r2}}{M_{r1}}\frac{M_{y1}}{M_{y2}}\right), \phi_{PM} = \cos^{-1}\left(\frac{P_r}{P_y \beta_r}\right)
\]

CROSS-SECTION STABILITY ANALYSIS
The required actions also induce a state of stress on the cross-section:

\[
M_{r1}, M_{r2}, P_r \rightarrow \\
f_r = \frac{P_r}{A} + \frac{M_{r1}y_{r\alpha}}{I_1} + \frac{M_{r2}x_{r\alpha}}{I_2}
\]
cross-section stability analysis (likely CUFSM) performed on \( f_r \) provides, buckling load factors \((\alpha')s\) for local (L), distortional (D), and global (G) buckling \( \rightarrow \)

\[
\alpha_{crL}, \alpha_{crD}, \alpha_{crG}
\]

these may be resolved back into the P-M-M space simply as

\[
\beta_{crL} = \alpha_{crL} \beta_r, \beta_{crD} = \alpha_{crD} \beta_r, \beta_{crG} = \alpha_{crG} \beta_r
\]

note that \( \beta \) is normalized (no units), also note these are not the same as finding the \( M_{cr} \) and \( P_{cr} \) separately, the whole point is to do the stability analysis on the actual P-M-M combo.

CROSS-SECTION YIELD STRENGTH
also we may use \( f_r \) to determine first yield in the section

\[
\text{if } \max(f_r(x_c, y_c)) > 0 \text{ then } \alpha_{yc} = F_y / \max(f_r(x_c, y_c)) \text{ else } \alpha_{yc} = \infty
\]

\[
\text{if } \min(f_r(x_c, y_c)) < 0 \text{ then } \alpha_{yt} = -F_y / \min(f_r(x_c, y_c)) \text{ else } \alpha_{yt} = \infty
\]

first yield is defined by \( \alpha_y = \min(\alpha_{yc}, \alpha_{yt}) \)

these may be resolved back into the P-M-M space simply as

\[
\beta_y = \alpha, \beta_r, \beta_{yc} = \alpha_{yc} \beta_r, \beta_{yt} = \alpha_{yt} \beta_r
\]

also we may use \( M_{r1}, M_{r2}, P_r \) and scale such that we find the fully plastic solution this is \( \alpha_p \) or in the P-M-M space \( \beta_p \)

note, this is not a simple calculation for full loading and may need further study (can be done by iteration but direct solution is complicated, see other notes)

DESIGN CHECK
We then seek to find the capacity \( \beta_n \) where

\[
\beta_n = \min(\beta_{ng}, \beta_{nl}, \beta_{nb})
\]

\[
\phi \beta_n \geq \beta_r \text{ or } \beta_n \geq \beta_r / \Omega
\]
GLOBAL (scratch work / development)
Column global strength calculation in beam-column format
\(\theta_{MM}=0, \phi_{PM}=0\) (compression)
\[
\beta_{ng} = \frac{p_{ng}}{P_y} = \left(0.658^{\lambda_G}\right)\frac{p}{P_y} = 0.658^{\lambda_G} \text{ for } \lambda_G \leq 1.5 \quad (\lambda_G = \sqrt{\frac{P_y}{P_{crG}} = \sqrt{1}/\beta_G})
\]
\[
\beta_{ng} = \frac{p_{ng}}{P_y} = \left(0.877\right)\frac{p}{P_y} = 0.877\frac{P_{crG}}{P_y} = 0.877\beta_G \text{ for } \lambda_G > 1.5
\]

Beam global strength calculation with additional notations (and including inelastic reserve)
\[
M_{ng} = M_p - (M_p - M_y)\sqrt{\frac{M_y}{M_{cr}} - 0.23}{0.37} \leq M_p \text{ for } M_{cr} > 2.78M_y \text{ or } \lambda_G = \sqrt{\frac{M_y}{M_{cr}} < 0.600}
\]
\[
M_{ng} = \frac{10}{9}M_y \left(1 - \frac{10M_y}{36M_{cr}}\right) \geq 2.78M_y \geq M_{cr} \geq 0.56M_y \text{ or } 0.600 \leq \lambda_G \leq 1.34
\]
\[
M_{ng} = M_{cr} \text{ for } M_{cr} < 0.56M_y \text{ or } \lambda_G > 1.34
\]

Beam global strength calculation in new beam-column format
\(\theta_{MM}=0\) (major-axis bending), \(\phi_{PM}=\pi/2\) (no axial load)
\[
\beta_{ng} = \frac{M_{ng}}{M_y} = \frac{M_p}{M_y} - \left(M_p - M_y\right)\sqrt{\frac{M_y}{M_{cr}} - 0.23}{0.37} \leq \frac{M_p}{M_y} \text{ for } \lambda_G = \sqrt{\frac{M_y}{M_{cr}} < 0.600}
\]
which simplifies to
\[
\beta_{ng} = \beta_p - \left(\beta_p - 1\right)\frac{\lambda_G - 0.23}{0.37} \leq \beta_p \text{ for } \lambda_G < 0.600
\]
\[
\beta_{ng} = \frac{M_{ng}}{M_y} = \frac{M_{cr}}{M_y} = \beta_G \text{ for } \lambda_G > 1.34
\]

Now combine the two to provide a full beam-column formulation using mixing with sin
\[
\frac{p_{ng}}{P_y} = 0.658^{\lambda_G} + (\beta_p - 0.658^{\lambda_G})\sin \phi_{PM} \text{ for } \lambda_G \leq 0.23
\]
\[
\beta_{ng} = 0.658^{\lambda_G} + (\beta_p - 1)\frac{\lambda_G - 0.23}{0.37} \cdot 0.658^{\lambda_G} \sin \phi_{PM} \text{ for } 0.23 < \lambda_G \leq 0.600
\]
\[
\beta_{ng} = 0.658^{\lambda_G} + \left(10\frac{10}{36\beta_G}\right) \cdot 0.658^{\lambda_G} \sin \phi_{PM} \text{ for } 0.600 \leq \lambda_G \leq 1.34
\]
\[
\beta_{ng} = 0.658^{\lambda_G} + (\beta_p - 0.658^{\lambda_G})\sin \phi_{PM} \text{ for } 1.34 < \lambda_G \leq 1.50
\]
\[
\beta_{ng} = 0.877\beta_{cr} + (\beta_p - 0.877\beta_{cr})\sin \phi_{PM} \text{ for } 1.5 < \lambda_G
\]
pretty similar to an interaction equation approach except \(\lambda_G\) and \(\beta_G\) are novel and the use of the sine expression for the interaction mixing is convenient and novel.
GLOBAL (proposed)

\[ \beta_{nG} = \beta_{nP} + (\beta_{nGM} - \beta_{nP}) \sin \phi_{PM} \]

Compression: \( 0 \leq \phi_{PM} < \pi / 2 \)
   for \( \lambda_G \leq 1.5 \)
   \[ \beta_{nP} = 0.658 \lambda_G^{1.5} \beta_y \]
   for \( \lambda_G > 1.5 \)
   \[ \beta_{nP} = 0.877 \beta_{crG} \]

Tension: \( \pi / 2 < \phi_{PM} \leq \pi \)
   \[ \beta_{nP} = \beta_y \]

Bending: \( 0 < \phi_{PM} < \pi \), \( 0 \leq \theta_{MM} \leq 2\pi \)
   for \( \lambda_G < 0.60 \)
   considering inelastic reserve capacity
   for \( \lambda_G \leq 0.23 \)
   \[ \beta_{nGM} = \beta_p \]
   for \( 0.23 < \lambda_G < 0.60 \)
   \[ \beta_{nGM} = \beta_p - (\beta_p - \beta_y) \frac{\lambda_G - 0.23}{0.37} \]
   ignoring inelastic reserve capacity
   \[ \beta_{nGM} = \beta_y \]
   for \( 0.60 \leq \lambda_G \leq 1.34 \)
   \[ \beta_{nGM} = \frac{10}{9} \beta_y \left( 1 - \frac{10}{36 \beta_{crG}} \right) \]
   for \( \lambda_G > 1.34 \)
   \[ \beta_{nGM} = \beta_{crG} \]

where: \( \lambda_G = \sqrt{\frac{\beta_y}{\beta_G}} \)

For member with holes \( \beta_{crG} \) should reflect the presence of the holes.
LOCAL (scratch work)
Column local strength calculation in new beam-column format
\( \theta_{\text{MM}}=\pi \), \( \phi_{PM}=0 \) (compression)

\[
\beta_{ul} = \frac{P_{ul}}{P_y} = \frac{P_{nG}}{P_y} = \beta_{nG} \quad \text{for} \quad \lambda_L \leq 0.776 \quad \text{and} \quad \lambda_L = \sqrt{\frac{P_{nG}}{P_{crL}}} = \sqrt{\beta_{nG}}
\]

\[
\beta_{ul} = \frac{P_{ul}}{P_y} = \left[ 1 - 0.15 \left( \frac{P_{crL}}{P_{nG}} \right)^{0.4} \right] \left( \frac{P_{crL}}{P_{nG}} \right)^{0.4} \frac{P_{nG}}{P_y} = \left[ 1 - 0.15 \left( \frac{\beta_{L}}{\beta_{nG}} \right)^{0.4} \right] \left( \frac{\beta_{L}}{\beta_{nG}} \right)^{0.4} \beta_{nG} \quad \text{for} \quad \lambda_L > 0.776
\]

Beam local strength calculation in new beam-column format
\( \theta_{\text{MM}}=\pi \) (major-axis bending), \( \phi_{PM}=\pi/2 \) (no axial load)

\[
\beta_{ul} = \frac{M_{ul}}{M_y} = \frac{M_{nG}}{M_y} = \beta_{nG} \quad \text{for} \quad \lambda_L \leq 0.776 \quad \text{and} \quad \lambda_L = \sqrt{\frac{M_{nG}}{M_{crL}}} = \sqrt{\beta_{nG}}
\]

\[
\beta_{ul} = \frac{M_{ul}}{M_y} = \left[ 1 - 0.15 \left( \frac{M_{crL}}{M_{nG}} \right)^{0.4} \right] \left( \frac{M_{crL}}{M_{nG}} \right)^{0.4} \frac{M_{nG}}{P_y} = \left[ 1 - 0.15 \left( \frac{\beta_{L}}{\beta_{nG}} \right)^{0.4} \right] \left( \frac{\beta_{L}}{\beta_{nG}} \right)^{0.4} \beta_{nG} \quad \text{for} \quad \lambda_L > 0.776
\]

Beam inelastic reserve
\( \theta_{\text{MM}}=0 \) (major-axis bending), \( \phi_{PM}=\pi/2 \) (no axial load)

For \( M_{nG} \geq M_y \) and \( \lambda_L \leq 0.776 \) where \( \lambda_L = \sqrt{\frac{M_y}{M_{crL}}} = \frac{1}{\sqrt{\beta_L}} \)

Symm. or first yield in compression: \( M_{ul} = M_y + \left( 1 - 1/C_{yL} \right) (M_p - M_y) \)

First yield in tension: \( M_{ul} = M_{yc} + \left( 1 - 1/C_{yL} \right) (M_p - M_{yc}) \leq M_{y3} \)

Parameters: \( C_{yL} = \sqrt{0.776/\lambda} \leq 3, \ M_{y3} = M_y + \left( 1 - 1/C_{yL} \right) (M_p - M_y), \ C_{yL} = 3 \)

Now combined for a full formulation
Here in inelastic reserve we use only the bending formulation

for \( \lambda_L \leq 0.776 \) \& \( \beta_{nG} \geq \beta_y \) comp y: \( \beta_{ul} = 1 + \left( 1 - 1/C_{yL}^2 \right) (\beta_p - 1) \) and \( \lambda_L = \sqrt{\frac{1}{\beta_L}} \)

for \( \lambda_L \leq 0.776 \) \& \( \beta_{nG} \geq \beta_y \) tens y: \( \beta_{ul} = \beta_{yc} + \left( 1 - 1/C_{yL}^2 \right) (\beta_p - \beta_{yc}) \) and \( \lambda_L = \sqrt{\frac{1}{\beta_L}} \)

for \( \lambda_L \leq 0.776 \) and \( \beta_{nG} < \beta_y \) then \( \beta_{ul} = \beta_{nG} \) and \( \lambda_L = \sqrt{\frac{\beta_{nG}}{\beta_L}} \)

\[
\beta_{ul} = \left[ 1 - 0.15 \left( \frac{\beta_{L}}{\beta_{nG}} \right)^{0.4} \right] \left( \frac{\beta_{L}}{\beta_{nG}} \right)^{0.4} \beta_{nG} \quad \text{for} \quad \lambda_L > 0.776 \quad \text{and} \quad \lambda_L = \sqrt{\frac{\beta_{nG}}{\beta_L}}
\]

To deal with holes we just need to limit to the net section \( \beta_{ul} \leq \beta_{\text{ynet}} \) need to think a little bit more on the beta_net.. but the idea is sound.
LOCAL (proposed)

for \( \lambda_L \leq 0.776 \)

\[ \text{and } \beta_{ng} > \beta_y \]

considering inelastic reserve capacity

symm. or first yield in comp.: \( \beta_{nl} = \beta_y + \left( 1 - 1/C_{yl}^2 \right) (\beta_p - \beta_y) \)

and \( C_{yl} = \sqrt{0.776 / \lambda_L} \leq 3 \)

first yield in tension: \( \beta_{nl} = \beta_{y2c} + \left( 1 - 1/C_{yl}^2 \right) (\beta_p - \beta_{y2c}) \leq \beta_{y3y} \)

\( \beta_{y3y} = \beta_y + \left( 1 - 1/C_{yt}^2 \right) (\beta_p - \beta_y) \), \( C_{yt} = 3 \)

\( \beta_{y2c} = \) when yielding reaches compression after tensile yielding. Conservatively may be taken as \( \beta_y \)

ignoring inelastic reserve capacity

\( \beta_{nl} = \beta_y \)

\[ \text{and } \beta_{ng} \leq \beta_y \]

\( \beta_{nl} = \beta_{ng} \)

for \( \lambda_L > 0.776 \)

\( \beta_{nl} = \left[ 1 - 0.15 \left( \frac{\beta_{crL}}{\beta_{ng}} \right)^{0.4} \left( \frac{\beta_{crL}}{\beta_{nl}} \right)^{0.4} \right] \beta_{ng} \)

where, for \( \beta_{ng} \leq \beta_y \) \( \lambda_L = \sqrt{\frac{\beta_{ng}}{\beta_{crL}}} \) and for \( \beta_{ng} > \beta_y \) \( \lambda_L = \sqrt{\frac{\beta_y}{\beta_{crL}}} \)

For members with holes \( \beta_{nl} \leq \beta_{ynet} \)
**DISTORTIONAL** (scratch work)

No Holes

\[ P_{nd} = \left( 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \left( \frac{P_{crd}}{P_y} \right)^{0.6} P_y \] for \( \lambda_D = \sqrt{\frac{P_y}{P_{crd}}} > 0.561 \)

\[ M_{nd} = \left( 1 - 0.22 \left( \frac{M_{crd}}{M_y} \right)^{0.5} \right) \left( \frac{M_{crd}}{M_y} \right)^{0.5} M_y \] for \( \lambda_D = \sqrt{\frac{M_y}{M_{crd}}} > 0.673 \)

generalize the expression:

\[ 0.25 + (0.22 - 0.25)\sin \phi_{PM} , 0.6 + (0.5 - 0.6)\sin \phi_{PM} , 0.561 + (0.673 - 0.561)\sin \phi_{PM} \]

simplifies to

\[ 0.25 - 0.03\sin \phi_{PM} , 0.6 - 0.1\sin \phi_{PM} , 0.561 + 0.112\sin \phi_{PM} \]

\[ \beta_{nd} = \left( 1 - c_1 \beta_D^{c_2} \right) \beta_D^{c_2} , c_1 = 0.25 - 0.03\sin \phi_{PM} , c_2 = 0.6 - 0.1\sin \phi_{PM} \] for \( \lambda_D > 0.561 + 0.112\sin \phi_{PM} \)

\[ \beta_{nd} = 1 \] for \( \lambda_D \leq 0.561 + 0.112\sin \phi_{PM} \)

need to be careful about \( \beta_y \) initial (to anchors) and potential other \( \beta_y \) at a given angle in PMM space.. this is corrected below…

Need to be careful with tension and biaxial bending too…
DISTORTIONAL (proposed)
for \( \lambda_D \leq 0.561 + 0.112 \sin \phi_{PM} \)
considering inelastic reserve capacity
symm. or first yield in comp.: \( \beta_{nD} = \beta_y + (1 - 1/C_{yD}^2) \left( \beta_p - \beta_y \right) \)
and \( C_{yD} = \sqrt{0.561 + 0.112 \sin \phi_{PM} / \lambda_D} \leq 3 \)
first yield in tension: \( \beta_{nD} = \beta_{y2c} + (1 - 1/C_{yD}^2) \left( \beta_p - \beta_{y2c} \right) \leq \beta_{y3} \)
\( \beta_{y3} = \beta_y + (1 - 1/C_{y2}^2) \left( \beta_p - \beta_y \right), \ C_{y2} = 3 \)
\( \beta_{y2c} \) when yielding reaches compression after tensile yielding. Conservatively may be taken as \( \beta_y \)
ignoring inelastic reserve capacity
\( \beta_{nD} = \beta_y \)
for \( \lambda_D > 0.561 + 0.112 \sin \phi_{PM} \)
\( \beta_{nD} = \left( 1 - c_1 \left( \frac{\beta_{yD}}{\beta_y} \right)^{c_2} \left( \frac{\beta_{yD}}{\beta_y} \right)^{c_2} \right) \beta_y \)
and \( c_1 = 0.25 - 0.03 \sin \phi_{PM}, \ c_2 = 0.6 - 0.1 \sin \phi_{PM} \)
where \( \lambda_D = \sqrt{\frac{\beta_y}{\beta_D}} \)

For members with holes
for \( \lambda_D \leq \lambda_{d1} \)
\( \beta_{nD} = \beta_{yD} \)
for \( \lambda_{d1} < \lambda_D \leq \lambda_{d2} \)
\( \beta_{nD} = \beta_{yD} - \left( \frac{\beta_{yD} - \beta_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) (\lambda_D - \lambda_{d1}) \)
for \( \lambda_D > \lambda_{d2} \)
\( \beta_{nD} = \) per previous section, no reduction due to hole

where
\( \lambda_{d1} = \lambda_{d1P} + (\lambda_{d1M} - \lambda_{d1P}) \sin \phi_{PM} \)
\( \lambda_{d1P} = 0.561 \beta_{yD} \)
\( \lambda_{d1M} = 0.673 (\beta_{yD})^3 \)
\( \lambda_{d2} = \lambda_{d2P} + (\lambda_{d2M} - \lambda_{d2P}) \sin \phi_{PM} \)
\( \lambda_{d2P} = 0.561 \left( 14 (\beta_{yD})^{0.4} - 13 \right) \)
\( \lambda_{d2M} = 0.673 \left( 1.7 (\beta_{yD})^{2.7} - 0.7 \right) \)
\( \beta_{d2} = \left( 1 - c_1 \left( 1 / \frac{\lambda_{d2}^2}{\lambda_{d2}} \right) \right) \left( 1 / \lambda_{d2} \right) \beta_y \)
\[ c_1 = 0.25 - 0.03 \sin \phi_{PM}, \quad c_2 = 0.6 - 0.1 \sin \phi_{PM} \]

RESISTANCE AND/OR SAFETY FACTOR
\[
\phi = \phi_P + (\phi_M - \phi_P) \sin \phi_{PM} \\
\phi_M = \phi_{M1} + (\phi_{M2} - \phi_{M1}) |\sin \theta_{MM}| \\
\Omega = \Omega_P + (\Omega_M - \Omega_P) \sin \phi_{PM} \\
\Omega_M = \Omega_{M1} + (\Omega_{M2} - \Omega_{M1}) |\sin \theta_{MM}| 
\]