a few thoughts on

Bracing and accumulation

AISI COFS meetings

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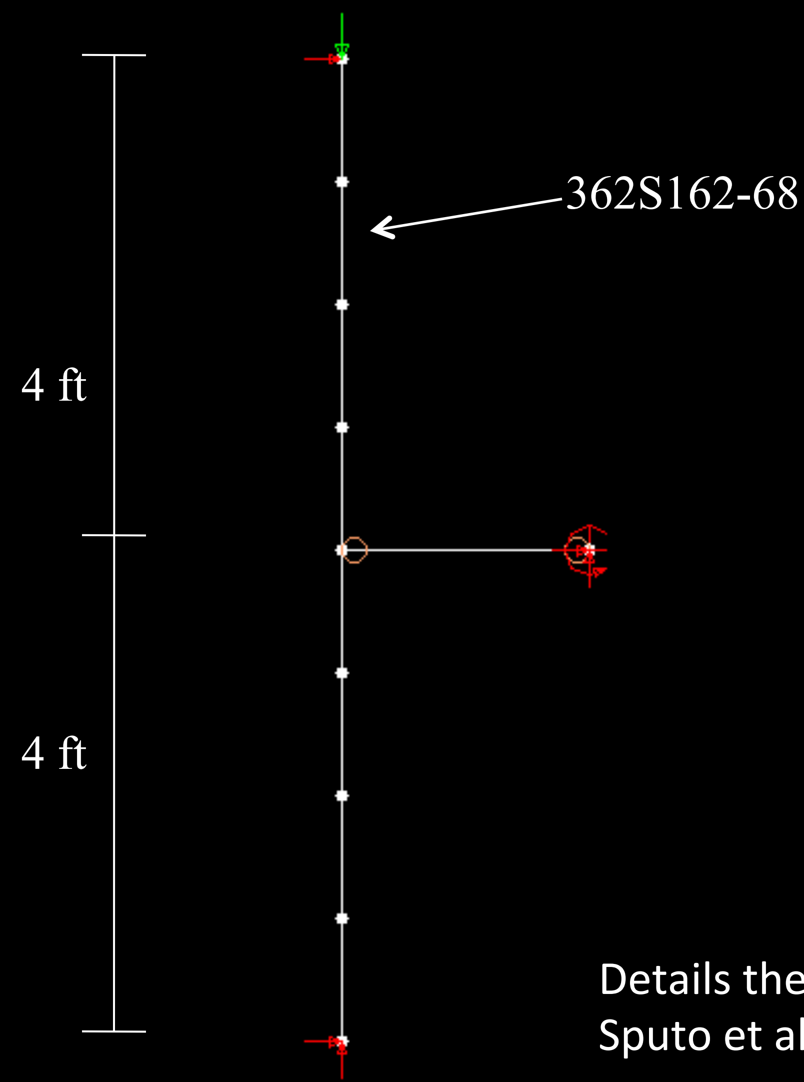
Introduction

- Sputo et al. provided brace force and stiffness expressions for walls with multiple studs
- The large accumulation of forces and high required stiffness have lead to some significant heal dragging in adopting these expressions
- With the help of a JHU undergraduate (Hannah Blum) we have been revisting Sputo et al.'s work to see if there is any way to mitigate or further understand these large accumulated demands on braces.

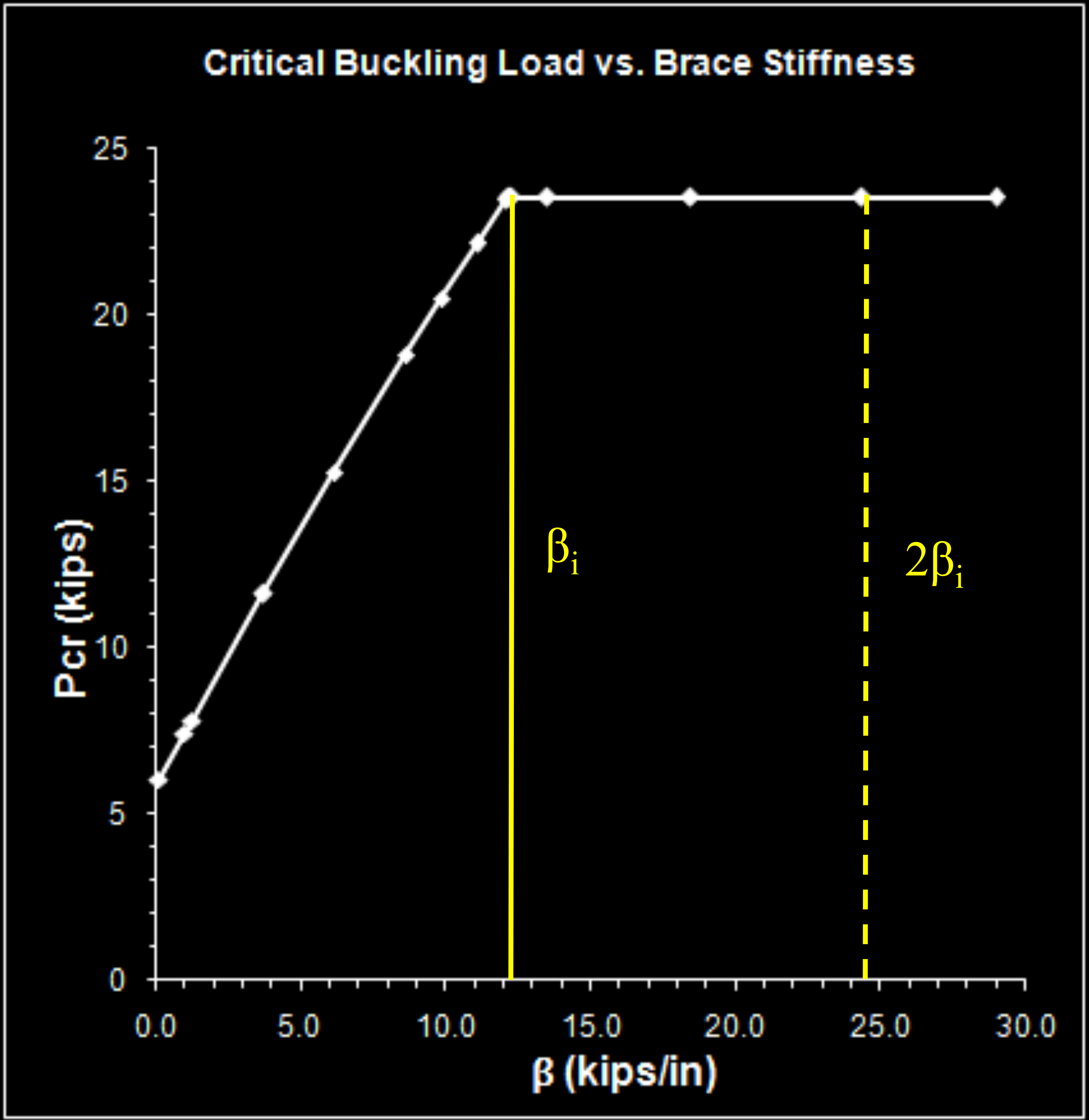
Begin at the beginning

single column (what we know)



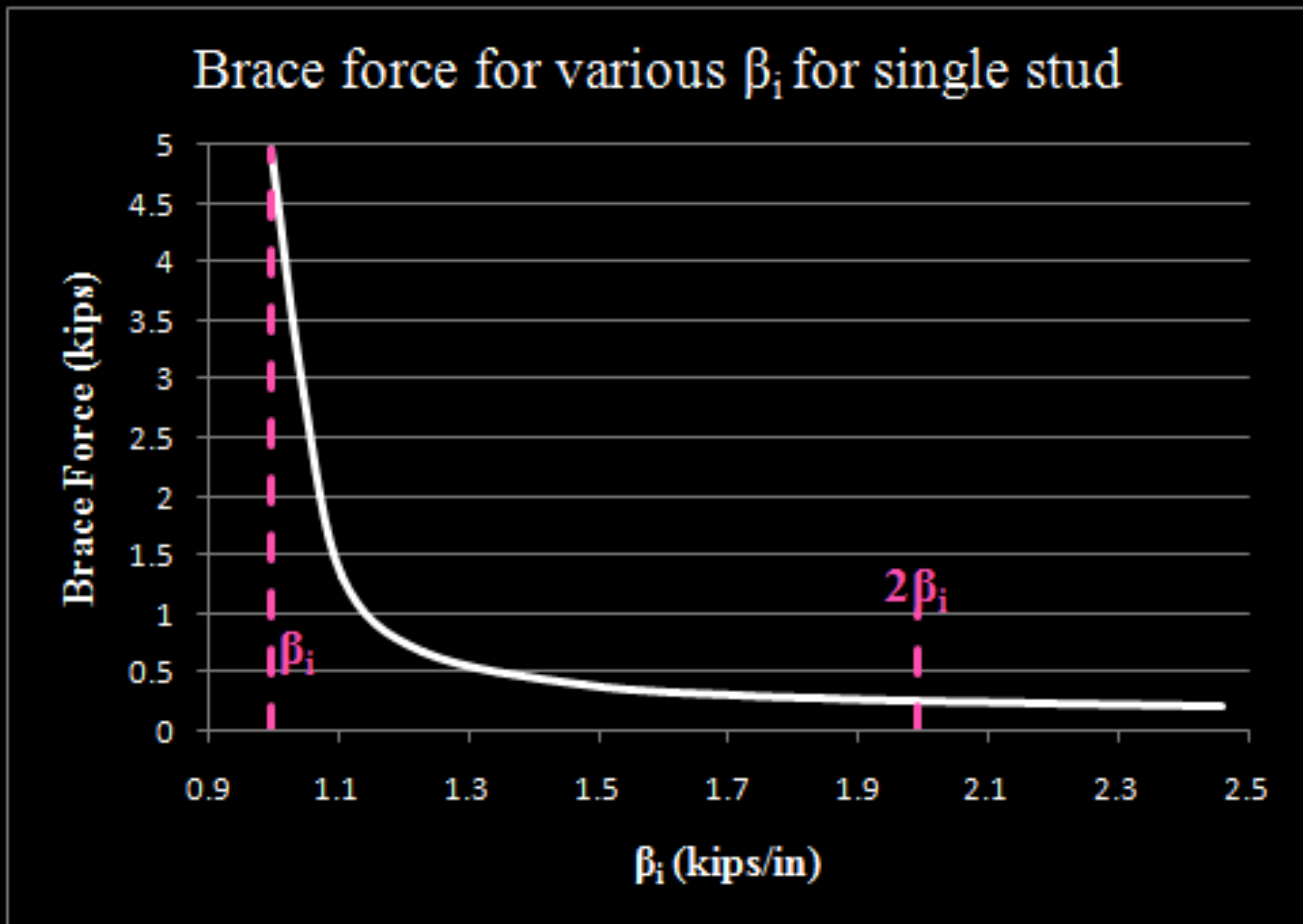
Details the same as
Sputo et al. modeling

Ideal brace stiffness (MASTAN)



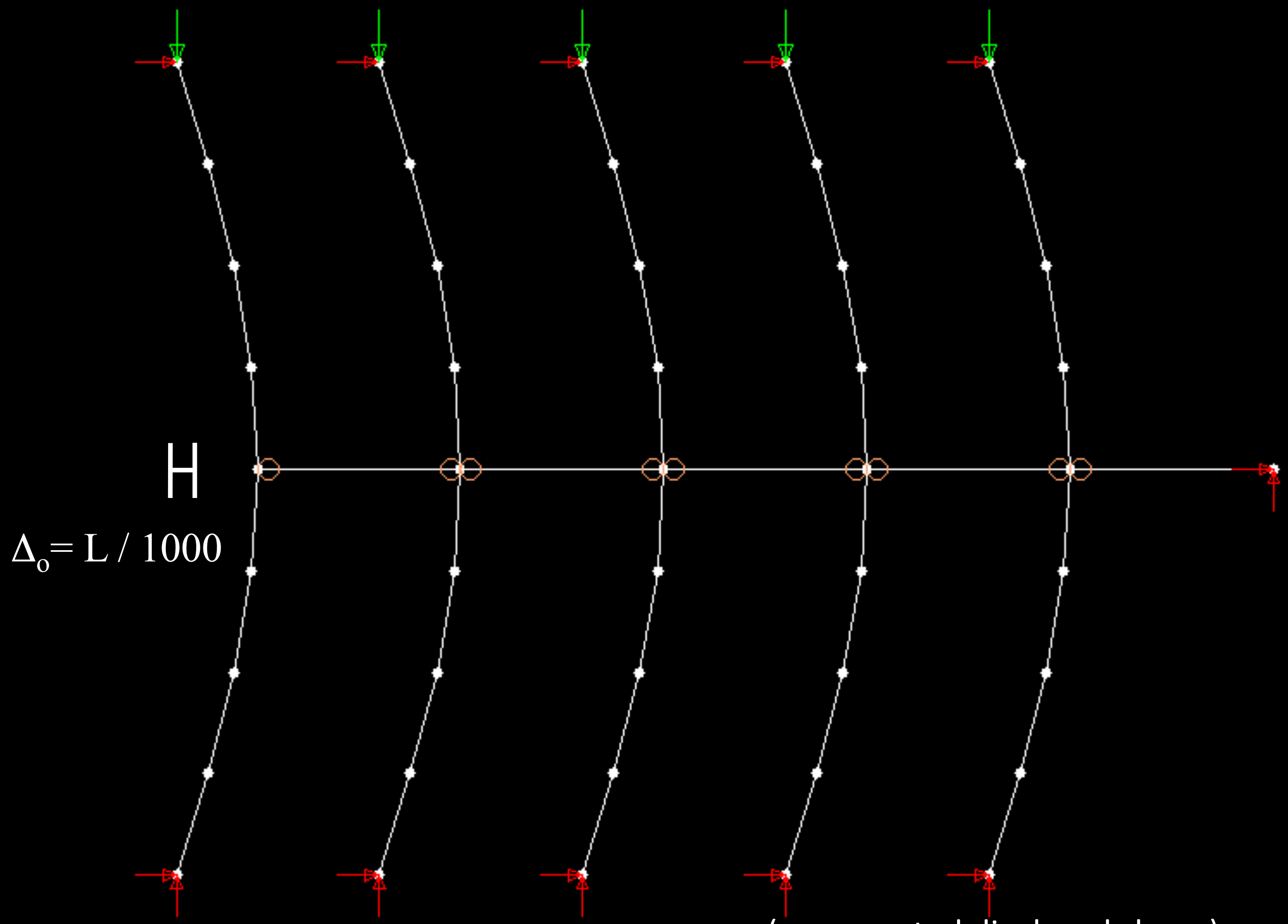
How to find brace stiffness using MASTAN.. (same as equations, but generalizable to multiple studs..)

MASTAN 2nd order: brace force



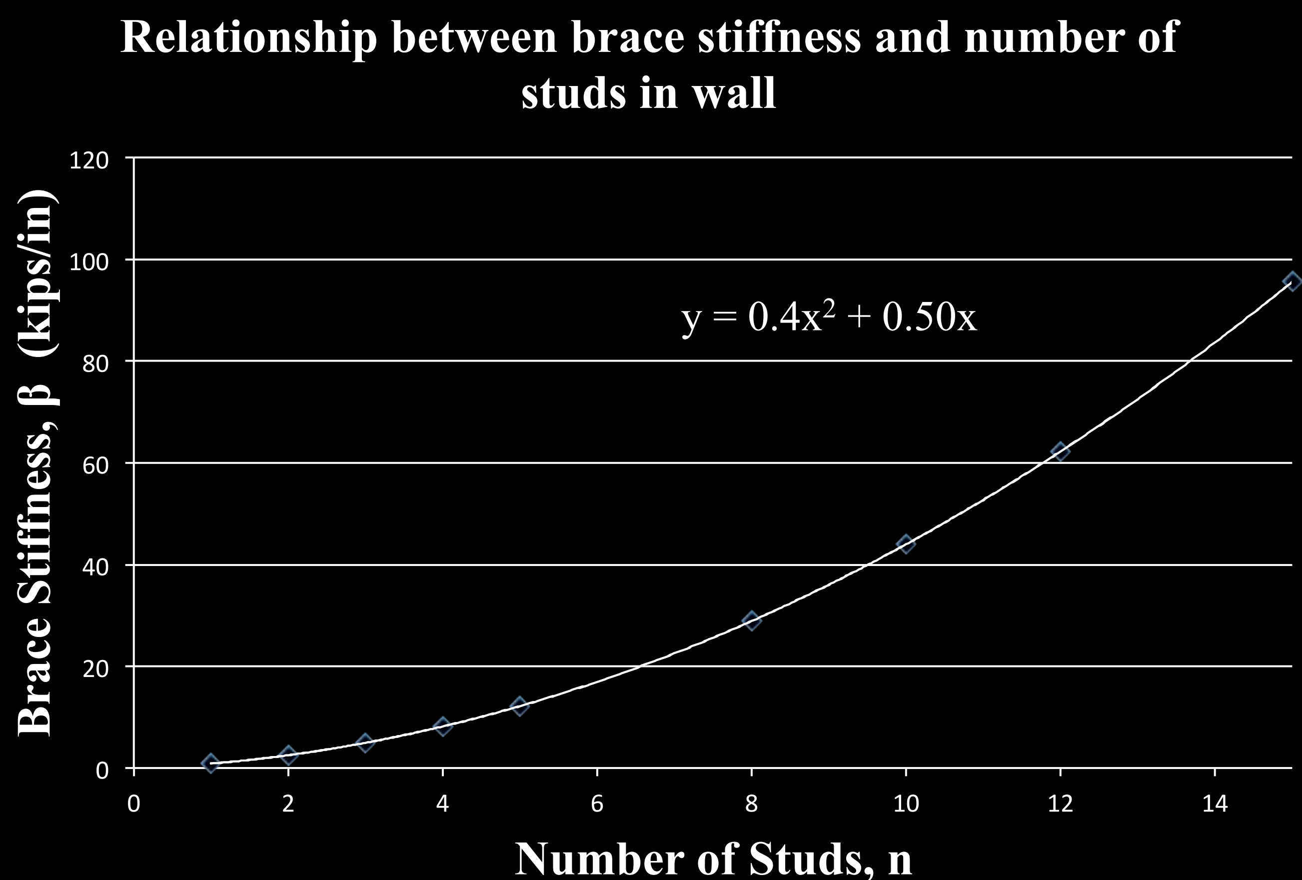
Some foreshadowing here.. Note, even $1.1\beta_i$ knocks forces down a lot…

Now turning to stud walls



(exaggerated displaced shape)

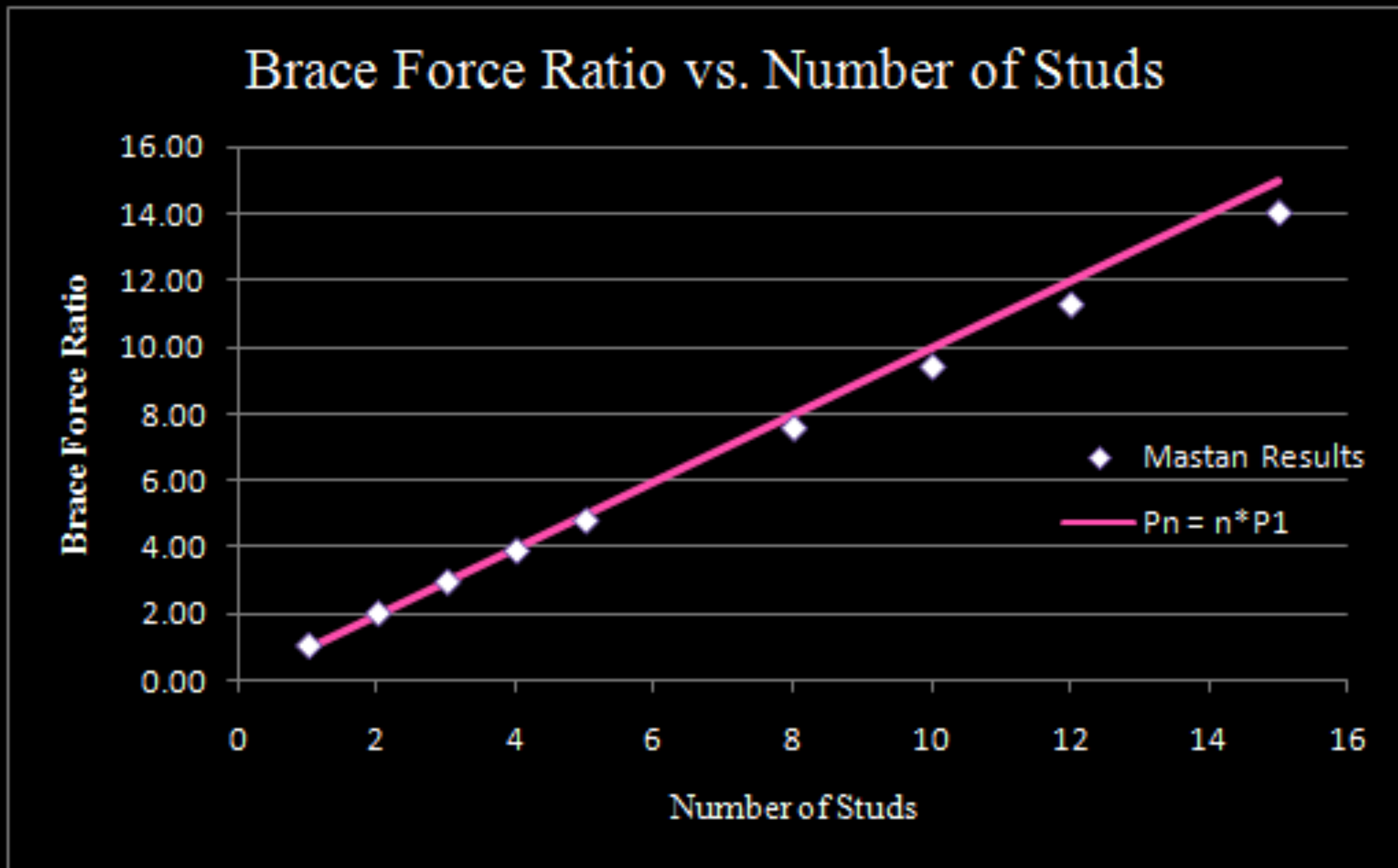
Recreated ideal brace stiffness expression of Sputo et al.



$$\beta_{br,n} = \beta_{br,1}[0.4n_s^{\,2} + \ 0.5n_s] \qquad \text{for } n_s > 1$$

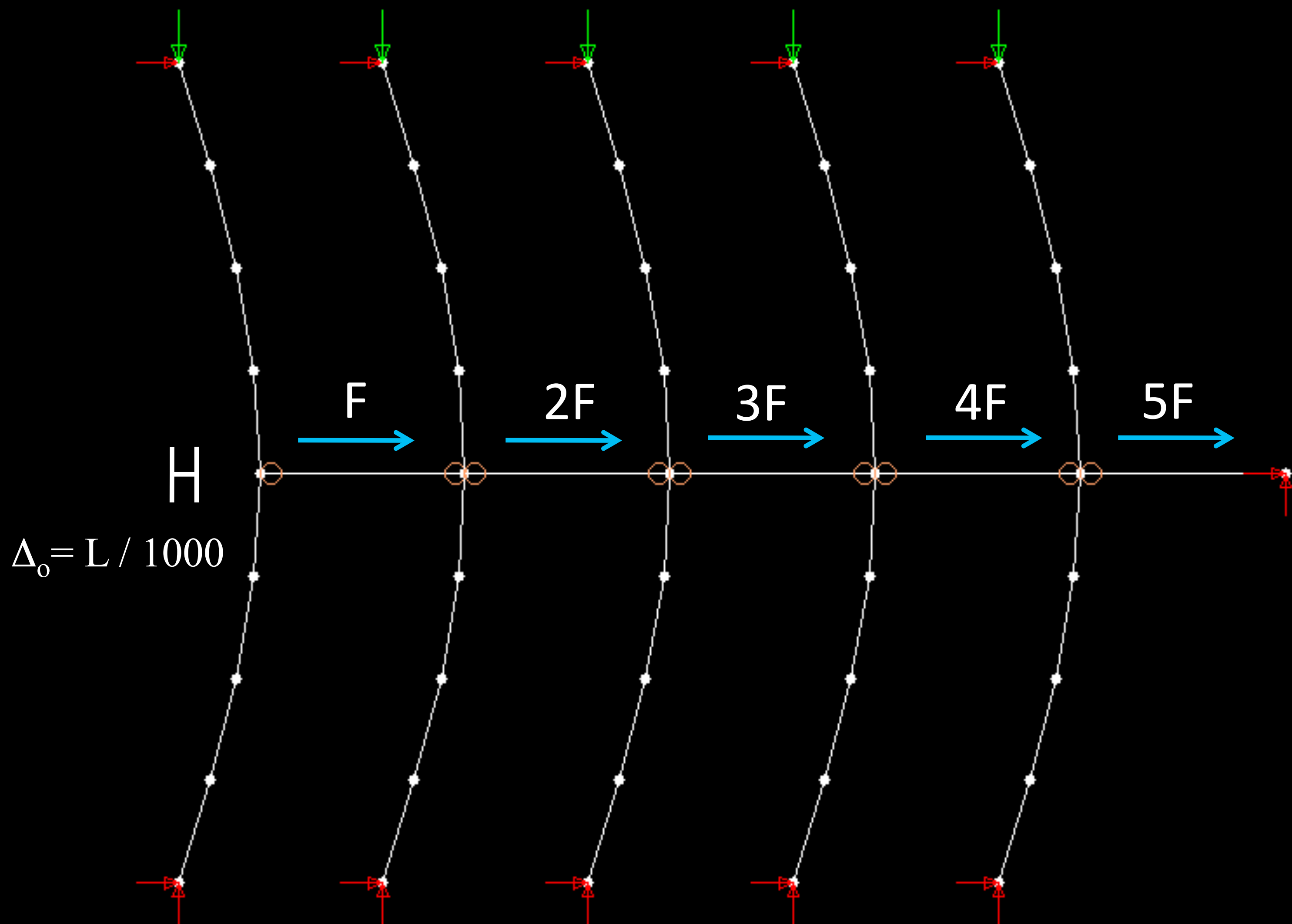
To recreate, the brace must be continuous and the same as sections in Sputo et al.

Observed linear accumulation

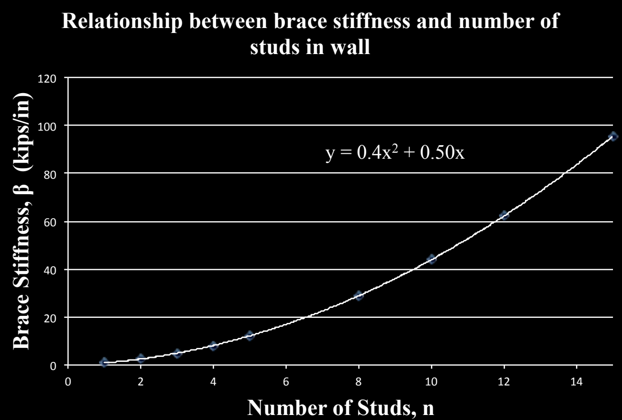


$$P_{br,n} = [n_s]P_{br,1}$$

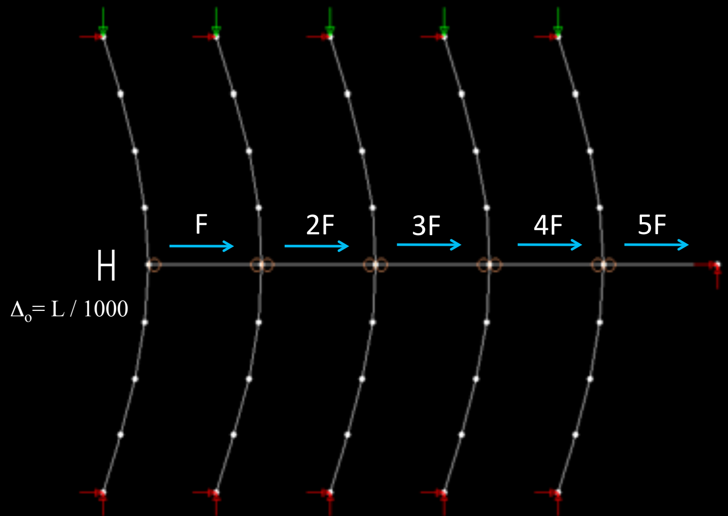
Accumulation-imperfection relation



Origin of the stiffness “blowup”



$$\beta_{br,n} = \beta_{br,1}[0.4n_s^2 + 0.5n_s] \quad \text{for } n_s > 1$$



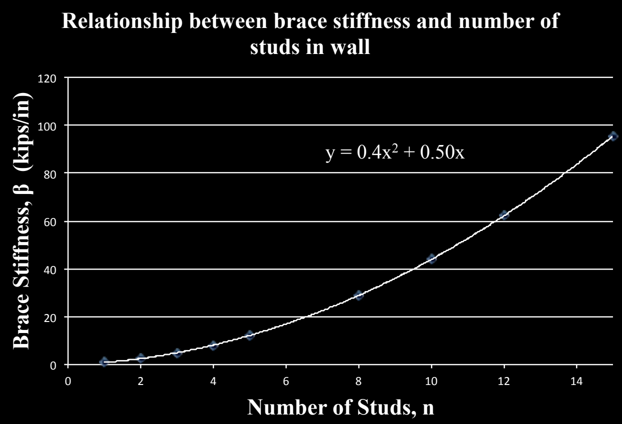
The first stud doesn’t need more stiffness.. It just needs the stiffness it wanted before, but now all the other studs deflect too…

$$\frac{F}{k_{eq}} = \frac{F}{k} + \frac{2F}{k} + \frac{3F}{k} + \frac{4F}{k} + \frac{5F}{k}$$

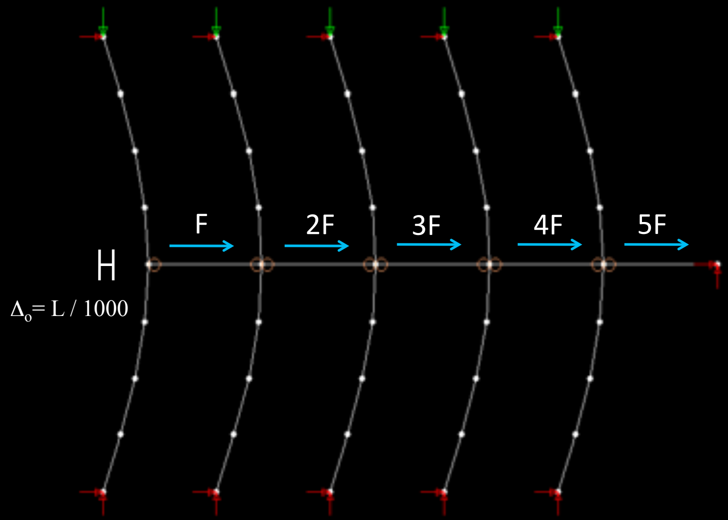
$$k_{eq} = (1+2+3+4+5)^{-1} k = (1/15)k$$

$$k_{eq} = \left(\sum_{i=1}^{nstuds} i\right)^{-1} k$$

Origin of the stiffness "blowup"



$$\beta_{br,n} = \beta_{br,1}[0.4n_s^2 + 0.5n_s] \quad \text{for } n_s > 1$$



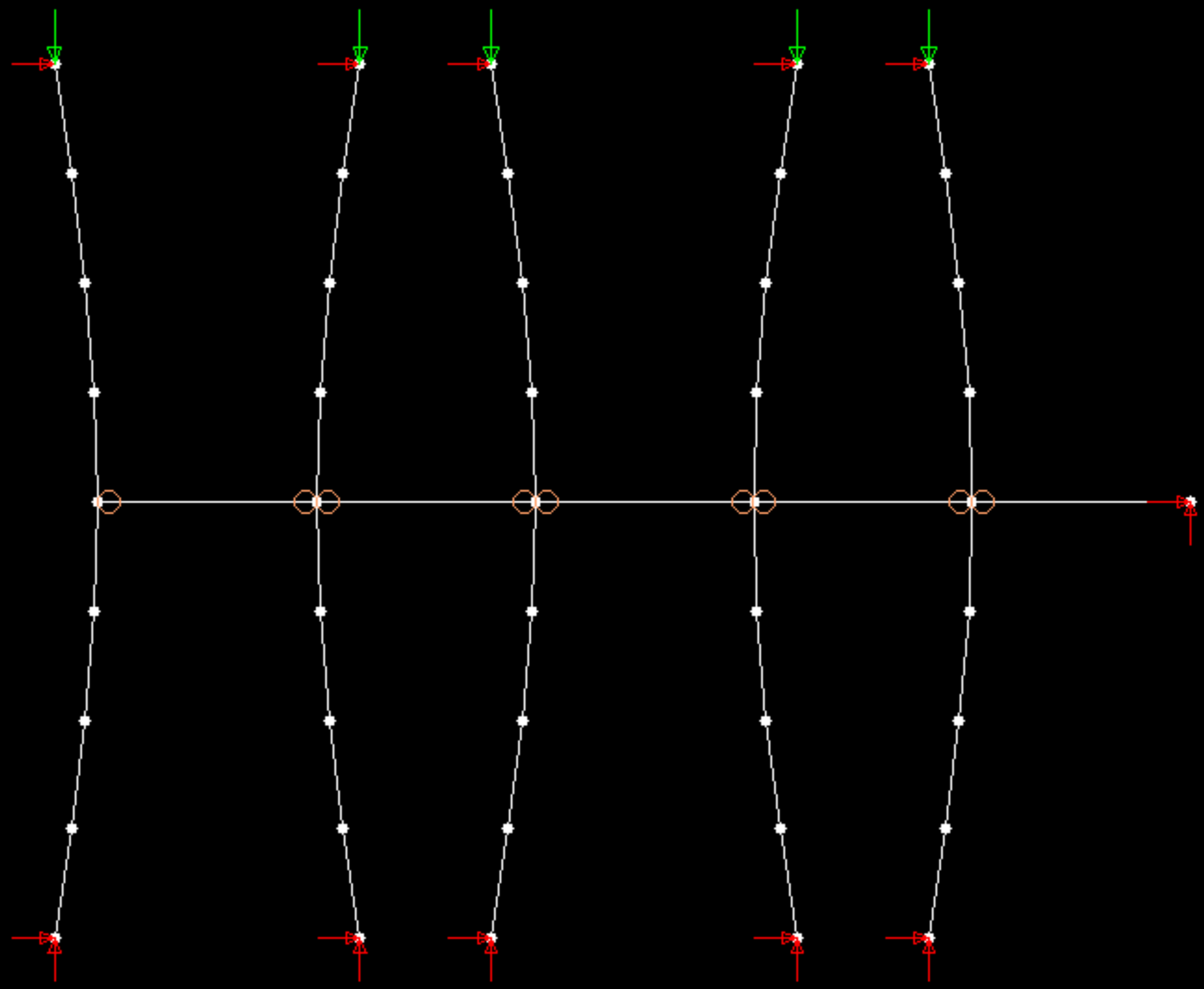
NO avoiding the blowup in ideal stiffness..

$$\beta_{br,n} = \beta_{br,1}\left(0.4n_s^2 + 0.5\right)$$ Sputo et al.

$$\beta_{br,n} = \beta_{br,1}\left(\sum_{i=1}^{n_s} i\right)$$ Present expression

They are nearly identical, but present expression, not detail dependent

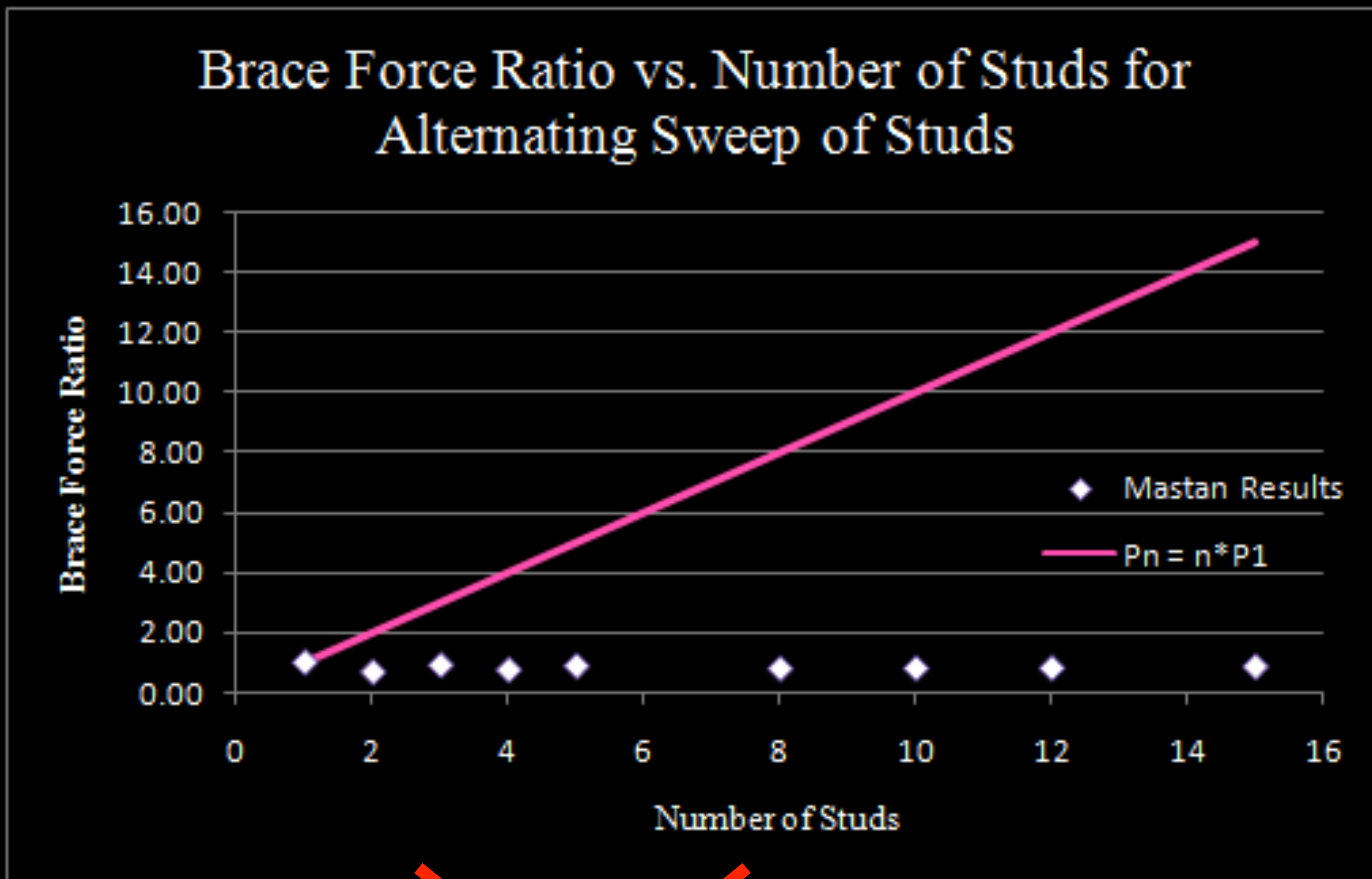
But,… what if the imperfections cancel?



Set **β=2βi for system**, Reverse sweep every other stud,
run 2[nd] order analysis….

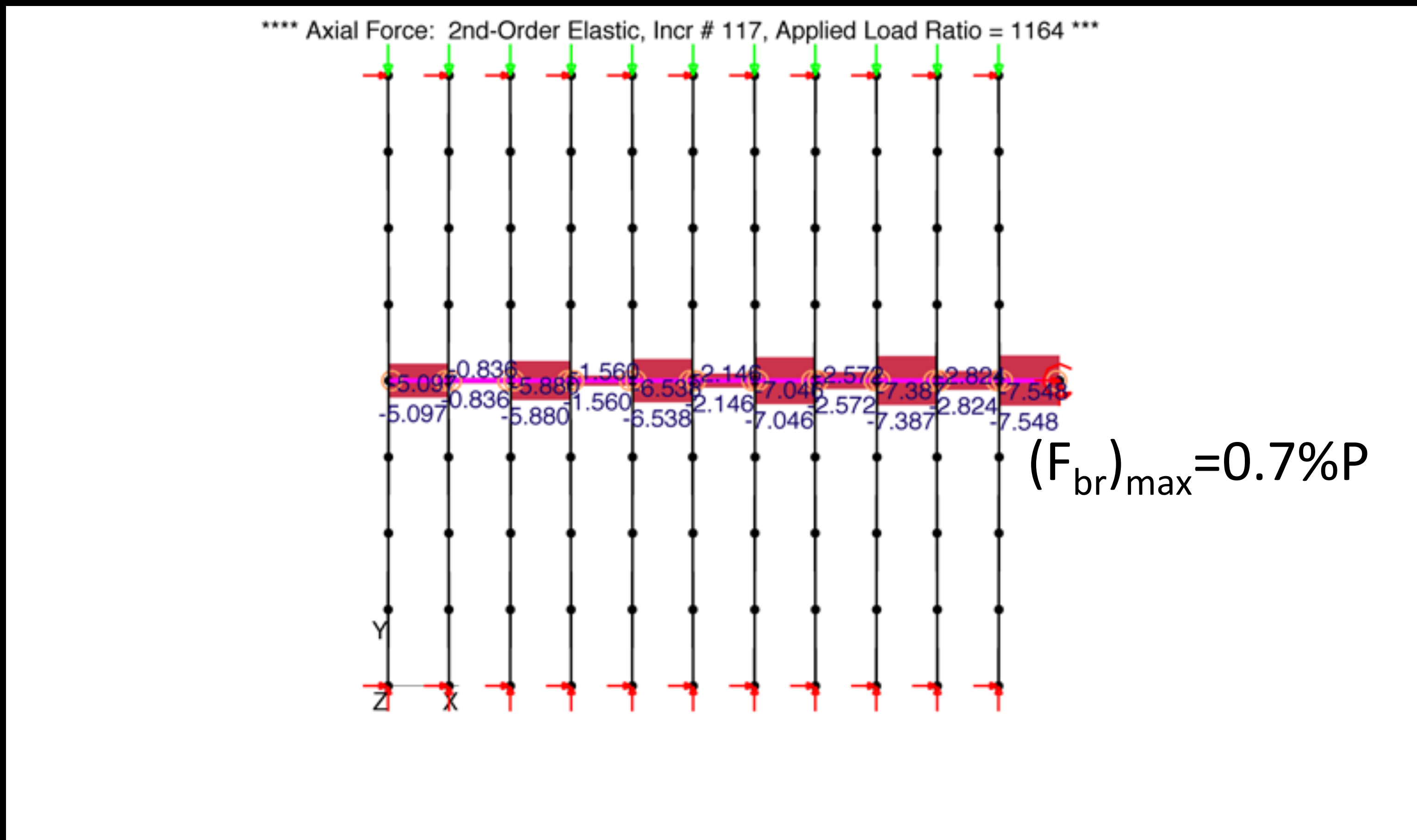
Credit to Steve Walker for planting the importance of this thought in my head in the past.

No accumulation of force



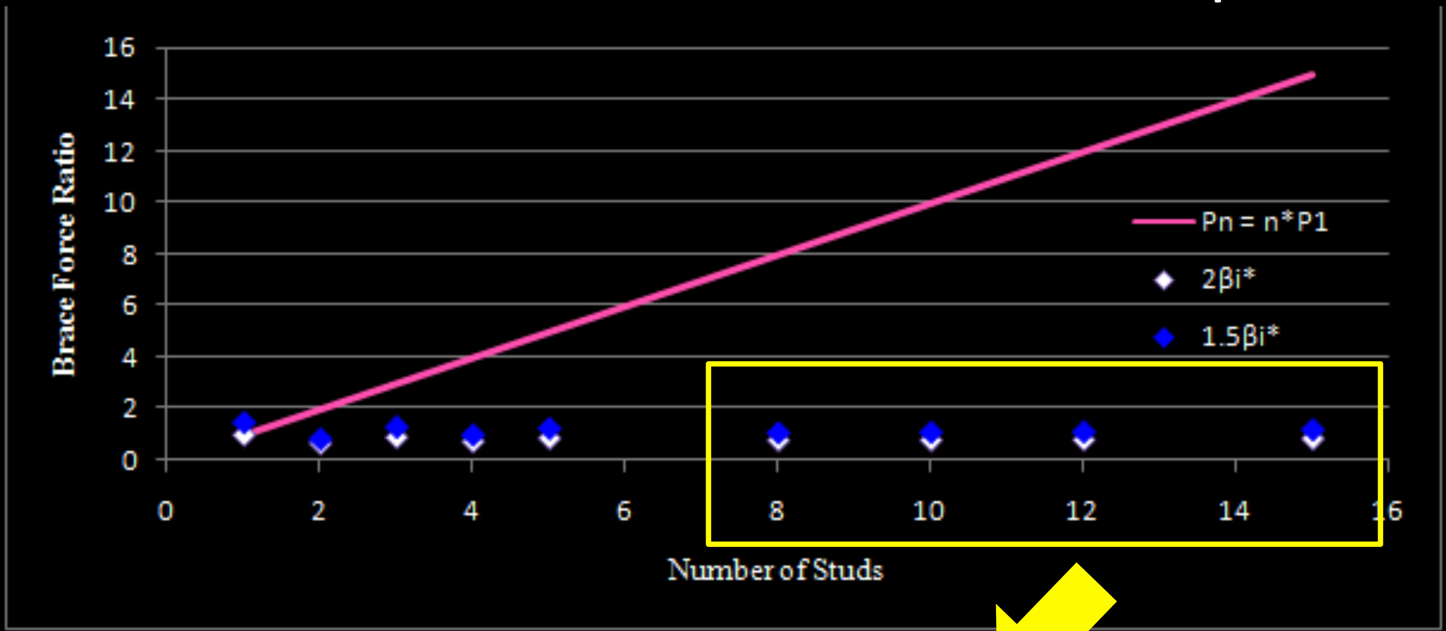
$$\cancel{P_{br,n} = [n_s]P_{br,1}}$$

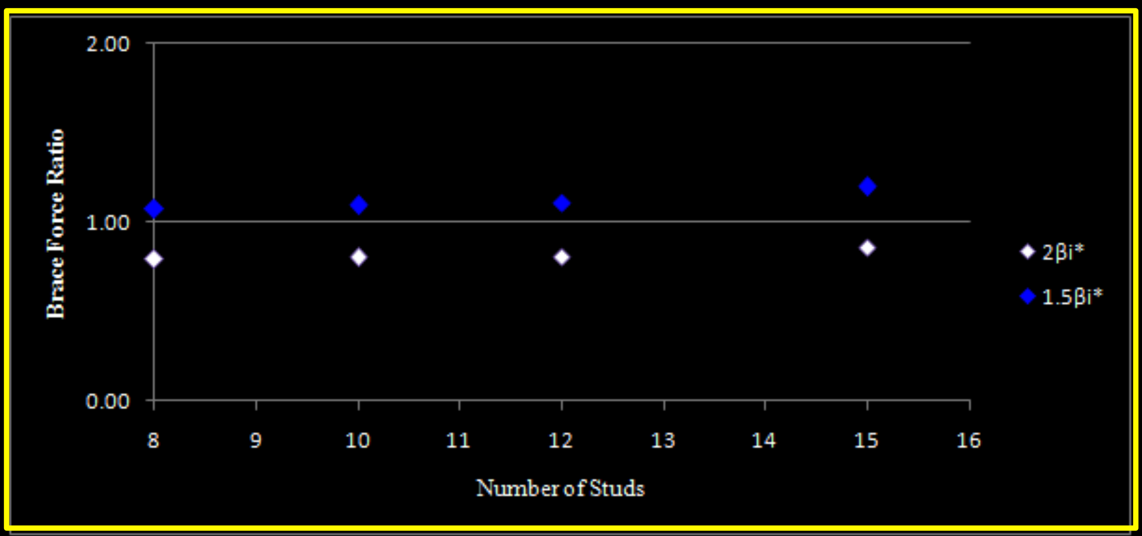
Example of brace forces ($\beta=2\beta i$)



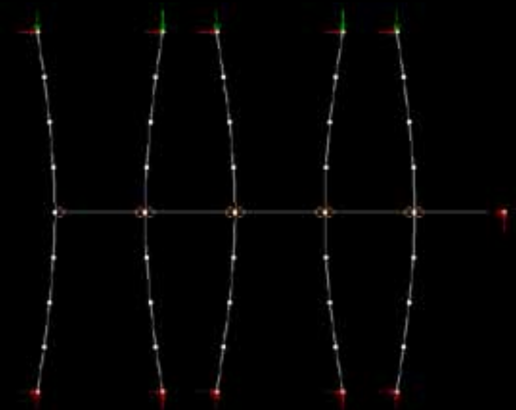
If system is stiff enough ($2\beta_i$) then imperfections rule and in this case can cancel each other out resulting in very mild brace forces.
(Note for imperfections all in 1 direction $(F_{br})_{max}$=10*0.8%P=8%P)

Could I go below $2\beta_i$?

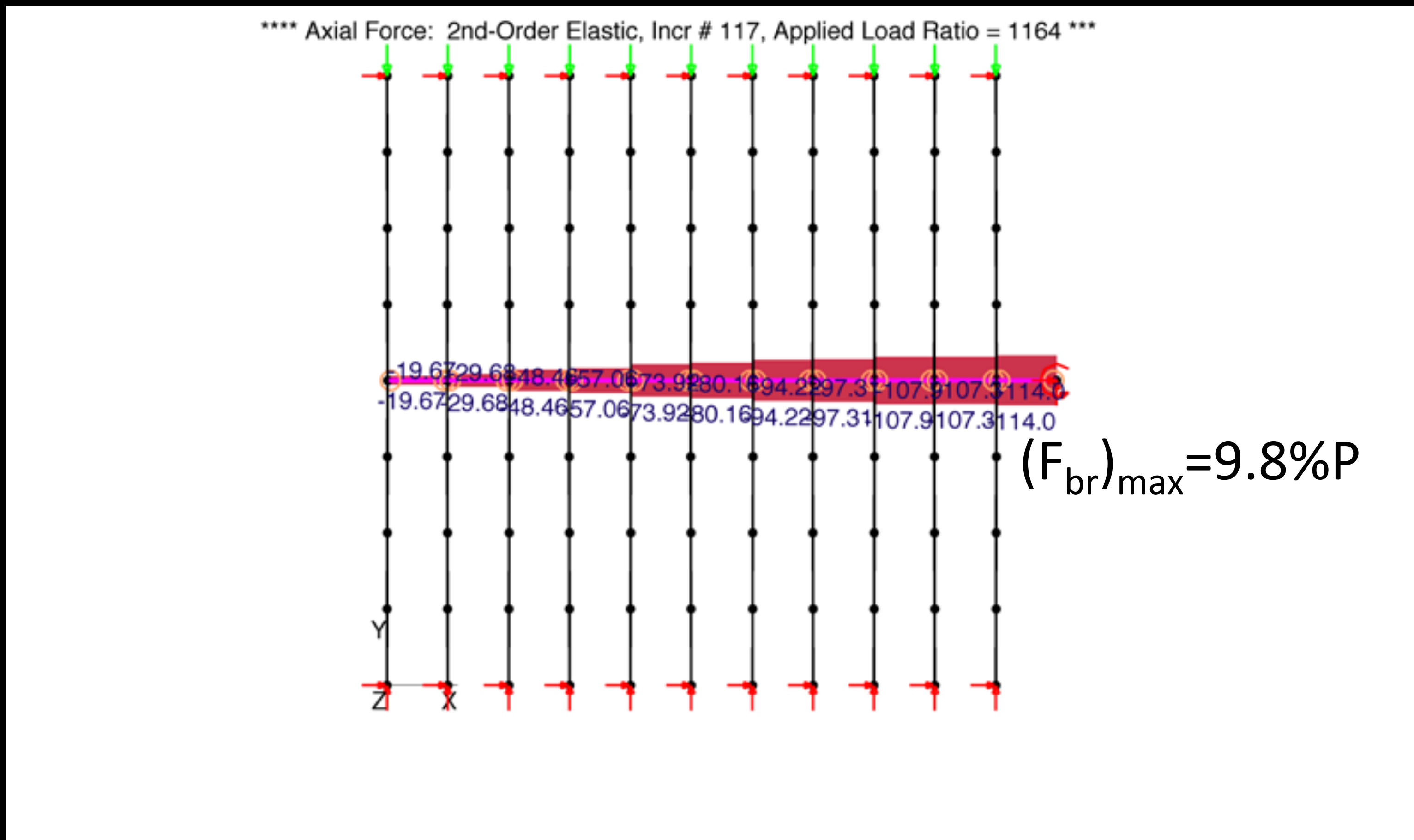




Weaken the brace?
You could…



Example of brace forces (β=βi)



$(F_{br})_{max}$=9.8%P

You can take the stiffness all the way down to βi without having the brace forces blowup! (Special to +/- imp case).

(Note for imperfections all in 1 direction $(F_{br})_{max}$=Infinity….)

Systems of studs

- Ideal stiffness does not change, but in the system providing that stiffness is more challenging because of relative displacements
- Simple expression exists for the ideal stiffness (it is based on single stud solution), its real, and must be used to achieve braced solutions
- Brace force magnitude comes from following sources
  - Stiffness
    - Must be above ideal stiffness (β_i) or results blow up. The higher we go above β_i the lower the brace forces, mandatory use of $2\beta_i$ may not make as much sense for systems as it does for single studs can be too much to achieve, may be willing to compromise on force.
  - Accumulation
    - Not as straightforward as previously thought as it is imperfection direction dependent. The two limits are:
      - Imperfection direction aligned = linearly accumulate forces
      - Imperfection direction cancels out = force from 1 brace only
  - Imperfection magnitude
    - Brace forces scale with imperfection magnitude (as always)

Chasing imperfection magnitude

- L/1000 leads to 1%P (actually 0.8%P) (at 2βi)
- Many studs at L/1000 leads to n*1%P
- As n increases, even if sweep all the same, the max bracing force comes from summation of the imperfections
- Summation of L/1000 increasingly conservative for each stud added
- What should the imperfection level be?

Imperfection for multiple studs

- If we take measured data from Zeinoddini et al. we know the (small) probability of exceeding L/1000.
- For n studs where the imperfections add up, let's use the same (small) probability on the summed imperfections, results in:

$$\delta_o = 1.69 \frac{L/3054}{\sqrt{n}} + L/2242$$

example, $n=10$, $\delta_o = L/1610$, $F_{br} = 0.5\%P$ (down from 0.8%P at L/1000)

Questions questions questions

- How high do we want the stiffness to be?
  It must be greater than ideal, but are we prisoners to the 2*ideal assumption?
- Is a variable brace force ok?
  If we allow variable stiffness we need to allow for variable brace force (this is doable)
- Account for imperfection direction?
  Accumulation of forces is between linear and none depending on this effect!
  Practice tells us what about direction?
- Account for imperfection magnitude?
  Accumulation of forces is summation of imperfections across studs, account for this system benefit?