ELSEVIER
Thin-Walled Structures 37 (2000) 127–145
THIN-WALLED STRUCTURES
www.elsevier.com/locate/tws

Torsion in thin-walled cold-formed steel beams

B.P. Gotluru [a], B.W. Schafer [b], T. Peköz [a,*]

[a] *Cornell University, Ithaca, NY 14853, USA*
[b] *Simpson, Gumpertz and Heger, Arlington, MA 02474, USA*

Received 7 September 1999; received in revised form 7 February 2000; accepted 7 February 2000

Abstract

Thin-walled cold-formed steel members have wide applications in building structures. They can be used as individual structural framing members or as panels and decks. In general, cold-formed steel beams have open sections where centroid and shear center do not coincide. When a transverse load is applied away from the shear center it causes torque. Because of the open nature of the sections, torsion induces warping in the beam. This paper summarizes the research on the behavior of cold-formed steel beams subject to torsion and bending. The attention is focused on beams subject to torque, because of the effect of transverse loads not applied at the shear center. A simple geometric nonlinear analysis method, based on satisfying equilibrium in the deformed configuration, is examined and used to predict the behavior of the beams. Simple geometric analyses, finite element analyses and finite strip analyses are performed and compared with experimental results. The influence of typical support conditions is studied and they are found to produce partial warping restraint at the ends. This effect is accounted for by introducing hypothetical springs. The magnitude of the spring stiffness is assessed for commonly used connections. Other factors that affect the behavior of cold-formed steel members, such as local buckling, are also studied.

1. Introduction

Thin-walled, cold-formed steel has wide applications in building structures. In general, cold-formed steel beams have open sections where the centroid and shear center do not coincide. When a transverse load is applied away from the shear center it causes torque. Because of the open nature of the sections, this torque induces

* Corresponding author. Tel.: +1-607-255-6366; fax: +1-607-255-4828.
E-mail address: tp26@cornell.edu (T. Peköz).

PII: S0263-8231(00)00011-2

warping in the beam. The magnitude of the warping stresses can be as high as the bending stresses in some cases. If the beam is not continuously restrained against torsion and lateral movement, it may fail in non-uniform torsion, that is, torsion combined with warping. Present design practice is based on lateral-torsional buckling or on linear bending and warping stress distributions, either of which is not completely realistic. Available advanced analysis techniques are computationally demanding and often not directly applicable to design. This paper, summarizes the studies [1] on the behavior and design of thin-walled steel beams subject to transverse loads applied away from the shear center.

There have been many experimental and analytical studies of torsion and lateral-torsional buckling. Barsoum and Gallagher [2] introduced the first derivative of the rotation about the axis of three-dimensional beam elements as the seventh degree of freedom at each node, representing warping deformation. Using this element they analyzed elastic-torsional and torsional-flexural buckling of symmetric sections. Rajasekaran [3] used the above element to solve the large displacement problem in the elastic–plastic range using a Lagrangian formulation. Attard [4] developed two finite element formulations for the calculation of the lateral buckling load for elastic straight prismatic thin-walled open beams under conservative static loads. Attard and Somervaille [5] presented a finite element formulation for the elastic nonlinear static analysis of thin-walled, open beams under combined bending and torsion. Chan and Kitipornchai [6] investigated geometric nonlinear analysis of structures comprising members of asymmetric thin-walled open section. Djugash and Kalyanaraman [7–9] developed a numerical method for the nonlinear and instability analysis of thin-walled members subject to biaxial bending. The nonlinear biaxial bending behavior of thin-walled zee and sigma sections subject to a transverse load is studied analytically and the results are compared with the experiments.

A study on the flexural strength and deflections of discretely braced cold-formed steel channel and zee sections was conducted by Ellifrit et al. [10,14,22] at the University of Florida. In the research, typical channel and zee sections were tested in flexure with various types of bracing. The load is applied at the web-top flange junction, i.e. not at the shear center. Therefore, this is a case of the combined bending and torsion acting on an unbraced beam. However, the effect of torsion was not considered in the analytical modeling. Pi and Trahair [11,12] developed a finite element model for the nonlinear large-deflection and rotation analysis of beam-columns. Put et al. [13] performed lateral buckling tests on unbraced, simply supported cold-formed lipped channel beams. A vertical load was applied at the shear center of the section, or at a point below the shear center. The beams were supported at the ends by connecting them to a steel block with two bolts at the web of the section. The steel block was supported against translation and rotation about the axis of the beam. The buckling test results revealed that these supports induced unexpected warping restraints. Hence, two torsion tests were conducted to determine the magnitude of the warping restraints induced by the supports. The warping restraint stiffness was measured.

Davies [21] gives an excellent introduction to the linear elastic theory of uniform and non-uniform torsion. Yu [23] provides numerical methods for calculating torsional properties of thin-walled sections.

2. CU-BEAM analysis

CU-BEAM is a program developed at Cornell University, for the analysis of continuous beams [1]. (This program can be downloaded from the thin-walled research web-site of Cornell University, Department of Civil and Environmental Engineering.) The analysis considers the nonlinearity due to the effect of transverse loads, applied away from the shear center. The stiffness matrix and load vector are derived based on linear finite element analysis [3].

A transverse force applied on the beam, away from the shear center of the beam cross-section, produces a torque. The torque acting on the beam can be calculated by the equation

$$m_z=(x-sc_x)f_y-(y-sc_y)f_x \tag{1}$$

where m_z is the torque produced, f_x and f_y are the x- and y-components of the force, respectively, (x, y) is the point of the cross-section where the force is applied, and (sc_x, sc_y) are the co-ordinates of the shear center. The torque, thus applied on the beam, causes a rotation about the shear center of the beam.

Due to the rotation, there is a change in the coordinates of the point at which the load is applied, as shown in Fig. 1. This displacement of the load-point, in turn, increases or decreases the torque applied on the beam (depending on the direction of the displacement). This leads to a problem where the torque causing the rotation is a function of the rotation itself. In mathematical terms,

$$K\theta=q(\theta) \tag{2}$$

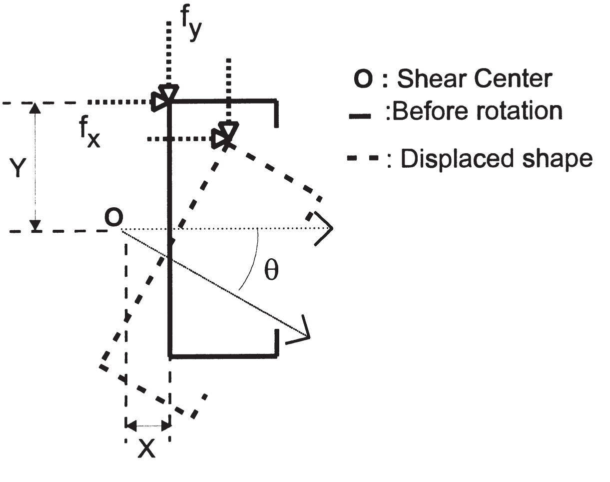

Fig. 1. CU-BEAM analysis.

where K is the stiffness matrix, q is the load vector (i.e., the torques) and θ is the unknown rotation vector. This is termed the 'nonlinear effects of load applied away from the shear center'.

The goal is to solve for the unknown rotation. Owing to the complex nature of the equations involved, it is not possible to derive a closed form solution for a general continuous beam analysis problem. Hence, a Newton–Raphson iterative procedure is implemented [20].

As can be seen from Rajasekaran [3], the linear stiffness matrix for the torsional problem is dependent only on J, the torsional constant, and C_w, the warping constant, which are section properties independent of the rotation. Thus, K is independent of the displacement vector, θ (rotations and derivatives of the rotation). Thus, CU-BEAM analysis can be viewed as satisfying 'linear' equilibrium in the deformed configuration.

This simple procedure is tested against complete nonlinear analysis by ABAQUS [16], a commercially available standard finite element software. The results matched closely, for the problems of interest, up until yielding takes place in the member. This can be seen in the results provided in the later sections and are detailed in Gotluru [1]. Therefore, it can be concluded that, for the problems of interest, the major contribution to nonlinearity, in the elastic range, is the nonlinearity due to the dependence of the torque on the rotation of the beam.

In the following sections, experimental and analytical studies that are conducted to understand and predict the behavior of cold-formed steel beams subject to bending and torsion are presented. The problems are described in the next section, followed by a brief description of the analysis methods. This is followed by a study of the partial warping restraint provided by the supports. Local buckling behavior and imperfection sensitivity are also studied to a limited extent, and this is presented at the end.

3. Problems analyzed and details of the experiments

The analytical studies are conducted on six problems. These problems are based on experiments at the University of Florida [14] and at Cornell University [1,15]. The problems are labeled C14U, C12U, C14TU, L, Rt and Rb. In this list, the first three experiments are conducted at the University of Florida and the latter three at Cornell University

3.1. Geometry and loading

All the experiments are performed on lipped channel sections tested as unbraced simply supported beams, with two-point loading. The load is applied at the web-top flange junction. The geometry of the test setup is shown in Fig. 2 and the dimensions are given in Table 1.

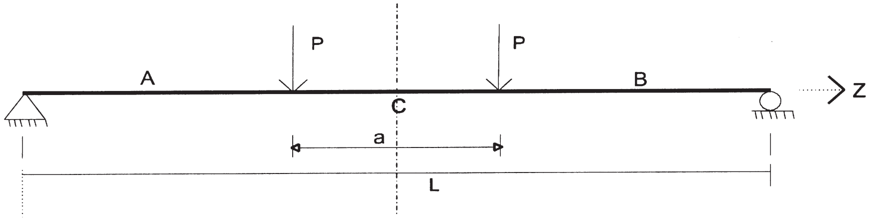

Fig. 2. Test setup.

Table 1
Test setup dimensions

No.	Designation	*L* (in)	*a* (in)
1	C14U	216.0	60.0
2	C12U	216.0	60.0
3	C14TU	216.0	72.0
4	L	147.5	36.0
5	Rt	111.5	36.0
6	Rb	111.5	36.0

3.1.1. Cross-section geometry

The cross-section geometry is similar for C14U, C12U, C14TU and L. The cross-sectional geometry is shown in Fig. 3 and the dimensions are listed in Table 2.

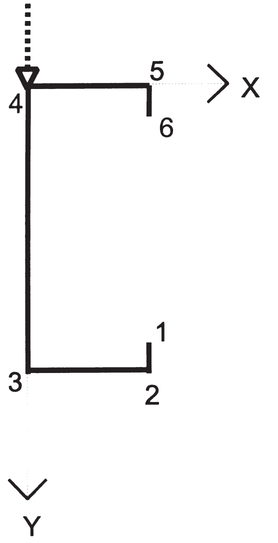

Fig. 3. Cross-sectional geometry for experiments C14U, C12U, C14TU and L.

Table 2
Cross-section dimensions

Section	Thickness (in)	Depth (34) (in)	Flange (23) (in)	Lip (12) (in)
C14U	0.0723	8.15	3.39	0.868
C12U	0.0954	8.24	3.44	0.827
C14YU	0.0728	8.12	3.18	1.055
L	0.0410	5.85	1.50	0.525

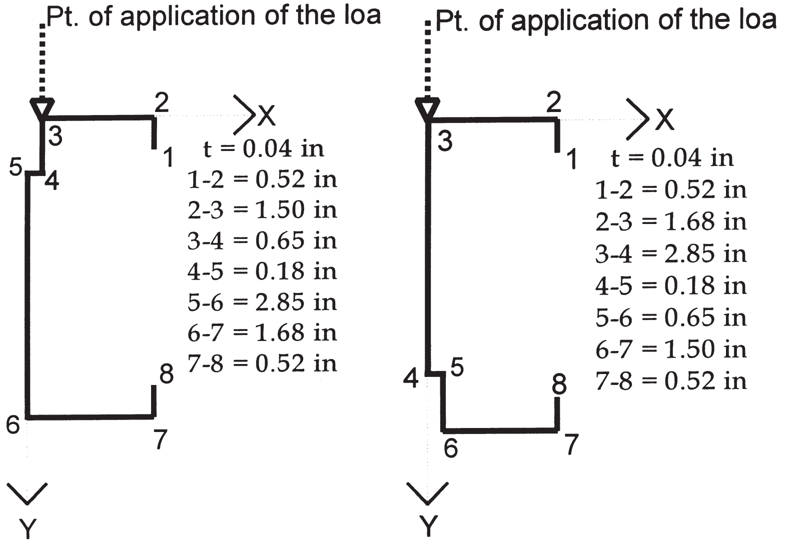

Fig. 4. Cross-section geometry for Rt and Rb.

The geometries of Rt and Rb are different, due to a small ridge in the cross-section. They are shown in Fig. 4.

3.1.2. Material

The measured yield strength of the studied sections are listed in Table 3.

Table 3
Yield stress of beams

Section	C14U	C12U	C14TU	L	Rt	Rb
Yield stress (Ksi)	63.6	60.4	60.0	48.3	46.9	44.7

3.1.3. Supports and loading

The supports restrain the displacements and torsional rotation, but the flexural rotations are not restrained. The support consists of two bolts connected to the web of the section, and hence it may provide a partial warping restraint. The loading arrangement is such that the loading point does not produce restraint against any of the displacements or rotations. Since the shear center does not coincide with the point of application of the vertical load, the load causes torque. In the above experiments, the beams failed due to large rotations and lateral displacements. The failure is triggered by a local mechanism in the web at midspan.

4. Analyses performed

4.1. ABAQUS analysis with beam elements

The problems are modeled using a beam element available in ABAQUS. The element used is an open-section linear beam element capable of modeling non-uniform torsion [16]. The modified-Riks method, an arc-length method, is used for tracing the equilibrium path. An elastic–perfectly plastic material model is used. The assumption inherent in the analysis is that the geometry of the cross-section remains undistorted throughout the analysis. The analysis does not capture local buckling or other local instabilities.

The beams analyzed are modeled with approximately 30 beam elements along the length. The vertical and horizontal displacements and torsional rotation are assumed to be restrained at both ends. The longitudinal displacement is restrained at one end. The warping is not restrained for the warping ‘free’ case and is fixed at both ends for the warping ‘restrained’ case.

4.2. ABAQUS analysis with shell elements

The beams are modeled using a thin shear flexible shell element [16]. The modified-Riks method, an arc-length method, is used for tracing the equilibrium path. An elastic–perfectly plastic material model is used. Geometric imperfections are incorporated in the modeling of the beam. The imperfections are introduced based on the eigenmode displacements corresponding to web local buckling (Fig. 5).

Previous researchers for the analysis of thin-walled steel structures successfully adopted this element as explained in Schafer [17]. The analysis is capable of capturing both local and global instabilities (local and overall buckling). The numerical modeling of geometric imperfections and residual stresses are discussed by Schafer and Peköz [18], and by Schafer [17].

The support conditions are modeled as follows:

1. All the end nodes on the web are restrained against horizontal and vertical displacements.

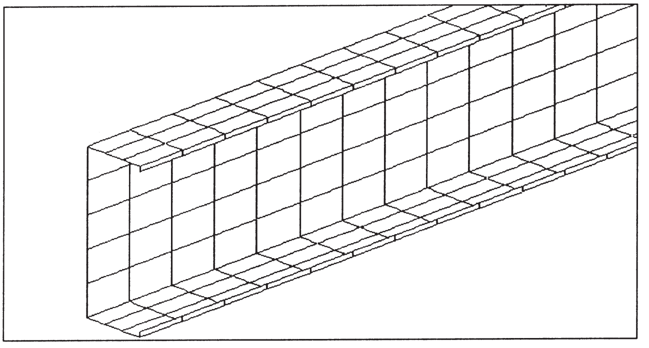

Fig. 5. Analytical modeling of beams by shell elements.

2. The node at the center of the web at one of the ends is restrained against longitudinal displacement.
3. For the warping 'free' case, none of the other nodes are restrained against longitudinal displacement. For the warping 'restrained' case, all the nodes at the ends are restrained against longitudinal displacement.
4. The support conditions for partial warping restraint cases are discussed in Section 6.

Convergence problems are encountered in the shell analyses that are performed. Therefore, the constraints for equilibrium are relaxed. The tolerance limits for the force residuals and the tolerance limits for the displacement corrections are higher than the default values for the tolerances provided by ABAQUS [16,17].

4.3. Elastic finite strip analysis

A thin-walled steel member subject to longitudinal stresses can undergo buckling, in various modes: such as local, lateral-torsional or distortional. Finite strip analysis provides a convenient and efficient way to determine the elastic buckling stresses and corresponding buckling modes. The elastic buckling stresses, buckling modes (local, overall and distortional) and the half wavelengths of the buckled shape can be determined by this analysis. The history and the implementation of the finite strip method are discussed by Schafer [17].

4.4. AISI specification approach

Ellifritt et al. [14] compared the experimental ultimate loads with those predicted by the AISI Specification. The AISI Specification overestimated the ultimate loads by 1%, 19% and 22%, for specimens C14U, C12U and C14TU, respectively. The AISI is based on lateral torsional buckling and the reduction in the ultimate due to the local buckling effects. Calculations show that the local buckling effects reduce the ultimate effects by 10–12%. The finite element studies show that the behavior is not one of lateral torsional buckling. The studies also show that the effect of local buckling on the overall behavior is negligible for the sections tested.

5. Comparison of various analyses and methods

The results obtained from the above analyses are compared with the available experimental results in Gotluru [1]. A few typical results are represented graphically in Figs. 6 and 7. In these figures, the load plotted is the magnitude of a single load of two-point loading.

The rotation behavior seen in Fig. 6 indicates that there is some warping fixity in the test. This fixity is considered in the analysis described below. The failure of the beam takes place immediately after (and in some cases not shown here, before) the yielding of the material. The yield load predicted by CU-BEAM and the failure load predicted by ABAQUS beam analysis are almost the same. The load–rotation behavior predicted by both of these methods is the same, until yielding.

It is also seen that the beam undergoes a large rotation before failure. Therefore, the serviceability limit state may be more critical than the strength limit state.

The effect of warping restraint is illustrated in Fig. 7. In this figure, 'Beam' indicates ABAQUS analysis by modeling as beam elements and 'Shell' indicates ABAQUS analysis by modeling as shell elements. 'Free' and 'Restrained' indicate warping boundary conditions at the ends.

It is seen in Fig. 7 that the ABAQUS analyses, using beam elements or shell elements, predict a similar behavior. The higher failure load in the case of shell analyses can be attributed to the boundary conditions. In the beam analyses, warping free and fixed boundary conditions at the ends are implemented directly. In the shell

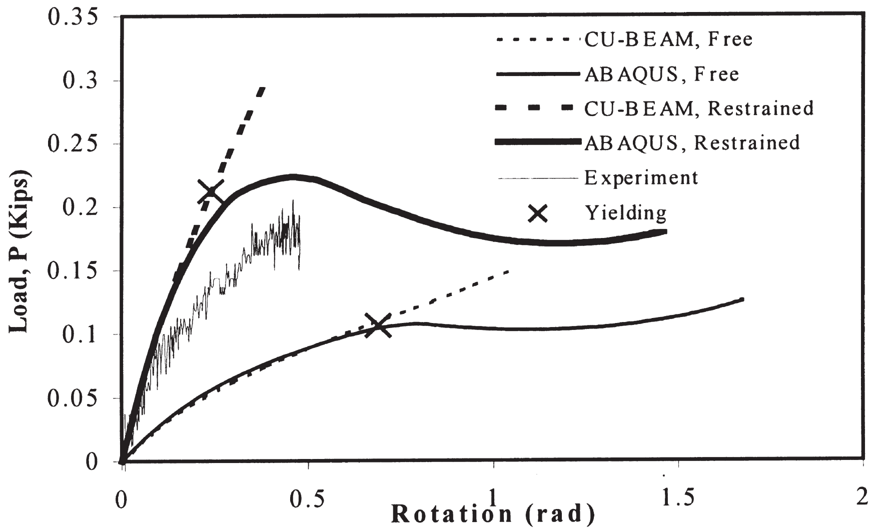

Fig. 6. Test Rb — rotation at midspan.

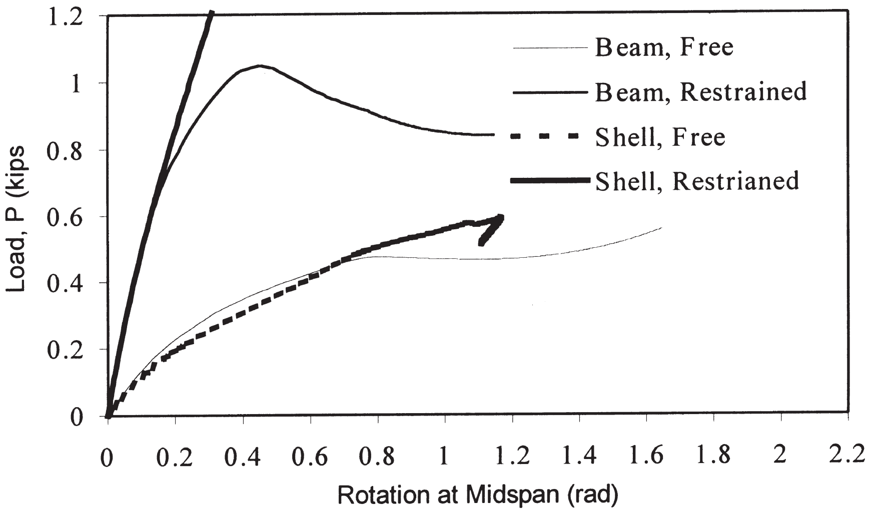

Fig. 7. Test C12U — effect of warping fixity.

analyses, they are only approximately implemented by restraining longitudinal displacements at select points.

In the analysis of all the tests, it is observed that the beam displaces horizontally and rotates, and *no* sudden lateral buckling of the beam takes place. This is consistent with experimental observations.

6. Study of partial warping restraint provided by the supports

The above comparison of the experimental results with the analytical results from CU-BEAM and ABAQUS shows that the supports induce a partial warping restraint at the ends. This can be seen in Fig. 6 and was also noted by Put et al. [13] in their experiments. A more realistic analysis should consider this effect. One of the ways that partial restraint can be accounted for is by means of springs. A hypothetical spring can be introduced instead of free or rigid boundary conditions. Here, the support can be assumed to be a spring providing partial restraint against warping. Put et al. conducted torsion tests to determine the magnitude of the restraint (equivalent spring stiffness) provided by the supports. The stiffness of a warping spring may be defined as follows:

$$\alpha_w = \frac{\text{Bimoment}}{\phi'} \tag{3}$$

where α_w is the spring stiffness in K-in^3/rad or N-mm^3/rad and ϕ' is the rate of change of rotation along the length of the member, or warping, in rad/in or rad/mm.

In this study [1], ABAQUS shell analyses are conducted, where more realistic support conditions can be specified. The following support conditions, which provide partial warping restraints are examined:

1. The longitudinal displacement is restrained at two points on the web, at distances $d/5$ from the flanges, where 'd' is the depth of the beam.
2. The longitudinal displacement is restrained at two points on the web, and at two points on each of the flanges (at 1/3rd of the flange width).

In the next section, some problems are analyzed using CU-BEAM, by introducing warping springs at the ends of the beam. A range of spring stiffnesses is used. The results from these analyses are compared to the results from the ABAQUS shell analysis and the experimental results. From these studies, an attempt is made to determine the equivalent warping spring stiffness provided by the supports, in terms of known parameters of the beam.

6.1. Relative warping spring stiffness

The restraint provided by a spring of a certain stiffness depends on its magnitude relative to the stiffness of the beam. The warping stiffness of the beam is related to the following parameters:

1. directly related to Young's modulus 'E' of the material;
2. directly related to the warping rigidity of the warping constant of the cross-section, 'C_w';
3. inversely related to the length, 'L', of the beam.

Therefore, by dimensional analysis, a non-dimensional parameter K_w is obtained as follows

$$K_w = \frac{\alpha_w}{(EC_w/L)} \tag{4}$$

where α_w is the spring stiffness, as defined by Eq. (3) and K_w is termed as the relative warping spring stiffness. Thus, K_w is defined as the ratio of the spring stiffness to the (EC_w/L) of the beam.

In the analyses conducted [1], the behavior of the beam is a function of the relative warping spring stiffness rather than the absolute magnitude of the spring stiffness. The absolute magnitude of the minimum spring stiffness required to obtain a warping fixed condition varies widely, but the K_w required is identical for all the problems ($K_w \cong 100$).

6.2. Analyses with springs

The problems treated in Section 3 are analyzed by introducing springs at the ends. These analyses are represented graphically in Figs. 8 and 9. The load vs. rotation

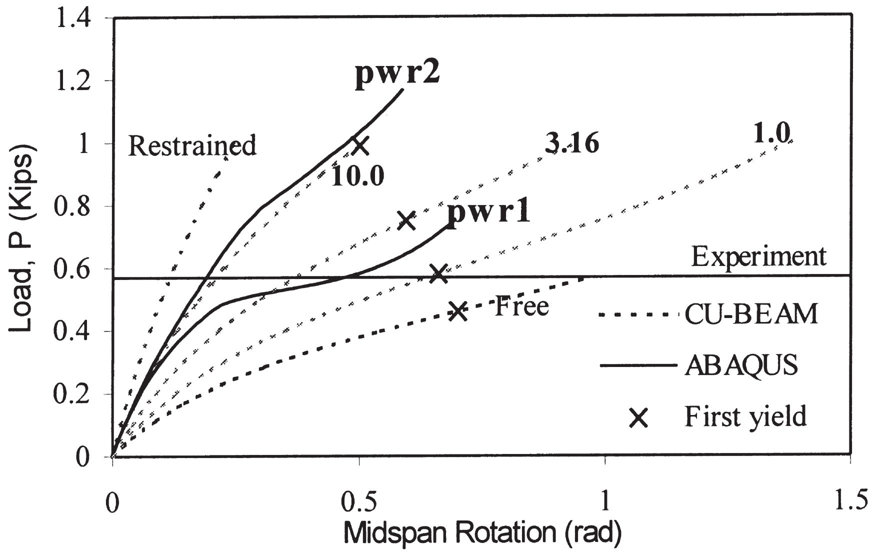

Fig. 8. Test C14U — CU-BEAM and ABAQUS results for support warping restraints.

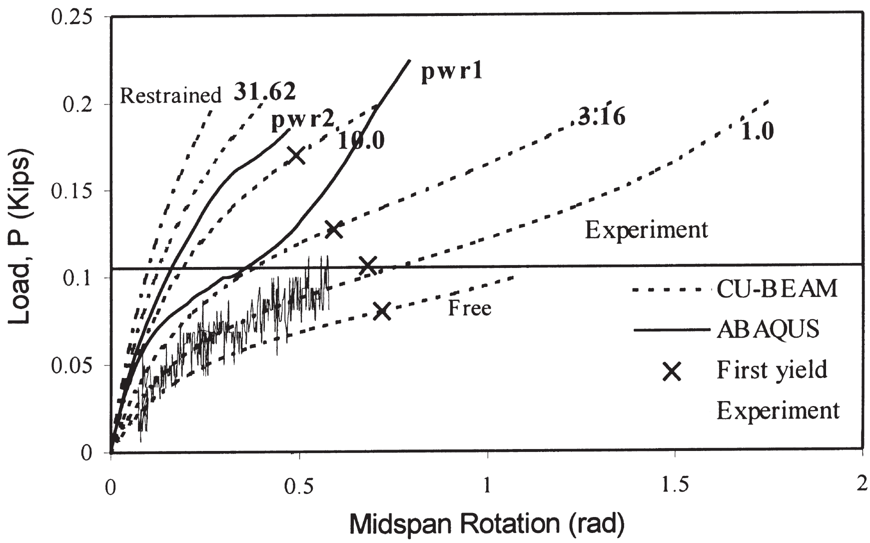

Fig. 9. Test L — CU-BEAM and ABAQUS results for support warping restraints.

curves, the load that causes the first yielding of the material and the experimental results are also presented.

In Figs. 8 and 9, the text next to the curve indicates the warping support conditions. A number indicates that a warping spring is present, with stiffness equal to the number times (EC_w/L) of the beam. In other words, the number is K_w, as defined by Eq. (4). ‘Free’ indicates a warping unrestrained case and ‘Restrained’ indicates a warping fixed case. Again, in these figures, ‘pwr1’ is ABAQUS shell analysis with partial warping restraint (longitudinal displacement is restrained at two points on the web) and ‘pwr2’ is ABAQUS shell analysis with partial warping restraint (longitudinal displacement is restrained at two points each on the web and flanges). In the figures, ‘Experiment’ indicates the failure load observed in the experiment. The ‘First yield’ is based on the maximum stress in the beam.

6.3. Observations and conclusions

6.3.1. Observed values of relative warping spring stiffness

The results from the CU-BEAM analyses with springs provide a way to recommend a value of K_w, the relative warping spring stiffness. (K_w is defined in Section 6.1, as ratio of the warping spring stiffness to the (EC_w/L) of the beam.) The value to be recommended for K_w is determined by the following criteria:

1. The load–rotation curves are compared with the curves obtained from the experimental results.
2. By assuming that yield load can be taken as failure load, the load at which first yield takes place, for different spring stiffnesses, is compared with the experimental failure load.
3. The load–rotation curves are compared to the curves obtained from ABAQUS shell analyses.

The values are compared with the results of previous researchers.

A summary of the above criteria is given in Table 4. (For the sake of simplicity, all the data are not presented in the above graphs.) For interpolation log(K_w) is used. It can be noted that the magnitude of the yield moment is not too sensitive to the variations in the value of K_w. (For example, for section C14U, increasing the value of K_w from 1.0 to 2.0 results in a 10% increase in the yield moment calculated.) The experimental load rotation curves for the University of Florida tests are not available, and the ABAQUS shell analyses for the Rt and Rb sections were not performed.

Table 4
Observed relative spring stiffness, K_w

Description	C14U	C12U	C14TU	L	Rt	Rb
Experimental curve	N/A	N/A	N/A	1.2	10.0	10.0
Yielding as failure	1.0	2.0	1.9	1.0	10.0	10.0
ABAQUS shell analysis	1.8	3.2	3.2	3.1	N/A	N/A

Table 5
Summary of observed relative spring stiffness

Description	C14U	C12U	C14TU	L
ABAQUS shell analysis	10	10	20	10

Case I. Partial warping restraint provided by two bolts connected to the web.

Case II. Partial warping restraint provided by two bolts connected to the web and two bolts each to the flange. (The results that are not shown are not available.)

From the results in Table 5, it is concluded that:

1. The values obtained by considering yielding as failure closely match those obtained from the experimental curves. Hence, the assumption that yielding triggers failure appears valid.
2. The ABAQUS shell analyses lead to a larger equivalent warping spring stiffness. This implies that the bolts in the experiments did not fix the longitudinal displacement completely, and a slip or local distortion may have occurred.
3. Higher stiffness values for Rt and Rb are observed. This is because they are shallow beams, and the bolts (which provide the restraint in the experiments) are much closer to the corners than for other beams.

6.3.2. Warping spring stiffness reported by previous researchers

Put et al. [13] conducted lateral buckling tests on cold-formed channel beams. As part of their study, they conducted torsion tests to obtain values of equivalent warping spring stiffness provided by the support. The support condition that they studied is the web of the beam connected by means of two bolts. In their study they conducted experiments with a single concentrated torque applied at the midspan of the beam. From the spring stiffness values, denoted as α_w in their study, K_w is calculated. This is summarized in the Table 6 (E=2×10^5 MPa). It can be seen from the table that the experimental values are in the range of the values obtained in this analytical research.

Table 6
Warping spring stiffness induced by supports [13]

Test	α_w (N-mm^3/rad)	C_w (mm^6)	L (Mm)	K_w
C10019	93.7×10^9	3.1×10^8	2500	3.7
C10010	10.2×10^9	1.6×10^8	2500	0.9

7. Torsion, local buckling and lateral-torsional buckling

Local plate buckling is an important phenomenon in the behavior of thin-walled steel members. No elastic local buckling is observed in the experiments or in the ABAQUS shell analyses conducted [1]. (The imperfections that induce local buckling were introduced in the shell analyses.) The local buckling observed in the experiments is in the plastic range, and failure immediately followed the local buckling. This is not the case, however, in elastic local buckling, where the beam resists the load after local buckling. The sensitivity of a member to local buckling depends on its width-to-thickness ratio. For the experimental sections, the width-to-thickness ratios of the members are not high and so it can be predicted that local buckling is not probable, and is only observed as a final failure mechanism.

Lateral-torsional buckling is the predominant mode of behavior of unbraced thin-walled steel beams, loaded through the shear center, and bending about the major axis. However, in the experiments or the analytical studies, a sudden lateral-torsional buckling was not observed, and instead beams underwent a gradual lateral displacement and rotation.

7.1. Finite strip analyses

Buckling analyses are conducted on the beams to study the influence of torsional rotation and warping stresses on local buckling and lateral-torsional buckling. CU-FSM (a finite strip analysis program developed at Cornell University [17]) is employed in these analyses. The procedure is as follows.

1. At every load step in CU-BEAM, the stresses in the constant moment and warping region (between the two loads) are noted.
2. These stresses are input into CU-FSM to calculate the load factor corresponding to the local plate buckling and overall buckling.

From the load factor, the corresponding buckling load was calculated and plotted. Thus, the local buckling curve and lateral-torsional buckling curves are plotted.
From the analyses it is observed that for lipped channel sections, with increasing rotation, the local buckling load decreased and the lateral-torsional buckling load increased. The local buckling mode observed was web local buckling in all cases. The distortional buckling mode is absent in most of the cases. The results are plotted in Fig. 10. In this figure, warping free support conditions are considered. 'LB' stands for local buckling and 'LTB' for lateral-torsional buckling. The ABAQUS analysis is beam analysis. The rotation is rotation at the midspan.

7.2. ABAQUS shell analysis of a section prone to local buckling

The sections used in the experiments are not prone to local buckling, because of low width-to-thickness ratios. Therefore, to study the influence of local buckling on the behavior of beams subject to transverse loads not applied at the shear center, a

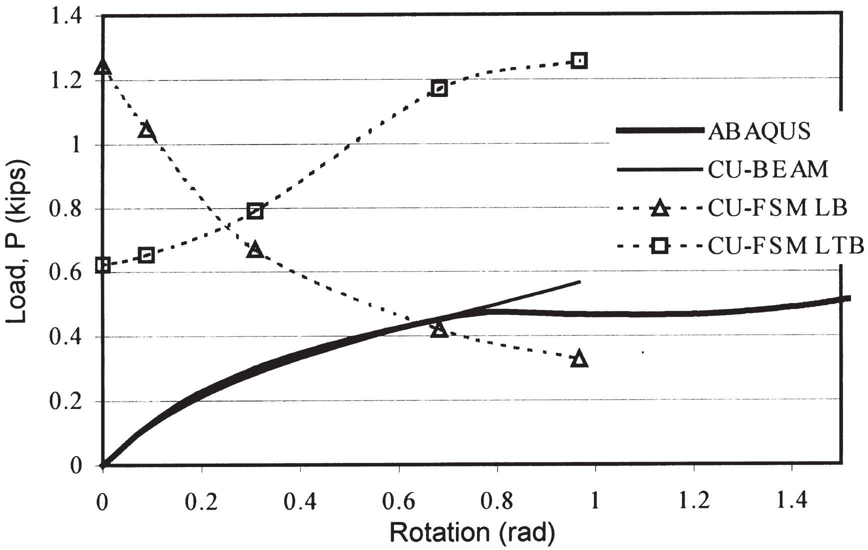

Fig. 10. C14U — local and lateral-torsional buckling analysis by finite strip method.

hypothetical problem is analyzed. The cross-section chosen is a lipped channel section with a high width-to-thickness ratio. This cross-section is taken from the cross-sections listed in the AISI Cold-Formed Steel Specification Design Manual [19]. The dimensions and other details of the cross-section are: lip (0.5 in), flange (3.75 in), depth (16.0 in), thickness (0.075 in), yield strength (65 ksi). The length of the beam is 216 in and the distance between the loads is 60 in.

The lipped channel is simply supported and loaded in two-point bending as shown in Fig. 2. The load is applied at the top flange–web junction. The observations from these analyses are described below:

1. The web buckles locally before yielding near the midspan.
2. The mode of failure is rotation of the beam, and yielding at the top flange–web junction (similar to the non-slender members).
3. The strength of the beam, as predicted by ABAQUS shell analysis, is lower than the strength predicted by ABAQUS beam analysis, or by the yielding load in CU-BEAM.

From Fig. 11, it can be noted that local buckling reduces the ultimate strength of the beam. The same is not observed in C14U, because C14U is not slender, and thus not prone to local buckling. In Fig. 10, the local buckling curve intersects the load–rotation curve only near the failure load for C14U. An interaction equation, to take into account the reduction in the ultimate strength due to local buckling, could potentially be developed. However, one should keep in mind that the local buckling curve in Fig. 11 is calculated with regard to the changing stress distribution due to

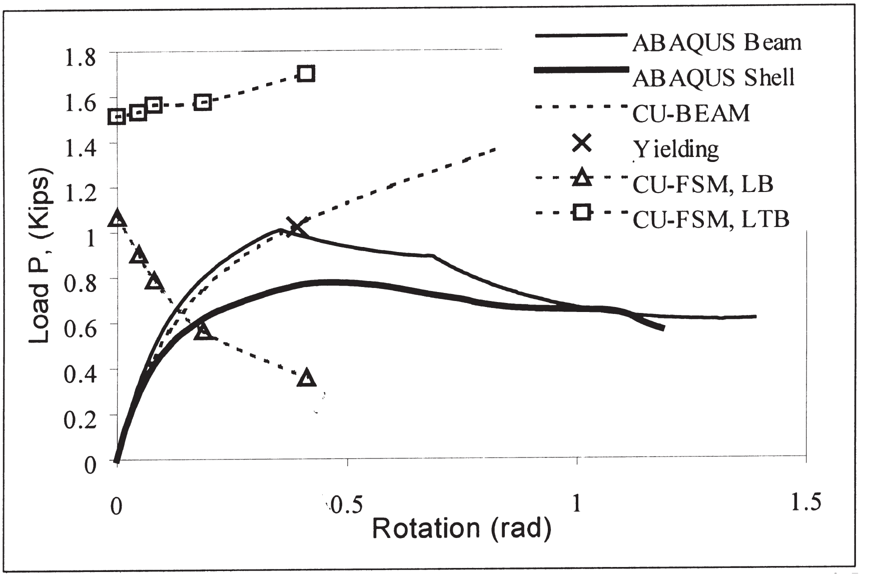

Fig. 11. ABAQUS and finite strip analyses of a beam with a cross-section prone to local buckling.

the additional torque induced by the load application point and the rotation of the member, not simply the elastic stress distribution.

8. Conclusions

In this paper, the studies on bending and torsion in thin-walled cold-formed steel members are presented. The theory behind CU-BEAM, a program for simple nonlinear analysis of continuous beams subject to eccentric transverse loads, is presented. Various experimental and analytical studies are described. The following conclusions can be reported about the behavior of unbraced thin-walled beams subject to transverse loads not applied at the shear center.

Under load, the beam displaces horizontally and rotates gradually, but no sudden lateral buckling of the beam takes place. The failure is started by yielding of the material. The AISI procedure for the estimation of the strength, based on lateral-torsional buckling, may under- or overestimate the strength.

The beam undergoes large rotation before failure. Therefore, the serviceability limit state may be more critical than the strength limit state.

In the case of beams with no slender elements, finite element analyses by modeling as beam elements or shell elements predict similar behavior.

The supports generally used induce a partial warping restraint at the ends. This partial restraint can be expressed in terms of equivalent warping spring stiffness. In the case of beams connected at the ends by means of two bolts in the web, the supports provide a warping stiffness ranging from 1.0 (EC_w/L) to 3.0 (EC_w/L). In the case of beams connected by two bolts each in the flanges and the web, the stiffness is approximately 10.0 (EC_w/L).

An examination of the elastic buckling in the deformed geometry indicates that, for lipped channel sections, the local buckling load decreases with the rotation of the beam, and the lateral-torsional buckling load increases.

In the case of beams with slender elements, local buckling influences the behavior. The mode of failure is not changed, but the strength of the beam is reduced. Therefore, in the design process, the effective section concept may be used to calculate the strength of the beam.

Acknowledgements

The sponsorship of the American Iron and Steel Institute in conducting this research is gratefully acknowledged.

References

[1] Gotluru BP. Torsion in thin-walled cold-formed steel beams. MS thesis, Cornell University, Ithaca, New York, 1998.
[2] Barsoum RS, Gallagher RH. Finite element analysis of torsional and torsional-flexural stability problems. International Journal for Numerical Methods in Engineering 1970;2(3):335–52.
[3] Rajasekaran S. Finite element method for plastic beam columns. In: Chen WF, Atsuta T, editors. Theory of Beam-Columns, vol. 2. New York: McGraw-Hill Inc, 1977:539–608.
[4] Attard MM. Lateral buckling analysis of beams by the FEM. Computers and Structures 1986;23(2):217–31.
[5] Attard MM, Somervaille IJ. Non-linear analysis of thin-walled open beams. Computers and Structures 1987;25(3):437–43.
[6] Chan SL, Kitipornchai S. Geometric nonlinear analysis of asymmetric thin-walled beam-columns. Engineering Structures 1987;9(4):243–54.
[7] Djugash ACR. Nonlinear biaxial bending behaviour of thin walled members. PhD thesis, Department of Civil Engineering, Indian Institute of Technology, Madras, India, 1988.
[8] Djugash ACR, Kalyanaraman V. Experimental study on the lateral buckling behaviour of cold-formed beams. In: Proceedings of the Tenth International Specialty Conference on Cold-Formed Steel Structures. St Louis, MO: University of Missouri — Rolla, 1994:363–80.
[9] Djugash ACR, Kalyanaraman V. Nonlinear analysis of thin-walled members under biaxial bending. J Construct Steel Research 1994;31:289–304.
[10] Ellifrit D, Sputo T, Haynes J. Flexural strength and deflections of discretely braced cold formed steel channel and zee sections. Project report, American Iron and Steel Institute, 1991.
[11] Pi YL, Trahair NS. Nonlinear inelastic analysis of steel beam-columns. I: Theory. Journal of Structural Engineering, ASCE 1994;120(7):2041–61.
[12] Pi YL, Trahair NS. Nonlinear inelastic analysis of steel beam-columns. II: Applications. Journal of Structural Engineering, ASCE 1994;120(7):2062–85.
[13] Put BM, Pi Y-L, Trahair NS. Lateral buckling tests on cold-formed channel beams. Research report no. R767, Center for Advanced Structural Engineering, Department of Civil Engineering, The University of Sydney, Australia, 1998.
[14] Ellifrit D et al. Flexural capacity of discretely braced C's and Z's. In: Yu W-W, LaBoube RA, editors. Proceedings of the Eleventh International Specialty Conference on Cold-Formed Steel Struc-

tures. St Louis, MO: Department of Civil Engineering, University of Missouri — Rolla, 1992:108–29.
[15] Kidokoro R, Kanda I. C-section analysis — a report on the experiments conducted on cold-formed steel lipped channel sections (Appendix C of [1]).
[16] Hibbit, Karlsson & Sorensen Inc.. ABAQUS/standard user's manual. Hibbit, Karlsson & Sorensen, Inc, 1995.
[17] Schafer BW. Cold-formed steel behavior and design: analytical and numerical modeling of elements and members with longitudinal stiffeners. PhD thesis, Cornell University, Ithaca, New York, 1997.
[18] Schafer B, Peköz T. Geometrical imperfections and residual stresses for use in the analytical modeling of cold-formed steel members. In: Proceedings of the 13th International Specialty Conference on Cold-Formed Steel Structures. St Louis, MO: University of Missouri — Rolla, 1996:649–64.
[19] American Iron and Steel Institute. Specification for the design of cold-formed steel structural members. American Iron and Steel Institute, 1996.
[20] Crisfield MA. Non-linear finite element analysis of solids and structures, vol. I. Chichester, UK: John Wiley & Sons Ltd, 1991.
[21] Davies JM. Torsion of light gauge steel members. In: Rhodes J, editor. Design of cold formed steel members. New York: Elsevier Applied Science, 1991:228–59.
[22] Ellifrit D, Glover B, Hren J. Distortional buckling of channels and zees not attached to sheathing. Report for the American Iron and Steel Institute, 1997.
[23] Yu W-W. Cold-formed steel design. 2nd ed. John Wiley and Sons, Inc, 1991.