CFSEI
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TECHNICAL NOTE

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On Cold-Formed Steel Construction

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DESIGN AIDS AND EXAMPLES FOR DISTORTIONAL BUCKLING

Summary: The objective of this Tech Note is to provide design examples and design aids specific to cold-formed steel framing systems that address the new distortional buckling limit states added to AISI-S100 in the 2007 edition. In addition, a method is provided for including rotational restraint, provided by sheathing to members, in the design calculations for distortional buckling. This method has been proposed for the next edition of AISI-S210 (floors and roofs) and AISI-S211 (walls studs) standards and partially mitigates the reduced capacity in the distortional buckling limit state.

INTRODUCTION

The latest edition of AISI-S100 (2007) has added new design checks for distortional buckling of cold-formed steel members in bending (Section C3.1.4) and compression (Section C4.2). As presented in the AISI-S100 commentary, distortional buckling is a mode of buckling in which the lip stiffener is insufficient to retard the compression flange and attached web from becoming unstable. The in-plane deformations that occur in distortional buckling are contrasted with those of local and lateral-torsional buckling for a member in bending in Figure 1.

DISTORTIONAL BUCKLING IN BENDING OR COMPRESSION

For many conventional bending members (e.g., floor joists) distortional buckling may now control the design strength (i.e., the provisions of C3.1.4 for distortional buckling provide a smaller predicted capacity than those of C3.1.1 for the nominal section strength or C3.1.2 for lateral-torsional buckling). This is particularly true for joists designed with a continuously braced design philosophy - in that case lateral-torsional buckling is fully restricted and only the local buckling (effective width) reductions are applied to the member. Unfortunately, checking distortional buckling to determine if it controls the design capacity can require significant effort. To simplify this process design aids and design examples are provided in this Tech Note.

Compression members are also subject to distortional buckling and must be checked per the provisions of C4.2 in AISI-S100. However, in compression, even when a sheathing braced design philosophy is detailed and flexural and flexural-torsional buckling are essentially restricted, local buckling of SSMA sections commonly provides a capacity lower than

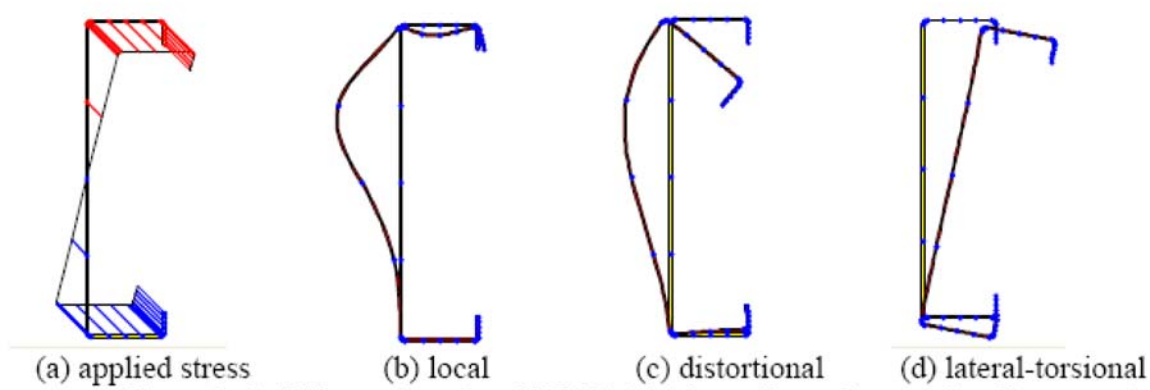


Figure 1: Stability modes of an 800S200-54 joist under major-axis bending stress*

* the applied stress is for major-axis bending where the load is applied through the shear center or bracing/sheathing/blocking has restricted the rotation of the member adequately such that the classical My/I bending stresses result. This implies warping stresses due to torsion are ignored.

FIGURE 1

distortional buckling. Thus, the focus of this Technical Note is on bending members, though the design aid does apply to compression members.

DETERMINING THE ELASTIC DISTORTIONAL BUCKING STRESS

The key step in the AISI-S100 distortional buckling provisions is the determination of the elastic distortional buckling stress, F_d. For standard SSMA sections F_d is tabled using the applicable AISI-S100 provisions in the design aid of this Note. Alternatively, as demonstrated in the design example, rational elastic buckling analysis for determining F_d may be performed using freely available open source software. However, the F_d reported in the design aid ignores a key benefit of typical cold-formed steel framing systems: resistance to distortional buckling provided by attached sheathing.

The AISI-S100 provisions for distortional buckling provide a means to include a supplemental rotational restraint, k_ϕ, in the prediction equations. However, little guidance is provided on what value to use for this stiffness. Recently, through AISI-COFS funding, Schafer et al. (2007, 2008) tested a variety of common sheathing details and proposed a design method for determination of k_ϕ. This design method, provided in the following sections and detailed in the design example, is recommended for use as a rational engineering analysis in the determination of k_ϕ until such time as it is adopted in the COFS standards.

PROPOSED METHOD FOR DETERMINING K_ϕ

The following provides the method for determining the rotational stiffness in "proposed" Specification language:

Calculation of the nominal distortional buckling strength in flexure per C3.1.4 of AISI S100, or per Appendix 1 of AISI S100 may utilize the beneficial system affect of sheathing fastened to the compression flange of *floor joists, ceiling joists, roof rafters, or wall studs* through the calculation of the rotational stiffness provided to the bending member, k_ϕ.

Calculation of the nominal distortional buckling strength in compression per C4.2 of AISI S100, or per Appendix 1 of AISI S100 may utilize the beneficial system affect of sheathing fastened to both flanges of *floor joists, ceiling joists, roof rafters, or wall studs* through the calculation of the rotational stiffness provided to the bending member, k_ϕ.

The rotational stiffness k_f shall be determined via

$$k_\phi = (1/k_{\phi w} + 1/k_{\phi c})^{-1} \quad (1)$$

where the sheathing rotational restraint $k_{\phi w}$ is calculated

for interior members (joists or rafters) with sheathing fastened on both sides as

$$k_{\phi w} = EI_w/L_1 + EI_w/L_2 \quad (2)$$

for exterior members, or members with sheathing fastened on one side as

$$k_{\phi w} = EI_w/L_1 \quad (3)$$

and:

EI_w = sheathing bending rigidity,

for plywood and OSB use APA (2004) as given in Table 1(a),

for gypsum board use min values of GA (2001) as given in Table 1(b);

note, gypsum may be used for serviceability, but not for ultimate strength

L_1, L_2 = one half the joist spacing to the first and second sides respectively, as illustrated in Figure 2

where the connection rotational restraint $k_{\phi c}$ is calculated

for fasteners spaced 12 in. o.c. or closer in plywood, OSB, or gypsum

$$k_{\phi c} = \text{values per Table 2} \quad (4)$$

PRACTICAL GUIDANCE REGARDING DISTORTIONAL BUCKLING

It is important to note, that even with the additional guidance provided here distortional buckling may still control the design strength of some commonly used SSMA sections, particularly in bending. The primary variable for improving distortional buckling resistance is a longer lip stiffener, but this is outside of the engineer's control.

Increasing k_ϕ with the goal of removing distortional buckling may also be impractical. The rotational restraint provided by the sheathing is commonly limited by the flange-to-sheathing connection stiffness ($k_\phi c$). This restraint is primarily influenced by the thickness of the member, and thus is not easily increased without significant cost. A fastener spacing tighter than 12 in. o.c. was shown to increase k_ϕ (Schafer et al 2007) but the testing was too limited to generalize the results.

Increased member thickness increases restraint, k_ϕ, and the distortional buckling stress, F_d. However, increased member thickness also increases local buckling resistance - typically at a faster rate. Thus, thicker members are often more likely to have a distortional buckling resistance which is less than the local buckling resistance.

TABLE 1: SHEATHING BENDING RIGIDITY

(a) Plywood and OSB bending rigidity per APA, Panel Design Spec. (2004)
divide table values by 12 to convert to lbf-in.^{2}/in. of panel width

RATED PANELS DESIGN CAPACITIES

Span Rating	Stress Parallel to Strength Axis				Stress Perpendicular to Strength Axis			
	Plywood				Plywood			
	3-ply	4-ply	5-ply	OSB	3-ply	4-ply	5-ply	OSB
PANEL BENDING STIFFNESS, EI (lb-in.^{2}/ft of panel width)								
24/0	66,000	66,000	66,000	60,000	3,600	7,900	11,000	11,000
24/16	86,000	86,000	86,000	78,000	5,200	11,500	16,000	16,000
32/16	125,000	125,000	125,000	115,000	8,100	18,000	25,000	25,000
40/20	250,000	250,000	250,000	225,000	18,000	39,500	56,000	56,000
48/24	440,000	440,000	440,000	400,000	29,500	65,000	91,500	91,500
16oc	165,000	165,000	165,000	150,000	11,000	24,000	34,000	34,000
20oc	230,000	230,000	230,000	210,000	13,000	28,500	40,500	40,500
24oc	330,000	330,000	330,000	300,000	26,000	57,000	80,500	80,500
32oc	715,000	715,000	715,000	650,000	75,000	165,000	235,000	235,000
48oc	1,265,000	1,265,000	1,265,000	1,150,000	160,000	350,000	495,000	495,000

(b) Gypsum board bending rigidity Gypsum Assoc., GA-235-01 (2001)

Effective Stiffness (EI)* (typical range)		
Board Thickness (in.)	Lb•in^2/in of width	N•mm^2/mm of width
1/2	1500 to 4000	220,000 to 580,000
5/8	3000 to 8000	440,000 to 1,160,000

* EI is dependent on board density, relative humidity, type of board, paper type, direction of board during testing and the amount of handling prior to measurement. In general the value of EI follows the following relationships:
Type X Gypsum Board > Regular Gypsum Board
Denser Gypsum Board > Less Dense Gypsum Board
Machine Direction > Cross Direction
Low Relative Humidity > High Relative Humidity

FIGURE 2: ILLUSTRATION OF L1, L2 FOR SHEATHING ROTATIONAL RESTRAINT

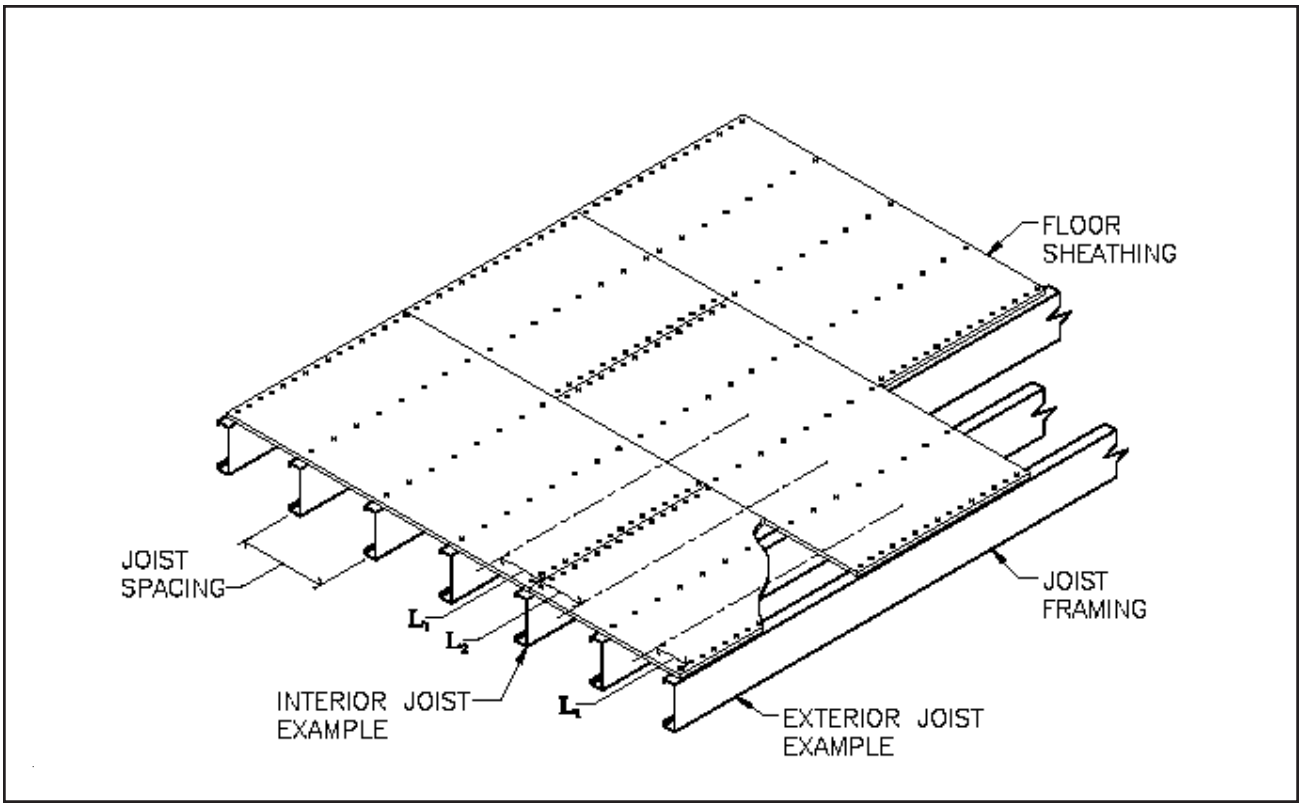
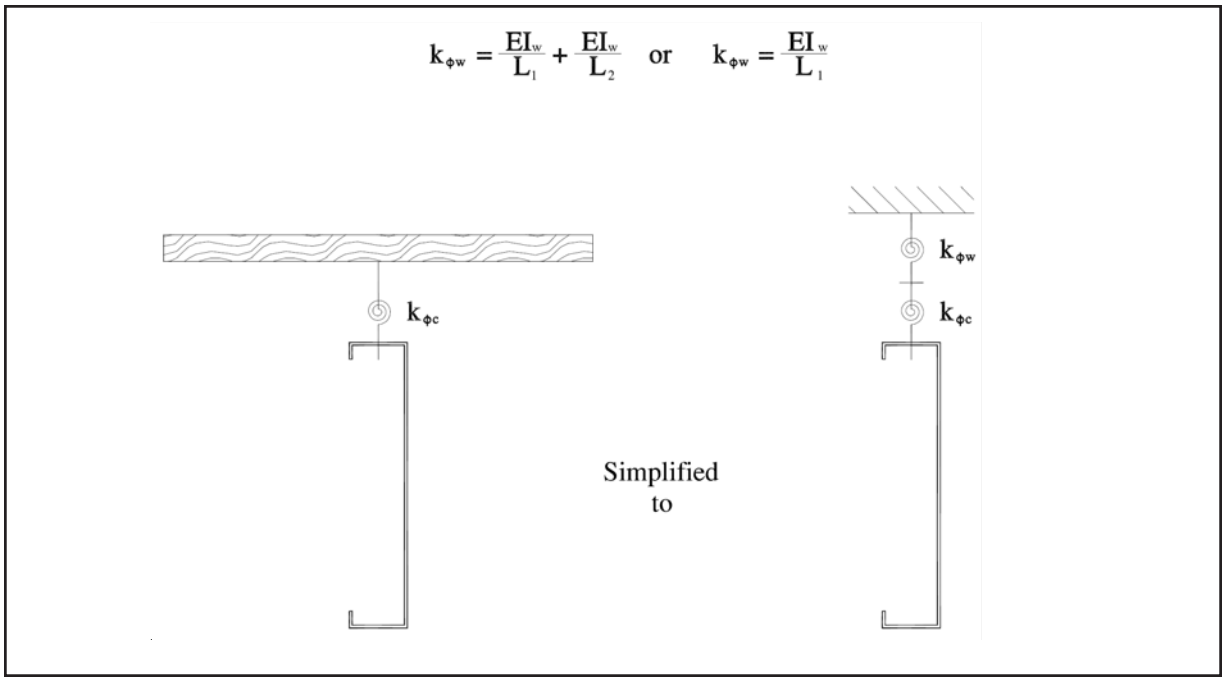


FIGURE 2: ILLUSTRATION OF L1, L2 FOR SHEATHING ROTATIONAL RESTRAINT



Engineers should be aware that currently there is a discontinuity between the application of resistance (ϕ) factors for beams in AISI-S100-07 that relates to distortional buckling. A fully effective beam per C3.1.1 of AISI-S100 uses a capacity of ϕM_y where ϕ=0.95, and M_y is the yield stress. The same beam has a maximum capacity per the distortional buckling provisions of C3.1.4 of ϕM_y where ϕ=0.90. Thus, the capacity is always limited to $0.9M_y$. (A similar discontinuity does not exist for safety factors, Ω, which uses 1.67 throughout). Correcting this discontinuity is being considered by the AISI-COS, but for now engineers should be aware that <u>all</u> fully effective beams are limited to $0.9M_y$ instead of $0.95M_y$ in AISI-S100-07 due to the distortional buckling provisions of C3.1.4.

t (mils)	t (in.)	$k_{\phi c}$ (lbf-in./in./rad)	$k_{\phi c}$ (N-mm/mm/rad)
18	0.018	78	348
27	0.027	83	367
30	0.03	84	375
33	0.033	86	384
43	0.043	94	419
54	0.054	105	468
68	0.068	123	546
97	0.097	172	766
(1) fasteners spaced 12 in. o.c. or less			
(2) values based on $k_{\phi c} = 0.00035Et^2 + 75$			
with E in psi, t in in., $k_{\phi c}$ in lbf-in./in./rad			

TABLE 2: CONNECTION ROTATIONAL RESTRAINT

DESIGN AID (TABLE)

A series of design tables to aid in the distortional buckling calculation of standard SSMA shapes have been generated. The tables follow the provisions of C3.1.4(b) for bending and C4.2(b) for compression.

DESIGN EXAMPLE

To facilitate understanding of the new distortional buckling provisions a design example has been prepared. The example provides the flexural capacity of an 800S200-54 (50ksi) per the AISI-S100 provisions and using the supplementary information regarding rotational restraint k_ϕ as provided in this Note.

CONCLUSIONS

The latest edition of AISI-S100 (2007) requires engineers to check distortional buckling as a potential limit state for compression and bending members. The new provisions are anticipated to limit the capacity in some common situations, particularly for bending members. To simplify the design check for distortional buckling a Design Aid has been provided. In addition, to account for the beneficial influence of sheathing connected to a member, a recently proposed method for determining rotational restraint against distortional buckling is summarized. A design example applying the distortional buckling provisions, and including the proposed method for including rotational restraint in distortional buckling calculations, is also provided.

TABLE 3: DISTORTIONAL BUCKLING DESIGN AID

Section	Beam distortional buckling per C3.1.4(b) with β=1, k_ϕ=0						Column distortional buckling per C4.2(b) with β=1, k_ϕ=0					
	L_{cr} (in.)	$k_{\phi fe}$ (kip)	$\tilde{k}_{\phi fg}$ (in^2)	$k_{\phi we}$ (kip)	$\tilde{k}_{\phi wg}$ (in^2)	F_d (ksi)	L_{cr} (in.)	$k_{\phi fe}$ (kip)	$\tilde{k}_{\phi fg}$ (in^2)	$k_{\phi we}$ (kip)	$\tilde{k}_{\phi wg}$ (in^2)	F_d (ksi)
250S137-33	10.6	0.158	0.0046	0.142	0.00014	63.3	11.7	0.109	0.0038	0.090	0.00065	45.0
250S137-43	9.2	0.366	0.0078	0.321	0.00024	85.6	10.2	0.254	0.0063	0.198	0.00112	60.5
250S137-54	8.1	0.759	0.0122	0.647	0.00038	112.0	9.0	0.532	0.0099	0.392	0.00182	78.6
250S137-68	7.1	1.613	0.0191	1.329	0.00062	149.1	7.9	1.143	0.0156	0.783	0.00296	103.7
250S162-33	14.3	0.150	0.0046	0.139	0.00008	62.2	15.8	0.102	0.0037	0.090	0.00036	47.0
250S162-43	12.4	0.343	0.0077	0.310	0.00013	83.1	13.7	0.236	0.0063	0.198	0.00062	62.6
250S162-54	10.9	0.703	0.0121	0.620	0.00021	107.2	12.1	0.486	0.0099	0.392	0.00099	80.6
250S162-68	9.6	1.469	0.0191	1.258	0.00034	140.1	10.7	1.024	0.0156	0.783	0.00161	104.9
350S162-33	15.5	0.109	0.0039	0.101	0.00018	52.1	17.2	0.075	0.0031	0.064	0.00083	34.9
350S162-43	13.5	0.251	0.0065	0.227	0.00030	70.0	14.9	0.173	0.0053	0.142	0.00143	46.5
350S162-54	11.9	0.517	0.0102	0.458	0.00049	90.8	13.2	0.359	0.0084	0.280	0.00230	59.9
350S162-68	10.5	1.087	0.0162	0.938	0.00079	119.6	11.6	0.763	0.0132	0.560	0.00374	78.1
362S137-33	11.6	0.112	0.0038	0.102	0.00034	51.5	12.9	0.078	0.0031	0.062	0.00163	29.4
362S137-43	10.1	0.262	0.0065	0.233	0.00059	70.3	11.1	0.183	0.0053	0.137	0.00284	39.5
362S137-54	8.9	0.547	0.0101	0.478	0.00094	92.8	9.8	0.387	0.0083	0.271	0.00458	51.2
362S137-68	7.8	1.171	0.0158	0.998	0.00151	125.0	8.6	0.840	0.0130	0.541	0.00748	67.5
362S162-33	15.7	0.106	0.0038	0.098	0.00019	51.1	17.3	0.072	0.0031	0.062	0.00090	33.6
362S162-43	13.6	0.243	0.0064	0.221	0.00033	68.7	15.0	0.168	0.0052	0.137	0.00156	44.8
362S162-54	12.0	0.501	0.0101	0.445	0.00053	89.2	13.3	0.349	0.0082	0.271	0.00250	57.7
362S162-68	10.6	1.055	0.0159	0.912	0.00085	117.6	11.7	0.741	0.0130	0.541	0.00407	75.2
362S200-33	20.7	0.102	0.0042	0.096	0.00011	46.2	22.9	0.069	0.0034	0.062	0.00052	33.5
362S200-43	17.9	0.233	0.0071	0.214	0.00019	61.5	19.9	0.159	0.0058	0.137	0.00089	44.4
362S200-54	15.9	0.474	0.0111	0.428	0.00031	79.1	17.6	0.326	0.0091	0.271	0.00143	56.9
362S200-68	14.0	0.985	0.0175	0.868	0.00049	102.8	15.5	0.683	0.0143	0.541	0.00232	73.5
400S137-33	11.9	0.102	0.0036	0.094	0.00044	48.2	13.2	0.071	0.0030	0.056	0.00209	25.1
400S137-43	10.3	0.239	0.0061	0.215	0.00075	66.0	11.4	0.168	0.0050	0.124	0.00364	33.7
400S137-54	9.1	0.501	0.0096	0.443	0.00120	87.4	10.1	0.356	0.0079	0.245	0.00588	43.7
400S137-68	8.0	1.075	0.0151	0.932	0.00192	118.2	8.8	0.774	0.0123	0.490	0.00960	57.6
400S162-33	16.1	0.096	0.0036	0.089	0.00025	48.2	17.8	0.066	0.0029	0.056	0.00115	29.8
400S162-43	13.9	0.222	0.0061	0.202	0.00042	64.9	15.4	0.154	0.0050	0.124	0.00200	39.7
400S162-54	12.3	0.458	0.0096	0.408	0.00067	84.4	13.6	0.319	0.0078	0.245	0.00321	51.1
400S162-68	10.8	0.965	0.0151	0.841	0.00109	111.5	12.0	0.680	0.0123	0.490	0.00523	66.5
400S200-33	21.2	0.093	0.0040	0.087	0.00014	43.8	23.4	0.063	0.0032	0.056	0.00066	30.5
400S200-43	18.4	0.212	0.0067	0.195	0.00025	58.4	20.4	0.145	0.0055	0.124	0.00114	40.5
400S200-54	16.3	0.432	0.0106	0.391	0.00039	75.1	18.0	0.298	0.0086	0.245	0.00183	51.9
400S200-68	14.4	0.899	0.0167	0.794	0.00063	97.8	15.9	0.625	0.0136	0.490	0.00297	67.1
550S162-33	17.4	0.072	0.0031	0.067	0.00054	38.5	19.3	0.049	0.0025	0.041	0.00255	17.8
550S162-43	15.1	0.166	0.0052	0.154	0.00092	52.2	16.7	0.115	0.0043	0.090	0.00443	23.7
550S162-54	13.3	0.344	0.0082	0.316	0.00146	68.5	14.8	0.242	0.0067	0.178	0.00712	30.4
550S162-68	11.7	0.729	0.0129	0.660	0.00234	91.4	13.0	0.518	0.0105	0.356	0.01159	39.5
600S137-33	13.2	0.071	0.0030	0.068	0.00117	33.7	14.6	0.050	0.0024	0.037	0.00577	10.6
600S137-43	11.4	0.167	0.0050	0.161	0.00199	46.8	12.6	0.119	0.0041	0.083	0.01004	14.2
600S137-54	10.1	0.351	0.0078	0.339	0.00315	63.1	11.1	0.254	0.0064	0.163	0.01620	18.5
600S137-68	8.9	0.756	0.0122	0.735	0.00500	86.9	9.8	0.559	0.0101	0.326	0.02645	24.2
600S137-97	7.3	2.501	0.0236	2.497	0.01014	148.3	8.0	1.929	0.0198	0.947	0.05658	37.6
600S162-33	17.8	0.066	0.0029	0.063	0.00067	35.6	19.7	0.046	0.0024	0.037	0.00318	14.8
600S162-43	15.4	0.153	0.0050	0.144	0.00114	48.5	17.1	0.107	0.0041	0.083	0.00550	19.8
600S162-54	13.6	0.318	0.0078	0.296	0.00180	63.8	15.1	0.224	0.0064	0.163	0.00885	25.4
600S162-68	12.0	0.675	0.0123	0.621	0.00288	85.4	13.3	0.482	0.0101	0.326	0.01441	33.0
600S162-97	9.8	2.194	0.0244	1.998	0.00594	138.1	10.8	1.607	0.0201	0.947	0.03077	50.2
600S200-33	23.4	0.063	0.0032	0.060	0.00039	33.9	25.9	0.043	0.0026	0.037	0.00183	18.0
600S200-43	20.4	0.145	0.0055	0.135	0.00067	45.5	22.5	0.100	0.0045	0.083	0.00315	23.9
600S200-54	18.0	0.297	0.0086	0.274	0.00106	59.0	19.9	0.206	0.0070	0.163	0.00506	30.6
600S200-68	15.9	0.623	0.0136	0.564	0.00170	77.5	17.6	0.437	0.0111	0.326	0.00820	39.5
600S200-97	13.0	1.983	0.0273	1.743	0.00353	120.8	14.4	1.414	0.0224	0.947	0.01739	59.4
600S250-43	23.2	0.143	0.0073	0.133	0.00052	35.2	25.7	0.098	0.0060	0.083	0.00242	21.6
600S250-54	20.6	0.293	0.0115	0.267	0.00082	45.4	22.8	0.203	0.0094	0.163	0.00388	27.6
600S250-68	18.2	0.613	0.0182	0.546	0.00132	59.3	20.1	0.428	0.0149	0.326	0.00627	35.6
600S250-97	14.9	1.938	0.0367	1.664	0.00273	91.4	16.5	1.375	0.0300	0.947	0.01323	53.7

TABLE 3: DISTORTIONAL BUCKLING DESIGN AID (CONTINUED)

Section	Beam distortional buckling per C3.1.4(b) with β=1, k_ϕ=0						Column distortional buckling per C4.2(b) with β=1, k_ϕ=0					
	L_{cr} (in.)	$k_{\phi fe}$ (kip)	$\tilde{k}_{\phi fg}$ (in^2)	$k_{\phi we}$ (kip)	$\tilde{k}_{\phi wg}$ (in^2)	F_d (ksi)	L_{cr} (in.)	$k_{\phi fe}$ (kip)	$\tilde{k}_{\phi fg}$ (in^2)	$k_{\phi we}$ (kip)	$\tilde{k}_{\phi wg}$ (in^2)	F_d (ksi)
800S137-33	14.2	0.054	0.0026	0.056	0.00232	22.7	15.7	0.039	0.0021	0.028	0.01184	4.8
800S137-43	12.3	0.129	0.0043	0.136	0.00393	32.2	13.6	0.093	0.0035	0.062	0.02060	6.4
800S137-54	10.9	0.271	0.0067	0.293	0.00615	44.0	12.0	0.201	0.0056	0.122	0.03325	8.3
800S137-68	9.7	0.582	0.0103	0.653	0.00970	61.6	10.5	0.446	0.0087	0.245	0.05431	11.0
800S137-97	8.0	1.894	0.0196	2.306	0.01945	107.5	8.6	1.561	0.0172	0.710	0.11614	17.0
800S162-33	19.1	0.051	0.0025	0.050	0.00134	25.8	21.1	0.035	0.0021	0.028	0.00652	7.3
800S162-43	16.6	0.118	0.0043	0.116	0.00227	35.5	18.3	0.083	0.0035	0.062	0.01130	9.8
800S162-54	14.7	0.246	0.0067	0.243	0.00359	47.3	16.2	0.175	0.0055	0.122	0.01817	12.6
800S162-68	12.9	0.523	0.0106	0.521	0.00569	64.1	14.2	0.379	0.0087	0.245	0.02958	16.3
800S162-97	10.6	1.704	0.0209	1.739	0.01160	106.0	11.6	1.282	0.0174	0.710	0.06317	24.7
800S200-33	25.2	0.048	0.0028	0.046	0.00079	26.3	27.9	0.033	0.0023	0.028	0.00375	10.1
800S200-43	21.9	0.111	0.0047	0.106	0.00135	35.6	24.2	0.077	0.0039	0.062	0.00648	13.4
800S200-54	19.4	0.229	0.0075	0.217	0.00213	46.5	21.4	0.160	0.0061	0.122	0.01038	17.1
800S200-68	17.1	0.481	0.0118	0.453	0.00340	61.6	18.9	0.340	0.0096	0.245	0.01683	22.1
800S200-97	14.0	1.543	0.0236	1.441	0.00699	97.7	15.5	1.115	0.0194	0.710	0.03570	33.1
800S250-43	25.0	0.110	0.0063	0.103	0.00105	28.8	27.6	0.076	0.0052	0.062	0.00498	13.6
800S250-54	22.1	0.225	0.0100	0.209	0.00166	37.4	24.5	0.157	0.0081	0.122	0.00796	17.3
800S250-68	19.5	0.473	0.0158	0.433	0.00265	49.2	21.6	0.332	0.0129	0.245	0.01288	22.4
800S250-97	16.1	1.508	0.0317	1.349	0.00544	76.9	17.8	1.081	0.0260	0.710	0.02717	33.7
1000S162-43	17.6	0.096	0.0038	0.100	0.00386	25.5	19.4	0.068	0.0032	0.050	0.01973	5.1
1000S162-54	15.6	0.201	0.0060	0.214	0.00606	34.3	17.1	0.145	0.0050	0.098	0.03174	6.6
1000S162-68	13.7	0.427	0.0094	0.467	0.00957	47.2	15.1	0.316	0.0078	0.196	0.05167	8.6
1000S162-97	11.4	1.383	0.0183	1.609	0.01933	79.5	12.3	1.079	0.0156	0.568	0.11036	13.1
1000S200-43	23.2	0.090	0.0042	0.089	0.00231	27.4	25.6	0.063	0.0035	0.050	0.01131	7.6
1000S200-54	20.5	0.187	0.0067	0.184	0.00364	36.0	22.7	0.131	0.0055	0.098	0.01813	9.7
1000S200-68	18.1	0.394	0.0105	0.391	0.00579	48.2	20.0	0.281	0.0086	0.196	0.02940	12.5
1000S200-97	14.9	1.266	0.0209	1.276	0.01179	77.7	16.4	0.930	0.0173	0.568	0.06237	18.8
1000S250-43	26.4	0.089	0.0057	0.086	0.00180	23.4	29.2	0.062	0.0046	0.050	0.00869	8.4
1000S250-54	23.4	0.184	0.0089	0.176	0.00285	30.6	25.9	0.129	0.0073	0.098	0.01391	10.7
1000S250-68	20.7	0.387	0.0141	0.368	0.00453	40.6	22.8	0.274	0.0115	0.196	0.02249	13.8
1000S250-97	17.0	1.241	0.0283	1.172	0.00924	64.4	18.8	0.899	0.0233	0.568	0.04746	20.8
1200S162-54	16.4	0.169	0.0054	0.196	0.00925	24.9	17.9	0.125	0.0045	0.082	0.05006	3.8
1200S162-68	14.5	0.359	0.0084	0.435	0.01456	34.6	15.8	0.273	0.0071	0.163	0.08150	4.9
1200S162-97	12.1	1.148	0.0162	1.537	0.02916	59.1	12.9	0.940	0.0142	0.474	0.17409	7.5
1200S200-54	21.5	0.158	0.0061	0.164	0.00562	27.6	23.7	0.112	0.0050	0.082	0.02860	5.8
1200S200-68	19.0	0.334	0.0095	0.352	0.00889	37.2	20.9	0.241	0.0079	0.163	0.04637	7.5
1200S200-97	15.7	1.073	0.0189	1.177	0.01798	61.0	17.1	0.804	0.0158	0.474	0.09839	11.2
1200S250-54	24.5	0.156	0.0081	0.154	0.00441	24.8	27.1	0.110	0.0066	0.082	0.02194	6.7
1200S250-68	21.7	0.329	0.0128	0.326	0.00698	33.1	23.9	0.235	0.0105	0.163	0.03548	8.6
1200S250-97	17.9	1.056	0.0256	1.063	0.01415	53.3	19.7	0.776	0.0212	0.474	0.07486	13.0

For the general case of distortional buckling when $\beta \neq 1$, $k_\phi \neq 0$:

where β is determined via C3.1.4 or C4.2 and k_ϕ is determined via proposed method

$$F_d = \beta \frac{k_{\phi fe} + k_{\phi we} + k_\phi}{\tilde{k}_{\phi fg} + \tilde{k}_{\phi wg}}$$

(F_d is the distortional buckling stress)

F_d in table 3 assumes $L_m > L_{cr}$, therefore $L = L_{cr}$

k_ϕ values in Table 3 assume continuous restraint, therefore $L = L_m = L_{cr}$

Example 1: Simple distortional buckling check using design aid

Objective: Determine the bending capacity, M_n, of an 800S20054 (50 ksi) in the distortional buckling limit state defined in AISI-S100-07 using the design aids provided in this Note.

Per AISI-S100-07 Section C3.1.4: to determine the nominal strength in the distortional buckling limit state, the elastic critical moment for distortional buckling must first be determined.

$M_{crd} := S_f F_d$ where S_f is the gross section modulus to the comp. flange, and F_d is the elastic crticial distortional buckling stress (Eq. C3.1.4-5)

$S_f := 1.643 \cdot in^3$ per SSMA tables

$F_d := 46.5 \cdot ksi$ per Table 3 of this Note (ignores potential benefit of moment gradient on the beam and of any attached sheathing providing restraint.)

therefore $M_{crd} := S_f F_d$ $M_{crd} = 76.4\, kip \cdot in$

noting $M_y := S_f 50 \cdot ksi$ $M_y = 82.15\, kip \cdot in$

Now determine the nominal bending moment in the distortional bucking limit state.

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\lambda_d = 1.037$ slenderness in the distortional buckling mode (Eq. C3.1.4-3)

$$M_n := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[\left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y\right] & \text{if } \lambda_d > 0.673 \end{cases}$$

(Eq. C3.1.4-1)
(Eq. C3.1.4-2)

$M_n = 62.415\, kip \cdot in$ note in this case inelastic buckling is predicted and the strength is reduced from the elastic distortional buckling moment. This nominal moment would need to be compared to that from local buckling (AISI-S100-07 C3.1.1) or lateral-torsional buckling (AISI-S100-07 C3.1.4) to determine the controlling moment for the joist.

Example 2: Distortional buckling check using design aid and k_ϕ

Objective: Determine the bending capacity, M_n, of an 800S20054 (50 ksi) in the distortional buckling limit state defined in AISI-S100-07 using the design aids provided in this Note and given that an attached sheathing provides a rotational restraint of 95.7 lbf-in./in.

Per AISI-S100-07 Section C3.1.4: to determine the nominal strength in the distortional buckling limit state the elastic critical moment for distortional buckling must first be determined.

$M_{crd} := S_f F_d$ where S_f is the gross section modulus to the comp. flange, and F_d is the elastic crticial distortional buckling stress (Eq. C3.1.4-5)

$S_f := 1.643 \cdot in^3$ per SSMA tables

per C3.1.4(b)

$$F_d := \beta \cdot \frac{k_{\phi fe} + k_{\phi we} + k_\phi}{k_{\phi fg} + k_{\phi wg}} \qquad \text{(Eq. C3.1.4-10)}$$

Table 3 of this note provides the rotational stiffness from the flange and the web

$k_{\phi fe} := 0.229 \cdot kip$ $\qquad k_{\phi we} := 0.217 \cdot kip$

$k_{\phi fg} := 0.0075 \cdot in^2$ $\qquad k_{\phi wg} := 0.00213 \cdot in^2$

$k_\phi := 95.7 \cdot \frac{lbf \cdot in}{in}$ (restraint from sheathing, given in this problem, calculated in later design examples)

$\beta := 1$ (asume constant moment)

$$F_d := \beta \cdot \frac{k_{\phi fe} + k_{\phi we} + k_\phi}{k_{\phi fg} + k_{\phi wg}} \qquad F_d = 56.251\,ksi$$

therefore $M_{crd} := S_f F_d$ $\qquad M_{crd} = 92.421\,kip \cdot in$

noting $M_y := S_f 50 \cdot ksi$ $\qquad M_y = 82.15\,kip \cdot in$

Now determine the nominal bending moment in the distortional bucking limit state.

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\quad \lambda_d = 0.943$ slenderness in the distortional buckling mode (Eq. C3.1.4-3)

$$M_n := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[\left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y\right] & \text{if } \lambda_d > 0.673 \end{cases}$$

(Eq. C3.1.4-1)
(Eq. C3.1.4-2)

$M_n = 66.802\,kip \cdot in$

Example 3: Full floor joist design considering distortional buckling

Objectives:
(1) Assess the bending capacity of a floor joist using the provisions of the main *Specification* (AISI-S100-07) focusing on the new C3.1.4 provisions regarding distortional buckling
(2) Demonstrate how to use the findings of Schafer et al. (2007) to include the beneficial rotational restraint (k_ϕ) provided by sheathing to partially restrain distortional buckling as ***proposed*** for the next edition of the AISI COFS Floor and Roof System Design Standard (AISI-S210-XX)

Given:
(1) Selected floor joist is an SSMA 800S200-54 (50 ksi)
(2) Single span with a span length of 12 ft
(3) Joists are spaced 24 in. apart
(4) 7/16 in. OSB is attached to the floor joist at 12 in. o.c. using #8 screws

Find:
(1) What is the nominal bending capacity of the specified floor joist system

Assumptions
(1) Assume shear capacity does not control and bearing stiffeners are designed as needed
(2) All additional assumptions are stated in the body of the example

Cross-section Dimensions and Properties

name := "800S200-54 (50 ksi)"

out-to-out dimensions

$h_o := 8 \cdot in$

$b_o := 2 \cdot in$

$d_o := 0.625 \cdot in \qquad D := d_o$

design thickness

$t := 0.0566 \cdot in$

centerline bend radius

$r := \max\left(\frac{3}{32} \cdot in, 2 \cdot t\right)$

$r = 0.113\ in$

material

$F_y := 50 \cdot ksi$

$E := 29500 \cdot ksi$

$\mu := 0.3$

$G := \frac{E}{2 \cdot (1 + \mu)}$

centerline dimensions (straight corners)

$h := h_o - t \qquad h = 7.943\ in$

$b := b_o - t \qquad b = 1.943\ in$

$d := d_o - 0.5t \qquad d = 0.597\ in$

dimensions of the flats

$h_i := h_o - t - 2 \cdot r \qquad h_i = 7.717\ in$

$b_i := b_o - t - 2 \cdot r \qquad b_i = 1.717\ in$

$d_i := d_o - \frac{t}{2} - r \qquad d_i = 0.484\ in$

angle of the lip stiffener

$\theta := 90 \cdot deg$

additional cross-section properties

$x_o := 1.296 \cdot in$

gross cross-section properties

$S_f := 1.643 \cdot in^3$

effective cross-section properties

$S_e := 1.475 \cdot in^3$

All properties listed per SSMA (2001) Product Technical Information Catalog.

Flexural capacity (Example 3 cont.)

Local buckling

The nominal local buckling capacity is determined via C3.1.1 of the *Specification* (AISI-S100-07).

$M_n := S_e \cdot F_y$ per Eq. C3.1.1-1 of the *Specification*

$M_n = 73.75\ \text{kip}\cdot\text{in}$ Using the S_e from the SSMA (2001) Product Technical Information Catalog results in the value given to the left. This S_e is based on an earlier version of the *Specification* and changes have been made to this calculation in the current AISI-S100-07 version. However, the purpose of this design example is to demonstrate the distortional buckling provisions, not the local buckling provisions. Therefore, in the absence of updated section properties this value for S_e is assumed adequate for the purposes of this example.

Let us call this capacity M_{nl} so that we can compare to other limit states.

$M_{nl} := M_n \qquad M_{nl} = 73.75\ \text{kip}\cdot\text{in}$

For reference, the moment at first yield provides a sense of the local buckling reduction

$M_y := S_f F_y \qquad M_y = 82.15\ \text{kip}\cdot\text{in} \qquad \dfrac{M_{nl}}{M_y} = 0.898$

Lateral-torsional buckling

The lateral-torsional buckling strength, including potential interaction with local buckling, is determined via C3.1.2 of the *Specification.* However, the AISI-COFS Floor and Roof System Design Standard (AISI-S210) provides additional guidance on the application of this section.

Per AISI-S210 designers must decide if the floor joist will follow a discretely braced design philosophy (and thus the floor sheathing is not considered as part of the structural system and discrete bracing/blocking must be selected and accounted for in design) or a continuously braced design philosophy whereby it is assumed that the floor sheathing acts, and is specificed as, part of the structural system.

Assume that a continuously braced design philosophy is employed. The continuously braced design philoshopy has prescriptive requirements for its application. These requirements are provided in B1, B1.2.1, B4.1, and B4.2 of the Floor and Roof System Design Standard (AISI-S210).

An 800S200-54 (50ksi) meets the section and material requirements of B1 of AISI-S210

Per B1.2.1 of AISI-S210 the bracing provisions of B4.1 and B4.2 must be met

Top flange: 7/16 in. OSB sheathing with #8 screws at 12 in. o.c. meets the requirements of B4.1 of AISI-S210

Bottom flange: Since the desired span length is greater than 8 ft. intermediate braces for the bottom (tension) flange are required. For uniform loading the braces (spaced no more than 8 ft. apart) must resist a brace force P_L of (1.5m/d)wa, where m is the distance from the shear center to the mid-plane of the web, d is the depth of the joist, w is the line load on the joist, and a is the brace spacing. For the purposes of this example the centerline depth of the joist has been previously defined as h, therefore the required bracing force is

$$P_L := 1.5\frac{m}{h}w\cdot a$$

"m" is not defined in the SSMA (2001) Product Technical Catalog, but x_o, the distance from the shear center to the centroid is. Determine the centroidal distance from the mid-plane of the web to the centroid (x_{cg}) and then $m=x_o-x_{cg}$.

$$x_{cg} := \frac{2\cdot b_o\cdot\frac{b_o - t}{2} + 2\cdot d_o\cdot\frac{b_o - t}{2}}{h_o + 2\cdot b_o + 2\cdot d_o} \qquad x_{cg} = 0.385\ \text{in}$$

this estimate ignores the corners for calc. of x_{cg}.

$$m := x_o - x_{cg} \qquad m = 0.911\ \text{in}$$

for our 12 foot span, let's specify one brace at mid length, thus the distance between braces is 6 ft and the tributary load for the center brace is w*6ft.

$a := 6\cdot\text{ft}$, for unit line load,w $\quad w := 1\frac{\text{lbf}}{\text{ft}}$

$$P_L := 1.5\cdot\frac{m}{h}\cdot w\cdot a \qquad P_L = 1.032\ \text{lbf}$$

obviously the brace must be designed for the actual load w, here it is **assumed** that an adequate brace is selected once the design load is known. With this assumption the conditions for the use of continuously braced design are met.

Since the continuously braced design conditions are met, it is assumed that lateral-torsional buckling is restricted, thus section C3.1.2 of the Specification predicts the full cross-section capacity as defined in Section C3.1.1.

$M_n := M_{nl} \qquad M_n = 73.75\ \text{kip}\cdot\text{in}$ (i.e. no lateral-torsional buckling)

Distortional Buckling

The nominal distortional buckling capacity, must be checked per C3.1.4 of the *Specification* (AISI-S100-07). The strength is a function of the elastic critical distortional buckling moment and the moment at first yield as defined in equations C3.1.4-1 through -5. Specifically:

$M_{crd} := S_f F_d$ where S_f is the gross section modulus to the comp. flange, and F_d is the elastic crticial distortional buckling stress (Eq. C3.1.4-5)

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ slenderness in the distortional buckling mode (Eq. C3.1.4-3)

$$M_n := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[\left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y\right] & \text{if } \lambda_d > 0.673 \end{cases}$$

(Eq. C3.1.4-1)
(Eq. C3.1.4-2)

The bulk of C3.1.4 (subsections a,b,c) is devoted to the determination of F_d, the elastic critical distortional buckling stress.Here we ***examine the predicted distortional buckling capacity by all 3 subsections of C3.1.4, i.e. C3.1.4(a), C3.1.4(b) and C3.1.4(c)***.

Distortional buckling via C3.1.4(a)

C3.1.4(a) provides a conservative, simplified method for determination of F_d, this method can be helpful in determining situations when distortional buckling will not control the design capacity.

The selected 800S200-54 (50ksi) meets the dimensional limits of C3.1.4(a)

$$F_d := \beta \cdot k_d \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \mu^2\right)} \left(\frac{t}{b_o}\right)^2$$ (where Fd is the distortional buckling stress) (Eq. C3.1.4-6)

β accounts for moment gradient as defined in C3.1.4-7 and is limited between 1 and 1.3. For a simply supported span with uniform load β=1.Justification for the lack of a beneficial moment gradient effect may be understood by comparing the length of a distortional buckling half-wave forms (L_{cr}) with the span length.

$$L_{cr} := 1.2 \cdot h_o \cdot \left(\frac{b_o \cdot D \cdot \sin(\theta)}{h_o \cdot t}\right)^{0.6} \qquad L_{cr} = 17.655\,\text{in}$$ (Eq. C3.1.4-8)

the moment gradient over the middle 18 in. (L_{cr}) of a uniformly loaded 12 ft. long beam is inconsequential, further explaining β=1 for this case.

now, turning to the calculation of the plate buckling coefficient for distortional buckling, k_d

$$k_d := 0.6 \cdot \left(\frac{b_o \cdot D \cdot \sin(\theta)}{h_o \cdot t}\right)^{0.7} \qquad k_d = 1.221$$ which is between 0.5 and 8 (Eq. C3.1.4-9)

$$F_d := \beta \cdot k_d \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \mu^2\right)} \left(\frac{t}{b_o}\right)^2 , \qquad F_d = 26.081\,\text{ksi}$$ (Eq. C3.1.4-6)

Now returning to the main body of C3.1.4 of the *Specification* and estimating the strength

$$M_{crd} := S_f F_d \qquad M_{crd} = 42.851\,\text{kip}\cdot\text{in}$$ (Eq. C3.1.4-5)

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 1.385$$ (Eq. C3.1.4-3)

$$M_n := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[\left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y\right] & \text{if } \lambda_d > 0.673 \end{cases}$$ (Eq. C3.1.4-1) (Eq. C3.1.4-2)

$M_n = 49.904\,\text{kip}\cdot\text{in}$ Let us call this estimate of the distortional strength M_{nda}

$M_{nda} = 49.904\,\text{kip}\cdot\text{in}$, but $M_{nl} = 73.75\,\text{kip}\cdot\text{in}$

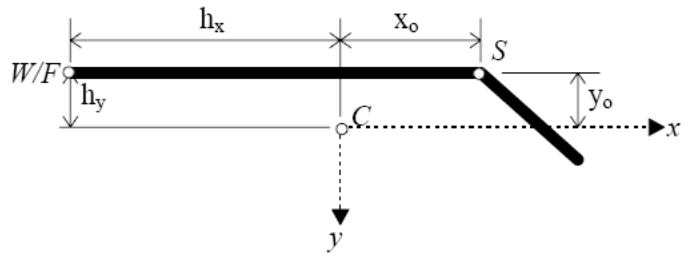
In this example the estimated capacity in the distortional buckling limit state is less than in the local buckling limit state. Therefore, the designer may accept this reduced capacity (if adequate for their purposes) or may use the more detailed and exact calculations of C3.1.4(b) or (c) which account for the beneficial restraint of the sheathing. Note, if C3.1.4(a) predicts a capacity greater than M_{nl} the additional calculations of C3.1.4(b) or (c) would be unnecessary.

Distortional buckling via C3.1.4(b)

C3.1.4(b) provides a more precise, but more involved calculation for the elastic distortional buckling stress, F_d. The method is quite involved and engineers are instead encouraged to use the C3.1.4(c) clause and use a computational analysis instead of this hand method. However, situtations exist where an agreed upon hand method still has benefits.

The analytical model for predicting distortional buckling in C3.1.4(b) considers flexural-torsional buckling of the flange as a column restrained at the web/flange juncture by the available rotational stiffness from bending/buckling of the web plate. As a result of this approach the cross-section properties of the flange itself are needed, these are not typically tabled and must be completed by hand. The Specification commentary provides formulae for the cross-section properties via Table C-C3.1.4(b)-1.

Flange properties for use in C3.1.4(b) per Table C-C3.1.4(b)-1



Properties of the Flange Only:

$A_f := (b + d)\cdot t$ $\qquad A_f = 0.144\ in^2$

$J_f := \frac{1}{3}\cdot b\cdot t^3 + \frac{1}{3}\cdot d\cdot t^3$ $\qquad J_f = 1.535 \times 10^{-4}\ in^4$

$$I_{xf} := \frac{t\cdot\left(t^2\cdot b^2 + 4\cdot b\cdot d^3 - 4\cdot b\cdot d^3\cdot\cos(\theta)^2 + t^2\cdot b\cdot d + d^4 - d^4\cdot\cos(\theta)^2\right)}{12\cdot(b + d)}$$ $\qquad I_{xf} = 0.003\ in^4$

$$I_{yf} := \frac{t\cdot\left(b^4 + 4\cdot d\cdot b^3 + 6\cdot d^2\cdot b^2\cdot\cos(\theta) + 4\cdot d^3\cdot b\cdot\cos(\theta)^2 + d^4\cdot\cos(\theta)^2\right)}{12\cdot(b + d)}$$ $\qquad I_{yf} = 0.059\ in^4$

$$I_{xyf} := \frac{t\cdot b\cdot d^2\cdot\sin(\theta)\cdot(b + d\cdot\cos(\theta))}{4\cdot(b + d)}$$ $\qquad I_{xyf} = 0.007\ in^4$

$$I_{of} := \frac{t\cdot b^3}{3} + \frac{b\cdot t^3}{12} + \frac{t\cdot d^3}{3}$$ $\qquad I_{of} = 0.143\ in^4$

$$x_{of} := \frac{b^2 - d^2\cdot\cos(\theta)}{2\cdot(b + d)}$$ x distance from the centroid to the shear center of flange. $\qquad x_{of} = 0.743\ in$

$$y_{of} := \frac{-d^2\cdot\sin(\theta)}{2\cdot(b + d)}$$ y distance from the centroid to the shear center of flange. $\qquad y_{of} = -0.07\ in$

$$h_{xf} := \frac{-\left(b^2 + 2\cdot d\cdot b + d^2\cdot\cos(\theta)\right)}{2(b + d)}$$ x distance from the centroid to the web/flange juncture. $\qquad h_{xf} = -1.2\ in$

$$h_{yf} := \frac{-d^2\cdot\sin(\theta)}{2\cdot(b + d)}$$ y distance from the centroid to the web/flange juncture. $\qquad h_{yf} = -0.07\ in$

$C_{wf} := 0\cdot in^6$ $\qquad C_{wf} = 0\ in^6$

Distortional buckling via C3.1.4(b) **(continued)**

$$F_d := \beta \cdot \frac{k_{\phi fe} + k_{\phi we} + k_{\phi}}{k_{\phi fg} + k_{\phi wg}} \qquad \text{(Eq. C3.1.4-10)}$$

where β accounts for moment gradient, and the distortional buckling stress F_d is found by determining how high the geometric flange and web stiffness ($k_{\phi fg}$ and $k_{\phi wg}$) must be for them to erode the elastic flange and web stiffness ($k_{\phi fe}$ and $k_{\phi we}$), as well as any externally provided restraint (k_{ϕ}) to cause buckling.

$\beta := 1$ see solution via C3.1.4(a) above for further discussion.

It is necessary to determine the length at which a distortional buckling half-wave forms over, as the related stiffnesses are a function of this length. Per Eq. C3.1.4-12:

$$L_{cr} := \left[\frac{4 \cdot \pi^4 \cdot h_o \cdot \left(1 - \mu^2\right)}{t^3} \cdot \left[I_{xf} \cdot \left(x_{of} - h_{xf}\right)^2 + C_{wf} - \frac{I_{xyf}^2}{I_{yf}} \cdot \left(x_{of} - h_{xf}\right)^2\right] + \frac{\pi^4 \cdot h_o^4}{720}\right]^{\frac{1}{4}} \qquad L_{cr} = 19.385\ \text{in}$$

L_{cr} is the length at which distortional buckling half-waves will naturally form. If discrete restraints exists that fully preclude distortional buckling at a length less than L_{cr}, then this length should be used in place of L_{cr}. For this example we are using the continuously braced design philosophy and thus per the notes of C3.1.4(b) this influence is captured through the supplemental rotational restraint k_{ϕ}, not through modifying L.

$L := L_{cr}$

Determine the elastic and "geometric" rotational spring stiffness of the flange:

Elastic rotational stiffness provided by the flange to the flange/web juncture:

$$k_{\phi fe} := \left(\frac{\pi}{L}\right)^4 \cdot \left[E \cdot I_{xf} \cdot \left(x_{of} - h_{xf}\right)^2 + E \cdot C_{wf} - E \cdot \frac{I_{xyf}^2}{I_{yf}} \cdot \left(x_{of} - h_{xf}\right)^2\right] + \left(\frac{\pi}{L}\right)^2 \cdot G \cdot J_f \qquad \text{(Eq. C3.1.4-13)}$$

$k_{\phi fe} = 0.229\ \text{kip}$

Geometric rotational stiffness (divided by the stress F_d) demanded by the flange from the flange/web juncture:

$$k_{\phi fg} := \left(\frac{\pi}{L}\right)^2 \cdot \left[A_f \left[\left(x_{of} - h_{xf}\right)^2 \cdot \left(\frac{I_{xyf}}{I_{yf}}\right)^2 - 2 \cdot y_{of} \cdot \left(x_{of} - h_{xf}\right) \cdot \left(\frac{I_{xyf}}{I_{yf}}\right) + h_{xf}^2 + y_{of}^2\right] + I_{xf} + I_{yf}\right]$$

$k_{\phi fg} = 0.007\ \text{in}^2$ (Eq. C3.1.4-15)

Distortional buckling via C3.1.4(b) **(continued)**

Determine the elastic and "geometric" rotational spring stiffness of the web:

Elastic rotational stiffness provided by the web to the flange/web juncture:

$$k_{\phi we} := \frac{E \cdot t^3}{12 \cdot \left(1 - \mu^2\right)} \cdot \left[\frac{3}{h_o} + \left(\frac{\pi}{L}\right)^2 \cdot \frac{19 \cdot h_o}{60} + \left(\frac{\pi}{L}\right)^4 \cdot \frac{h_o^3}{240} \right] \qquad \text{(Eq. C3.1.4-14)}$$

$k_{\phi we} = 0.217\,\text{kip}$

Geometric rotational stiffness (divided by F_d) demanded by the web from the flange/web juncture:

The stress gradient in the web (ξ_{web}) must be defined, as $k_{\phi wg}$ is a function of this gradient. If f_1 is the stress at the top of the web and f_2 at the bottom (relative values are all that matter for these stresses, not the absolute magnitudes) then:

$$f_1 := 1 \qquad f_2 := -1 \qquad \xi_{web} := \frac{f_1 - f_2}{f_1} \qquad \xi_{web} = 2$$

$$k_{\phi wg} := \frac{h_o \cdot t \cdot \pi^2}{13440} \cdot \frac{\left[45360 \cdot \left(1 - \xi_{web}\right) + 62160\right] \cdot \left(\frac{L}{h_o}\right)^2 + 448 \cdot \pi^2 + \left(\frac{h_o}{L}\right)^2 \cdot \left[53 + 3 \cdot \left(1 - \xi_{web}\right)\right] \cdot \pi^4}{\pi^4 + 28 \cdot \pi^2 \cdot \left(\frac{L}{h_o}\right)^2 + 420 \cdot \left(\frac{L}{h_o}\right)^4}$$

$k_{\phi wg} = 0.002\,\text{in}^2$

Rotational stiffness provided by a restraining element (i.e. sheathing)

For now, let us assume that this k_ϕ is zero. This will allow us to compare to the earlier C3.1.4(a) prediction and also to directly understand the influence of the rotational restraint on the solution.

$k_\phi := 0 \cdot \text{lbf}$

Determine the distortional buckling stress:

$$F_d := \beta \cdot \frac{k_{\phi fe} + k_{\phi we} + k_\phi}{k_{\phi fg} + k_{\phi wg}} \qquad \text{(Eq. C3.1.4-10)}$$

$F_d = 46.497\,\text{ksi}$ which we may compare with the prediction of C3.1.4(a): $F_{da} = 26.081\,\text{ksi}$

Now returning to the main body of C3.1.4 of the *Specification* and estimating the strength

$$M_{crd} := S_f F_d \qquad M_{crd} = 76.394\,\text{kip} \cdot \text{in} \qquad \text{(Eq. C3.1.4-5)}$$

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 1.037 \qquad \text{(Eq. C3.1.4-3)}$$

Distortional buckling via C3.1.4(b) **(continued)**

$$M_n := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[\left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y\right] & \text{if } \lambda_d > 0.673 \end{cases}$$

(Eq. C3.1.4-1)
(Eq. C3.1.4-2)

$M_n = 62.413\,\text{kip}\cdot\text{in}$ Let us call this estimate of the distortional strength $M_{ndb\_no\_k\phi}$ comparing to results of previous methods:

$M_{ndb\_no\_k\phi} = 62.413\,\text{kip}\cdot\text{in}$ (C3.1.4(b) with k_ϕ=0)

$M_{nda} = 49.904\,\text{kip}\cdot\text{in}$ (C3.1.4(a))

$M_{nl} = 73.75\,\text{kip}\cdot\text{in}$ (C3.1.1 local bucking for continuously braced design)

So, the more exact hand calculation of C3.4.1(b) is useful, but still the strength is lower than the local buckling predicted strength, let us consider the influence of the sheathing on the solution.

Distortional buckling via C3.1.4(b) with nonzero k_ϕ

Estimate the rotational restraint k_ϕ provided by the sheathing

Although the *Specification* (AISI-S100-07) provides a means to include the beneficial influence of restraint on members the *Specification* does not prescribe how that restraint, k_ϕ, should be determined. The Floor and Roof System Design Standard (AISI-S210-XX) provides guidance on bracing for lateral-torsional buckling, but no guidance on distortional buckling. Testing and analysis provided in Schafer et al. (2007) provides a means of estimating the rotational restraint provided by sheathing. A design method based on Schafer et al. (2007) has been proposed for adoption in the next edition of the Floor and Roof System Design Standard (AISIS-S210-XX). This proposed design method is emplyed in this example.

The external rotational stiffness has contributions from the sheathing and from the joist-sheathing connectors, which may be combined to provide the total rotational stiffness as follows:

$$k_\phi := \frac{1}{\frac{1}{k_{\phi w}} + \frac{1}{k_{\phi c}}}$$

The connection rotational stiffness has been found to be most strongly infuenced by the thickness of the joist. Based on the results of Schafer et al. (2007) summarized to the right,

$$k_{\phi c} := 105 \cdot \frac{\text{lbf} \cdot \text{in}}{\text{in}}$$

Table from Schafer et al. (2007)

t (mils)	$k_{\phi c}$ (lbf-in./in./rad)	$k_{\phi c}$ (N-mm/mm/rad)
18	78	348
27	83	367
30	84	375
33	86	384
43	94	419
54	105	468
68	123	546
97	172	766

Distortional buckling via C3.1.4(b) with nonzero k_ϕ **(continued)**

The sheathing component of the rotational restraint depends simply on the EI/L bending of the sheathing, i.e.:

$$k_{\phi w} := \frac{EI_w}{L_1} + \frac{EI_w}{L_2}$$

where EI_w is the bending rigidity of the specified sheathing, and L_1 and L_2 are half of the joist spacing to either side of the joist.

The bending rigidity of plywood and OSB sheathing may be found in APA (2004)

RATED PANELS DESIGN CAPACITIES

Span Rating	Stress Parallel to Strength Axis: Plywood 3-ply	Plywood 4-ply	Plywood 5-ply	OSB	Stress Perpendicular to Strength Axis: Plywood 3-ply	Plywood 4-ply	Plywood 5-ply	OSB
PANEL BENDING STIFFNESS, EI (lb-in.²/ft of panel width)								
24/0	66,000	66,000	66,000	60,000	3,600	7,900	11,000	11,000
24/16	86,000	86,000	86,000	78,000	5,200	11,500	16,000	16,000
32/16	125,000	125,000	125,000	115,000	8,100	18,000	25,000	25,000
40/20	250,000	250,000	250,000	225,000	18,000	39,500	56,000	56,000
48/24	440,000	440,000	440,000	400,000	29,500	65,000	91,500	91,500
16oc	165,000	165,000	165,000	150,000	11,000	24,000	34,000	34,000
20oc	230,000	230,000	230,000	210,000	13,000	28,500	40,500	40,500
24oc	330,000	330,000	330,000	300,000	26,000	57,000	80,500	80,500
32oc	715,000	715,000	715,000	650,000	75,000	165,000	235,000	235,000
48oc	1,265,000	1,265,000	1,265,000	1,150,000	160,000	350,000	495,000	495,000

Plywood and OSB bending rigidity per APA, Panel Design Spec. (2004)

RATED PANELS DESIGN CAPACITIES

Span Rating	Stress Parallel to Strength Axis: Plywood 3-ply	Plywood 4-ply	Plywood 5-ply	OSB	Stress Perpendicular to Strength Axis: Plywood 3-ply	Plywood 4-ply	Plywood 5-ply	OSB
PANEL BENDING STIFFNESS, EI (lb-in.²/ft of panel width)								
24/0	66,000	66,000	66,000	60,000	3,600	7,900	11,000	11,000
24/16	86,000	86,000	86,000	78,000	5,200	11,500	16,000	16,000
32/16	125,000	125,000	125,000	115,000	8,100	18,000	25,000	25,000
40/20	250,000	250,000	250,000	225,000	18,000	39,500	56,000	56,000
48/24	440,000	440,000	440,000	400,000	29,500	65,000	91,500	91,500
16oc	165,000	165,000	165,000	150,000	11,000	24,000	34,000	34,000
20oc	230,000	230,000	230,000	210,000	13,000	28,500	40,500	40,500
24oc	330,000	330,000	330,000	300,000	26,000	57,000	80,500	80,500
32oc	715,000	715,000	715,000	650,000	75,000	165,000	235,000	235,000
48oc	1,265,000	1,265,000	1,265,000	1,150,000	160,000	350,000	495,000	495,000

TABLE 5

NOMINAL THICKNESS BY SPAN RATING

(The nominal thickness is given. The predominant thickness for each span rating is highlighted in **bold** type.)

Span Rating	Nominal Thickness (in.) 3/8	7/16	15/32	1/2	19/32	5/8	23/32	3/4	7/8	1	1-1/8
APA Rated Sheathing											
24/0	**.375**	.437	.469	.500							
24/16		**.437**	.469	.500							
32/16			**.469**	.500	.594	.625					
40/20					**.594**	.625	.719	.750			
48/24							**.719**	.750	.875		
APA Rated Sturd-I-Floor											
16 oc					**.594**	.625					
20 oc					**.594**	.625					
24 oc							**.719**	.750			
32 oc									**.875**	1.000	
48 oc											**1.125**

Note: 1 inch = 25.4 mm.

Assume the 7/16 in. OSB is span rated as 24/16 per APA nomenclature

$$EI_w := 78000 \cdot \frac{lbf \cdot in^2}{ft}$$

per the table, but we desire the results per inch of panel instead of per ft. of panel, therefore:

$$EI_w = 6500 \frac{lbf \cdot in^2}{in}$$

Based on joists evenly spaced at 24 in. (as given on page 1 of the example)

$$L_1 := 12 \cdot in \qquad L_2 := 12 \cdot in$$

$$k_{\phi w} := \frac{EI_w}{L_1} + \frac{EI_w}{L_2} \qquad k_{\phi w} = 1083.333 \frac{lbf \cdot in}{in}$$

Distortional buckling via C3.1.4(b) with nonzero k_ϕ **(continued)**

The supplemental rotational stiffness provided by the sheathing-connection assembly is:

$$k_\phi := \frac{1}{\frac{1}{k_{\phi w}} + \frac{1}{k_{\phi c}}}$$

where:

$$k_{\phi w} = 1083.333\, lbf \cdot \frac{in}{in}$$

$$k_{\phi c} = 105\, lbf \cdot \frac{in}{in}$$

$$k_\phi = 95.722 \frac{lbf \cdot in}{in}$$

(note in this example that the connection stiffness is dominating the result, in essence the connector stiffness is the weak link in this configuration. If more stiffness is desired a tighter fastener spacing may help, but current provisions only provide results for 12 in. o.c.)

Returning now to C3.1.4(b) Eq. C3.1.4-10 to determine the distortional buckling stress:

$$F_d := \beta \cdot \frac{k_{\phi fe} + k_{\phi we} + k_\phi}{k_{\phi fg} + k_{\phi wg}} \qquad \text{(Eq. C3.1.4-10)}$$

$F_d = 56.482\,\text{ksi}$ which we may compare with the prediction of C3.1.4(a): $F_{da} = 26.081\,\text{ksi}$

or when k_ϕ=0 from C3.1.4(b): $F_{db\_no\_k\phi} = 46.497\,\text{ksi}$

Now returning to the main body of C3.1.4 of the *Specification* and estimating the strength

$$M_{crd} := S_f F_d \qquad M_{crd} = 92.8\,\text{kip}\cdot\text{in} \qquad \text{(Eq. C3.1.4-5)}$$

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 0.941 \qquad \text{(Eq. C3.1.4-3)}$$

$$M_n := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \begin{matrix}\text{(Eq. C3.1.4-1)} \\ \text{(Eq. C3.1.4-2)}\end{matrix}$$

$M_n = 66.897\,\text{kip}\cdot\text{in}$ Let us call this estimate of the distortional strength M_{ndb}

comparing to results of previous methods:

$M_{ndb} = 66.897\,\text{kip}\cdot\text{in}$ (C3.1.4(b))

$M_{ndb\_no\_k\phi} = 62.413\,\text{kip}\cdot\text{in}$ (C3.1.4(b) with k_ϕ=0)

$M_{nda} = 49.904\,\text{kip}\cdot\text{in}$ (C3.1.4(a))

$M_{nl} = 73.75\,\text{kip}\cdot\text{in}$ (C3.1.1 local bucking for continuously braced design)

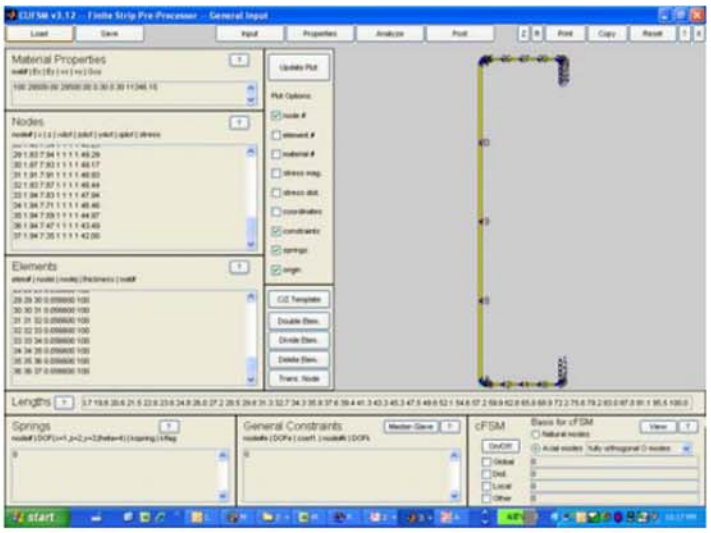
The results show the benefit of considering the rotational stiffness provided by the sheathing on the distortional buckling solution. However, distortional buckling is still predicted to control the strength in a continuously braced design situation. Rational elastic buckling analysis of C3.1.4(c) provides a final way to examine this problem. (Note the Specification specifically allows the use of C3.1.4(a),(b) or (c)).

Distortional buckling via C3.1.4(c)

Per C3.1.4(c) of the Specification a "rational elastic buckling analysis. . .. shall be permitted to be used in lieu of the expression of given in Section C3.1.4(a) or (b)."

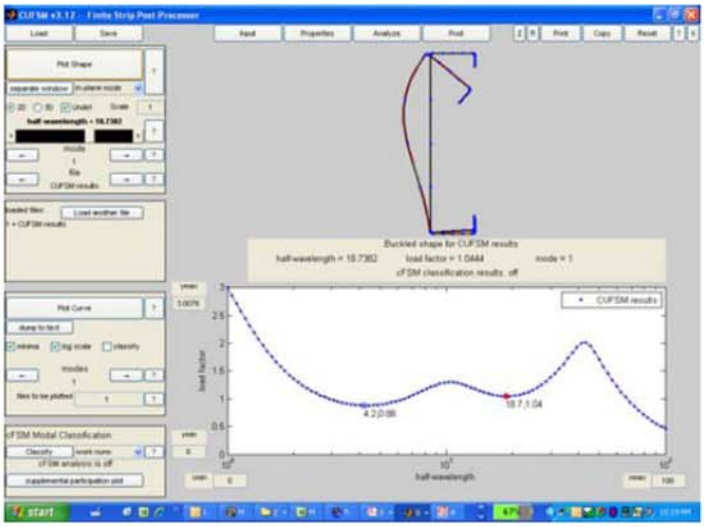
In this case it is decided to use CUFSM, an open source program using the semi-analytical finite strip method for determination of thin-walled member stability. The program, along with tutorials, etc., may be found at www.ce.jhu.edu/bschafer/cufsm.

The model is built using the C/Z template which requires that the dimensions of the flats provided on page 1 of this example be used. The resulting model has A,I, etc, that agree with the SSMA Product Technical Catalog.



CUFSM model of 800S200-54 (50ksi)

Note in the model shown to the left no rotational restraint has been included.



Finite strip analysis results

For the section without rotational restraint (k_ϕ) distortional buckling is found to occur at a half-wavelength of 18.7 in. and at a load factor of 1.04. The applied reference load is the moment at first yield*. So, distortional buckling is found to occur at $1.04M_y$.

$$M_{crdFSM} := 1.04 \cdot M_y$$

$$F_d := \frac{M_{crdFSM}}{S_f} \qquad F_d = 52\,\text{ksi}$$

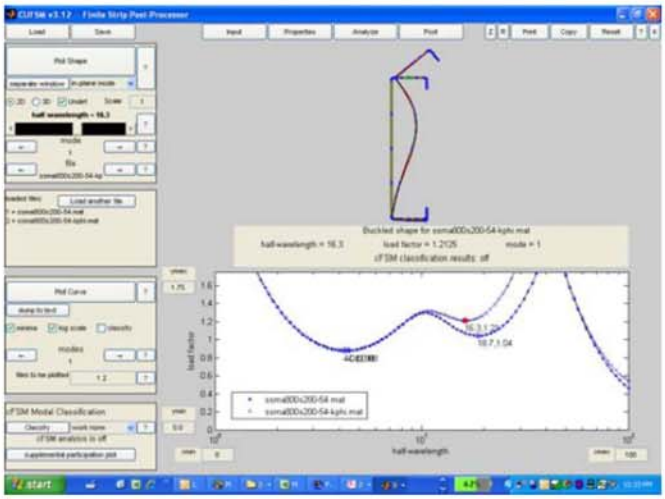
Comparing this distortional buckling stress with earlier predictions: $F_{da} = 26.081\,\text{ksi}$

$F_{db\_no\_k\phi} = 46.497\,\text{ksi}$

*in CUFSM stresses are referred to the centerline of the plates, to generate a distribution that has an extreme fiber stress at 50 ksi (consistent with typical M_y definitions) the stress at mid-thickness: $50*(h_o/2-t/2)/(h_o/2)$ ksi is used as the reference stress.

Distortional buckling via C3.1.4(c) **(continued)**

The rotational restraint provided by the sheathing may be directly modeled in CUFSM, the k_ϕ determined in C3.1.4(b) is connected to the node at mid-width of the compression flange and the model re-analyzed as shown:



Finite strip analysis results

Distortional buckling analysis with k_ϕ increases the buckling load factor from 1.04 up to 1.21. (The reference load is the yield moment, M_y)

$$M_{crdFSM} := 1.21 \cdot M_y$$

$$F_d := \frac{M_{crdFSM}}{S_f} \qquad F_d = 60.5\,\text{ksi}$$

Now returning to the main body of C3.1.4 of the *Specification* and estimating the strength

$$M_{crd} := S_f F_d \qquad M_{crd} = 99.401\,\text{kip}\cdot\text{in} \qquad \text{(Eq. C3.1.4-5)}$$

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 0.909 \qquad \text{(Eq. C3.1.4-3)}$$

$$M_n := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \begin{matrix}\text{(Eq. C3.1.4-1)} \\ \text{(Eq. C3.1.4-2)}\end{matrix}$$

$M_n = 68.497\,\text{kip}\cdot\text{in}$ Let us call this estimate of the distortional strength M_{ndc}

comparing to results of previous methods:

$M_{ndc} = 68.497\,\text{kip}\cdot\text{in}$ (C3.1.4(c) with k_ϕ=96 lbf-in./in.)

$M_{ndb} = 66.897\,\text{kip}\cdot\text{in}$ (C3.1.4(b) with k_ϕ=96 lbf-in./in.)

$M_{ndb\_no\_k\phi} = 62.413\,\text{kip}\cdot\text{in}$ (C3.1.4(b) with k_ϕ=0)

$M_{nda} = 49.904\,\text{kip}\cdot\text{in}$ (C3.1.4(a))

$M_{nl} = 73.75\,\text{kip}\cdot\text{in}$ (C3.1.1 local bucking for continuously braced design)

Distortional buckling controls the bending strength of the joist and the nominal moment capacity is 69 kip-in. The design capacity may be found from C3.1.4 for LRFD using ϕ=0.9 or for ASD using Ω=1.67.

References

1. *AISI-S100-07North American Specification for the Design of Cold-Formed Steel Structural Members.* American Iron and Steel Institute, Washington, D.C., 2007.
2. *AISI-S210-07 North American Standard for Cold-Formed Steel Framing - Floor and Roof Systems Design.* American Iron and Steel Institute, Washington, D.C., 2007.
3. *AISI-S211-07 North American Standard for Cold-Formed Steel Framing - Wall Stud Design*. American Iron and Steel Institute, Washington, D.C., 2007.
4. Schafer, B.W., Sangree, R.H., Guan, Y. "Experiments on Rotational Restraint of Sheathing: Final Report", American Iron and Steel Institute - Committee on Framing Standards, Washington, D.C., 2007.
5. Schafer, B.W., Sangree, R.H., Guan, Y. "Rotational restraint of distortional buckling in cold-formed steel framing systems." International Conference on Thin-walled Structures. Brisbane, Australia, 18-20 June 2008.
6. Schafer, B.W., Ádány, S. "Buckling analysis of cold-formed steel members using CUFSM: conventional and constrained finite strip methods." Proceedings of the Eighteenth International Specialty Conference on Cold-Formed Steel Structures, Orlando, FL. 39-54, 2006.
CUFSM is available at www.ce.jhu.edu/bschafer/cufsm

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This Technical Note was approved by the CFSEI Technical Review Committee in 2008. Rob Madsen, P.E., Devco Engineering, Inc., chairman.

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