INELASTIC BENDING CAPACITY IN
COLD-FORMED STEEL MEMBERS

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Acknowledgments

• National Science Foundation
• Cheng Yu
• American Iron and Steel Institute
• American Institute of Steel Construction
Overview

• Background and motivation on inelastic bending capacity in cold-formed steel (CFS) members
• Mechanisms for inelastic reserve
• Observed inelastic reserve in CFS tests
• Recent testing on CFS beams & summary of test results
• FE modeling
  – Modeling assumptions
  – Material models
  – Parametric FE study
• Design Methods
  – Strength as a function of strain
  – Strain as a function of slenderness
  – DSM style inelastic strength prediction: Strength as a function of slenderness
• Future work
• Conclusions
Inelastic reserve

$M_y < M < M_p$

hot-rolled

Limit $b_f/2t_f$ (FLB)
Limit $h/t_w$ (WLB)
$M \rightarrow M_p$

cold-formed

C and Z sections
$M \rightarrow M_y$
Hats
(stiffened element compression flange)
$M$ can be $> M_y$
Motivation

“Stub-column interaction diagram”

- $P_n$
- $P_y$
- $M_n$
- $M_p$

**hot-rolled steel**

**cold-formed steel**

including local, distortional, and LTB via Direct Strength Method

(DSM for hot-rolled steel?)

DSM including Inelastic reserve?
Elementary mechanics:
mechanism 1 for inelastic reserve

\[ \varepsilon_{\text{max}} = \varepsilon_y, \ C_y = 1 \]
\[ \varepsilon_{\text{max}} = 2\varepsilon_y, \ C_y = 2 \]
\[ \varepsilon_{\text{max}} = 3\varepsilon_y, \ C_y = 3 \]
\[ \varepsilon_{\text{max}} = \infty, \ C_y = \infty \]

(a) \[ M = M_y \]
(b) \[ M_y < M < M_p \]
(c) \[ M = M_p \]

symmetric section
Elementary mechanics:
mechanism 2 for inelastic reserve

\[
\begin{align*}
C_y = 1/3 & \quad \text{or} & \quad M = M_y \\
C_y < 1 & \quad \text{or} & \quad M_y < M < M_p \\
C_y = 1.2 & \quad \text{or} & \quad f_y \\
C_y = \infty & \quad \text{or} & \quad f_y
\end{align*}
\]

first yield in tension
Observed inelastic reserve in CFS tests

<table>
<thead>
<tr>
<th>Section and Researcher</th>
<th>count</th>
<th>$M_{\text{test}} &gt; 0.95M_Y$</th>
<th>$M_{\text{test}}/M_Y$</th>
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<td><strong>Hats and Deck Sections</strong></td>
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shape factor
Back-calculated strain at failure

- $C_y$ (compression)
- $\epsilon_{\text{max-compression}} / \epsilon_y$

- $C_y$ (tension)
- $\epsilon_{\text{max-tension}} / \epsilon_y$

- $\epsilon_{\text{max}}$

- Yielding
- First yield in tension
- Comp.
- Inelastic buckling
- No inelastic reserve
Recent testing on CFS beams

- Cold-formed steel beams may buckle instead of exhibiting inelastic reserve.
- Recent testing has investigated local and distortional buckling as separate limit states on typically used CFS beams, even in the inelastic regime.

Yu and Schafer (2003, 2006, 2007)
buckling modes in a thin-walled beam

- local buckling
- distortional buckling
- lateral-torsional buckling
Local buckling tests

25 C and Z beam pairs

Standard panel fastener configuration, test and FE showed distortional buckling failures even though panel stiffness should be enough to restrict DB

Yu and Schafer (2003)
Local buckling tests

Total 25 C and Z beams

Panel fastener configuration to insure panel is engaged and distortional buckling restricted

Yu and Schafer (2003)
Distortional buckling tests

Total 24 C and Z beams

Distortional mode shape

Yu and Schafer (2006)
Typical test results

Test 8.5Z092

Local buckling test
99% of NAS*

Distortional buckling test
83% of NAS*

*North American Specification for the Design of Cold-Formed Steel Structural Shapes
2007 version adopted new rules to account for distortional buckling in beams
Summary test results

\[ M_{test} = M_{y} \]

inelastic reserve

\[ \frac{M_{test}}{M_{y}} = (\frac{M_{y}}{M_{cr}})^{0.5} \]

- DSM-Local
- DSM-Distortional
- Local buckling tests
- Distortional buckling tests
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Finite element modeling

• **Objective**
  To study inelastic reserve in typical (symmetric) CFS beams for local and distortional limit states.

• **Method**
  – Parametric study motivated from testing of Yu and Schafer (2003, 2006)
  – Material and geometric nonlinear analysis in ABAQUS using S9R5 shell element models
  – Models generated from purpose-built Matlab code (performs elementary mechanics, $C_y$, FSM and FEM models)
Boundary conditions
(distortional buckling)

ends are constrained to act rigidly about tie point rotation (moment) enforced at ends

Warping restraint (a la FSM)

compression flange is unrestrained.
Boundary conditions
(local buckling)

ends are constrained to act rigidly about tie point rotation (moment) enforced at ends

Warping restraint (a la FSM)

compression flange rot. Restrained at mid width of flange
Sections considered

3.62 in. to 6 in.

8.5 in.

SSMA sections

MBMA member company section

Vary t

Thickness (in):

0.0538
0.0566
0.0598
0.0673
0.0713
0.0747
0.0897
0.1017
0.1046
0.1196
0.1345
Geometric imperfections

- Shape = local or distortional buckling mode shape
- Magnitude = 50% probability of exceedance based on statistics gathered by Schafer and Peköz (1998) as summarized below

<table>
<thead>
<tr>
<th>P(X &lt; x)</th>
<th>Type 1 d/t</th>
<th>Type 2 d/t</th>
</tr>
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<tr>
<td>0.25</td>
<td>0.14</td>
<td>0.64</td>
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<tr>
<td>0.50</td>
<td>0.34</td>
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<td>0.75</td>
<td>0.66</td>
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<td>0.95</td>
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<td>3.44</td>
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<tr>
<td>0.99</td>
<td>3.87</td>
<td>4.47</td>
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μ = 0.50, σ = 0.66

CDF of Maximum Imperfection

• Shape = local or distortional buckling mode shape
• Magnitude = 50% probability of exceedance based on statistics gathered by Schafer and Peköz (1998) as summarized below
Material models

\[\sigma - \varepsilon \text{ curves measured, Yu and Schafer (2007)}\]
• Direct DSM-style relationship is difficult to justify with initial results
• Sensitivities to imperfection magnitude (direction) observed
  
  (Camotim et al. 2006)
Strain as function of slenderness?

- Strain demands greater in distortional buckling
- Slenderness where inelastic $C_y > 1$ occurs much lower for local
Strain as function of strength?

- Strain to strength similar for local and distortional buckling
- Elementary mechanics is driving inelastic reserve
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Strain as function of reserve strength

\[ \frac{M_n - M_y}{M_p - M_y} = 1 - \frac{1}{C_y} \]
Strain as function of slenderness

if $\lambda_\ell < \lambda_{ty}$, $C_{y\ell} = \left(\frac{\lambda_\ell}{\lambda_{ty}}\right)^{0.9}$ where $\lambda_{ty} = 0.776$

if $\lambda < \lambda_{dy}$, $C_{yd} = \left(\frac{\lambda_d}{\lambda_{dy}}\right)^2$ where $\lambda_{dy} = 0.673$

$\lambda_\ell = \left(\frac{M_y}{M_{cr\ell}}\right)^{0.5}$ or $\lambda_d = \left(\frac{M_y}{M_{crd}}\right)^{0.5}$
Direct Strength Method for Beams

\[ \lambda_{\text{max}} = \sqrt{\frac{M_y}{M_\sigma}} \]
Inelastic reserve strength prediction

\[ \text{slenderness: } \lambda = \left( \frac{M_y}{M_{cr}} \right)^{0.5} \text{ or } \lambda_d = \left( \frac{M_y}{M_{crd}} \right)^{0.5} \]
Inelastic reserve strength prediction

**LOCAL**

\[ if \, \lambda_l < \lambda_{ty}, M_{nl} = M_y + (M_p - M_y) \left(1 - \left(\frac{\lambda_l}{\lambda_{ty}}\right)^{0.9}\right) \]

**DISTORTIONAL**

\[ if \, \lambda_d < \lambda_{dy}, M_{nd} = M_y + (M_p - M_y) \left(1 - \left(\frac{\lambda_d}{\lambda_{dy}}\right)^2\right) \]

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<th>( M_{test}/M_n ) for ( M_n &gt; M_y )</th>
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<td></td>
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<tr>
<td>Distortional models</td>
<td>1.10</td>
<td>0.06</td>
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* these statistics are provided only when Eq. 14 and 15 are employed for prediction, i.e., when the predicted \( M_n \) is greater than \( M_y \) (or equivalently \( \lambda_l < \lambda_{ty} \) or \( \lambda_d < \lambda_{dy} \))
Why cutoff strain?

- Local demands can be far in excess of average (elementary mechanics) strain demands

<table>
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<tr>
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<th>$C_{ymax}$-membrane/$C_Y$</th>
<th>$C_{ymax}$-surface/$C_Y$</th>
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<td>3.2</td>
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* $C_Y$ without any modifier is taken as the average of the flange membrane strain
Future work

- Impact of strain hardening
- Further study on sensitivity to inward vs. outward inelastic buckling collapse mechanisms
- Extent to which LTB, and unbraced length influence solution particularly in distortional buckling
- Comprehensive verification of FE model
- Further study on using elastic slenderness to correlate to inelastic strength
Conclusions

• Inelastic bending reserve exists in CFS beams developing from one of two distinct mechanisms.
• From FE models, average strain demands in inelastic distortional buckling were found to be much greater than in inelastic local buckling at equivalent slenderness.
• Peak strain demands exceed average member strains predicted via elementary mechanics.
• Simple expressions for strain demand in inelastic local and distortional buckling as function of elastic cross-section slenderness as well as inelastic bending reserve are established.
• Preliminary Direct Strength Method design expressions for inelastic bending reserve in local and distortional buckling as a function of elastic section slenderness are established.
Any Questions?
Observed inelastic reserve in CFS beams

transition in inelastic regime, where $M_n > M_y$ has not been explored for cold-formed sections.

$$\frac{M_{\text{test}}}{M_y}$$

cross-section slenderness $\lambda_{\max} = \sqrt{\frac{M_y}{M_{cr}}}$
Finite Element Modeling – by ABAQUS

- Shell element S4R for sections, panels and tubes, solid element C3D8 for load beam.
  - Geometric imperfection and material nonlinearity are considered.

- Residual stress is not considered.

- The auto Stabilization method in ABAQUS were used for postbuckling analysis.
Finite Element Modeling - geometric imperfections

Geometric imperfection is obtained by the superposing two eigenmodes which are calculated by finite strip method (software CUFSM).

Local buckling shape + Distortional buckling shape = Input geometric imperfection
Basic test setup
Formulas for strain at failure

\[ \int_{0}^{y'} \int_{0}^{h} \sigma \, dx \, dy + \int_{y'}^{y} \int_{0}^{h} \sigma \, dx \, dy = 0 \]

\[ \sigma = f(\varepsilon) \text{ and } \varepsilon = \left( \frac{y}{y_{\text{lim}}} \right) \varepsilon_{\text{max}} \]

\[ M = \int_{A} f(\varepsilon) y \, dA = \int_{A} f(\left( \frac{y}{y_{\text{lim}}} \right) \varepsilon_{\text{max}}) y \, dA \]

\[ \varepsilon_{\text{max-compression}} = C_y \varepsilon_y \text{ and } \varepsilon_{\text{max-tension}} = C_y \text{-tension} \varepsilon_y \]