Computational Modeling of Cold-Formed Steel

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Thoughts on...

Computational Modeling of Cold-Formed Steel

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Disclaimer and acknowledgments

The opinions presented herein are my own, but are influenced by my experiences under the guidance of

- Teoman Peköz, and

  as an advisor of former and current Ph.D. students:

- Cheng Yu, Rachel Sangree, Cris Moen, Vahid Zeinoddini, Mina Seif, Yared Shifferaw, Luiz Vieria, Jr., and Zhanjie Li,

  as well as a collaborator with visiting researchers:

- Sandor Ádány in particular.

  Kim Rasmussen suggested the topic and provided the forum.
overview

introduction

elastic buckling

collapse modeling

discussion

conclusions
overview

introduction
elastic buckling
collapse modeling
discussion
conclusions
Computational modeling is growing

Consider ICTWS 2008, just completed
- 33 papers in the Cold-Formed Steel track
- 18 papers (55%) had major computational modeling
- 13 of the 18 modeling papers had significant shell FE work

Computing power feels slow, but
- 24 hours was required for a shell model of a single member on a $30,000 Digital VMS workstation in 1997
- 10 minutes is required for a shell model of a single member on a $2,000 PC today (2008)

We are starting to learn how to use aspects of computational modeling within the design process
- rational elastic buckling analysis clauses
- Direct Strength Method
overview

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elastic buckling

\[(K_e - \lambda K_g(\sigma))\phi = 0\]

- Marvelous power of shell FE comes with surprisingly little insights, and huge limitations
- This reality has lead to significant use of other methods for elastic buckling of cold-formed steel
semi-analytical FSM for a column

- FSM mode 1
- FSM mode 2
- FSM mode 3 and higher

- Local
- Distortional
- Global

- 100x60x10x2 (mm)
- E = 210 GPa
- ν = 0.3
- f_y = 345 MPa
and higher modes

Local
Distortional
Global

FSM mode 1
FSM mode 2
FSM mode 3 and higher

P_{c,t}/P_y

100x60x10x2 (mm)

E=210 GPa
ν=0.3
f_y=345 MPa

half-wavelength (mm)
and higher number of half-waves
and comparing to FE

Consider FE at 3m
semi-analytical FSM

半波长 (mm)

FSM 模式 1
FSM 模式 2
FSM 模式 3 及更高

100x60x10x2 (mm)

E = 210 GPa
ν = 0.3
f_y = 345 MPa

局部

扭曲

全局

P_c/P_y

0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0
constrained FSM: modal decomposition

Local
Distortional
Global

Pcr/Py

E=210GPa
ν=0.3
fy=345MPa

100x60x10x2(mm)
constrained FSM: modal identification

- Global
- Distortional
- Local
- Other

Material properties:
- $E = 210 \text{ GPa}$
- $\nu = 0.3$
- $f_y = 345 \text{ MPa}$

Dimensions:
- $100 \times 60 \times 10 \times 2 \text{ (mm)}$
**heart of cFSM is mechanical definitions of modes**

<table>
<thead>
<tr>
<th></th>
<th>G modes</th>
<th>D modes</th>
<th>L modes</th>
<th>O modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vlasov’s hypothesis</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Longitudinal warping</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>-</td>
</tr>
<tr>
<td>Undistorted in-plane section</td>
<td>Yes</td>
<td>No</td>
<td>-</td>
<td>-</td>
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Definitions allow deformations to be constrained for example:

\[
\{d\} = [R_L] d_L
\]

Constrains \( \{d\} \) to \( L \) space.

The stability problem may be similarly constrained:

\[
[R_L]^T [K_e(a)] [R_{LV}] \{\phi_L\} = \lambda_L [R_L]^T [K_g(a)] [R_L] \{\phi_L\}
\]
AISI-S100-07 buckling mode definitions

- **Local buckling**: limit state of buckling of a compression element where the line junctions between elements remain straight and angles between elements do not change.

- **Distortional buckling**: A mode of buckling involving change in cross-sectional shape, excluding local buckling.

- **Flexural-torsional buckling**: Buckling mode in which a compression member bends and twists simultaneously without change in cross-sectional shape.
Mode definitions are important

AISI-S100-07 buckling mode definitions

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without change in cross-section shape?

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- **Flexural-torsional buckling**: Buckling in which a compression member bends and simultaneously without change in cross-sectional shape

- cFSM has motivated new insights on global buckling and the relationship between beam and plate theory, recently illuminated and deserving far greater attention, namely:
  - Ádány (2006) on flexural buckling at SDSS in Lisbon and
  - Ádány (2008) on torsional buckling at ICTWS (last week!)

- **Global bucking does in fact involve change in cross-sectional shape** (subtle point, but worth knowing)!
Example: Euler buckling of a strip

- cross-section remains rigid
  \[ w_1 = w_2 \]
  \[ \theta_1 = \theta_2 = 0 \]
  \[ u_1 = u_2 = 0 \]

- transverse bending (\( \theta \)) allowed
  \[ w_1 = w_2 \]
  \[ \theta_1 = -\theta_2 \]
  \[ u_1 = u_2 = 0 \]
Euler buckling of a strip

\[ \frac{2EI}{(L/r)^2} \]

Euler buckling

FSM

FSM rigid

FSM \( \theta \) allowed
Euler buckling of a strip

\[ f_{cr} = \frac{\pi^2 E}{(1-\nu^2)(L/r)^2} \]

| \( E - E/(1-\nu^2) / E \) | = 0.0989 for \( \nu = 0.3 \)

transverse extension may be restricted, but transverse bending (even though small) must be allowed in plate theory based solutions.
overview

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elastic buckling

collapse modeling

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collapse modeling

Largely the domain of shell FE analysis
(SFSM and some recent GBT work not withstanding)

• Imperfections
• Residual stresses
• Yield criteria
• Boundary conditions
• Element selection
• Solution scheme
collapse modeling

Largely the domain of shell FE analysis
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Geometric imperfections

are imperfections included as a modeling convenience, or as a physical reality?

Modeling convenience

- **distribution**
  - first mode
  - representative local, distortional, & global modes
- **magnitude**
  - %t (e.g. 0.1t, 0.5t)
  - %t selected to fit to a test
  - written as function of plate slenderness

Physical reality

- **distribution**
  - as measured
  - simulation from measured data (hopefully x times)
- **magnitude**
  - as measured
  - %t to match expected probability of exceedance
  - regression of observed data with magnitude $f(t, ..)$
Geometric imperfections are imperfections included as a modeling convenience, or as a physical reality?

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**TABLE 1**

CDF Values for Maximum Imperfection

<table>
<thead>
<tr>
<th>P((\Delta &lt; d))</th>
<th>Type 1</th>
<th>Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.14</td>
<td>0.64</td>
</tr>
<tr>
<td>0.50</td>
<td>0.34</td>
<td>0.94</td>
</tr>
<tr>
<td>0.75</td>
<td>0.66</td>
<td>1.55</td>
</tr>
<tr>
<td>0.95</td>
<td>1.35</td>
<td>3.44</td>
</tr>
<tr>
<td>0.99</td>
<td>3.87</td>
<td>4.47</td>
</tr>
<tr>
<td>mean</td>
<td>0.50</td>
<td>1.29</td>
</tr>
<tr>
<td>st. dev.</td>
<td>0.66</td>
<td>1.07</td>
</tr>
</tbody>
</table>

(Schafer and Peköz 1998)

**Fig. 2.** Histogram of Type 1 Imperfection

**Fig. 3.** Histogram of Type 2 Imperfection
multiple imperfection magnitudes a must


Section Slenderness = \( (M_y/M_{cr})^{0.5} \)

- Local buckling FEM
- Distortional buckling FEM

25% CDF
75% CDF
Average

FEM-to-Test Ratio

200% 120%
110%
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
multiple imperfection magnitude = new insights
(a dated example from Schafer and Peköz 1998)

understanding the impact of slenderness on imperfection sensitivity

examining interactions and sensitivity in different modes/mechanisms

final note.. need to do plus/minus imperfections if only have a few half-waves
Residual stresses

- Some obvious limitations of existing guidance that some guy called Schafer summarized
- Disconnect between residual stresses and cold work of forming completely illogical
- Mben and Schafer (2008) to be presented in the Hancock symposium is our latest on this...
collapse modeling

Largely the domain of shell FE analysis
(SFSM and some recent GBT work not withstanding)

- Imperfections
- Residual stresses
- Yield criteria
- Boundary conditions
- Element selection
- Solution scheme

Let us examine these through an example/case study
Consider ICTWS 2008, just completed

- 33 papers in the Cold-Formed Steel track
- 18 papers (55%) had major computational modeling
- 13 of the 18 modeling papers had significant shell FE work
  - 9 ABAQUS, 3 ANSYS, 1 LUSAS
- 12 of the 13 shell FE papers used linear elements. All but one of the ABAQUS users used reduced integration elements (i.e., the S4R not the S4)

Conclusion?

Today it is common in research to use nonlinear shell finite element analysis in ABAQUS with linear elements.
Demonstration of modeling sensitivity

- Cold-formed steel column modeled through collapse
  \(L=1200\text{mm}, \text{web}=100\text{mm}, \text{flange}=80\text{mm}, \text{lip}=10\text{mm}, t=2\text{mm}\)

- Imperfections
  local and distortional at 50% CDF values from Schafer and Peköz (1998)

- Residual stresses
  ignored

- Material model
  von Mises, isotropic hardening, elastic-perfectly plastic (eng. \(\sigma-\varepsilon\))
  \(E=210\text{GPa}, \nu=0.3, F_y=345\text{MPa}\)

- Boundary conditions
  all end nodes fully pinned (thus warping fixed)

- Element selection
  varied, linear and quadratic w/ w/o reduced integration

- Solution scheme
  Riks (arc-length), Artificial damping
element and mesh sensitivity

coarse          medium          fine

S4/S4R

S9R5

N/A
Linear element w/o reduced integration

Displacement, mm

P, kN

- S4 element- coarse
- S4 element- medium
- S4 element- fine

10%
Plastic strain at peak for S4 with coarse mesh
Plastic strain in collapse for S4 with coarse mesh
Plastic strain in collapse for S4 with coarse mesh
Plastic strain in collapse for S4 with fine mesh

![Graph showing plastic strain](image)

- S4 element - coarse
- S4 element - medium
- S4 element - fine
Linear element with reduced integration

Displacement, mm

P, kN

S4R element - coarse
S4R element - medium
S4R element - fine

7%
Quadratic element with reduced integration

Displacement, mm

P, kN

S9R5 element - medium
S9R5 element - fine

7%
Plastic strain in collapse for S9R5 with fine mesh
Medium mesh density across elements

- S4 element - medium
- S4R element - medium
- S9R5 element - medium

<table>
<thead>
<tr>
<th>Displacement, mm</th>
<th>P, kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>1.5</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>2.5</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
</tr>
</tbody>
</table>

2%
Fine mesh density across elements

![Graph showing displacement vs. force for S4, S4R, and S9R5 elements.](image)

- S4 element - fine
- S4R element - fine
- S9R5 element - fine

Displacement, mm

P, kN

2%
element and mesh sensitivity

<table>
<thead>
<tr>
<th></th>
<th>coarse</th>
<th>medium</th>
<th>fine</th>
</tr>
</thead>
<tbody>
<tr>
<td>S4/S4R</td>
<td>115kN</td>
<td>112kN</td>
<td>105kN</td>
</tr>
<tr>
<td>S9R5</td>
<td>N/A</td>
<td>110kN</td>
<td>103kN</td>
</tr>
</tbody>
</table>
Mesh sensitivity in elastic buckling

- Mesh types: S4, S4R, S9R5
- Buckling coefficient, k
- Number of elements per buckled half-wave
- Critical load, f_{cr}
- Dimensions: Length (L), Height (h)

Graph showing the relationship between number of elements per buckled half-wave and buckling coefficient for different mesh types.
Demonstration of modeling sensitivity

- **Cold-formed steel column modeled through collapse**
  L=1200mm, web=100mm, flange=80mm, lip=10mm, t=2mm

- **Imperfections**
  local and distortional at 50% CDF values from Schafer and Peköz (1998)

- **Residual stresses**
  ignored

- **Material model**
  von Mises, isotropic hardening, elastic-perfectly plastic (eng. $\sigma$-$\varepsilon$)
  $E=210\text{GPa}$, $\nu=0.3$, $F_y=345\text{MPa}$

- **Boundary conditions**
  all end nodes fully pinned (thus warping fixed)

- **Element selection**
  varied, linear and quadratic w/ w/o reduced integration

- **Solution scheme**
  Riks (arc-length), Artificial damping
Riks solver and initial step size

![Graph showing the Riks solver and initial step size](image-url)

- **Displacement, mm**: 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5

- 5 steps
- 10 steps
- 20 steps
- 50 steps
- 100 steps

**S9R5**
Demonstration of modeling sensitivity

- **Cold-formed steel column modeled through collapse**
  \[ L=1200\text{mm}, \text{web}=100\text{mm}, \text{flange}=80\text{mm}, \text{lip}=10\text{mm}, t=2\text{mm} \]

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- **Boundary conditions**
  all end nodes fully pinned (thus warping fixed)

- **Element selection**
  varied, linear and quadratic w/, w/o reduced integration

- **Solution scheme**
  Riks (arc-length), Artificial damping
what's the impact of introducing a small reduction in stiffness at $50\%F_y$?
Plasticity sensitivity

(an issue deserving further study, currently underway...)

Displacement, mm

P, kN

12%

S9R5 fine E
S9R5 fine 0.99E
S9R5 fine 0.95E
S9R5 fine 0.90E
### Collapse Modeling

Typical model sensitivity for peak load:

- Imperfections: 20%+
- Residual stresses: -
- Yield criteria: 10%
- Boundary conditions: -
- Element selection: 15%
- Solution scheme: 5%

This makes for a lot of ways to match a test when that is the only question you ask.
overall observation

• I am a true believer in the long-term potential for computational modeling to
  — elevate our research,
  — improve fundamental understanding, and
  — enable novel new design

• But! our new capabilities do not relieve us of the need to think, we are still required to
  — ask good questions
  — use engineering judgment as an analyst
  — be skeptical, not cynical
Shortcomings and Opportunities

(Focusing on collapse modeling)

• model results which are more sensitive than experimental results –
  — too sensitive to solution controls,
  — too sensitive to imperfections (distribution & magnitude)
  — too sensitive to boundary conditions,

• solvers which over-predict the potential for elastic stability when compared with testing, and

• significant challenges in the accurate modeling of as-built conditions.

The list is longer, but these represent major issues.
Shortcomings and Opportunities

(Focusing on collapse modeling)

- systems
  - modeling non-metallic materials, modeling connections
- ductile fracture
  - (seriously we can’t even model a tensile coupon to collapse)
- metal forming
  - Note recent work of H. Saal and Quach reported at ICTWS
- dynamics
  - blast, crashworthiness, wind, seismic – so much room to grow here
- multi-physics
  - fluid-structure, thermo-mechanical
- multi-scale
  - implicit and explicit schemes of interest
- stochastic FE and simulation
conclusions

- In elastic buckling FSM and other specialty computational methods have special importance in modeling of cold-formed steel - deservedly so as they provide far more useful information.

- In collapse modeling the “field” has settled on ABAQUS models, with linear shell elements, imperfections based on mathematical convenience, and the reporting of a single result for a given member.

- In collapse modeling greater diversity in FE implementations is likely good, quadratic shell elements are so much more robust we really should use them, imperfections deserve to be treated as a physical reality whenever possible, and model sensitivity is real and should be addressed in a more upfront manner when reporting results.

- Computational modeling advances in other fields are not being transferred to cold-formed steel modeling and this is limiting our capabilities, and worse yet, limiting the questions we are asking.
Computational modeling of cold-formed steel: characterizing geometric imperfections and residual stresses

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School of Civil and Environmental Engineering, Cornell University, Ithaca NY, USA
Received 7 October 1997, received in revised form 12 December 1997, accepted 6 January 1998

Abstract
This-wallled, cold-formed steel members exhibit a complicated post-buckling regime that is difficult to predict. Today, advanced computational modeling supplements experimental investigation. Accuracy of computational models relies significantly on the characterization of selected inputs. No assumptions exist on distributions or magnitudes to be used for modeling geometric imperfections and for modeling residual stresses of cold-formed steel members. In order to provide additional information existing data is collected and analyzed and new experiments performed. Simple rules of thumb and probabilistic concepts are advanced for characterization of both quantities. The importance of the modeling assumptions are shown in the examples. The ideas are summarized in a preliminary set of guidelines for computational modeling of imperfections and residual stresses. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: Cold-formed steel; Imperfections; Residual stresses; Computational modeling

1. Introduction
Post-buckling of cold-formed steel members is difficult to predict due to material and geometric nonlinearities. Nonetheless, numerical techniques, such as finite element or spline finite strip, have reached a level of maturity such that many are now successfully undertaking ultimate strength analysis of cold-formed steel members

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more on admitted bias

- This paper was initially rejected because it was noted to be of too narrow a topic to be of real interest.
- This is, by far, my most cited work. It seems some of you have found it useful.
- Let us revisit the topics of
  - geometric imperfections
  - residual stresses