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The Strength of Thin Plates in Compression

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The stability of thin plates has been investigated by many authors. However, in aeronautical structures, thin metal sheets are often used beyond the stability limits, and the load which can be carried by the structure is determined by the ultimate strength in compression. A recent series of experiments by the Bureau of Standards showed the ultimate load to be independent of width and length of the plate and approximately proportional to the square of the thickness. In the present paper an approximate theoretical analysis of this problem is developed, by which the "effective width" and the ultimate strength can be found. The result of this analysis shows the ultimate strength of a plate to be proportional to the square roots of the modulus of elasticity and the yield point of the material, and to the square of the thickness. This result gives a good check with the experiments mentioned.

The problem of stability of thin plates has been investigated by many authors, S. Timoshenko having been especially successful in obtaining simple results for most of the cases which are of importance in engineering. However, in aeronautical structures, thin metal sheets are often used beyond the stability limits, and the load which can be carried by the structure is determined by the ultimate strength in compression. Recently a series of interesting experiments has been made by the Bureau of Standards, in cooperation with the Bureau of Aeronautics, Navy Department, for the purpose of determining the strength of plates under "edge compression." In these tests the edges parallel to the load were supported and the load was applied by rigid or nearly rigid beams perpendicular to the load. The ultimate load was found to be independent of the width and length of the plate, and approximately proportional to the square of the thickness.

Denoting the ultimate load by P and the thickness by t, the result of these experiments can be expressed approximately by the simple formula

$$P = Kt^2$$

where K is a constant depending on the physical properties of the material. From the theoretical point of view the question occurs how the physical characteristics of the material enter in the constant K, which obviously has the dimension of a stress. The following approximate analysis is a simple method of attacking this question.

Consider a thin sheet of length L and width b under a compressive load P. To simplify the calculations, let it be assumed that the entire load is carried by two strips of width w, one on each side of the sheet, and that the loading is uniform across

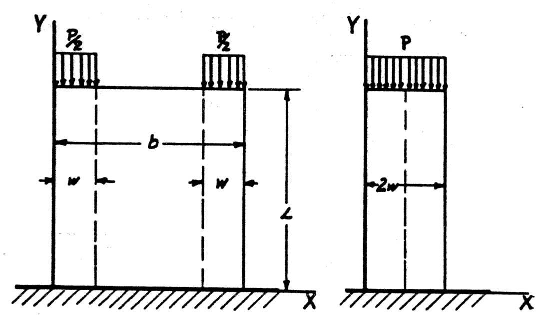

Fig. 1

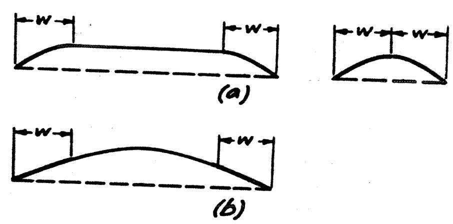

Fig. 2 Assumed Cross-Sections of Buckled Plates

these strips (Fig. 1). Each strip carries the load $P/2$. Then if σ is the stress in the material,

$$P = 2wt\sigma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [1]$$

Consider now the plate to be simply supported on the sides, so that the lateral deflection z along the sides is zero. For simplicity assume that the deflection is such that horizontal tangents at the inner edges of the two load-supporting strips are parallel to the X-direction, Fig. 2(a). Then the center of the sheet can be disregarded and the two strips can be handled as if they were together.

The lateral deflection z of such a strip under compression is given by the equation

$$\frac{Et^2}{12(1-\mu^2)}\left(\frac{\partial^4 z}{\partial x^4} + 2\frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4}\right) + \sigma\frac{\partial^2 z}{\partial x^2} = 0 \ldots [2]$$

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Presented at the National Applied Mechanics Meeting, Purdue University, Lafayette, Ind., June 15 and 16, 1931, of The American Society of Mechanical Engineers. (The paper proper has been prepared by Dr. Von Kármán; the Appendix, by Messrs. Sechler and Donnell.)

where E is the modulus of elasticity and μ is Poisson's ratio for the material. A solution of this equation is

$$z = z_0 \sin \frac{\pi x}{2w} \sin \frac{\pi y}{l}$$

which satisfies the conditions that $z = 0$ and $\partial^2 z/\partial x^2 = 0$ when $x = 0$ or $2w$, and $z = 0$, $\partial^2 z/\partial y^2 = 0$ when $y = 0$ or any multiple of l. l then is the length of a half-wave of the deformation along the line $x = w$. Substituting this value for z in [2] and dividing through by $\sin \frac{\pi x}{2w} \sin \frac{\pi y}{l}$ gives:

$$\frac{Et^2}{12(1-\mu^2)}\left(\frac{\pi^4}{16w^4} + \frac{2\pi^4}{4l^2w^2} + \frac{\pi^4}{l^4}\right) - \sigma\frac{\pi^2}{l^2} = 0$$

from which

$$\frac{\sigma}{E} = \frac{\pi^2 t^2}{12(1-\mu^2)}\left[\frac{l}{4w^2} + \frac{1}{l}\right]^2 \ldots\ldots\ldots\ldots [3]$$

Assume now that the two strips carrying the load are at the limit of stability, corresponding to "the most dangerous wave length." This means that we must find the minimum value of σ/E. Finding the derivative of $\left[\frac{l}{4w^2} + \frac{1}{l}\right]$ with respect to l, and equating it to zero, we find

$$\frac{1}{4w^2} - \frac{1}{l^2} = 0$$

$$l = 2w$$

therefore

$$\frac{\sigma}{E} = \frac{\pi^2 t^2}{12(1-\mu^2)}\left[\frac{1}{2w} + \frac{1}{2w}\right]^2 = \frac{\pi^2 t^2}{12(1-\mu^2)w^2} \ldots . [4]$$

from which

half of effective width

$$w = \frac{\pi}{\sqrt{12(1-\mu^2)}}\sqrt{\frac{E}{\sigma}}\,t \ldots\ldots\ldots\ldots [5]$$

and

$$P = 2wt\sigma = \frac{\pi}{\sqrt{3(1-\mu^2)}}\sqrt{\frac{E}{\sigma}}\,t^2\sigma = \frac{\pi}{\sqrt{3(1-\mu^2)}}\sqrt{E\sigma}\,t^2 \ldots [6]$$

Equations [5] and [6] give us the relation between effective width and the load carried by the plate. The equations hold in this form below the elastic limit. Beyond the elastic limit, it is necessary to replace E by the slope of the stress-strain curve E', and determine the maximum of the product $E'\sigma$. However, it is easy to see that this maximum value will not differ greatly from the product of the value of E below the elastic limit and the yield point σ_y. Hence the maximum load that can be sustained by the plate is:

$$P_{\text{ult.}} = \frac{\pi}{\sqrt{3(1-\mu^2)}}\sqrt{E\sigma_y}\,t^2 \ldots\ldots\ldots\ldots [7]$$

In the experiments mentioned above, the average values of E and σ_y for the duralumin sheets tested were 10,500,000 lb. per sq. in. and 41,000 lb. per sq. in., respectively. Using these values and $\mu = 0.25$ in [7],

$$P_{\text{ult.}} = 1.23 \times 10^6 t^2$$

J. S. Newell[4] gives, as an empirical formula based on these experiments, the value of the maximum load for duralumin as

$$P_{\text{ult.}} = 1.2 \times 10^6 t^2 - 3.0 \times 10^6 t^3$$

1932!

where the second term represents only a small correction for usual thicknesses. A more exact checking of the experimental results is given in the Appendix by Messrs. Donnell and Sechler.

These results have also been applied to the case of plates supported by longitudinal stiffeners. It is assumed that the stresses in the stiffener and in the effective strip of the plate are the same when buckling takes place. It is then possible to determine the width w of the effective strips on each side of the stiffener and to calculate a strengthening factor, by which the strength of the stiffener alone is multiplied, to obtain the strength of the stiffener with its adjacent effective strips. However, it is desired first to check these results by experiments, and the authors expect to do this very soon. It is felt that the experiments on stiffened sheets mentioned by Newell did not have sufficiently definite end conditions for this purpose.

Stiffeners combined with curved plates can be investigated in the same way. Analyzing the effect of a small curvature $1/R$ of the plate, it is observed that the strengthening factor should increase with the curvature due to the increased stability of the curved strips. However, if the stiffeners are located on the concave side of the plates, as must usually be the case with aeronautical structures, the curvature also has an opposite influence on the strengthening factor, because portions of the curved plates are nearer to the neutral axis and hence contribute less to the resultant moment of inertia than corresponding portions of flat plate.

It is easy to show that for small curvatures, i.e., for large values of R, the increase of the strengthening factor is proportional to $1/R^2$, while the loss due to the last-mentioned effect is proportional to $1/R$. It is therefore concluded that if plates supported by stiffeners are curved, the strength decreases with the curvature at first. This fact, which is rather contrary to what one would expect at first sight, was found experimentally in the tests cited by Mr. Newell.

Appendix

THE formula derived by Dr. von Kármán for the ultimate compressive load on a sheet simply supported at the sides can be written

$$P_{\text{ult.}} = C\sqrt{E\sigma_y}\,t^2 \ldots\ldots\ldots\ldots\ldots\ldots [8]$$

Experience with other types of stability problems indicates that the ultimate compressive load on a sheet with other edge conditions—such as a sheet clamped along the sides or the portion of a sheet between stiffeners—can be expressed by the same formula [8], but with a different value of the constant C. In making his analysis, Dr. von Kármán, for simplicity, assumed somewhat artificial conditions, and hence it is not to be expected that the value of C given above is exactly correct even for the case of simply supported sides. The purpose of the derivation was to prove that the ultimate load was proportional to $\sqrt{E\sigma_y}\,t^2$, and it is intended to obtain the correct value of C, for any given edge conditions, from experimental evidence.

It is interesting, however, to compare the value of C found in experiments with the values calculated for certain extreme assumptions.

If a perfectly rectangular flat sheet is freely supported along the sides and compressed between rigid flat plates at the ends, the compressive stress will obviously be uniformly distributed. As the end plates are brought together, this compressive stress will increase, and when it reaches a certain critical value σ_k, lateral buckling deflections will occur all across the sheet, except at the edges, which are constrained from lateral movement. Just

[4] "The Strength of Aluminum Alloy Sheets," *Airway Age*, November, 1930.

as the buckling begins, it can be assumed that the compressive stress is still uniformly distributed across the sheet. This is the classical theory of buckling; there is no reason to suppose anything wrong with it.

The thing that Dr. von Kármán has investigated is what takes place when the end plates are brought still further together. It is known from general experience with buckling that, in the middle of the sheet, the compressive stresses will remain substantially constant as the buckling deflections increase. On the other hand, at the very edges buckling is prevented by the lateral support, so that the stress there must increase according to Hooke's law until it reaches the yield point of the material σ_y; general experience indicates that the maximum resistance of the sheet will be realized at about this time. At points near the edges the stress will be somewhere between the limits σ_b and σ_y. The distribution of stress along the sheet will now be something like that shown in Fig. 3(a). The total area under the curve is a measure of the total force on the sheet, while the shaded area shows the total force according to the classical theory.

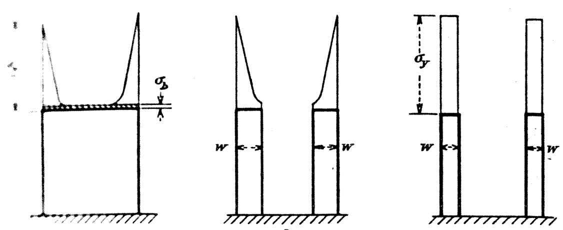


FIG. 3

For very thin sheets such as used in aeronautical work, σ_b is so small compared to σ_y that the middle portion can be neglected entirely. This reduces the problem to that of the buckling of two strips of unknown width w under a stress distribution something like that shown in Fig. 3(b). To facilitate calculations Dr. von Kármán makes a further simplification by assuming that the strips are under uniform stress σ_y, as in Fig. 3(c). This does not affect the final result as much as might be thought, because the width w is still to be solved for, and with the assumption of uniform distribution we will get a smaller value for w than we would with the distribution of Fig. 3(b); consequently the total force will be more nearly the same in the two cases.

We can now calculate w by exactly the same methods that are used in the classical theory to calculate the buckling stress; in that case the stress is unknown, but the width is known, while here the stress is known and the width is unknown. To make this calculation it is necessary to find or assume the shape of the buckled surface. In the derivation given in the main part of the paper it is assumed that buckled cross-sections of the sheet are as shown in Fig. 2(a), so that if the two effective portions are joined they form half a sine wave. With this assumption $C = \frac{\pi}{\sqrt{3(1-\mu^2)}} = 1.90$ for $\mu = 0.3$.

Another and perhaps more likely assumption would be that buckled cross-sections are as shown in Fig. 2(b) such that the portions w can be considered straight. It is shown later that with this assumption the ultimate load is found to be approximately

$$P_{ult.} = \sqrt{\frac{2}{1+\mu}}\sqrt{E\sigma_y}t^2 = 1.24\sqrt{E\sigma_y}t^2, \text{ for } \mu = 0.3 \quad [9]$$

The actual buckled shape is probably somewhere between these two extremes and so one would expect to find experimental values of C somewhere between 1.90 and 1.24.

The Bureau of Standards experiments[5] previously mentioned were made on sheets of four materials of widely different physical properties: duralumin, stainless steel, monel metal, and nickel. Several widths were tested for each thickness (or nearly the same thickness), all widths giving nearly the same results. Table 1 shows the average ultimate load found for each average thickness, and also values of E and σ_y computed from the stress-strain curve given in the report for each thickness (or the nearest thickness). The yield point σ_y was taken as the stress for which the stress-strain slope is one-third the original elastic slope, as suggested in the report. In the last column of the table the values of $C = \frac{P_{ult.}}{\sqrt{E\sigma_y t^2}}$ (from Equation [8]) are calculated. These values are also shown plotted against the thickness in Fig. 4.

TABLE 1 TESTS MADE ON SHEETS OF VARIOUS MATERIALS

	t	$P_{ult.}$	$E \times 10^{-6}$	σ_y	$C = \frac{P_{ult.}}{\sqrt{E\sigma_y t^2}}$
Duralumin	0.017	310	10.6	39,000	1.67
	0.030	930	10.2	41,000	1.60
	0.045	2,190	10.6	40,500	1.65
	0.060	3,840	10.5	39,000	1.67
	0.075	5,180	10.6	44,500	1.34
	0.090	7,500	10.6	42,000	1.39
Stainless steel	0.015	400	26.5	52,500	1.51
	0.034	1,990	27.3	52,000	1.45
	0.050	4,000	24.3	42,000	1.58
	0.060	5,650	34.0	45,000	1.27
	0.078	8,340	25.8	26,000	1.71
	0.095	12,400	27.0	35,000	1.41
Monel metal	0.020	520	26.0	36,000	1.34
	0.030	1,080	22.7	34,500	1.35
	0.044	2,250	24.3	27,000	1.43
	0.063	4,650	25.3	27,500	1.41
	0.079	7,350	22.8	46,000	1.15
	0.093	9,400	22.0	34,000	1.26
Nickel	0.020	620	28.4	40,000	1.46
	0.032	1,540	27.6	33,000	1.58
	0.042	2,550	29.0	30,000	1.55
	0.060	4,500	27.5	36,500	1.25
	0.080	5,000	27.6	15,500	1.20
	0.094	7,900	27.0	21,300	1.18

If Equation [8] is correct these values of C should all be the same. Considering the uncertainties of such tests and the fact that the physical properties varied very widely even for the same material and the indefiniteness of the yield point, the values of C obtained show remarkable constancy. It is also notable that there seems to be no systematic difference between the different materials, the average values for duralumin and stainless steel, for instance, being very nearly the same, in spite of the great difference in physical properties. This is convincing evidence of the correctness of Dr. von Kármán's conclusions that the effect of the physical properties of the material is measured by $\sqrt{E\sigma_y}$ (neglecting the slight influence of Poisson's ratio). There seems to be a slight systematic decrease in the value of C with the thickness. However, this might easily be explained by the fact that the flexibility of the testing jig used in the experiments would play a more important part when testing the thicker, and therefore stiffer, sheets. ?

The average value of C for smaller thicknesses is about 3/2. It is hoped soon to make a similar experimental determination of the value of C for the more important case, in practice, of portions of a plate between stiffeners.

Derivation of Equation 9: If it be assumed that cross-sections of the effective strip remain straight, the lateral deflection can be taken as

$$z = z_0 x \sin\frac{n\pi y}{L} \quad [10]$$

Longitudinal elements of the strip shorten, due to buckling, by the amount $\int_0^L \frac{1}{2}\left(\frac{\partial z}{\partial y}\right)^2 dy$. The external work done by the load during buckling is therefore

[5] N.A.C.A. Report No. 356.

TABLE 2

Type of stiffener	a	a	a	a		b		c	d
Number of stiffeners	2	3	4	2		2		3	2
Distance between lines of rivets, in.	11.25	5.625	3.75	11.25	10.25	+ 2	at 0.875	5.5	11.0
Sheet thickness	0.019	0.032	0.052	0.019		0.032		0.031	0.020
Length of specimen, in.	12	12	12	6		12		12	12
Allowable stress on sheet	1900	5800	13,400	1900	3250		20000[a]	6000	1850
Allowable load on sheet	434	2230	8280	434	1070	+	1120	2235	445
Load on stiffeners	2600	3900	5200	3100		4700		2790	3400
Predicted maximum load	3034	6130	13,480	3534		6890		5025	3845
Observed maximum load	2960	6100	13,200	3300		6500		5240	3800

[a] Stress on section between rivets attaching sheet to stiffener estimated from Bureau of Standards data on Flat Sheet.

$$V_1 = \int_0^L \int_0^w \sigma t dx \, \frac{1}{2}\left(\frac{\partial z}{\partial y}\right)^2 dy$$

The internal energy of bending and torsion in the strip after buckling is

$$V_2 = \frac{Et^3}{12(1-\mu^2)} \int_0^L \int_0^w \frac{1}{2}\left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^2 + 2(1-\mu)\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + 2\mu \frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2}\right] dxdy$$

Substituting [10] in these equations and setting $V_1 = V_2$,

$$\frac{\sigma}{E} = \frac{t^2}{12(1-\mu^2)w^2}\left[\frac{n^2\pi^2 w^2}{L^2} + 6(1-\mu)\right]$$

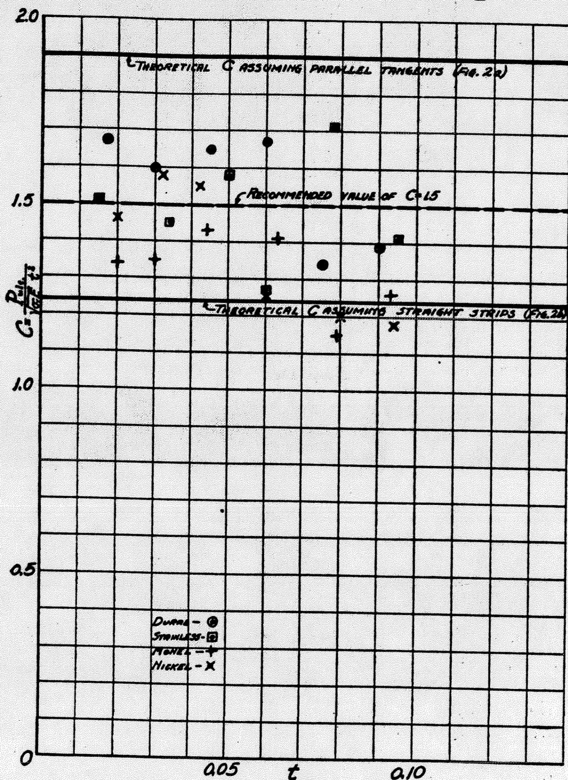


FIG. 4 VALUES OF C PLOTTED AGAINST THICKNESS t

To make σ/E a minimum, n should evidently have its minimum possible value, namely, unity. The first term in the brackets, $\pi^2 w^2/L^2$, can then be neglected in comparison with the second term if w is small compared to L, giving

$$\frac{\sigma}{E} = \frac{1}{2(1+\mu)}\frac{t^2}{w^2} \qquad [11]$$

Equation [9] can then be obtained from [11] in the same manner that [7] was obtained from [4] in the body of the paper.

Discussion

JOSEPH S. NEWELL.[6] This paper is a timely and interesting contribution to the problem of all-metal airplane construction which is so vexing to designers at present. Equation [5] furnishes a rational means for determining the effective width of sheet which functions with the stiffener on a stiffened thin plate and will provide the numerous investigators who are cooperating on this problem a standard method for analyzing their data from this point of view.

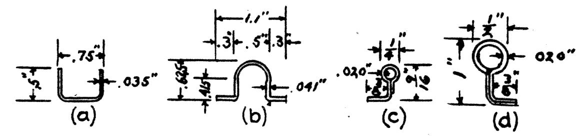
It is, however, suggested that another assumption be considered by investigators of this problem of the stiffened thin plate in compression. The assumption is that the load which the combination of sheet and stiffener will carry is equal to the load which the sheet alone will take plus the load which the stiffener alone will take. To determine the load on the sheet one may use the data obtained by the Bureau of Standards[7] to determine the stress at maximum load on a sheet whose width is equal to the clear distance between rows of rivets connecting the sheet to the stiffeners.

This method of attacking the problem obviates the assumption that the stress in the sheet and in the stiffener are equal when maximum load is approached. In view of the fact that the effective EI of an assembly of sheet and stiffener, when tested as a beam, is not equal to but is normally appreciably less than the product of the modulus of elasticity of the material times the moment of inertia of the cross-section of the combined section, it would appear difficult to justify any assumption based on the homogeneity of the composite section. Furthermore, thin sheets normally wrinkle between the rivets attaching them to a stiffener as the load approaches the maximum that the combination will take. Under such conditions the strains of sheet and stiffener are obviously unequal; hence an assumption of equality of stress is hardly justifiable. The assumption of "effective widths" has long been employed in naval architecture and mechanical engineering. It may be the solution to this problem, but in view of the results shown in Table 2 the proposed scheme of taking the strength of the composite section equal to the sum of its parts appears to be worthy of further consideration.

Fig. 5 shows four types of stiffeners used on flat aluminum-alloy sheets for which more or less data are available. All of the tests have been on panels 12 in. wide. Most of them have been 12 in. long, but there are a few at 6 in. The data in Table 2 are a few taken from the results of investigations made at the Massachusetts Institute of Technology. Attention is invited to the agreement between "Predicted" and "Observed" values where the sheet carries about 12 per cent of the load on the combination, as well as where it carries nearly 75 per cent of the load. The agreement is too close to be the result of mere coincidence, hence appears worthy of the closest investigation.

The scheme in Table 2 is readily applicable for use in design, if later investigations justify its use at all, since one may determine

[6] Associate Professor of Aeronautical Structural Engineering, Massachusetts Institute of Technology, Cambridge, Mass.
[7] N.A.C.A. report No. 356.



	(a)	(b)	(c)	(d)
Allowable load for 12 in. length, lb	1300	2300	930	1700
Allowable load for 6 in. length, lb	1550	..	...	..

Fig. 5

the strength of a stiffener by test or analysis, add to it the load carried by a sheet of width equal to the distance between attachments of sheet to stiffener, and so obtain the strength of the combination. This strength, or total load, may be divided by the total area to obtain an "average" or "apparent" stress for the combination which may be used in analyzing the composite section.

Determining the maximum load to be taken by a stiffener involves the question of end conditions. Tests to date have been made on flat-ended sections as it has been felt advisable to obtain some working data which engineers having judgment could utilize. They are more in the nature of "guessing points" to be tempered by good judgment than exact scientific data. To obtain the latter means the use of knife edges or similar devices to get a true, pin-ended condition on the combination of sheet and stiffener. Since it is impossible to determine the effective centroid of the composite cross-section analytically, due to non-homogeneity of riveted assemblies, the determination of the axis of loading involves a trial and error method in the testing machine. This is practicable, but slow. Eventually it may be necessary to carry out investigations in which such a method will be used, if it proves difficult to obtain a correlating coefficient between the flat end conditions of these tests and the conditions obtained during static tests of actual airplanes, but the demand at present is for a set of approximate values obtained from tests to serve as values which a designer may modify to suit his problem.

Such values are admittedly temporary, and further investigations over a greater range of specimens may necessitate revising them completely. The present demand is for data with the limitations of the tests carefully stated in each case. Next comes the correlation of the data and the development of rational or empirical methods for predicting strengths of thin plates such as are used in aircraft. The present paper covers a possible method which is worthy of careful consideration, and the authors are to be congratulated on the clarity as well as the timeliness of their presentation.

Authors' Closure[8]

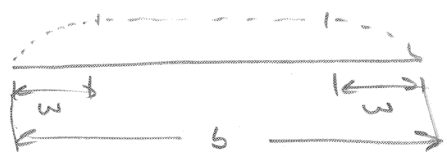
In answering Professor Newell's interesting discussion of the stiffened-sheet problem, it could be pointed out that rivet connections between sheet and stiffener may function quite differently when the sheet is tested in bending and in compression, and that if the rivets are not spaced closely enough to prevent wrinkling when testing in compression, then no simple theory can apply. However, this does not answer the arguments completely, and it is felt that there is not at present enough experimental data to decide what theory will fit most closely the conditions found in practice.

The objection to the experiments mentioned by Professor Newell was that flat-ended columns seem to lie somewhere between columns with perfect end fixity and pin-ended columns; this is also indicated by the results given for compression of the stiffeners alone. As fixed-end columns are supposed to have four times the resistance of corresponding pin-ended ones, it would seem that the effect of the sheet in increasing the width of the end, and hence the degree of end fixity, might be quite large compared to the effect which it is wished to measure. It may be possible to eliminate this factor more easily than by the method mentioned in the discussion, by arranging for a high degree of end fixity which will not be influenced by the presence of the sheet.

[8] In the absence in Europe of Dr. von Kármán, this was written by L. H. Donnell.

VON KARMAN'S "DERIVATION" OF EFF WIDTH.

- ASSUMES THE EXISTENCE OF A NOTIONAL PLATE OF EFFECTIVE WIDTH, IN WHICH ALL THE DEFORMATION OCCURS. CROSS-SECTION OF BUCKLED/DEFORMED CONFIGURATION IS:



$b_e = 2w$

- USING SMALL DEFLECTION THEORY HE FINDS THE BUCKLING LOAD OF THIS NOTIONAL PLATE OF WIDTH b_e

$$(f_{cr})_n = 4\frac{\pi^2 D}{b_e^2 t}$$

- ASSUMES THAT THIS IS THE CORRECT RELATIONSHIP AT ALL TIMES IN THE NOTIONAL PLATE. SO THAT

$$(f)_n = 4\frac{\pi^2 D}{b_e^2 t}$$

- FAILURE OCCURS WHEN f REACHES f_y THUS DEFINING b_e

$$f_y = 4\frac{\pi^2 D}{b_e^2 t} \qquad b_e = \sqrt{4\frac{\pi^2 D}{f_y t}}$$

- MODERN TREATMENTS WOULD THEN SAY $\dfrac{b_e}{b} = \sqrt{4\dfrac{\pi^2 D}{f_y b^2 t}} = \sqrt{\dfrac{f_{cr}}{f_y}}$

WINTER'S CORRECTION

$$P = C\sqrt{E f_y}\, t^2$$

$$C = 1.9 - 1.09\sqrt{\frac{E}{s}}\left(\frac{t}{b}\right)$$

$$b_e = 1.9\, t \sqrt{\frac{E}{s}}\left[1 - 0.574\left(\frac{t}{b}\right)\sqrt{\frac{E}{s}}\right]$$

$$\frac{b_e}{b} = \frac{1.9\, t}{b}\sqrt{\frac{E}{s}}\left[1 - 0.574\left(\frac{t}{b}\right)\sqrt{\frac{E}{s}}\right]$$

$$f_{cr} = \frac{k\,\pi^2 E t^2}{12(1-\nu^2) b^2} \qquad E t^2 = \frac{f_{cr}\cdot 12\cdot(1-\nu^2)\cdot b^2}{k\,\pi^2}$$

$$\frac{b_e}{b} = \sqrt{\frac{1.9^2\, E t^2}{b^2 s}}\left[1 - 0.22\sqrt{\frac{6.807\, E t^2}{b^2 s}}\right]$$

$$\frac{b_e}{b} = \sqrt{\frac{1.9^2\, f_{cr}\, 12\cdot(1-\nu^2)\cdot b^2}{k\pi^2 b^2 s}}\left[1 - 0.22\sqrt{\frac{6.807\cdot f_{cr}\cdot 12\cdot(1-\nu^2)\cdot b^2}{k\pi^2 b^2 s}}\right]$$

$$\frac{b_e}{b} = \sqrt{\frac{f_{cr}}{s}}\left[1 - 0.22\sqrt{1.88\frac{f_{cr}}{s}}\right]$$

if $k = 4$

$\boxed{0.3} = \nu$!!

• HOWEVER IN Y.U. PAPER

$$b_e = \sqrt{4\frac{\pi^2 D}{f_y t}}$$

$$P_u = b_e t f_y = \sqrt{4\frac{\pi^2 D}{f_y t}}\, t\, f_y$$

$$= \sqrt{4\frac{\pi^2 E t^3 f_y}{12(1-\nu^2) t}}$$

$$= \sqrt{4\frac{\pi^2}{12(1-\nu^2)}}\, \sqrt{E f_y} \cdot t^2$$

$$\uparrow$$

$$= C\sqrt{E f_y}\, t^2$$