NUMERICAL ANALYSIS OF COLD-FORMED STEEL PURLIN-SHEETING SYSTEMS

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Abstract: The structural behavior of a cold-formed steel purlin-sheeting system is surprisingly complex. To adequately model the response using computational methods, the numerical analysis must consider material and geometrical nonlinearity, and in addition the influence of contact between the purlin, sheeting, and connection should also be considered. Models proposed in the literature were analyzed and compared with a finite element model that was developed considering the essential nonlinearities including contact. The developed finite element model was successfully validated against experimental results conducted in Brazil using a standard vacuum rig. With the success of the model a parametric study was conducted to better understand the impact of the purlin, sheeting, and connection variables on the purlin demands in wind uplift. Empirical design expressions were generated using the parametric study to provide engineers a more convenient means to predict the bending demands on the purlin as a function of the displacement due to wind uplift.

Keywords: Steel Structures, Cold-formed Steel Structures, Steel Purlin, Purlin-sheeting System.

1. INTRODUCTION

Purlins are structural elements that serve to link roof sheeting to trusses, thus transferring the loads applied on the sheeting to the trusses (or the main structure). The same elements are called girts when they support the lateral sheeting and transmit the loads to the columns.

In the last few years, several researchers have analyzed and suggested models for designing beams connected to sheeting based on theoretical and experimental models. Peköz & Soroushian (1982) laid down the foundation for the procedure adopted by Eurocode 3 – part 1.3 (1996). LaBoube (1988, 1991 and 1992) developed the method incorporated by NAS (2001). In their research, Ye et al. (2002, 2004) presented important models for understanding the modes of instability, and other researchers have put forward models for numerical analysis using the Generalized Beam Theory (GBT), the Finite Strip Method (FSM) and the Finite Element Method (FEM).

Among the researchers who have used the FEM to study purlin-sheeting systems are Lucas et al. (1997a,b) and Basaglia (2004). Lucas et al. (1997a,b) proposed two models of nonlinear analysis: a complete model, which discretizes the purlin-sheeting system with shell elements and couples nodes to simulate the purlin-sheeting contact and sheeting continuity; and a simplified model, which substitutes spring elements for the sheeting.

Basaglia (2004) simulated the vacuum rig test developed by Javaroni (1999), adding to the complete model of Lucas et al. (1997a,b) the contact elements between the purlin and the sheeting. He obtained very satisfactory results, but at a high computational cost.

Thus, the present work building on the existing efforts, proposes a numerical model for nonlinear analysis. This model was applied in a parametric study of the structural behavior of purlins supported at two points on a lipped channel section, which enabled us to determine expressions relating load (wind uplift) to displacements.
2. NUMERICAL ANALYSIS

2.1 Finite elements used

The numerical analysis was carried out with ANSYS (1997). The shell element, SHELL181, which ANSYS reports is suitable for the nonlinear analysis of thin shells under large strains and rotations, was used to model both the sheeting and purlin. The element has four nodes with six degrees of freedom per node, translation in the direction of the x, y and z axes, and rotation around those same axes. This element allows for the use of materials with nonlinear behavior.

Contact elements were used to represent the interaction between the sheeting and purlin shell elements. Contact elements can be used in 3-D analyses to simulate the contact pressure between the purlin and the sheeting, as well as their separation. Contact elements may also consider the friction between the two surfaces, although inclusion of friction changed the results of the cases analyzed here only slightly.

The COMBIN39 unidirectional spring element, which has two nodes and allows for the input of load-displacement or nonlinear moment-rotation curves, was used to simulate the restriction provided by the support angle (also known as the anti-roll clip) to the purlins.

2.2 Criteria for the nonlinear analysis

The nonlinear response was analyzed admitting geometric, physical and contact nonlinear behavior of the purlin-sheeting. Contact nonlinearity is included so that as the purlin rolls or buckles when the purlin face moves away from the sheeting no load is added to the purlin, but when the purlin rolls into the sheeting, causing compression, pressure is generated in the contact between purlin and sheeting creates an addition path (beyond the fasteners) for transmitting load in the system.

All the values predefined by ANSYS were used as parameters to simulate the contact, except the coefficient of friction between the two surfaces, which was altered to 0.3, corresponding to a value compatible with zinc-coated surfaces.

The constitutive multilinear elastic-plastic model with isotropic hardening and the von Mises yield criterion were adopted for the purlin and the sheeting. The stress-strain curve was limited to three sections, the first corresponding to a linear elastic model considering the modulus of elasticity of the steel (E) up to the proportional stress ($f_p$), which is equivalent to 70% of the yield stress; the second following a straight line up to the point corresponding to the yield stress ($f_y$) and a strain of 5%, and lastly, the third following a straight line up to the point equivalent to the tensile strength ($f_u$) and strain of 20%.

The nonlinear system was solved using the full version of the Newton-Raphson incremental-iterative method, which updates the tangent stiffness matrix at each iteration. The Stress Stiffness tool was also used concomitantly. It should be noted that in some models where convergence was difficult, the system was solved using the full unsymmetrical Newton-Raphson method. The load was applied incrementally with the ANSYS Automatic Load Stepping tool, and the convergence criterion in terms of displacements was used. The Line-Search tool was employed to improve the convergence of the model, since, according to Lourenço (1999), incremental-iterative processes are limited to being convergent for the solution of the system of nonlinear equations starting from practically any initial solution. Hence, the Line-Search tool is used to reach an estimated solution outside the radius of convergence of the Newton-Raphson method. The method consists of multiplying the displacement increment vector by a factor determined by the minimization of the system’s energy.

2.3 Proposed model

In his simulations, Basaglia (2004) adopted a model like Javaroni’s (1999) vacuum rig test (Figure 1), using only an axis of symmetry situated at mid-span of the purlin.
However, it was found that (a) Basaglia’s model was quite computationally expensive, and (b) the vertical reactions of the end purlins were influenced by the test boundary conditions and vary from 45% to 55% of the value of the vertical reaction at the support of the intermediate purlin. Based on these observations it was decided to focus on the intermediate purlin alone, and the sheeting contained in the tributary width, as well as to increase the model efficiency to use two axes of symmetry located at mid-span between purlins (Figure 2).

Thus, according to the system of axes presented in Figure 1, the axis of symmetry in the section at mid-span of the purlin restricts displacement in the direction of the x axis and rotations around y and z axes \( \phi_y = \phi_z = 0 \), while the axis of symmetry simulating the continuity of the sheeting restricts displacements in the direction of the z axis and the rotations around the x and y axes \( u_z = \phi_x = \phi_y = 0 \).

The purlin-sheeting connection is normally made with screws. Based on this detail, we chose to couple the displacements of the nodes that demarcate the projection of the screw shaft. This modeling choice is expected to be adequate for approximating the forces delivered by the fastener between the purlin and sheeting, but does not incorporate fastener deformation or failure.

The contact elements were placed along the entire top flange of the member (CONTA173 element) and along the bottom part of the sheeting (TARGE170).

The purlin is usually connected to the roof truss at the web by means of bolts and a support angle which, in turn, is welded or bolted to the truss. The bolts that connect the purlin to the support angle were simulated by restricting all the displacements \( u_x = u_y = u_z = 0 \) of the nodes positioned in the same site as the respective bolts (Figure 3). Thus simulating an ideal pinned connection at the bolt locations.

The connection provided by the support angle, as illustrated in Figure 3, restricts the displacement of the bottom flange of the purlin in the negative \( u \) direction, but imposes no restriction in the positive \( u \) direction. To simulate this contact boundary condition, the COMBIN39 spring element was employed with 0 stiffness for positive \( u \) (displacement), and near infinite stiffness for negative \( u \) displacement.

The uplift was applied by means of a uniformly distributed force perpendicular to the sheeting surfaces, disregarding self-weight.

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Figure 1 – Vacuum rig test of the purlin-sheeting system

Figure 2 – Strained configuration and tributary width of the purlin (After. Basaglia, 2004)

Figure 3 – Modeling of the connection simulating the support angle (After. Basaglia, 2004)
2.3 Validation of numerical model

The vacuum rig tests conducted by Javaroni (1999) were adopted as reference for the numerical analyses of this work (Figure 1).

The tested prototypes consisted of three 5,620 mm long purlins spaced equally 1,780 mm from each other, and roof sheeting with 40 mm height and 0.65 mm thickness. The numerical models were calibrated considering a 127x50x17x3 lipped channel with the sheets connected to all the ridges.

To validate the numerical models, comparisons with the experiments of Javaroni (1999) were used, in particular the “Rig 2 Test”. The results obtained at mid-span of the intermediate purlin were compared between the model and test. The vertical (Figure 4a) and horizontal (Figure 4b) displacements were measured at the junction of the bottom flange with the web.

Figure 4a shows that satisfactory agreement between the experimental model and the numerical analysis with respect to vertical displacement is achieved. However, for horizontal displacement (which also approximates the twisting that is occurring) as shown in Figure 4b, the agreement is adequate up to a pressure of approximately 0.25 kN/m², beyond this pressure the numerical model is “stiffer” than the experimental one.

Several factors that may influence the results are: initial imperfections, settling of the bolts, and the difficulty of experimentally recording the displacements, particularly the horizontal displacement.

![Vertical Displacement](image)

a) Pressure versus vertical displacement

![Horizontal Displacement](image)

b) Pressure versus horizontal displacement

Figure 4 – Vertical and horizontal displacement versus pressure.

Figure 5 illustrates the longitudinal stresses, at the middle of span, under the pressure of 0.93 kN/m². According to Javaroni (1999), this pressure indicates the value of the pressure applied in experimental tests corresponding to a flexure equivalent to 1/100th of the span of the purlin.

As can be seen in Figure 5, the maximum difference between the numerical model and experimental results (via strain gages, converted to stress) is 20% (point 4). This relatively small difference is considered acceptable given the relatively large deformations and twist occurring, as well as the difficulty in measuring local strain/stress values.

![Stresses](image)

Figure 5 – Comparison of the stresses along the plane of the transverse section (positive = tension)
3. PARAMETRIC ANALYSIS

Based on the proposed model, a parametric analysis was completed for purlins supported at two points on a lipped channel section without sag rods (Vieira Jr., 2007). The purlins, spans and mechanical properties adopted here reflect the usual practices of the Brazilian market (Tables 1 and 2).

Table 1 – Dimensions of the shapes and range of spans adopted in the parametric analysis

<table>
<thead>
<tr>
<th>Section</th>
<th>C - b_w x b_f x d x t</th>
<th>Span (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150x60x20x1.5</td>
<td>4,788 – 6,498</td>
<td></td>
</tr>
<tr>
<td>150x60x20x2.65</td>
<td>4,788 – 6,840</td>
<td></td>
</tr>
<tr>
<td>200x75x20x2</td>
<td>5,814 – 8,208</td>
<td></td>
</tr>
<tr>
<td>250x85x25x2</td>
<td>7,524 – 9,576</td>
<td></td>
</tr>
<tr>
<td>250x85x25x3</td>
<td>7,524 – 9,576</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 – Mechanical properties adopted.

<table>
<thead>
<tr>
<th>Element</th>
<th>Commercial name</th>
<th>Yielding Strength (f_y) (MPa)</th>
<th>Ultimate Strength (f_u) (MPa)</th>
<th>Modulus of Elasticity (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purlin</td>
<td>USI-SAC 300 (USIMINAS)</td>
<td>300</td>
<td>400</td>
<td>205,000</td>
</tr>
<tr>
<td>Sheeting</td>
<td>ZAR-230 (CSN)</td>
<td>230</td>
<td>310</td>
<td>205,000</td>
</tr>
</tbody>
</table>

Initially, models with sheets measuring 40mm and 25mm in height and 0.43mm and 0.65mm of nominal thickness, corresponding to 0.394mm and 0.614mm, respectively, discounting the thickness of the coating were analyzed. Comparative studies indicate that different types of sheeting produce only slight differences in vertical displacements, but that sheeting thickness significantly affects the system’s response in horizontal displacements, which indirectly effects the distortion of the purlin. Therefore, it was decided to analyze only models with a 25mm high, 0.43mm thick sheet, i.e., the one presenting the smallest contribution to the purlin-sheeting system and, hence, the greatest distortion of the section of the purlin. In addition to verifying failures (plastic moment and instabilities), the design of purlin-sheeting systems should include a careful evaluation of the service moment capacity corresponding to excessive displacements, since the load-displacement response may be strongly nonlinear and, in many cases, may dictate the section designed.

Thus, the parametric analysis conducted here focused mainly on the load-displacement response. The value of L/120 (L is the span of the purlin) was established as the limit for the vertical displacements. For reference, for the service load combinations considered here, this corresponds approximately to the limit of L/180 recommended by the ABNT NBR 14762:2001 standard.

Table 3 lists the values of pressure applied to the sheeting (wind uplift) corresponding to the bending moment resistance, M_R, obtained based on six analyses, P1 to P6, also defined in Table 3. Table 3 only provides the values of pressure for the shortest and longest spans considered for each section.

It should be noted that, among the models presented in Table 3, the only numerical model of finite elements in which local instability and yield were identified was the model with the 250x85x25x2 lipped channel and 7,524 mm span. However, the ultimate pressure also corresponds to excessive displacement, reinforcing the importance of the study of displacements in purlin-sheeting systems.

Table 3 reveals the considerable difference between the ultimate displacement reached by the elastic analysis of the isolated purlin (analysis P3) and that given by the numerical analysis of the purlin-sheeting system (analysis P4), a difference that can reach 60%, as in the 200x75x20x2 lipped channel and 5.814 mm span model.

Another point to highlight is the values resulting from the normative procedures, which are sometimes too conservative and at other times unconservative.
The last column in Table 3 also presents the R factor required to limit the displacements to L/120, indicating that R = 0.6 satisfactorily evaluates this limit state for purlins supported at two points on lipped channels without sag rods. A study of the influence of sag rods on the structural behavior of the purlin-sheeting system is given in Basaglia (2004).

### Table 3 – Values of pressure (kN/m²) for the ends of each span length analyzed

<table>
<thead>
<tr>
<th>C - Section</th>
<th>Span (mm)</th>
<th>Pressure (^a) (kN/m²)</th>
<th>R (^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P1</td>
<td>P2</td>
</tr>
<tr>
<td>150x60x20x1.5</td>
<td>4,788</td>
<td>1.06</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>6,498</td>
<td>0.58</td>
<td>0.60</td>
</tr>
<tr>
<td>150x60x20x2.65</td>
<td>4,788</td>
<td>1.86</td>
<td>1.86</td>
</tr>
<tr>
<td></td>
<td>6,840</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>200x75x20x2</td>
<td>5,814</td>
<td>1.58</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>8,208</td>
<td>0.83</td>
<td>0.87</td>
</tr>
<tr>
<td>250x85x25x2</td>
<td>7,524</td>
<td>1.39</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>9,576</td>
<td>0.86</td>
<td>0.91</td>
</tr>
<tr>
<td>250x85x25x3</td>
<td>7,524</td>
<td>2.12</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td>9,576</td>
<td>1.33</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Where:
- P1: Pressure correspond to \(M = S_{ef} f_y\) (\(S_{ef}\) is the elastic section modulus of effective section);
- P2: Pressure correspond to \(M = S f_y\) (\(S\) is the elastic section modulus of full unreduced section);
- P3: Pressure correspond to a vertical displacement L/120, elastic analysis of the purlin section as a isolated beam (\(L\) is the purlin’s span);
- P4: Pressure correspond to a vertical displacement L/120, using the finite element model proposed;
- P5: Pressure correspond to \(M = R S_{ef} f_y\) as defined by the Brazilian standard ABNT NBR 14762:2001;
- P6: Pressure correspond to \(M = R S_{ef} f_y\) as defined by NAS (2001).

Notes: a - Values of pressure for a tributary width of 2,000 mm.
- b - Values of factor R based on the limit of vertical displacement acquired by the finite element model proposed.

Due to the significant nonlinear response of the purlin-sheeting system and based on the results of the parametric analysis, equations are proposed to evaluate the vertical displacement \((d\) in cm) as a function of the other parameters involved: \(L/S\) (\(L\) is the span of the purlin and \(S\) is the elastic section modulus of the full unreduced section) and the load distributed uniformly along the purlin \((\rho\) in kN/m), i.e., the product of the pressure applied on the sheeting by the tributary width.

Among various satisfactory equations, equations (1) and (2) are relatively simple and are in good agreement with the numerical analysis, with a correlation coefficient, \(r^2\), equal to 0.96 and 0.93, respectively. Figure 6 presents the surfaces adjusted by equations (1) and (2).

\[
\ln(d) = -17.59 + 1.20 \cdot \ln\left(\frac{L^3}{S}\right) + 1.41 \cdot \ln(\rho) \quad (1)
\]

\[
d = 1/(0.025 - 7.87 \cdot 10^{-5} \cdot \sqrt[3]{\frac{L^3}{S}} + 0.41/\sqrt{\rho}) \quad (2)
\]
4. CONCLUSIONS

The purlin-sheeting system shows a complex structural behavior characterized by a nonlinear load-displacement response differing substantially from that obtained based on the elastic analysis of the isolated purlin, since wind uplift involves bending, torsion and lateral distortion.

For sections of steel purlins and sheeting commonly designed, the ultimate limit states of yield and instability were not characterized; rather, their design is limited by conditions of excessive displacement.

The model proposed here, which considered geometric, physical and contact nonlinearity, but only includes a tributary portion of the sheeting and takes advantages of symmetry in the modeling, displayed good correlation with the experimental results of a vacuum rig test, especially with respect to the vertical displacement and stresses developed in the section.

A parametric study completed using the validated model lead to a series of proposed equations for determining the vertical displacements as a function of the span of the purlin and the elastic section modulus of the full unreduced section and the applied load. These empirical expressions provide a better approximation of the purlin displacements than elastic analysis and are recommended as a design aid.
5. ACKNOWLEDGEMENTS

The authors are indebted to the company USIMINAS and to CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) (Brazil) for the grants awarded. The work reported herein was conducted by the first author as part of his master’s thesis at Universidade de São Paulo under Prof. Malite’s guidance and during a two months stay at Johns Hopkins University as a visiting student scholar.

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