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Paper No. 2305

STRENGTH OF THIN STEEL COMPRESSION FLANGES

BY GEORGE WINTER,¹ M. ASCE

With DISCUSSION BY MESSRS. FRED T. LEWELLYN, JACOB KAROL, ROBERT L. LEWIS AND DWIGHT F. GUENDER, L. C. MAUGH AND L. M. LEGATSKI, BRUCE G. JOHNSTON, EDWARD L. BROWN AND DON S. WOLFORD, AND GEORGE WINTER.

SYNOPSIS

The production of light structural steel shapes from sheet steel, by cold forming and spot welding, necessitates the development of special design methods adapted to the peculiarities of such members. One of the questions of most practical importance is that of the strength and behavior of thin, wide flanges in compression. This paper reports the results of, and conclusions drawn from, an extensive experimental investigation of this problem. The strength, general behavior, and deformation of two types of structural elements are investigated—(A) members with compression flanges both of whose longitudinal edges are stiffened adequately (such as top flanges of inverted U-beams); and (B) members with compression flanges only one of whose longitudinal edges is stiffened (such as either half of a flange of an I-beam). Results of these tests are evaluated in terms of formulas and charts by which the strength, deformation, and general behavior of such members can be predicted under load conditions.

GENERAL

The thickness of ordinary hot-rolled structural steel sections cannot be decreased beyond a certain minimum, because of limitations inherent in the rolling process. This fact, in the past, has all but prevented the economical use of steel for small-scale structures such as residences, small commercial and manufacturing buildings, and for many secondary members such as floors with moderate loads in larger structures. In view of the low loads occurring in such members, available rolled sections are unduly heavy for the purpose and therefore are unable to compete with other materials more adaptable to such conditions.

Note.—Published in February, 1946, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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However, great advances have been made in forming structural shapes from sheet steel which can be obtained in all desirable thicknesses.² Outstanding examples of this type are the many kinds of steel roof and floor decks that are manufactured in various shapes and with thicknesses ranging from 14 gage (0.0766 in.) to 24 gage (0.0245 in.). The development of automatic spot welding has further increased the possibilities of using sheet steel for structural purposes. It is now possible, technically and economically, to form extremely thin I-shapes by joining two cold-formed, sheet-steel, channels by automatic spot welding in the web. Other shapes, such as thin-walled U-beams and box beams, likewise can be formed from cold-bent sheet steel. This development opens new applications for the use of steel in buildings and other structures.

The possibilities of mass production members and of spot welding make such members particularly suitable for prefabrication. A number of design practices developed over the years in connection with the usual rolled sections will need revision in their application to thin-walled members. Although the problems of this kind are familiar in airplane manufacturing, they are new to the designer of the conventional type of steel structures. One of the most important of these problems is that of the behavior of thin steel members in compression. Thickness is a factor of very minor importance in members subject to tension; but thin sheets in compression will buckle at rather low stresses. This fact must be taken into account in the design of the compression flanges of thin-walled beams or in the design of the entire cross section of thin-walled columns. An investigation of the behavior and strength of such thin-walled compression members is reported in this paper. The findings are based in part on those of other investigators in this field, particularly in airplane design, and in part on tests reported in detail.

Two types of compression flanges were investigated: (A) Flanges that are stiffened along both longitudinal edges such as the compression flanges of inverted U-beams and box beams which are prevented by the two webs from buckling out of their initial plane along both joints (that is, along both longitudinal edges); and (B) flanges that are stiffened only along one longitudinal edge such as either half of the compression flange of a thin-walled I-beam, stiffened only by the web, but free to distort at the outer, unsupported edge. The ranges of dimensions for the specimens (identified herein as types A and B) are given in Table 1. Since the behavior of these two types of flanges was found to be fundamentally different, the two types will be discussed separately.

TYPE A: COMPRESSION FLANGES STIFFENED ALONG BOTH

LONGITUDINAL EDGES

Notation.—The letter symbols introduced in this paper are defined where they first appear and are assembled alphabetically, for convenience of reference, in the Appendix. Discussers are requested to adapt their comments to the

¹⁰ "Light-Gage Steel for Peacetime Building," by Milton Male, *Engineering News-Record*, October 18, 1945.

TABLE I.—RANGE OF FLANGE DIMENSIONS (IN INCHES) FOR THE TWO TYPES OF STRUCTURAL ELEMENTS INVESTIGATED

Type	Width, b		Thickness, t		Depth, λ		Ratio, b/t	
	From:	To:	From:	To:	From:	To:	From:	To:
A	3.40	10.10	0.0237	0.1478	1.49	8.00	14.3	170.0
B	1.43	12.0	0.0368	0.1077	2.0	8.00	9.3	108.0

Equivalent Width of Thin Flanges.—The thin rectangular plate subjected to compressive forces in the plane of the plate has been analyzed extensively by G. H. Bryan and others. The theoretical critical stress at which buckling occurs was thus found to be³

$$S_C = K \frac{\pi^2 E}{12(1 - \alpha^2)} \left(\frac{t}{b}\right)^2 \dots \quad (1)$$

in which E is the modulus of elasticity; μ is Poisson's ratio; t is the thickness of plate; b is the width of plate perpendicular to the direction of compression; and κ is the numerical factor depending upon the ratio of length to width of the plate and the conditions of edge support.

beams, likewise can be formed from cold-rolled sections, opens new applications for the use of steel in buildings and other structures. The possibilities of mass production inherent in the processes of cold forming and of spot welding make such members particularly suitable for prefabrication. A number of design practices developed over the years in connection with the usual rolled sections will need revision in their application to thin-walled members. Although the problems of this kind are familiar in airplane manufacturing, they are new to the designer of the conventional type of steel structures. One of the most important of these problems is that of the behavior of thin steel members in compression. Thickness is a factor of very minor importance in members subject to tension; but thin sheets in compression will buckle at rather low stresses. This fact must be taken into account in the design of the compression flanges of thin-walled beams or in the design of the entire cross section of thin-walled columns. An investigation of the behavior and strength of such thin-walled compression members is reported in this paper. The findings are based in part on those of other investigators in this field, particularly in airplane design, and in part on tests reported in detail

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Experiments, with plates in compression, conducted by several investigators,^{4,5} reveal, however, a fundamental difference between the practical significance of the Euler critical stress for columns and the critical stress given by Eq. 1 for plates. Whereas long columns actually fail at, or slightly below, the Euler stress, plates supported longitudinally along both edges have been found to carry compressive stresses considerably above the critical. L.

Sotouhman and G. Back summarize their experiments qualitatively as follows:

"It is seen that for only the very narrow and thick plates do the Bryan loads approach or exceed the maximum loads found in the tests. For the wide, thin plates, the Bryan load is as low as 1/30 of the maximum load * * *. [In most of the plates the buckling, that is, the formation of waves] was gradual * * showing no sudden change. In some of the thick and narrow specimens, however, there was no appreciable buckling until the load approached the maximum. Owing to lack of ideal conditions, such as initial curvature, all plates buckled [but did not fail] before the Bryan load was reached."

³"Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 324.

⁴"Strength of Rectangular Flat Plates Under Edge Compression," by L. Schuman and G. Back, Technical Report No. 356, National Advisory Committee for Aeronautics, 1931.

⁵"The Ultimate Strength of Thin Flat Sheet in Compression," by F. E. Soehler, Publication No. 27

Jugenheim Aeronautics Laboratory, California Inst. of Technology, Pasadena, 1933.

⁴ "Strength of Rectangular Flat Plates Under Edge Compression," by L. Schuman and G. Back, Technical Report No. 356, National Advisory Committee for Aeronautics, 1931, p. 523.

Thus, while in the column field the Euler load is actually the maximum load a column can reach, for thin plates the Bryan load⁸ is found to be of little practical consequence, except for very narrow ones.

Physically, the difference between the significance of the critical loads for free columns and for edge-supported plates is easily visualized as follows: Once a column starts deflecting at or near the Euler load, the deflection continues to increase because of the increasing bending moment caused by that same deflection. The column, then, fails very quickly, literally "getting away from its load" by unrestrained motion. On the other hand, in an edge-supported plate deflections are possible only in the central parts, the edges and parts adjacent to them being kept straight by the supports. Thus, the plate cannot "get away from its compression load," by unrestrained deflection such as that in columns. The central, more highly distorted, regions of the plate decrease in their resistance, thus throwing more of the total compressive force toward the stiffened edges. The plate reaches the limit of its carrying capacity only when these stiffened outer parts are stressed to the yield point. This action occurs at a load higher than the critical—that is, higher than that load at which, theoretically, small deflections start to occur in the plate. For these reasons, in columns, a small deflection at the Euler load produces almost immediate failure, whereas in plates small deflections merely result in a redistribution of the compressive stresses across the width of the plate.

The "small deflection" (Bryan) theory of plate buckling, therefore, gives results of only very limited practical significance. An exact mathematical treatment of plates in compression on the basis of the "large deflection theory" is extremely difficult. Attempts in this direction have failed to give practically significant results, except for the one case of the circular plate uniformly compressed around the perimeter.⁹ However, in 1932 Theodor von Kármán,^{10,11} M. ASCE, suggested a semi-empirical approach to this question which, subsequently elaborated by others, proved to be extremely useful. Professor von Kármán took account of the fact that, in a plate compressed beyond the Bryan stress, the central strip (Fig. 1), most heavily distorted by wave formation, cannot be counted on to carry an appreciable part of the compressive load. On the other hand, two strips, adjacent to the edges, are held in line by the edge stiffeners and will withstand most of the compression. Thus, with only two strips of width b_s , each being effective, the actual plate of width b is equivalent to the fully effective narrower plate of equivalent width $b_e = 2b_s$ (Fig. 1). By rather intuitive

⁸"On the Stability of a Plane Plate Under Thrust in Its Own Plane," by G. H. Bryan, *Proceedings, London Mathematical Soc.*, Vol. 22, 1890, pp. 54-67.

⁹"Buckling of the Circular Plate," by K. O. Friedrichs and J. J. Stoker, *Journal of Applied Mechanics*, March, 1932, p. A-7.

¹⁰"The Strength of Thin Plates in Compression," by Theodor von Kármán, E. E. Sechler, and L. H. Donnell, *Transactions, A.S.M.E.*, Vol. 54, 1932, p. 53.

¹¹"Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 353.

analytical reasoning this width was assumed to be

$$b_e = 1.9 t \sqrt{\frac{E}{s_y}} \quad \dots \dots \dots \quad (2a)$$

in which s_y is the yield point of the material. To test the validity of this hypothesis, tests were made by E. E. Sechler,¹² on thin plates of various metals. He found Eq. 2a to hold except that instead of the fixed constant 1.9 a variable coefficient C resulted in better agreement with the tests. Thus:

$$b_e = C t \sqrt{\frac{E}{s_y}} \quad \dots \dots \dots \quad (2b)$$

This coefficient was found to depend on the parameter $\sqrt{E/s_y} (t/b)$. Fig. 2(a) shows the results of Mr. Sechler's tests, representing the experimental values of C plotted against this parameter. (The considerable scattering

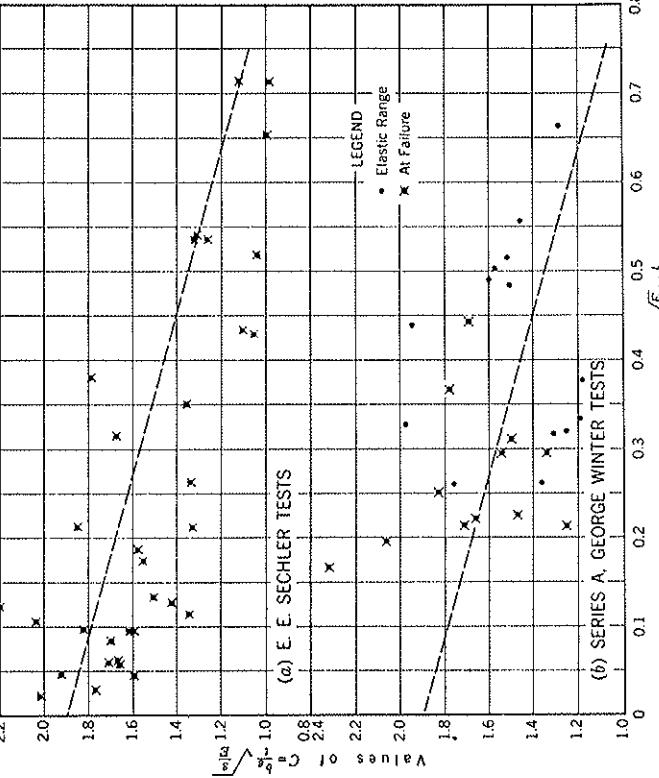


Fig. 2.—EXPERIMENTAL DETERMINATION OF EQUIVALENT WIDTH

noticeable in Fig. 2(a) is a feature common to all compression tests on thin plates and is probably caused by the influence of inevitable deviations from true shape of such specimens.) Only for the very small values of the parameter (that is, for extremely wide and thin plates) does C approach 1.9; for the

¹²"Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 402.

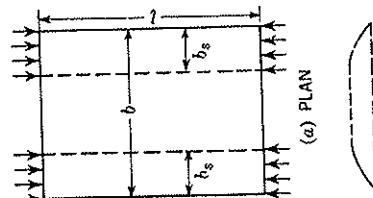


Fig. 1.—THIN PLATE IN COMPRESSION (ACCORDING TO THEOREM VON KÁRMÁN, M. ASCE)

The remainder of the range a smaller C is applicable. In airplane design it is common practice either to use an average value of $C = 1.7$ or to determine an appropriate value from the results of Mr. Sechler's tests.¹⁵

The tests by Messrs. Schuman and Back and those by Mr. Sechler were made on individual thin compression plates. To determine whether the equivalent width approach could be recommended for wider use in the field of structural engineering, it appeared desirable to institute further tests to investigate: (1) Whether thin compression plates representing component parts of structural members (such as compression flanges of thin-walled beams) performed in the same way as disjointed, individual plates; (2) whether a simple sufficiently accurate expression could be found for determining the equivalent width for design purposes; and (3) whether such an expression applied to the state of ultimate failure only or whether the same, or similar, expressions could be used at stresses below

venience of computation; and y_n , the distances of the centroids of these areas from the neutral axis. In Eq. 3a all quantities are known except the effective area of the compression flange and, consequently, it is possible to determine the equivalent width of that flange. For determining the equivalent width at the failure load the position of the neutral axis was again determined from strain measurements. In this case strains were measured as close to failure as possible, at loads which were 5% or less below the ultimate. However, since the stress distribution at failure is that

the yield point. (The original information by Messrs. von Kármán and Sechler leading to Eqs. 2 pertained to failure stresses only.) The test results are summarized in Fig. 2(b).

In test series A twenty-five inverted U-beams representing thirteen different types of sections of the general shape shown in Fig. 3 were tested as simple beams with quarter-point loading. In these specimens b varied from 3.40 in. to

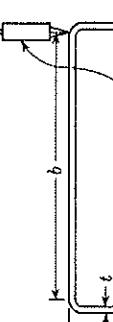


Fig. 3.—SHAPE OF INVERTED U-BEAMS OF TEST SERIES A

5.33 in., k from 1.49 in. to 2.46 in., and t from 0.0237 in. to 0.0589 in. The span was 80 in. for all tests, equal loads being applied 20 in. from either support. Two 8 in. strain gages were used for strain measurements, mounted on the outer surfaces of the specimens, as close as possible to the web (see Fig. 3). In addition to strains, midpoint deflections were recorded. Mechanical properties of the various sheet steels were determined from tension tests on coupons cut from the steels from which the beams were formed.

The effective width of the compression flanges of each specimen was determined at two different loads: (a) In the wholly elastic range—that is, before the bottom (tension) flange began to yield; and (b) at, or as close as possible to, the failure load. For loads in the elastic range, the position of the neutral axis was determined from the strain readings. Once the location of the axis is known, the equivalent area, and subsequently the equivalent width, of the top flange is determined from the usual condition that the moment of the cross section about the axis is zero; that is,

(3a)

in which A_n represents the subareas into which the section is divided for construction.

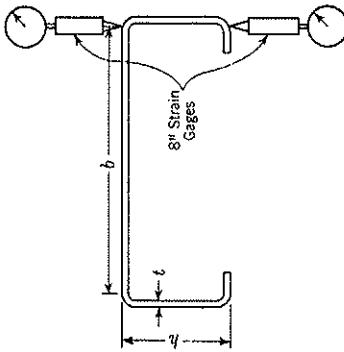
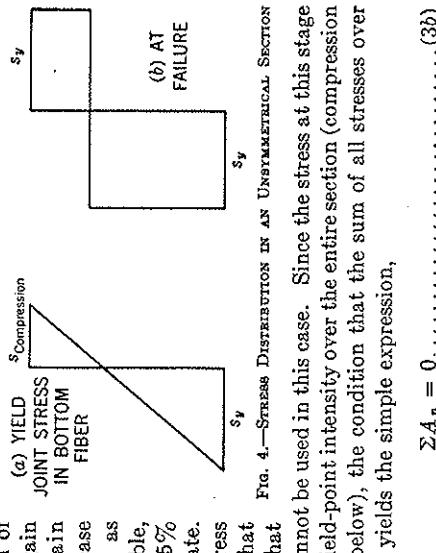


FIG. 3.—SHAPE OF INVERTED U-BEAMS OF
TEN SERIES.

8-in. strain gages were used for strain measurements, mounted on the outer surfaces of the specimens, as close as possible to the web (see Fig. 3). In addition to strains, midpoint deflections were recorded. Mechanical properties of the various sheet steels were determined from tension tests on coupons cut from the steels from which the beams were formed.

in which A_n represents the subareas into which the section is divided for computation.



in which A_n is positive for the areas below the axis and negative for those above. Eq. 3b, then, can be solved for the effective areas of the compression flange.

Once the equivalent widths were obtained experimentally, the coefficients C were computed from Eq. 2a. These coefficients are plotted in Fig. 2(b) in the same manner as are Mr. Sechler's data in Fig. 2(a). Comparing these two sets of data, the following conclusions can be drawn:

1. The experimental points in Fig. 2 were obtained by three fundamentally different methods. Mr. Sechler's results (Fig. 2(a)) refer to ultimate loads of individual plates. On the other hand, the points in Fig. 2(b) refer to compression flanges representing integral parts of structural shapes, and, whereas half of the latter refer to ultimate loads (at the yield point), the other half are obtained at stresses far below the yield point, ranging from 7,700 lb per sq in. to 21,500 lb per sq in., depending on the shape of the specimen. Despite this diversity, the agreement between the results of the two investigations is remarkably close.

2. Whereas Mr. Decurtins' results (Fig. 2(b)) refer to failure stresses only, the fact that the points in Fig. 2(b) obtained at low stresses are located in the same general way as those obtained at the ultimate load indicates that Eq. 2b

"Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N.Y., 2d Ed., 1941, Pt. 2, FIG. 234, p. 371.

¹⁵ Discussion by George Winter of "Theory of Limit Design," by J. A. Van den Broek, *Transactions, ASCE*, Vol. 105, 1940, Fig. 24, p. 574.

can be used not only at the yield point but likewise in the elastic range, that is:

in which s is any stress at or below the yield point. (It is likely that at very low stresses the equivalent width so obtained is somewhat on the conservative side.)

Once the relationship between the coefficient C and the parameter indicated in Fig. 2 is rather firmly established experimentally, it appears desirable to express the equivalent width b_e in a simple explicit formula rather than to determine it by using Eq. 2 in conjunction with Eq. 4. For this purpose the averaging straight line was drawn in Fig. 2. This line was so drawn as to represent the average values of both Mr. Schlier's tests (Fig. 2(a)) and the present tests (Fig. 2(b)). The experimental values scatter rather regularly around that line. The equation of this averaging straight line is

By substitution of this expression for C in Eq. 4, the following equation is obtained for the equivalent width b_e :

$$b_e = 1.9 t \sqrt{\frac{E}{s}} \left[1 - 0.574 \left(\frac{t}{6} \right) \sqrt{\frac{E}{s}} \right] \quad (6)$$

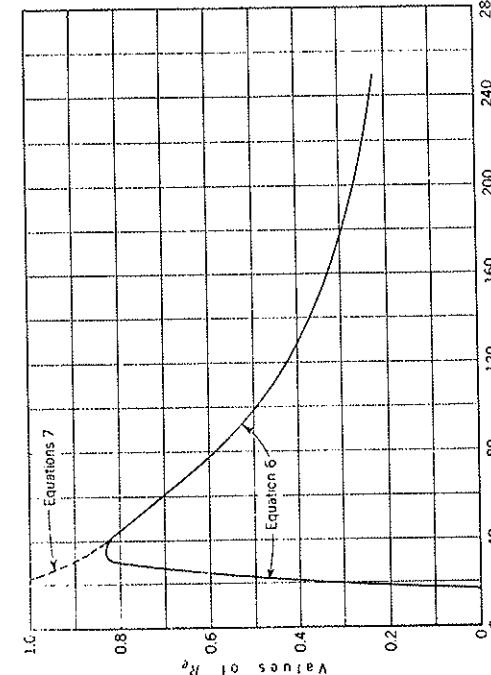


FIG. 5.—Ratio $R_s = b_s/b$ for the particular case of $s = 30,000 \text{ lb per sq in.}$ and
 $b_s = 30,000,000 \text{ in. per sq in.}$

Thus, b_s , expressed as a multiple of the plate thickness t , depends on two parameters, the thickness-width ratio t/b of the flange and the strain δ at the unsupported edge (since $E/s = 1/\delta$).

The relationship expressed by Eq. 6 is shown graphically in Fig. 5 for the particular case $E = 30,000,000$ lb per sq in. and $s = 30,000$ lb per sq in. In Fig. 5 the ratio of the equivalent to the actual width, $R_e = b_e/b$, is plotted against the width-thickness ratio of the flange, b/t . In the medium and high range of b/t the equivalent width ratio decreases with increasing b/t , as would be expected. For the low range of b/t , however, b_e computed from Eq. 6 decreases with decreasing b/t , which is obviously impossible physically. It must be concluded, therefore, that Eq. 6, which was established from tests in the medium and high range of b/t , is limited in its application to that same range. (In series A the range of b/t tested extended from 64 to 170.)

To investigate the behavior of compression flanges with rather small width-thickness ratios b/t , a special series B of tests was instituted. The specimens were built-up I-beams formed by two channels, of the general shape shown in Fig. 6, with width-thickness ratios ranging from 14.3 to 56.0. The depths of

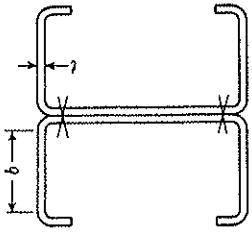
at distances of about $0.4L$ from the respective supports rather than at the quarter points of the span L . The magnitudes of the equivalent widths were determined experimentally from strain-gage readings in the same manner as discussed previously. The results of this investigation are given in Table 2(a) where average results of two tests of each type of beam are recorded. In Table 2, b is the free, horizontal width of half of compression flange measured between toes of transition radii; t is the thickness of metal; and s is the stress, in pounds per square inch, at which strain measurements were made to determine the test values of b . The "test" values of b , (Col. 4, Table 2(a)) are those determined from strain measurements; and the "chart" values are those determined from strain measurements and the constants of the tables.

determined from fig. 7 for unit b/t -values and unit s -values of one-tenth.

The accuracy of the determination of b_s is somewhat impaired in this series by the following fact: In these beams the compression flanges represent a rather small percentage of the total cross-sectional area. For this reason a moderate change in the equivalent area of the compression flange will change the position of the neutral axis by a much smaller percentage than the area itself changes, and will therefore cause a very small change in the strains recorded by the gages. Consequently, the accuracy of determining b_s from such tests is much smaller

The most important conclusion to be drawn from the b_e -values in Col. 4, Table 2(*w*), is that flanges of this type are fully effective ($b_e = b$) for values of b/e smaller than about 20 to 30. From these values upward, the equivalent widths b_e are consistently smaller than the actual width b . For small

FIG. 6.—SERIES B

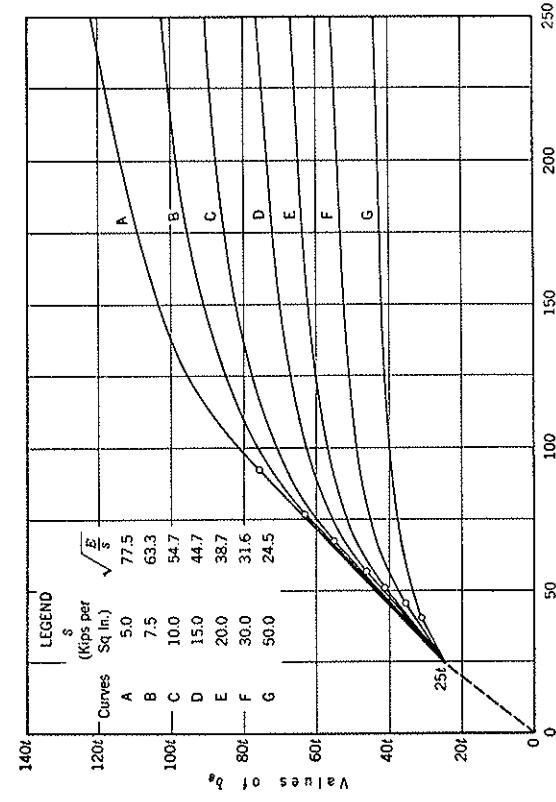


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TABLE 2.—TESTS ON I-BEAM SECTIONS IN THE LOW RANGE OF b/t

$\frac{b}{t}$	Type*	(a) EQUIVALENT WIDTHS (SERIES B)			(b) ULTIMATE MOMENTS (SERIES B AND C)			
		s (lb per sq in.)	b_s	Col. 5 Col. 4	s_s	M_s/M_t	M_t/M_t	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
14.3	I-1	26.300	14.11	14.3 t	1.01	35.700	0.98	0.98
16.3	I-2	27.200	16.5 t	16.3 t	0.99	33.700	0.94	0.94
16.4	I-3	26.600	16.6 t	16.4 t	0.99	33.100	0.93	0.93
19.2	I-4	27.600	19.2 t	19.2 t	1.00	35.100	0.90	0.90
22.9	I-5	27.200	21.8 t	22.9 t	1.05	33.100	0.95	0.95
23.6	I-6	26.300	23.4 t	23.6 t	1.01	36.300	0.95	0.95
24.0	I-7	29.200	22.6 t	24.0 t	1.06	35.100	0.95	0.95
26.9	I-8	27.400	27.4 t	27.4 t	0.98	30.900	0.93	0.93
32.0	I-9	27.200	31.7 t	29.0 t	0.91	36.200	1.06	0.99
33.5	I-14*	37.300	1.07	1.09
36.0	I-15	25.600	37.2 t	32.5 t	0.87	36.400	0.84	0.73
38.3	I-10	27.600	39.6 t	35.0 t	0.88	30.200	1.02	0.92
42.6	I-11	27.400	40.8 t	37.0 t	0.90	37.300	1.14	1.02
45.0	I-12	23.500	30.300	1.04	0.92
49.5	I-16*	37.900	1.20	1.04
49.9	I-17	36.400	0.98	0.84
51.6	I-18*	32.200	1.08	0.95
56.0	I-13	28.000	48.8 t	41.5 t	0.85	37.300	1.17	1.02
77.7	I-19*	37.900	1.55	1.10
86.6	I-20*	32.200	1.47	1.10

* Average results of two tests of each type of beam, except types I-14, I-16, I-18, I-19, and I-20, which are the average of three tests each.

FIG. 7.—CHART FOR DETERMINING THE EQUIVALENT WIDTH b_s .

values of b/t it appears reasonable, therefore, to draw a transition curve from $b/t = 25$, for $R = 1$, tangent to the curve representing Eq. 6 as is done in broken lines in Fig. 5. It is not unlikely that a more extensive investigation of b , in this range of b/t would show a somewhat different shape of this transition curve. However, since in this region compression flanges are nearly fully effective (b_s close to b), the details of the shape of this transition line are of little practical consequence.

In practical design the use of Eq. 6 in the medium and high range of b/t in conjunction with the transition line in the low range just discussed is somewhat cumbersome. For this reason the chart in Fig. 7 was drawn for design purposes. The curves in Fig. 7 allow the equivalent width b_s to be read directly for any ratio b/t and for any practically important value of the stress s . The curved part of each line in this chart represents the values obtained from Eq. 6 and the straight parts in the low range were drawn as lines tangent to the curves and ending at $b_s = 25$ for $b/t = 25$. (The point of tangency, indicated by small circles on Fig. 7, is found from

$$\left(\frac{b}{t}\right)_s = \frac{1.0906 \frac{E}{s} + \sqrt{\left(1.0906 \frac{E}{s}\right)^2 - 27.265 \frac{E}{s} \left(1.9 \sqrt{\frac{E}{s}} - 25\right)}}{1.9 \sqrt{\frac{E}{s}} - 25} \quad (7a)$$

and the equation of the straight lines in the range of low b/t is

$$b_s = \left[\frac{1.0906 \frac{E}{s}}{(b/t)_s^2} \left(\frac{b}{t} - 25 \right) + 25 \right] t \quad (7b)$$

in which $(b/t)_s$ is defined in Eq. 7a.) The chart is drawn for steel members, with $E = 30,000,000$ lb per sq in., but it can also be used for compression flanges made of other metals. In this case, instead of using the stress s indicated by the respective curves, the value $\sqrt{E/s}$ should be determined for the particular case and the corresponding curve found in the chart.

For the medium and high range of b/t the validity of Eq. 6, and of the curves derived from it, appears to be well established by the tests summarized in Fig. 2. Nevertheless, it is desirable to obtain more information on the low range—that is, on the validity of the straight-line parts of the respective curves. For this reason for all tests given in Table 2(a), the equivalent width ratios b_s , as obtained from Fig. 7, were determined and entered in Table 2(c). A comparison of these values (Col. 5, Table 2(a)) with those obtained from the tests (Col. 4, Table 2(a)) shows, with a very few exceptions, good agreement throughout this range (see Col. 6, Table 6, Table 2(a)). Any significant deviations from Fig. 7 are conservative side; that is, the equivalent width, determined from Fig. 7, is equal to or somewhat smaller than that obtained from tests. For this reason it is believed that the chart, over its entire range, can safely be recommended for design purposes.

For practical use the findings incorporated in Fig. 7 should enable the designer to determine: (a) The ultimate load for a member of given configuration

and material; and (b) for such a member, the deformation for a given loading, particularly the deflections of transversely loaded beams. In Tables 2(b) and 3 test results are reviewed in the light of these practical requirements.

Ultimate Loads.—To ascertain whether the equivalent-width concept permits a satisfactory prediction of ultimate loads, Table 2(b) gives the comparison of ultimate loads obtained from tests with those computed on the basis of the full, unreduced cross section and with those obtained by considering the equivalent width of the compression flanges. Tension tests were made on

specimens cut from the steel from which the beams were formed. Equivalent widths were then determined from Fig. 7 for the yield points so obtained. Section moduli S were computed both for the full, unreduced cross section and for the reduced cross section—the latter by using the equivalent instead of the actual width of the compression flanges. The ultimate moments were then obtained from

$$M_{\star} = s_{\star} S \quad (8)$$

Table 2(b) contains not only the results of series B but also those of a special, additional series C of I-beam tests (types I-14 to I-20). These beams were shaped in a manner similar to those of series B except that the tension flange was made about 20% wider than the compression flange and the beams were loaded at the third points. The depth of all beams of series C was 8 in.; the widths of the top flanges varied from 4.50 in. to 10.10 in.; and the thicknesses, from 0.0447 in. to 0.0755 in.

In Cols. 8 and 9, Table 2(b), M_1 is the ultimate moment computed from the section modulus of the full, unreduced cross section; M_2 is the ultimate moment computed from the section modulus obtained by substituting the equivalent width of the compression flange for the actual width; M_3 is the average test-failure moment of the "identical" beams of the given type.

It is to be noted from Col. 9, Table 2(6), that the behavior of beams I-15 and I-17 is rather different from that of all others in that these beams failed at moments considerably above both M_1 and M_2 . These two beams were formed of the same steel and this steel was the only one tested which showed the following peculiarity in the tension tests: Instead of revealing a definite yield point by the usual criterion of arrest of load or drop of the beam, this steel showed only a more rapid increase of strain with increasing stress at about 36,400 lb per sq.in. In other words, the stress-strain curve of this steel had no horizontal part at the yield point. The absence of this horizontal part seemingly accounts for the fact that the outer fibers were stressed considerably higher than the rather indefinite yield point, before the beams actually failed.

A study of Table 2(6) reveals that, in the low range of b/t , the difference between M_1 and M_2 for this type of I-beams is rather insignificant. This is caused by the fact that the equivalent width in this range is very close to the actual (b_e is close to b in Fig. 7). Since, in addition, the compression flanges are only a small part of the total cross section, a small change in width changes the section modulus only insignificantly. In this range (say, to $b/t = 40$) both M_1 and M_2 are rather close to M_t and somewhat on the conservative side. (For other types of cross sections, if the compression flange represents a more

substantial part of the entire area, the reduction in width may become practically important even in the low range of b/t .

For b/t larger than about 40, M_1 becomes significantly and increasingly larger than M_2 , whereas M_2 agrees with M_1 rather satisfactorily throughout this range. If the values for the beams with b/t larger than 30 are averaged (with I-15 and I-17 omitted for reasons previously cited) the following average values are obtained: $M_1/M_2 = 1.18$ and $M_2/M_1 = 1.00$. In other words, if beams in this range were designed on the basis of the full, unreduced cross section, they would be underdesigned by an average amount of 18%, whereas, if they are designed on the basis of the equivalent width, a satisfactory and safe design would be obtained.

similar commonalities of religious monotheism from Asia.

A similar comparison or ultimate moments for the inverted U-beams of series A is somewhat complicated by the nonsymmetry of the cross sections. Since the neutral axes of such sections are rather close to the compression flange, the stress in this flange will be rather low at loads at which the tension flange reaches yield-point stress. For this reason the beam will continue to carry increasing loads until the top flange, and with it practically all the cross section, is stressed to the yield point. The stress distribution is that of Fig. 4(b) and only then does the beam fail. (This somewhat novel concept of failure of steel beams has gained wide acceptance in recent years and was verified extensively by E. Volterra⁶ and others. For the I-beams of series B and C discussed previously, this concept is of no importance because (a) these sections are closer to symmetry so that both flanges reached yield-point stress at about the same load; and, more important, (b) once the compression flange and a considerable part of the turned-down lips begin to yield, the latter become unstable and cause the ultimate breakdown of the beam. In the beams of series A, however, restraint of both edges of the compression flange is provided by complete webs which, therefore, do not become unstable until practically all the cross section reaches yield-point stress.) For these reasons the ultimate moments for series A are computed on the basis of the stress distribution of Fig. 4(b) by methods

The symbols in Table 3(a) are the same as those in Table 2(b) except that M_1 , the ultimate moment computed for the stress distribution of Fig. 4(b) for the full, unreduced cross section; and M_2 , is the ultimate moment computed in the same manner but by substituting the equivalent width of the compression flange for the actual width. A study of Table 3(a) reveals that M_1 in all cases is considerably larger than M_2 , whereas M_2 in all cases is in rather satisfactory agreement with the test value. The average values are: $M_1/M_2 = 1.36$; $M_1/M_{\text{test}} = 1.02$.

Summarizing the results of these seventy-one tests on thirty-three different types of I-beams (Table 2) and inverted U-beams (Table 3), it is notable that a conventional design procedure based on the full, unreduced cross section would result in definitely, and in many cases highly, unsafe values whereas an analysis based on the equivalent width concept (Fig. 7) results in very satisfactory agreement between computed and observed values.

"Results of Experiments on Metallic Beams Bent Beyond the Elastic Limit," by E. Volterra, *Journal Institution Civ. Eng.* (London) Vol. 20, 1919-1922 ";

Deflections.—Any rational method should enable the designer to predict not only the strength of his structure, but also the magnitude of its deformation under load. In this case, then, the equivalent width approach should furnish a method of predicting deflections of beams with thin, wide compression flanges.

TABLE 3.—TESTS ON THE U-TYPES OF BEAMS; COMPARISON OF ACTUAL AND COMPUTED VALUES, SERIES A

Type	No.	$\frac{b}{t}$	(a) FAILURE MOMENTS			(b) DEFLECTIONS		
			s_0 (lb per sq in.)	M_1 M_t	M_2 M_t	In Elastic Range	At Yield Point	d_1/d_t
U-1	1	64.0	39,100	1.16	0.99	0.96	0.99	0.93
U-2	1	78.2	36,400	1.23	1.01	0.95	1.04	1.05
U-3	1	98.5	32,800	1.39	1.06	0.95	1.04	1.02
U-3	3	103.5	30,300	1.20	0.97	0.88	1.00	0.98
U-5	1	104.7	42,910	1.47	1.08	0.92	1.05	1.00
U-6	3	107.5	40,080	1.29	0.88	0.93	0.98	0.84
U-7	3	111.0	30,350	1.22	0.96	0.90	0.96	0.91
U-8	3	144.0	31,880	1.18	0.91	0.95	1.02	1.02
U-9	3	143.0	35,770	1.53	1.09	0.94	1.11	0.91
U-10	2	147.0	27,820	1.49	1.07	0.96	1.00	0.96
U-11	2	148.0	28,470	1.44	1.07	0.92	0.93	0.95
U-12	1	147.0	28,470	1.47	1.07	0.95	0.92	0.92
U-13	2	170.0	34,300	1.65	1.15	0.89	0.91	0.87

flanges. For such a prediction, it is only logical to compute the moment of inertia on the basis of the equivalent width of the compression flange, rather than on the basis of the full width, the equivalent width being determined from Fig. 7 for the stress corresponding to the load at which the deflection is desired. This approach, however, is somewhat approximate for the following reason: The stress in the compression flange of a freely supported beam is not constant along the span; it varies from zero at the supports to a maximum at or near the center. Fig. 7 indicates that the equivalent width decreases with increasing stress. Consequently, a beam with constant, thin-walled cross section actually has a varying effective moment of inertia, depending on the local magnitude of the compression stress, with a maximum at the supports and a minimum at the place of maximum moment. A full analysis of the test data taking account of this variation not only would be cumbersome but would also exceed the amount of effort a designer may reasonably be expected to devote to determining deflections. For this reason, deflections for the reduced cross section in Table 3(b) were determined for the minimum equivalent width—that is, the width corresponding to maximum moment. In the tests of series A, because of quarter-point loading, since the entire center half of the beam is stressed uniformly by the maximum moment, the error so introduced is rather small and the results obtained should be expected to be somewhat on the conservative side; that is, the computed deflections should be somewhat larger than the measured ones.

In Table 3(b) actual and computed deflections are compared at two different loads: (a) In the purely elastic range (at loads of from about 75% to 90% of those which cause the bottom fibers to reach yield-point stress); and (b) at the

loads that cause the bottom fiber to yield. It will be remembered that at this latter load practically all the cross section including the top flange is stressed considerably below yield-point intensity.

Table 3(b) gives average results of the same number of specimens of each type as does Table 3(a). The deflections d are defined as follows: d_1 is the deflection computed on the basis of the moment of inertia of the full, unreduced cross section; d_2 is the deflection computed on the basis of the moment of inertia, determined by substituting the equivalent width of the compression flange for the actual width; and d_t is the deflection measured in the test.

Table 3(b) shows that deflections computed from the full, unreduced cross section throughout are smaller than measured deflections, particularly for the larger values of b/t . On the other hand, deflections computed on the basis of the equivalent width agree very satisfactorily with the measured deflections throughout the entire investigated range of b/t .

A similar comparison for the tests of series B and C is less revealing in its details. The b/t -ratios for these series are rather low (see Table 2(b)), causing the equivalent widths to be only slightly smaller than the actual widths. Also, the compression flanges of the I-beams represent a much smaller part of the cross section than do those of the U-beams of series A. For both these reasons the moments of inertia of the beams are little affected by the decrease of the equivalent width—indeed, less than the section moduli of these same beams. Therefore, only the average values of the deflections in the elastic range are given for these two series. For the beams of series B and C with b/t larger than 30 (that is, those for which the difference between equivalent and actual width is at all significant), the following averages were found: $d_1/d_t = 0.925$ and $d_2/d_t = 0.996$. For these beams, too, the deflections computed from the equivalent widths agree much better with the measured ones than do those computed from the unreduced cross section. (In analyzing the deflections of series B and C, account was taken of the part of the total deflections caused by shear stresses. This part amounted in some beams to as much as 6.7% of the total deflection. In the tests of series A the part of the deflection caused by shear is negligible because of the large span:depth ratios of these beams.)

Summarizing, the determination of deflections from the unreduced cross section would lead to a design which (just as with regard to strength) is in error on the dangerous side, whereas deflections determined from the equivalent width of the compression flanges are confirmed very satisfactorily by experiment.

Distortion Under Load and at Failure.—The compression flanges of all beams in the range of medium and large b/t developed very slight waviness at loads considerably less than the ultimate. At loads of the order of half of the ultimate the amplitude of these approximately quadratic waves, however, was so small that the distortion produced could not be considered of any structural consequence. In fact, so minute was the distortion that it escaped photographic recording, although wave formation could be detected by touch and was visible by careful inspection.

Failure occurred when one of the waves developed into a definite wrinkle; usually such development occurred within a very small load increment. Two

views of such beams in the medium range of b/t (I-15 and I-17, Table 2) after failure are given in Fig. 8. The local, wavelike distortion of the compression flange is clearly visible. It is also noticeable that adjacent parts of the flanges are practically undistorted: The minute waves, which, before failure, were

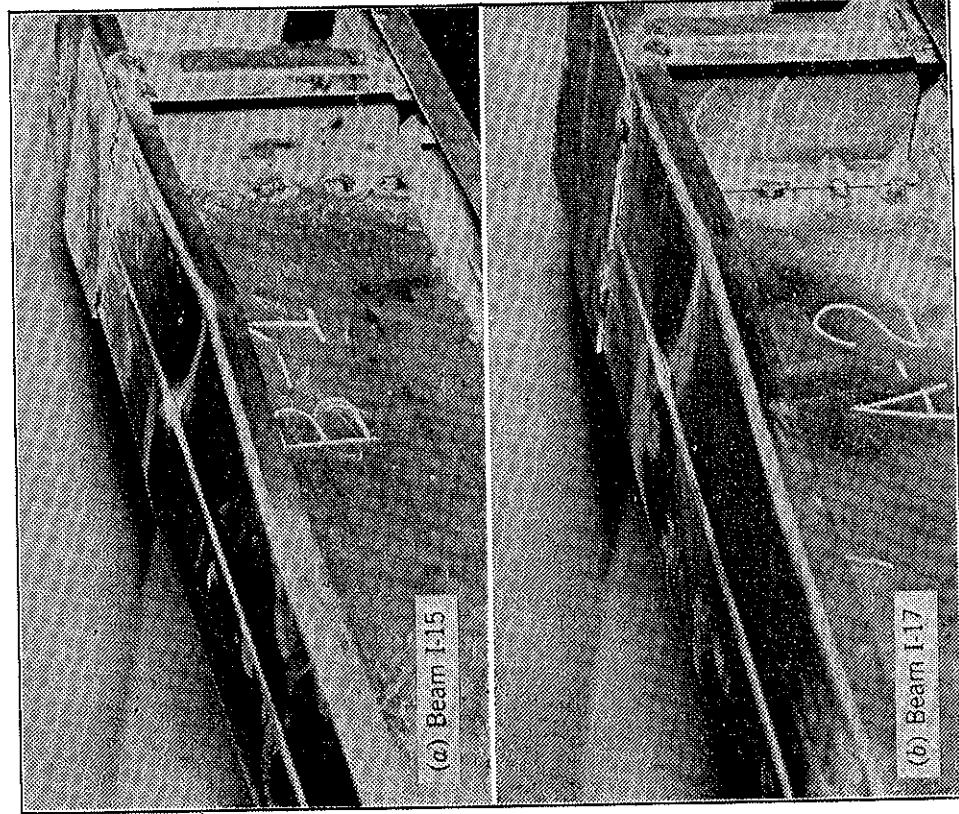


FIG. 8.—EXAMPLES OF FAILURE CAUSED BY LOCAL BUCKLING

present over the entire flange of the beam between loads did not cause any permanent distortion and disappeared after unloading, except for the one wave that resulted in failure. Turned-down lips remained straight up to failure loads, except for the widest of the I-beams (I-19 and I-20) in which the lips showed some slight wavelike departure from the initially straight shape all

along the center parts of the beams at loads below the ultimate. Possibly the lips of these two types of beams were not rigid enough to fully stiffen the flanges and somewhat stiffer lips may have raised the ultimate loads slightly (see Table 2(b)).

TYPE B. COMPRESSION FLANGES SUPPORTED ALONG ONE LONGITUDINAL EDGE

Elements of this type occur in I-beams or channels not furnished with stiffening lips at the outer edges, so that flanges are stiffened by the webs along the joints of these two elements, but are free to distort out of their planes at the other, unstiffened edge.

To investigate the behavior of such flanges, two series of tests were made:

(1) I-beams (designated by I-B); and (2) struts (designated by I-S).

1. In series D, seventeen types of beams of the general shape shown in Fig. 9(a) were tested—principally three “identical” specimens of each type. Most of the beams were 4 in. deep; a few of them were 4 in. deep, within shaping tolerances. Widths of top flanges ranged from 1.43 in. to 9.90 in., widths of bottom flanges from 1.93 in. to 10.18 in., and thicknesses from 0.0368 in. to 0.1035 in. The beams were tested on various lengths of span, depending on their width, to avoid beams so slender that they might buckle laterally before reaching full local strength. The beams were loaded by two equal loads, symmetrically located with respect to the center of the beam, at a distance from each other varying from 21 in. to 36 in. Ultimate loads, deflections, and strains were measured in these tests in the same manner as in the preceding series.

2. In series E, twenty-two types of struts (designated by I-S) of the general shape shown in Fig. 9(b) were tested in compression—mostly three “identical” specimens of each type. The narrower struts were 2 in. deep and the wider ones 4 in. deep, within shaping tolerances. Widths of flanges ranged from 1.56 in. to 12.06 in.; thicknesses, from 0.0368 in. to 0.1077 in.; and lengths of struts, from 15 in. for those with the narrowest flange widths to 63 in. for those with largest flange widths. The struts were tested with end attachments providing knife-edge support centered as precisely as possible in the plane of the web; and the aforementioned lengths were so adjusted as to keep the L/r -ratio in the direction of the least radius of gyration within the range of from 35 to 50. This was done to prevent failure of the struts due to column buckling rather than due to local failure. As a further safeguard against failure by bending or column buckling, deflections were measured at midlength both in the direction of the web and perpendicular to it. Thus, eccentric loading and conse-

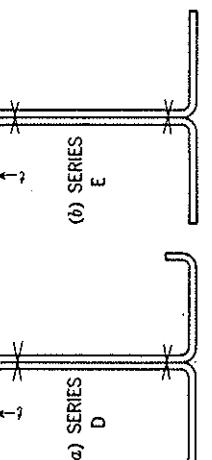


FIG. 9.—SHAPES OF I-SECTIONS WITH UNSTIFFENED OUTER EDGES OF COMPRESSION FLANGES

quent bending, if they occurred, were detected at low stresses; and, in such cases, the loading was realized before the strut was tested to failure. This procedure, in conjunction with the small L/r -ratios insured the full development of local compressive strength rather than over-all buckling strength in these tests. In addition to ultimate loads and deflections, strains were measured by applying two 8-in. gauges opposite each other in the center line of the web (see Figs. 10 and 11).

Since the results of both series of tests with regard to the behavior of the compression flanges were identical, the strut tests will not be separated from the beam tests in the following discussion, except for the nomenclature.

General Behavior and Types of Failure.—The tests revealed three rather different types of behavior under load and of final failure, depending on the ratio b_w/t of the compression flanges, in which b_w is the free projection of compression flange, measured from toe of radius to outer edge.

Flanges in the lowest range of b_w/t , up to about 12, as expected, failed by simple gradual yielding with little or no distortion perpendicular to the plane of the flange. Flanges with b_w/t in the range of from 12 to 33 behaved and failed in a manner illustrated by Figs. 10(a), 10(b), and 10(c), in which a strut specimen I-S-10 with $b_w/t = 27.1$ is shown in three consecutive stages of loading. Fig. 10(a) shows the strut under a low load of 1 kip; flange faces are plane and their edges straight except for slight inaccuracies obtained in the forming process. Fig. 10(b) shows the strut under a high load of 14 kips, with no perceptible distortion of the flanges. At a somewhat higher load (16 kips), definite distortions are seen in the upper part of the front flange of Fig. 10(c). These "kinks" are purely localized, the remainder of the flanges being essentially undistorted. The strut failed finally at a load of 18,600 lb after developing some more kinks of the same character. This strut and other struts (particularly in the range of b_w/t from about 20 to 33) carried considerably larger loads at failure than those in which the first kinks developed. Nevertheless, the load at which such kinks first appear must be regarded as the limit of structural usefulness for the particular element, and the subsequent discussion, therefore, is based on this local buckling rather than on the ultimate strength.

Finally, compression flanges in the range of b_w/t from 42 to 109 (the largest ratio of all specimens tested) behaved in the manner illustrated in Fig. 11. (No data are available for the range of b_w/t from 33 to 42.) Fig. 11 presents views of a strut specimen I-S-15 with $b_w/t = 42.6$ under increasing load; and Fig. 11(a), with a load of only 1 kip applied, shows the flanges being plane except for slight manufacturing distortion. However, at one half of the ultimate load, Fig. 11(b), taken at a load of 21 kips, shows slight regular waves beginning to develop in the flanges along the entire length. At further increased loads (31 kips and 39 kips), Figs. 11(c) and 11(d) show the amplitude of these waves gradually increasing, the regularity of the pattern being preserved. Finally, the strut failed at a load of 42 kips; and Fig. 11(e), taken after failure, shows that one of the previously developed waves had suddenly enlarged into a definite "kink" and produced failure. Whereas this particular strut did not

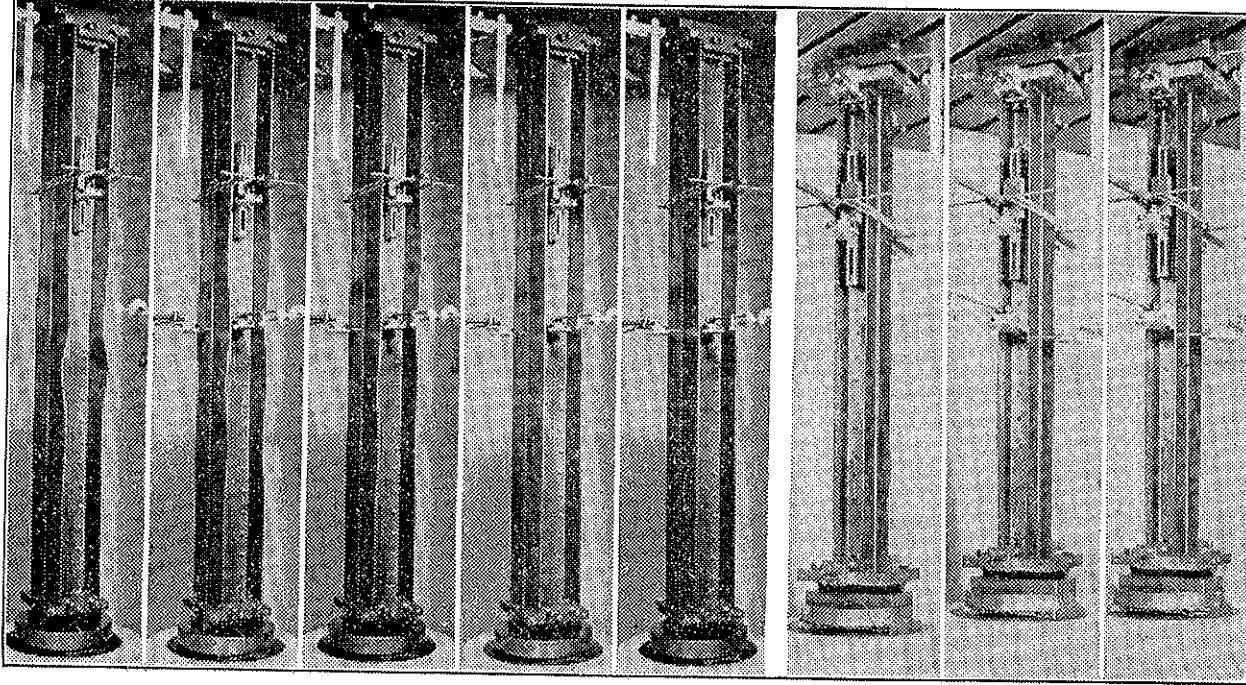


Fig. 10.—Distortion of Strut I-S-10 ($b_w/t = 27.1$) Under Increasing Compression Load P

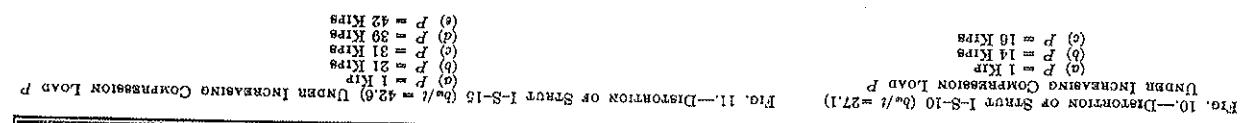


Fig. 11.—Distortion of Strut I-S-15 ($b_w/t = 42.6$) Under Increasing Compression Load P

develop any perceptible distortions at loads less than about 50% of the ultimate, struts in the higher range of b_w/t developed distinct regular wave patterns at much smaller fractions of the ultimate load, down to less than 20% of the ultimate for the largest values of b_w/t .

The compression flanges of the beams of series D behaved in precisely the same manner in the respective ranges of b_w/t as did those of the struts of series E.

Three practical qualitative conclusions are to be drawn from these findings:

1. Flanges with b_w/t smaller than 12 do not buckle; they fail by yielding and can therefore be designed for full yield-point strength (for quantitative verification see the next section).

2. Flanges with b_w/t from 12 to about 30 (to stay slightly on the conservative side since no data are available for the range of from 33 to 42) will remain undistorted and serviceable until sudden buckling (kinking) occurs in the flanges. Despite the fact that such flanges may carry higher loads after kinking, the design criterion in this range of b_w/t should be the local buckling stress.

3. Flanges in the still higher range of b_w/t , although capable of carrying considerable stress, become distorted so seriously at loads far below the ultimate that they cannot be regarded as structurally useful except if used at extremely low design stresses.

The series of photographs in Fig. 11 is interesting in still another respect: It illustrates pictorially the concept of the equivalent width of thin flanges. Regular, wavelike distortion does not preclude considerable carrying capacity of the flange. On the other hand, a flange so distorted is not able to carry as much stress as the same flange if it were kept straight, say, by a system of stiffeners. After distortion occurs, moreover, the parts of the flange close to the web, kept essentially plane by the latter, will carry the main part of the total compression force in the flange, whereas the most highly distorted outer regions carry a rather small part of this force. Precisely this situation is idealized in the von Kármán concept of the equivalent width (Fig. 1). Compression flanges with both edges stiffened, like those discussed in the preceding section, behave in exactly the same manner, the only difference being that here the amplitude of the waves is limited to a much smaller magnitude by the combined action of both stiffeners as compared with that of flanges with one edge free. The idealization of this situation by the von Kármán concept is, simply, as follows: In the case of a flange with, say, both edges stiffened, the actual distribution of compression stresses over the width is of the character shown in Fig. 12. For convenience of design computation, the area under this stress curve is replaced by the equal area of the sum of the two dotted rectangles, whose altitude is the maximum actual stress at the stiffeners. Consequently,

the maximum computed stress of a beam designed or analyzed according to this concept is equal to the actual maximum, and therefore governing stress of the real, nonuniform stress distribution—all that is required for a rational design procedure.

Determination of Buckling Stresses.—In all cases in which formation of kinks was observed, the term "limiting stress" in this paper pertains to that stress at which the kink or kinks were first noticed. In the low range of b_w/t , failure occurred either by simple yielding, or by buckling (bending) of the entire specimen, or, in the case of specimens I-B-4, by an intermediate action between kinking and yielding. In these cases the limiting stress was determined as that at which irregularity of behavior appeared as a marked departure from the straight line in the load-deflection curve and the load-strain curve. In all cases stresses were computed by standard procedure—that is, by dividing the actual load (for struts) or moments (for beams) by the area or section modulus, respectively. These cross-sectional properties were computed from the full, unreduced dimensions of the sections. The justification for this procedure is given subsequently. Table 4, gives the results of the tests for the specimens

TABLE 4.—LIMITING STRESSES, s_u , AND ULTIMATE STRENGTHS, s_u , OF COMPRESSION FLANGES SUPPORTED ALONG ONE EDGE ONLY

Type	No.	$\frac{b_w}{t}$	s_u	Stresses, in Pounds Per Square Inch			$\frac{s_{uc}}{s_u}$
				s_u	s_u	s_{uc}	
I-S-2	3	9.3	25,900	34,600	33,400	35,400	1.38
I-B-3	3	10.1	19,800	35,800	35,800	49,400	1.38
I-B-3	3	10.1	37,900	30,200	29,600	37,300	1.26
I-S-4	3	11.7	36,800	40,300	30,400	30,200	0.99
I-S-6	3	18.5	35,400	31,800	25,600	28,000	1.09
I-S-7	3	19.0	34,500	26,100	22,800	27,000	1.18
I-S-8	3	19.1	49,400	38,800	36,500	36,500	1.03
I-B-5	2	20.3	37,300	29,400	23,600	27,000	1.14
I-B-5	2	20.8	30,000	29,200	26,700	26,700	0.95
I-B-6	3	21.6	32,600	28,800	23,400	23,100	0.99
I-S-9	3	21.6	34,000	25,500	23,900	23,700	0.99
I-B-8	3	25.2	38,000	30,000	21,200	21,300	1.00
I-S-10	3	27.1	34,500	22,900	16,700	18,000	1.08
I-S-11	3	27.8	34,900	23,900	15,600	17,200	1.10
I-S-12	3	27.8	34,500	29,200	23,700	17,300	0.73
I-S-13	3	28.3	49,400	29,200	17,600	18,500	1.03
I-B-9	1	33.9	32,900	29,200	26,200	19,700	0.53
I-B-10	3	30.9	34,500	24,600	15,200	15,200	1.00
I-B-11	3	30.6	34,500	25,700	17,400	14,800	0.88
I-B-12	3	31.2	37,300	23,800	14,200	13,800	0.81
I-B-14	3	33.1	34,000	23,000	15,200	12,300	0.81

with b_w/t less than 33.1—that is, for those which showed sudden formation of kinks (flange buckling) without preceding noticeable wave distortion. Table 4, therefore, covers the structurally useful range of b_w/t .

In Table 4, b_w is the free projection of compression flange measured from toe of radius to the outer edge; s_u is the ultimate strength computed for struts from $s_u = P_u/A$ and for beams from $s_u = M_u/S$; A and S , respectively, being the area and the section modulus of the full, unreduced cross section; s_{uc} is the limiting stress, computed in the same manner as s_u but from the test loads at which local buckling occurred; and s_{uc} is the limiting stress computed from Fig. 13 which is discussed in detail subsequently.

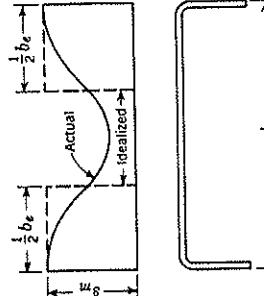


FIG. 12.—Stress Distribution Over the Width of a Stiffened Compression Flange

Type I-S-3 was the smallest in cross section of all those given in Table 4—its total flange width being only 1.56 in.; its depth, 2.01 in.; and its length, 15 in. These small dimensions made an accurate centering of the load extremely difficult with the available apparatus and, in addition, made it im-

Average yield point was computed from Table 4—specimens with $s_y = 49,400$ lb per sq in. being excluded from the averaging because of the high value of this yield point as compared with all others. The average value of s_y so obtained is 34,800 lb per sq in. Since flanges with b_{av}/t less than about 12 failed by

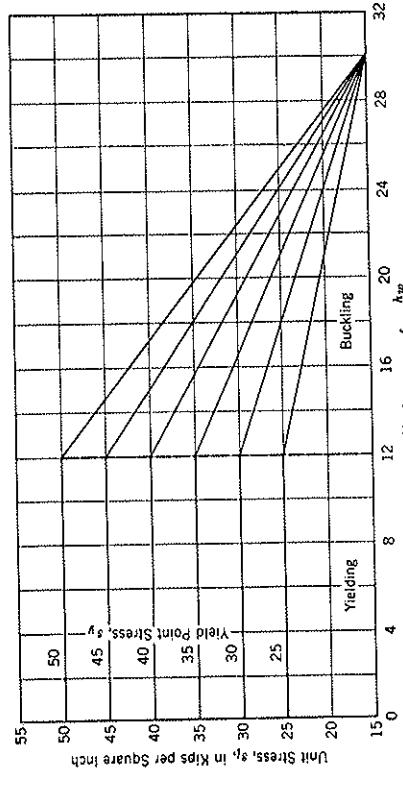


FIG. 13.—CHART FOR DETERMINING THE LOCAL BUCKLING STRENGTH FOR A GIVEN YIELD POINT, FOR WIDTH OF PROTECTION b_c , AND FLANGE THICKNESS t , AS GIVEN BY EQ. 10

possible to control the accuracy of loading by measuring deflections in the direction of the least radius of gyration because the strain gages (see Fig. 10) left no space for the deflection-measuring device. The low value of s_u as compared with that of s_v seems to indicate that this strut was subject to considerable bending caused by eccentric loading, which provoked premature

failure as compared with pure concentric loading.

In Fig. 14, the values of s_{11} are plotted against b_{∞} . The curve representing the formula,

is shown by the solid line. Eq. 9 gives the theoretical, critical buckling stress for a long, narrow plate simply supported along one longitudinal edge and supported along the other.³ Fig. 14 suggests that, in the higher range of b_w/t —from about 25 upward, local buckling (kinking) occurs at stresses equal to, or somewhat larger than, those given by Eq. 9. At lower values of b_w/t , however, buckling occurs at stresses considerably below s_c . This situation is analogous to that in the case of column buckling where failure loads are found to be considerably below the Euler loads for values of L/r less than about 100. To develop a procedure for predicting limiting stresses in this range of b_w/t ,

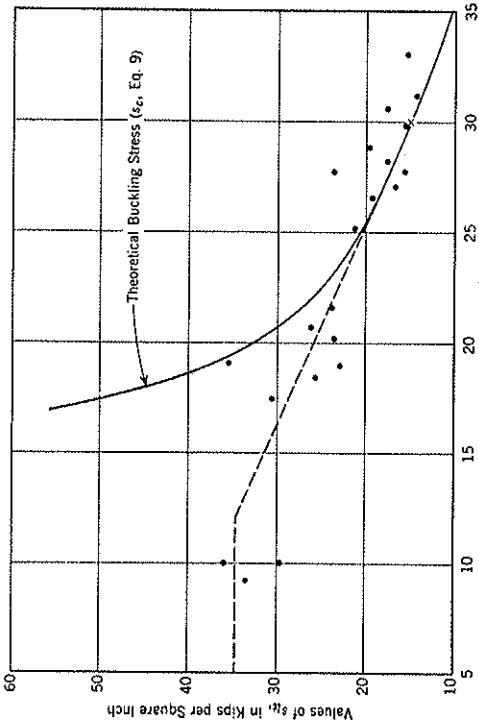


FIG. 14.—OBSERVED VALUES OF LOCAL BUCKLING STRENGTH s_u PLOTTED AGAINST THE RATIO b/H . Series D AND E.

yielding rather than by buckling, it seemed logical to presume that the dotted straight line in Fig. 14 (starting with the average s_y at $b_w/t = 12$ and ending with s_y at $b_w/t = 30$) would approximately represent the values of s_u . The distribution of experimental points around that line indeed suggests that such may be the case. Fig. 14, however, is not sufficient evidence to prove the accuracy of this approach, because the yield points of the test specimens varied over a wide range from 29,200 lb per sq in. to 49,400 lb per sq in., whereas the dotted line in Fig. 14 refers to the average yield point only.

If the assumption is correct that a straight line, drawn from s_y at $b_w/l = 30$ to s_y at $b_w/l = 12$, determines the limiting stresses satisfactorily, then the limiting stress for each specimen should be found, according to its particular yield point, from Fig. 13. (The equation of the straight lines in Fig. 13, between $b_w/l = 12$ and $b_w/l = 30$ is

from which the limiting stress can be computed in that range. For the few specimens with b_*/t larger than 30, the limiting stress s_c entered in Table 4 is the critical stress according to Eq. 9 (see also Fig. 14). The values so computed from Fig. 13 are entered under the heading s_{lc} in Table 4 and a comparison of these with the test values is given in the last column of Table 4.

This last column (Table 4) reveals rather satisfactory agreement, except for types I-S-3 and I-B-3. The reasons for the peculiar behavior of these two types was discussed previously. The influence of the yield point on the limiting stress, very pronounced in the low range of b_w/t , but gradually decreasing in the higher range (see Fig. 13), is well demonstrated by types I-S-8 and I-S-13 with their unusually high values of s_y . This indicates that a prediction of failure stresses solely on the basis of the theoretical s_s , as often attempted, is certain to lead to erroneous results.

It should be noted that the agreement between test results of "identical" specimens of the same type is not as satisfactory for flanges of this kind as for flanges stiffened along both edges. Of the twenty types of sections for which more than one specimen of each type was tested (mostly three specimens of a type, see Table 4), the maximum deviation from the mean for a given type did not exceed $\pm 15\%$ for fifteen types. In the remaining five types the maximum deviation on the safe side was $+30\%$; and, on the unsafe side, -17% . The data refer to s_u . This scattering of test results, no doubt, is caused by inevitable inaccuracies of forming and possibly nonuniformity of material. (To give only one practical comparison, the scattering is of about the same order of magnitude as that found from strength tests on concrete specimens mixed to identical specifications and cut out of a finished structure.¹¹)

Equivalent Width.—From the discussion of Fig. 11, it is apparent that the concept of equivalent width must be expected to apply not only to flanges stiffened along both edges but also to those with one edge free. Therefore, should not the evaluation of test series D and E be based on the equivalent areas of the compression flanges rather than on the unreduced cross sections as was done in Table 4?

In an academic thesis prepared under the writer's supervision,¹² E. A. Miller, Jun. ASCE, has computed the equivalent widths from the strain observations of series D and E in a manner similar to that previously discussed in connection with series A. This investigation was not limited to the practically useful range of b_w/t but included all specimens of these series with ratios of b_w/t up to 109.

Mr. Miller showed that expressions similar to Eq. 6 could also be developed in this case to express the equivalent width b_e . He found that the expression,

$$b_e = 1.25 t \sqrt{\frac{E}{s}} \left(1 - 0.333 \frac{t}{b_w} \sqrt{\frac{E}{s}} \right) \dots \dots \dots \quad (11a)$$

which is valid to about $t/b_w \sqrt{E/s} = 1.55$, represented rather accurately the average values found experimentally. Since results showed considerable scattering, a more conservative expression was developed, which, with a very few exceptions, gives a good approximation for the lowest values of b_e obtained

¹¹"Properties of Job-Cured Concrete at Early Ages," Report of Committee 107, *Journal, A.C.I.*, September-October, 1936, Tables 2 and 5, pp. 46 and 51.

¹²"A Study of the Strength of Short, Thin Walled Steel Studs," by E. A. Miller, a thesis presented to the faculty of the Graduate School of Eng., Cornell Univ., Ithaca, N. Y., in October, 1943, in partial fulfillment of the requirements for the degree of Master of Civil Engineering.

experimentally. This expression is

$$b_e = 0.8 t \sqrt{\frac{E}{s}} \left(1 - 0.202 \frac{t}{b_w} \sqrt{\frac{E}{s}} \right) \dots \dots \dots \quad (11b)$$

which is valid to about $t/b_w \sqrt{E/s} = 1.75$.

On the basis of these findings it is possible to decide whether, in determining buckling stresses by the procedure of the preceding section, the equivalent or the full, unreduced flange width should be used. This question, obviously, is of greater practical significance for large than for small values of b_w/t . It will therefore be investigated for the suggested upper limit of the useful range of b_w/t —namely, for $b_w/t = 30$. For this value the local buckling stress, according to Figs. 13 and 14, is found from Eq. 2b to be 15,100 lb per sq in. For this value of s , Eqs. 11a and 11b for $b_w/t = 30$ give, respectively, $b_e = 28.2 t$ and $b_e = 25.2 t$. In other words, in the practically important range of b_w/t the maximum reduction of flange width is only 6% on the average (Eq. 11a) with a maximum of 16% from the more conservative expression Eq. 11b.

Hence, in the range of b_w/t to about 30, computations based on the full, unreduced flange width are as accurate as can reasonably be expected—considering that (1) the degree of scattering of the values of the experimental buckling stresses makes a high accuracy of computation illusory; and (2) a reduction in width of 6% or even 16% results in a much smaller reduction of the significant values of A and S for ordinary sections in which the compression flanges of type B represent only a moderate part of the entire cross section. The introduction of an equivalent width, in this case, would unduly complicate computations without any significant improvement of the validity of results. Thus, Eqs. 11 appear to be practically valuable only in establishing the foregoing conclusion. They may become significant in connection with further analytical research in this field, in testing future theoretical expressions for the equivalent widths of flanges of this type.

CONCLUSION

The experimental evidence presented in this paper indicates that the strength and general behavior of thin steel compression flanges is satisfactorily accounted for in the following manner:

1. *Flanges Stiffened Along Both Longitudinal Edges.*—Flanges with values of b/t to about 25 were found to fail by simple yielding, the full area of the flange being effective—that is, the stress is of uniform magnitude across the width. For flanges with larger values of b/t , deflections and ultimate loads computed by considering the compression flanges fully effective were found to occur on the dangerous side.

Replacement of the actual flange width by an equivalent width determined from Fig. 7 resulted in very satisfactory agreement between observed and computed ultimate loads and deflections. Locations of neutral axes, as determined experimentally from strain measurements, showed good agreement with locations computed on this basis.

Within the investigated range of dimensions, distortions of such flanges at loads below the ultimate were very limited and of little, if any, practical significance.

2. Flanges Stiffened Along One Longitudinal Edge.—Flanges with ratios of b_w/t to about 12 failed by simple yielding, the full area of the flange being effective. In the range of b_w/t from 12 to about 30, local sudden and pronounced flange buckling was found to occur at stresses determined from Fig. 13. Local flange buckling did not precipitate immediate failure; indeed, ultimate loads of some specimens were as high as twice the load that produced local buckling, stresses being computed in all cases from the full, unreduced flange width. For practical purposes, however, it was held that loads resulting in local buckling must be regarded as reaching the limit of structural usefulness.

In the range of b_w/t larger than about 30, wavelike gradual flange distortion was observed to occur at stresses rapidly decreasing with increasing values of b_w/t . The distortions, even at rather small fractions of the ultimate loads, were so pronounced as to make such members useless for most structural applications, except at extremely low design stresses.

From strain measurements it was found that the area of such flanges was only partly effective. This phenomenon resembles the behavior of the flanges under conclusion 1. An expression for the equivalent width, as determined from strain measurements, is given in the text.

It is not contended that the findings of this investigation, as expressed chiefly in Figs. 7 and 13 necessarily represent the final answer to the question of the strength of thin compression flanges. Being rather analytically inclined himself, the writer is perfectly aware of the limitations inherent in such purely experimental results, unsupported as they are by any more rigorous theoretical analysis.

It so happens, however, that the rigorous treatment of buckling of thin plates along classical lines of elastic stability¹⁹ proved to be of little practical value. The reasons for this situation have been stated in the paper. Attempts by various investigators to go beyond the simple, classical approach have so far failed to give practically significant results and have led, in most cases, to rather contradictory conclusions.

Under these circumstances the only feasible approach seemed to be that of a rather extensive experimental investigation and an attempt to express the results of such test work by formulas or graphs that would agree reasonably well with test results. The situation is comparable with that of earlier column investigations in the range in which the Euler formula is invalid. The simple formulas then developed, mostly of straight-line character, served their purpose rather well until such time as more accurate and more involved approaches, such as the seant formula, could be developed.

It is hoped that a similar situation may obtain with regard to the topic of this paper and that the limited results of this investigation may serve their practical purpose until such time as more general and exact treatments will become available.

THIN FLANGES

ACKNOWLEDGMENTS

The tests and conclusions reported herein are part of an extensive investigation of thin-walled steel structures sponsored jointly by the American Iron and Steel Institute and Cornell University, in Ithaca, N. Y.,² under the general supervision of Prof. W. L. Malcolm, M. ASCE, director, School of Civil Engineering, the writer being in active charge of the work. The writer wishes to express his sincere appreciation of the unfailing patient cooperation and valuable suggestions of Director Malcolm and of the members of the technical subcommittee of the Committee on Building Codes of the Steel Institute—in particular that of Milton Male, M. ASCE, chairman; B. L. Wood; and F. E. Fahy, Assoc. M. ASCE.

In conducting the test work and the computations the writer was ably assisted by the following McMullen Graduate Scholars (in chronological order): C. A. Dunn, Assoc. M. ASCE; Capt. G. G. Green, R. L. Lewis, and E. A. Miller, Juniors, ASCE; and R. H. J. Pian. Their individual contributions, overlapping as they are as to time and topic, cannot readily be separated. The writer wishes to thank all these collaborators for their conscientious and devoted cooperation and their many helpful suggestions.

APPENDIX. NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A	= area of cross section; A_n = area of one of n subareas;
b	= width of a plate of type A perpendicular to the direction of compression;
b_e	= equivalent width;
b_s	= width of a strip, part of the total width;
b_w	= width of the free projection of a compression flange of type B, measured from the toe of the radius to the outer edge;
C	= a variable experimental coefficient; as a subscript, C denotes "computed";
d	= deflection, with subscripts t denoting "by test," etc.;
E	= Young's modulus of elasticity;
h	= depth of beam section;
L	= span lengths;
M	= moment, with subscript t denoting "by test" and u denoting "due to ultimate loads";

¹⁹ "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936.

P_u = axial compression load (P_u = load at ultimate strength);

r = least radius of gyration;

s = section modulus;

s = unit stress;

s_c = computed stress;

s_c = critical stress;

s_l = limiting stress;

s_m = maximum stress;

s_u = ultimate strength;

s_y = yield-point stress;

t = thickness of plate;

y = coordinate distance; y_n = centroidal distance to an area A_n ;

δ = strain, or unit elongation;

κ = a numerical factor depending upon the ratio of length to width of the plate and the conditions of edge support; and

μ = Poisson's ratio.

DISCUSSION

FRED T. LLEWELLYN,²⁰ M. ASCE.—This scholarly, yet not pedantic, study is peculiarly interesting to the writer because, prior to 1939, he was a member of the American Iron and Steel Institute committee sponsoring the Cornell University (at Ithaca, N. Y.) tests on which the paper is largely based. In 1933, the writer suggested the concept of effective (or equivalent) widths as applied to thin metal sections, such as the one illustrated by Fig. 12. He first collated the published proportions of a number of such sections, whose efficiency was fairly well established by industrial use; and next he endeavored to harmonize the results with the nonorderly and all-too-few tests then available.²¹

A comparison of the values then tentatively suggested by the writer, with the much more adequately supported values now given by Professor Winter, is offered in Table 5. The conditions of fixity are not quite identical in both cases, but an effort has been made to classify them along comparable lines.

The author is asked to state whether the surprising value, $\frac{E_t}{s_0}$, is correctly assigned in Table 5.

Although this rough comparison tends to confirm the safety of many miscellaneous sections in commercial use, the design of all thin-walled sections, of course, should be placed on a common and broad foundation. The present paper is welcomed as a valuable step in this direction—the development of a standard design specification for the products in question.

In view of the author's expressed inclination toward analytical methods, his restraint in, and happy application of, their use are to be applauded. Professor Winter's academic citations are few, but pertinent. In former years, when the writer was tempted to try and go "nighbrow," he used to recall a warning from the field of art criticism. On one page of a revised edition of John Ruskin's works, the text was footnoted about as follows:

"In the first edition of this book there was here a quotation from Aristotle, in the Greek. I inserted it to show I had read Aristotle. Having accomplished that purpose, it is now omitted."

The present paper affords a very suitable balance of the academic with the practical, to which Mr. Ruskin's self-irony could not possibly apply. The author does not need to apologize for approaching the subject from an experi-

²⁰ Baton Rouge, La.

²¹ "Light-Gage Flat-Rolled Steel in Housing," by F. T. Llewellyn, A.I.S.C., 1937, p. 29.

ment rather than a classical standpoint. Is it not true that all mathematical theory must be converted into conventional forms that are valid only within the boundaries of everyday practice? Thus, the writer would expect Figs. 7 and 13 to receive most attention from actual designers of thin-walled sections.

It is hoped that these casual, but quite appreciative, comments by an "old timer" may be of as much interest to the author, and fellow engineers, as the paper was to the writer.

It is hoped that these casual, but quite appreciative, comments by an "old timer" may be of as much interest to the author, and fellow engineers, as the paper was to the writer.

JACOB KAROL,² Assoc. M. ASCE.—Despite the fact that millions of formed sheet channels with and without stiffening lips have been used as bending members in airplane control surfaces, there has been no definite design procedure whereby a stress analyst could determine their ultimate strength. The paper by Professor Winter, therefore, is a timely and important addition to the literature.

The writer's procedure in determining the strength of symmetrical formed sheet channels in bending has been to compute the ultimate strength of the

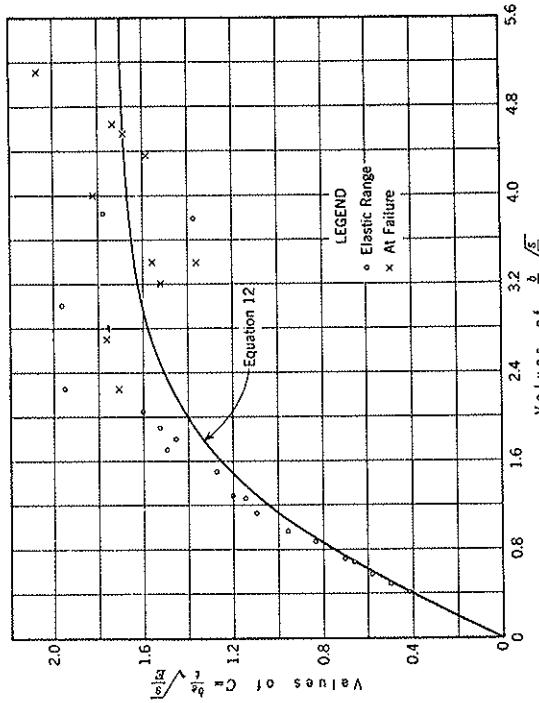


FIG. 15.—EXPERIMENTAL DETERMINATION OF EQUIVALENT WIDTH

component elements of the section on the compression side, to obtain the resisting moments about the centroidal axis, and then to double their sum to obtain the total resisting moment. It is interesting to note that this procedure is similar to that presented in the paper.

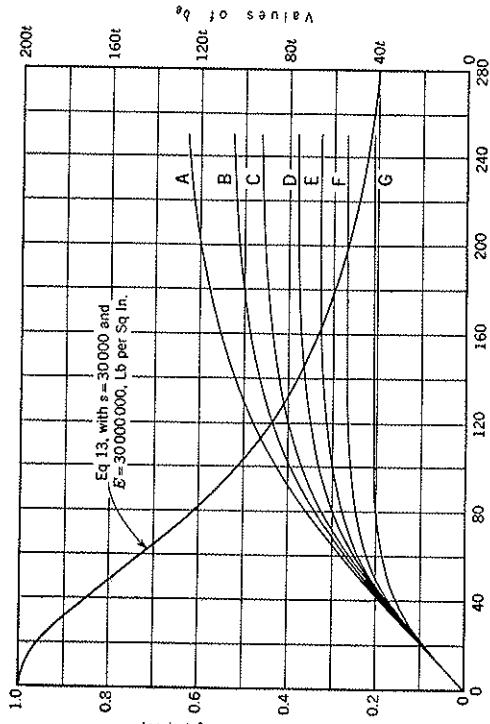


Fig. 16.—CHART FOR DETERMINING EQUIVALENT WIDTH b .

whereas, for the high ranges of the inverse parameter, the value of C is practically constant. The following expression fulfills these conditions and gives results practically identical with Eq. 6:

■ Design: Howard Needles Tammen & Bergendoff, Kansas City, Mo.

Eq. 12 is shown as a solid line in Fig. 15. The ratio of the ordinates to the abscissas in Fig. 15 is

$$R_e = \frac{b_e}{b} = \frac{\tanh\left(\frac{1}{1.7}\frac{b}{t}\sqrt{\frac{s}{E}}\right)}{\frac{1}{1.7}\frac{b}{t}\sqrt{\frac{s}{E}}} \dots \dots \dots \quad (13)$$

Eq. 13 is plotted for the particular case of $s = 30,000$ lb per sq in. and $E = 20,000,000$ lb per sq in. in Fig. 16 using b as abscissas and B_1 as ordinates.

This curve (marked "Eq. 13") corresponds to Fig. 5 and shows that the value of R approaches 1 for small values of $\frac{b}{\lambda}$, as it should.

Eq. 13 has been used to plot a family of curves similar to those in Fig. 7 (the legend in Fig. 7 applies to Fig. 16), and the resulting chart for equivalent width is also shown in Fig. 16. Although the author recommends the use of the chart for other metals than mild steel, it should be noted that, for metals with an indefinite yield point such as aluminum and magnesium, the value of E is a function of the stress s and is only equal to the initial modulus of elasticity for stresses below the proportional limit of the material. Hence, for stresses beyond the proportional limit, an effective modulus of elasticity corresponding to the particular stress must be used in determining the value of the parameter

The behavior of beams I-15 and I-17 indicates that, for material with an indefinite yield point, the ultimate stress in the curved corners of the flange is considerably beyond the yield point, and this must be considered in calculating the parameter $\sqrt{\frac{E}{s}}$ for such materials.

(\bar{t}), an expression similar to Eq. 12 can undoubtedly be developed which would be applicable to the entire range of $\frac{b}{\bar{t}}$. Since sufficient data are not presented in Table 4 from which to do so, it is suggested that the author derive the expression in his closing discussion.

**ROBERT L. LEWIS,²² JUN., ASCE., AND DWIGHT F. GUNDER,²⁴ Assoc. M.
ASCE.—A basis for the design of structural members fabricated from thin
sheets of steel has been presented by Professor Winter. Any design procedure
should be kept as simple as is consistent with safe and economical practice.
With this thought in mind the writers have examined the proposed develop-
ment, hoping to find a somewhat simpler approach for the designer. It should**

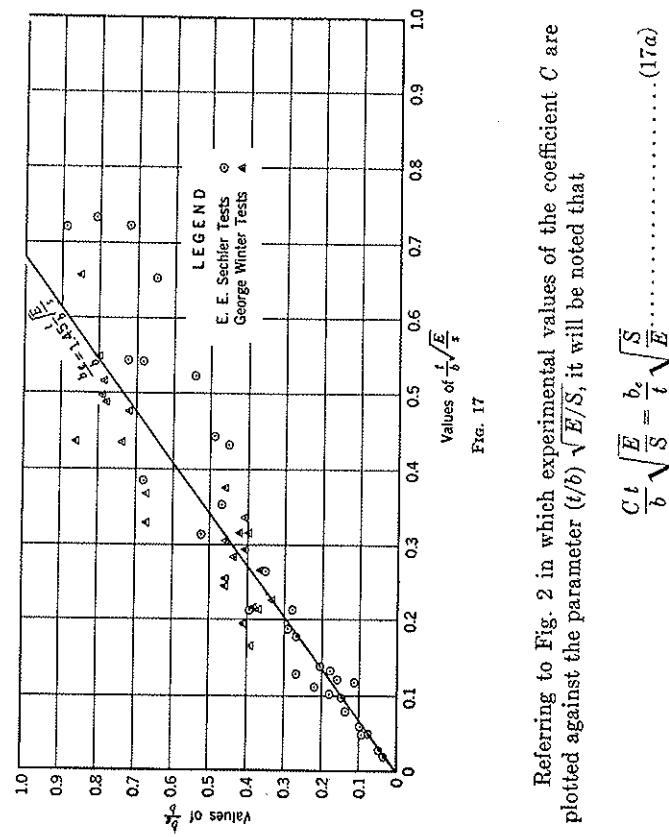


Fig. 17

Referring to Fig. 2 in which experimental values of the coefficient C are plotted against the parameter $(t/b)^{\sqrt{E/S}}$, it will be noted that

in which η is the lateral displacement and $D = \frac{E l^3}{12(1 - \mu^2)}$. Furthermore, there is no fundamental reason why the differential equation cannot be satisfied by superimposing the wave displacements involved in the edge resistance on the sine series ordinarily used for deriving the primary resistance of the plate. The question arises, therefore, as to whether the ultimate strength of a thin element does not also depend on the primary resistance, as well as on the edge

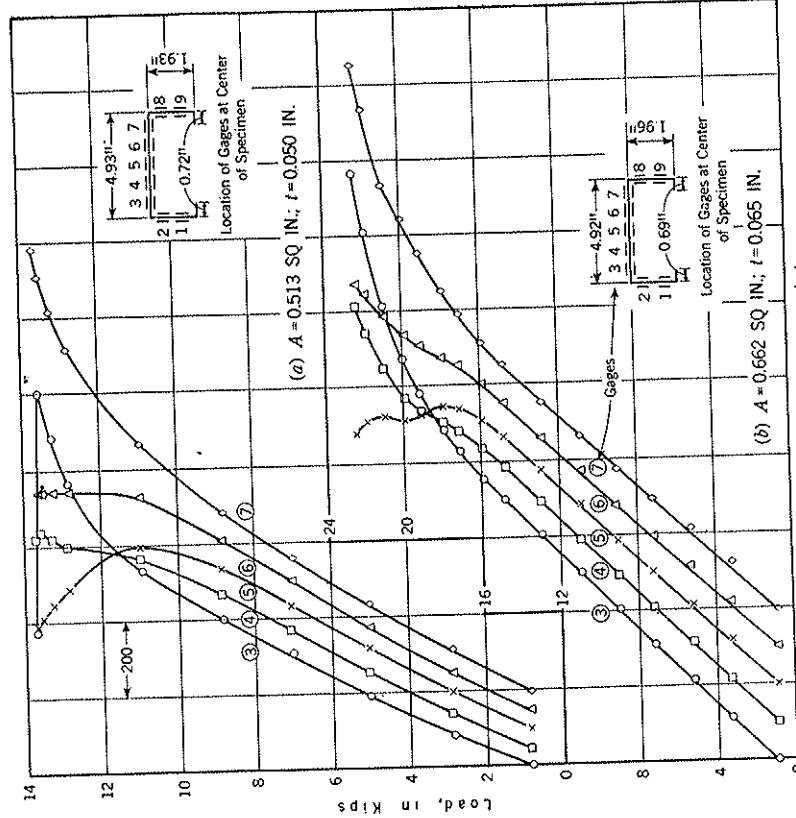


FIG. 18.—STRAIN MEASUREMENTS ON STEEL C-SECTIONS
 $(L = 24$ IN. AND $L/f = 31.2)$

effect. Strain measurements have been made by various investigators which indicate a considerable increase in axial strain at the edges after the element has buckled. Strain measurements that were made by the writers on steel C-sections, 24 in. long, and that were tested in direct compression are shown in Fig. 18. These measurements were taken with standard electrical-resistance gauges fastened in pairs to both faces of the plate elements and connected in

$$F' = 2(s_p - s_c) t^2 \sqrt{K_1} \sqrt{\frac{E}{s_p}} \dots \dots \dots \quad (24)$$

The total additional resistance F' of both edges is, therefore,

and the average increase in stress over the element, due to the edge effect, is

$$s_r = \frac{P'}{\delta t} = \frac{2(s_p - s_e)}{(b - \frac{b}{t})} \sqrt{K_1} \sqrt{\frac{E}{s_p}} \dots \dots \dots \quad (25a)$$

The critical stress σ_c is given by the Brewar formula:

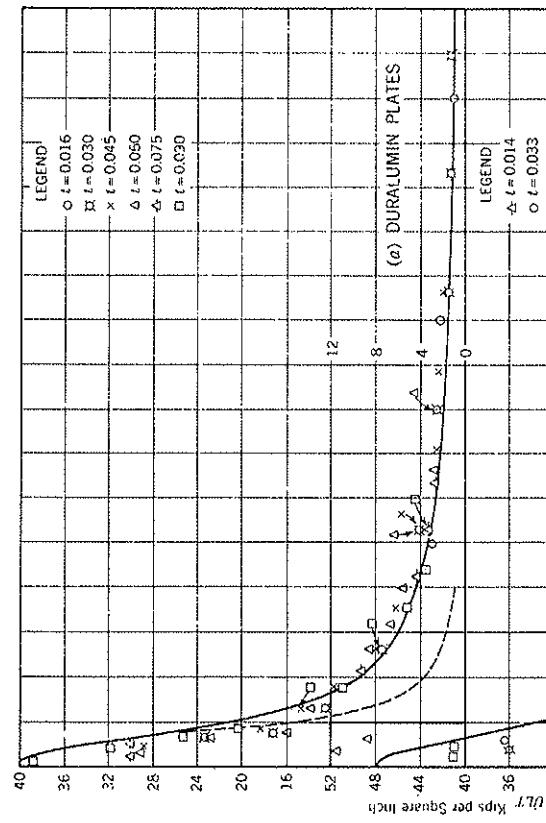
The values of K and K_1 vary with the degree of restraint at the edges. For simply supported edges, $K = 3.62$ and $K_1 = 0.452$. These values give the following equation for the ultimate strength of an element that is simply supported at the edges:

ט

It is suggested that the proportional limit, s_p , be taken as five eighths of the

The values obtained from Eq. 26b have been compared with the experimental results that were obtained by L. Schuman and G. Back in their comprehensive series of tests. The theoretical values as shown by the solid lines¹⁷ in Fig. 19 agree satisfactorily with the test results for large values of b/t , but are somewhat high for the smaller values. This condition is not surprising as the initial curvature in the specimens would have the greatest effect on the primary resistance of the stiffer elements. For steel plates with b/t -ratios less than 80, the edge effect could well be ignored. The dotted lines in Fig. 19 represent the critical stress s_c . The equations of the solid lines in Fig. 19 are, for

and, for $s_c < s_p$,



It is suggested that the proportional limit, s_p , be taken as five eighths of the mean.

The yield-point stress, s_y , to allow for variation of material in the specimens. The values obtained from Eq. 26b have been compared with the experimental results that were obtained by L. Schuman and G. Back in their comprehensive series of tests. The theoretical values as shown by the solid lines¹⁷ in Fig. 19 agree satisfactorily with the test results for large values of b/t , but are somewhat high for the smaller values. This condition is not surprising as the initial curvature in the specimens would have the greatest effect on the primary resistance of the stiffer elements. For steel plates with b/t -ratios less than 80, the edge effect could well be ignored. The dotted lines in Fig. 19 represent the critical stress s_c . The equations of the solid lines in Fig. 19 are, for

Values of $\frac{i}{100}$

FIG. 18

The author has based his design procedure on the use of Eq. 2b in which the coefficient C is represented by a function of $\sqrt{\frac{E}{s}} \times \frac{t}{b}$. To the writers, this variation is to be commended to assure that the coefficient must correct the

equation for other factors as well as for imperfections in the specimens and apparatus. The scatter of points in Fig. 2 could indicate many things besides deviation from shape such as variation in edge restraint of the element, inaccuracies in the determination of the neutral axis and of stress distribution over the beam, and neglect of the primary resistance of the element.

Without accepting the effective width concept as the most logical approach to the stability problem in light-gage members, the writers would question several points in the author's method of determining the effective width experimentally. The instrumentation appears to be inadequate for the purpose of locating the neutral axis. With the gages on only one side of the member, as shown in Fig. 3, the effect of lateral bending and twisting action, which could introduce considerable error, is ignored. The fact that these effects were not nullified by duplicate gages on the opposite edge casts some doubt on the accuracy with which the neutral axis was located from strain measurements. The author states that at the failure load the neutral axis is again located from strain measurements. The only possibility of securing this result from the strains measured is by assuming a triangular strain distribution under conditions that approach the assumed rectangular stress distribution.

For unsymmetrical sections such as those of test series A, the writers cannot agree that the neutral axis can be located accurately in this manner. In calculating deflections of light-gage beams, the use of an effective section appears to offer the best solution when the compression elements are stressed beyond the critical buckling stress. Fig. 20 shows a typical load-deflection curve for such a beam. The ordinate P_c indicates the load at which the flange elements reach the critical stress. Up to this load the properties of the full section should be used in calculating deflections. For higher loads an additional increment of deflection should be added to d_e , and this increment should be calculated using a reduced moment of inertia. For steel sections, sufficiently accurate results can be obtained by calculating the reduced moment of inertia using a flange width of $20 t$ adjacent to each element that stiffens the flange. Perhaps the author was led to the erroneous conclusion that the Bryan formula cannot be used to determine the ultimate buckling strength of thin plates—by attempting too close an analogy between the Bryan formula and the Euler formula for columns. Both formulas give a critical buckling load.

The Euler load is the ultimate load for the column since there is no other element of strength to prevent failure. The Bryan load is the load at which the plate element forms into buckling waves, but it is not the ultimate load because a boundary or edge effect remains as an additional element of strength. If the Bryan load alone is compared to the ultimate load, the agreement is good for b/t -values up to about 50 for duralumin and 80 for steel, nickel, and monel metal. For larger b/t -values, the Bryan load is low (as indicated by the statement of Messrs. Schuman and Beck, referred to by the author under the heading, "Type A. Compression Flanges Stiffened Along Both Longitudinal Edges":

Equivalent Width of Thin Flanges"). This condition occurs because the boundary effect is small as compared to the effect of the Bryan load within the foregoing values of b/t , whereas, for larger b/t -values, the reverse is true.

Bruce G. JOHNSTON,² M. ASCE.—The tests³ made at Lehigh University at Bethlehem, Pa., by Lloyd Cheney, Jun. ASCE, and the writer, supplement the results presented in Fig. 14 of the paper by Professor Winter. In the Lehigh University tests, the flanges of both carbon and silicon steel 10WF49 sections, each from a single rolling, were planed to different thicknesses, thereby providing a range of width to thickness ratios with a minimum variation in yield point.

Details of the tests will not be discussed, since they have already been reported,² but the results are summarized in Fig. 21.

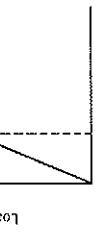


Fig. 20

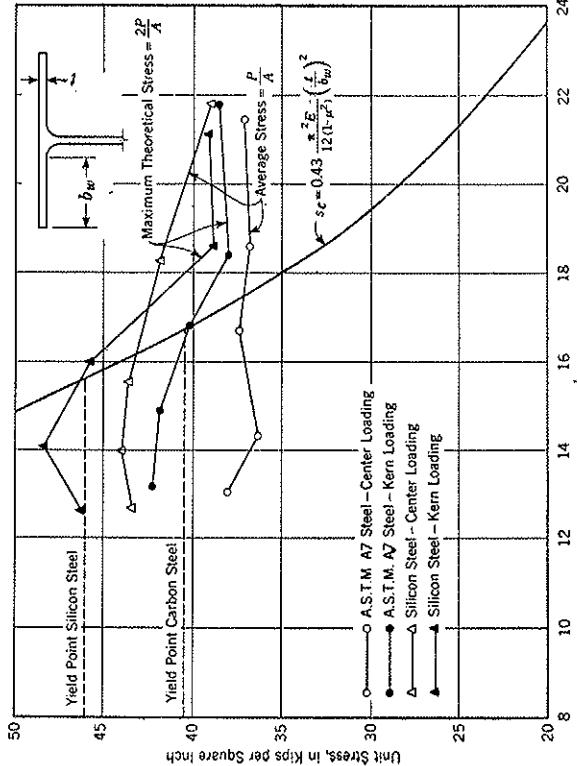


FIG. 21.—Stress in Buckled Flange at Maximum Load

For a very long plate, with one longitudinal edge simply supported and the other free, S. Timoshenko³ gives a coefficient of 0.46 in the elastic buckling formula, whereas the author quotes a factor of 0.50 in Eq. 9. Professor Timoshenko's coefficient is based on Poisson's ratio of 0.25, corresponding

² Associate Director, Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

³ "Steel Columns of Rolled Wide Flange Section," by Bruce Johnston and Lloyd Cheney, Publication No. 180, A.I.S.C., November, 1942.

"Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 340.

to neither steel nor aluminum, for which coefficients of 0.43 and 0.41 are obtained³² for Poisson's ratios of 0.30 and 0.33, respectively. These differences are somewhat academic, since the buckling strength will be affected by the torsional stiffness of the thick section at the juncture of the web and flange, and by the bending stiffness of the web, which may be negative or positive. However, if no account is to be taken of these effects, the question arises as to whether a coefficient of 0.43 is not preferable to one of 0.50.

In the tests at Lehigh University, in the case of large values of $\frac{b_w}{t}$ the web was thicker than the planed flanges, providing a partly fixed edge, and yielding effective coefficients even greater than 0.5; in fact, the buckling in all the tests was plastic rather than elastic. The results presented in Fig. 21 are self-explanatory and show that flanges of rolled shapes may be expected to develop at least 90% of the yield point of the material, within the range of $\frac{b_w}{t}$ in which plastic buckling is predicted by the elastic buckling formula.

EDWARD L. BROWN,³³ Esq., AND DON S. WOLFORD,³⁴ Esq.—More accurate methods for computing strength and other properties of structural sections formed of thin flat-rolled metals have been needed for some time. Previous methods were mainly deficient because they did not adequately evaluate the strength of compression flanges. Designers have become accustomed to assuming certain maximum multiples of the thickness to be fully effective in stiffened flanges, which has been a fairly successful practice in reasonably compact sections. However, it was evident that such simple relations did not adequately define behavior for all cases. Professor Winter presents relations in his paper by which such flanges may be evaluated, taking both b/t -ratio and stress level into account. Sections discussed in this paper cover flanges with b/t -ratios up to 1/10. The purpose of this discussion is to present test data made on sections containing compression flanges with b/t -ratios ranging from 242 to 420.

These sections were formed of 18-gage and 20-gage mild steel. All were 3 in. deep with compression flanges of 12-in. and 16-in. widths. They were similar to the U-beam in Fig. 3, except that the lower flanges were lipped and one flange was turned outward to provide a joint for adjacent sections. These tests were sponsored jointly by the American Iron and Steel Institute and Cornell University at Ithaca, N. Y., and were made under Professor Winter's supervision. They were witnessed by the writers of this discussion who also made the calculations and analysis of the results.

Four identical pairs of each type of section were subjected to quarter-point beam loading using an 80-in. span. Deflections and strains in top and bottom

³² "Theory of Elastic Stability Applied to Structural Design," by Leon S. Moiseiff and Frederick Lienhard, *Transactions, ASCE*, Vol. 101, 1941, p. 1058.

³³ "Chart for Critical Compressive Stress of Flat Rectangular Plates," by H. N. Hill, *Technical Note No. 773*, National Advisory Committee for Aeronautics, August, 1940.

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fibers were observed as load was applied and released at successively higher values. The yield-point load was assumed to be that which first caused a large increase in residual deflection upon release of load.

Section properties were computed for each type of section using (1) gross section and (2) effective section. The effective widths were determined from curves computed by Eq. 6, carried out to b/t -ratios in the range needed. The measured tensile yield strengths of the steels used were taken into account. Computed and observed deflections and loads are compared in Table 6.

TABLE 6.—COMPARISON OF ACTUAL AND COMPUTED VALUES

Type	DIMENSIONS, IN INCHES			DEFLECTION, ³⁵ IN INCHES			COMPUTED	OBSERVED IN TEST	LOAD, IN POUNDS
	Width	Thickness	Ratio ³⁶ b/t	Observed in test	Effective width	Gross section			
320	12	0.037	321	0.25	0.30	0.16	1.267	1.212	1,912
328	12	0.039	242	0.25	0.28	0.17	1.725	1.610	2,070
360	16	0.037	429	0.24	0.30	0.15	1.350	1.179	1,880
368	16	0.039	323	0.28	0.28	0.16	1.725	1.633	2,168
Average ratio, $\frac{\text{computed value}}{\text{test value}}$... 1.14	... 0.63	... 0.935	1.310		

³⁵ The ratio of width (between webs) to thickness. ³⁶ Deflections are at working stress level, assumed as the yield point divided by 1.55. ³⁷ Loads are for a single section and are at the yield point.

Computed loads based on effective-width properties averaged 6.5% lower than observed loads, indicating good correlation tantamount to safe and efficient design. Gross section properties led to expected loads that were higher than actually obtained, and were therefore misleading. Deflections shown in Table 6 are at working stress levels, determined by dividing the yield point by 1.85. It is the deflection at this stress level that is pertinent in design. The moment of inertia used to determine deflection is based on the effective width of the compression flange at the working stress level. A larger part of the flange is effective at this level than is effective at the yield-point stress level.

The calculated deflections averaged 14% greater than those observed in the test, which is reasonable. Deflections calculated by gross section properties were only 65% of those observed in test, which shows that such an approach is not reliable.

Top and bottom strains were approximately equal at the yield loads, placing the neutral axes near mid-depth where computations using properties based on effective widths indicated they should be, rather than near the compression flange as indicated by gross properties.

In summary, these additional tests and correlations show definitely that the relations given by Professor Winter for determining the effective widths of compression flanges, stiffened along both edges, enable the designer to compute design loads and deflections quite accurately at b/t -values at least as high as 429.

GEORGE WINTER,⁴³ M. ASCE.—Since the publication of this paper, the concept of the effective width has been adopted in two structural design specifications: The Specifications for the Design of Light-Gage Steel Structural Members, American Iron and Steel Institute, April, 1946, and the February, 1946, edition of the Specifications for the Design, Fabrication, and Erection of Structural Steel in Buildings, American Institute of Steel Construction. In the latter code, Sections 18c and 18d make use of an effective width, although in a greatly simplified manner. The writer does not claim any credit for these latter sections. He merely wishes to note the increasing acceptance of this concept in structural design.

The gracious comments of Mr. Llewellyn are deeply appreciated, particularly coming, as they do, from an "old-timer," with such thorough and longstanding interest in this particular field. To Mr. Llewellyn, credit is due for having first proposed the use of an effective width in structural design of light-gage members.⁴⁴ Although the values originally suggested by Mr. Llewellyn underwent inevitable correction by subsequent investigation, they were amazingly close to over-all averages, considering the dearth of information on the subject more than a decade ago. Although Mr. Llewellyn differentiated between edges supported by webs, and those stiffened by lips, the writer found that flanges stiffened either way developed the same strength, provided the rigidity of the lip proper was sufficient to furnish full support. This statement holds at least for the range of b/t of the beams of Table 2—that is, for the lipped flanges tested in this investigation.

The only contribution that undertakes to challenge the writer's approach to the problem (merely with regard to flanges stiffened along both edges) is that by Messrs. Maugh and Legatski. To evaluate the contentions contained therein, it appears necessary to clarify some obvious misunderstandings, the source of which the writer is unable to trace.

In summarizing their discussion, Messrs. Maugh and Legatski state that "Perhaps the author was led to the erroneous conclusion that the Bryan formula cannot be used to determine the ultimate buckling strength of thin plates * * *." Three lines farther, however, they write:

"The Bryan load is the load at which the plate element forms into buckling waves, but it is not the ultimate load because a boundary or edge effect remains as an additional element of strength."

This is exactly what the writer maintained in his discussion of fundamentals (paragraph preceding Eqs. 2), where he stated that,

"The central, more highly distorted, regions of the plate decrease in their resistance, thus throwing more of the total compressive force toward the stiffened edges * * *. This action [failure] occurs at a load higher than the critical—that is, higher than that load at which, theoretically, small deflections start to occur * * *."

These two statements are exactly identical in content, and the writer is at a loss to understand wherein lies his "erroneous conclusion."

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In the concluding paragraphs, the statement, "Without accepting the effective width concept as the most logical approach * * *," seems to indicate that Messrs. Maugh and Legatski know of, or have developed, a method which dispenses with that concept. Actually the approach they propose is likewise based on an effective width, implicitly for stress determinations (see Eq. 23), and explicitly for deflection computations. The writer, again, is at a loss to understand the quoted statement.

From an analytical point of view, contrary to the contributors' contention, Eq. 20, from which the Bryan formula is derived, is no longer applicable, once the critical stress is exceeded. Under such conditions, it is necessary to consider the so-called large deflection theory; that is, Theodor von Kármán's differential equation,⁴⁵ in which F is the stress function:

$$\frac{\partial^4 \eta}{\partial x^4} + 2 \frac{\partial^4 \eta}{\partial x^2 \partial y^2} + \frac{\partial^4 \eta}{\partial y^4} = \frac{t}{D} \left(\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 \eta}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 \eta}{\partial x \partial y} \right) \dots (29)$$

which depends not only on the external compression force, but also on the deformations. It is the extreme complexity of Eq. 29 which is the reason that a solution, so far, has been obtained for circular plates, only. In other words, once the critical stress is exceeded, the plate assumes a new state of equilibrium, which is governed by factors greatly different from those which determine the Bryan load. Eq. 20 does not hold for this state of equilibrium, because, in this state, the stress s is not uniaxial and constant throughout the flange.

An analytical investigation⁴⁶ of the stresses above the critical, by means of approximate strain energy methods, not only shows the stress distribution to be exactly of the type of Fig. 12, it also indicates that, contrary to the assumption made by Messrs. Maugh and Legatski, the stress in the center is not constant, and equal to the Bryan stress. For very wide and thin plates (as Professor Timoshenko has shown⁴⁷), the center strip of an edge-compressed plate is subject to tension, rather than to the Bryan compression stress. The same result, in a rigorous manner, was obtained by Messrs. Friedrichs and Stoker for a circular plate.⁴⁸

To summarize: The writer did not maintain, as is stated by Messrs. Maugh and Legatski, that the center strip is free of stress. This is evident from Fig. 12, whose validity is borne out by the contributors' own strain measurements (for which the writer is grateful indeed as collateral evidence). However, he does maintain that the stress at the center strip is not constant and equal to the Bryan stress, which is again confirmed by the contributors' own stress measurements (Fig. 18(a)). These show a decrease of the center stress by about 40% of its maximum value with increasing loads, even at a b/t -ratio as moderate as 98.

In view of the theoretical complexity of the problem, the writer pursued a frankly semi-empirical method. Messrs. Maugh and Legatski propose instead

⁴⁴ "Encyclopädie der Mathematischen Wissenschaften," Vol. IV/4, 1910, p. 349.

⁴⁵ "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 323.

⁴⁶ *Ibid.*, pp. 390-395.

⁴⁷ *Ibid.*, p. 395, Fig. 207.

a semi-analytical approach, based on a number of arbitrary assumptions, one of which is discussed herein (that is, center-strip stress equals Bryan stress). Another such assumption is that of taking the proportional limit as five eighths of the yield point, which was apparently necessary to make the equations fit the Schuman and Back results. Even for steels, the so-called "proportional limit," is a completely fictitious quantity, whose value depends primarily on the investigator and his instrumentation.⁴⁶ In sharply yielding steels, in addition, the error introduced by equating the proportional limit to the yield point of a mean stress, s_y , related to the entire unreduced flange area.

For designing members in uniform compression, it may be irrelevant whether the actual stress distribution, Fig. 12, is replaced by a uniform mean stress, or by the writer's equivalent width, related to the maximum stress.

For designing members in flexure or eccentric compression, however, it is necessary to determine the actual maximum edge strain; because it is that strain which governs the location of the neutral axis, without which stresses

cannot be computed. The writer's method allows this determination, the contributors' does not. The latter, therefore, would determine flange stresses by the ordinary flexure formula, using the centroidal axis. How far the actual neutral axis, as determined by strain measurements, can deviate from the

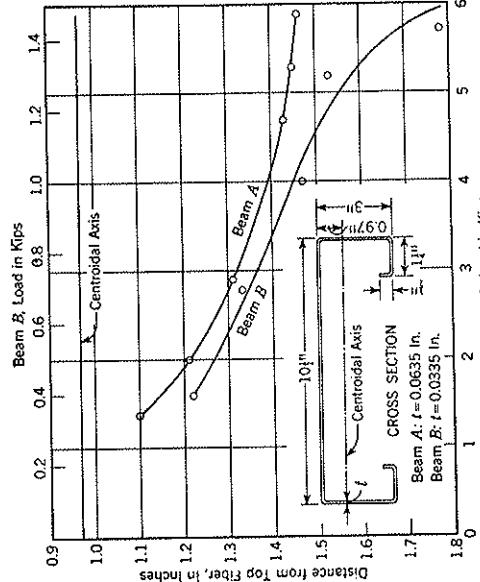


Fig. 22

cannot be computed. The writer's method allows this determination, the contributors' does not. The latter, therefore, would determine flange stresses by the ordinary flexure formula, using the centroidal axis. How far the actual neutral axis, as determined by strain measurements, can deviate from the

⁴⁶ "Stress, Strain and Structural Damage," by H. F. Moore, *Bulletin No. 10, Univ. of Illinois, Urbana, Vol. 37, 1939, pp. 17-18.*

centroid is shown in Fig. 22. The results there given were obtained on two beams of practically identical over-all dimension, but with different sheet thicknesses. The error that would result in using the centroidal axis instead of the actual neutral axis for design computations is evident from the figure. This situation is implicitly conceded by Messrs. Maugh and Legatski in their statement that, for deflection computations, an equivalent width must be used. For reasons not indicated this width, for deflection computations, is different from the width used implicitly by the contributors for stress computations, as given in Eq. 23. Effective widths, locations of neutral axes, and deflections are interrelated purely geometrically, inasmuch as they are all functions of the magnitudes of the extreme fiber strains. If an effective width approach is necessary for determining deflections (that is, neutral axes), it is physically contradictory to maintain, on the other hand, that stresses can be computed from unreduced widths. This is tantamount to stating that a beam has two neutral axes—one, referred to the unreduced section, governing stresses, and another, referred to the reduced section, governing deflections. In the writer's proposed method, the same approach applies to both stress and deflection determinations.

Parenthetically, the contributors' method (computing deflections by (a) using an unreduced moment of inertia to find part of the deflection, (b) computing a critical stress, and (c) determining a second, reduced moment of inertia for finding the additional deflection) is certainly more cumbersome than that proposed in the paper. It results, qualitatively, in the same behavior as that found by the writer; that is, the effective moment of inertia (in the contributors' case, the weighted mean between the full and the reduced one) decreases with increasing b/t and increasing stress. It is rather doubtful, however, whether the arbitrary effective width of $40t$ for the reduced moment of inertia, given without justification or empirical verification, leads to reasonably accurate results. For the wider flanges (as used in many commercial decks with high b/t) where the critical stress is very low, the use of $40t$ will be found to lead to erroneous results on the conservative (uneconomical) side in determining deflections at design loads—that is, at stresses of from 10 to 20 kips per sq in. (For such conditions effective widths in this investigation were found to be of the order of $60t$ to $80t$ as compared with the contributors' $40L$.)

With regard to the contributors' criticism of test methods, the writer will agree to the extent only that (1) strain measurements on light-gage sections are plagued by various disturbing influences not present in more solid members; and (2) that it is always possible to think of other methods of instrumentation that could have been used. With regard to the latter, however, the time element is an important consideration. In this investigation, it was necessary to cover a very wide range of dimensions, which can be done on a large number of specimens, only. The investigator is limited, therefore, to such measurements as will furnish the most pertinent data. This, it is believed, was achieved by the methods selected. The collateral, confirmatory evidence provided by Messrs. Maugh and Legatski, by different test methods, is therefore doubly welcome.

The contributors' statement that twist and lateral deflection may have distorted the measurements, can be discounted, except for possible microscopic effects. Beams of the shape of Fig. 3 have no tendency toward lateral deflection, particularly if loaded through rollers with axes perpendicular to the axis of the specimen. In fact, to deflect laterally, they would have to overcome the contact friction at the load points. Not only can it be shown by computation from known friction coefficients that this is impossible, but any such sliding motion would immediately manifest itself by a scraping sound, as the writer observed frequently in tests of an entirely different nature. Twist, on the other hand, was prevented by a loading arrangement, which forced the beam into parallel vertical displacement. It is evident that effects of the magnitude of those of Fig. 22, consistently obtained on a great number of specimens, cannot be ascribed to details of instrumentation. Finally, the problem of the linearity of strain distribution in such specimens, even if it were open to question, appears to be irrelevant. The location of the neutral axis, as used in the proposed methods, is merely a means to determine, in design computations, the relative magnitude of the top and bottom strains. Since these strains were measured directly, even a curvilinear transition would not affect the results of such design computations. In addition, however, no reason for a curvilinear strain distribution is given by the contributors; nor is any apparent to the writer, particularly since strain measurements were made in the center part of the quarter-point loaded beams, that is, in a region of pure bending.

Two of the contributions (Messrs. Lewis and Gunder's, and Mr. Karol's) do not question the findings of this investigation, but concern themselves mainly with the mathematical form of Eqs. 6 and 7. Formulas such as these are developed with due consideration to simplicity of application. In this particular case, it was imperative, for simplification of design, to delimit a range of b/l , for which the full width can be used (up to $b/l = 25$). Although, theoretically, the establishment of such a definite limit is questionable, it is completely justified practically. This is why seemingly elaborate mathematical expressions had to be chosen, which result in $b_s = b$ for this limit, independently of the stress. The mathematical complexity of the formulas is relatively irrelevant, since designers will work from graphs such as Fig. 7, rather than from equations. In addition, Mr. Karol's Eq. 13, which involves a hyperbolic function, for that reason, will not be found very convenient by most designers.

Mr. Karol's cautioning remark regarding the use of the writer's chart (Fig. 7) for materials other than steel is to the point, and should be considered in such applications.

The writer gladly accepts Messrs. Lewis and Gunder's justified criticism of the statistical method used in this paper. It is quite true that more sensitive devices, such as standard deviations, should be used in determining the accuracy of fit of empirical formulas.

The writer appreciates the painstaking work of Messrs. Lewis and Gunder in replotting Fig. 2. He does not believe that Eq. 18 should be used practically, despite its simplicity. On the one hand, as the contributors state, goodness of fit of Eq. 18 within the tested range is not as satisfactory as that of Eqs. 6

and 7. Also, tests conducted since the publication of the paper showed Eq. 6 to be applicable satisfactorily for values of b/l up to 400 and more (see contribution by Messrs. Brown and Wolford). In this range of high b/l , Eq. 18 would err on the conservative side up to about 25%.

Eq. 19 is identical in structure with Eq. 6, except for minor differences in the constants, and, as stated by the contributors, is not significantly better. The writer is grateful for the fact, established by Messrs. Lewis and Gunder, that, despite his somewhat cursory methods of curve fitting, he apparently managed to arrive at expressions which are as accurate as those developed by more sensitive statistical means.

Professor Johnston furnishes interesting additional material on flanges stiffened along one edge, by reference to his and Professor Cheney's earlier tests on wide flange sections.

The failure criterion used by Messrs. Johnston and Cheney differs from the writer's in that they noted merely the ultimate loads, whereas the writer considered also the stress at which local wrinkling was first noted. Therefore, Professor Johnston's results should be compared with the values of the ultimate stress, s_u , in Table 4. In the range of b/l , common to both investigations (that is, up to about 21), it is seen from Table 4 that the writer obtained ultimate stresses of 0.8 to 0.9 of the yield point, with very few exceptions. This is in general agreement with Professor Johnston's findings of a 90% yield-point strength. The somewhat lower values obtained by the writer on thin gage specimens are probably due (a) to greater edge restraint in the H-sections, as noted by Professor Johnston, and (b) to the better forming accuracy of the milled H-beam flanges, as compared with cold formed sheet steel elements.

The writer's colleague, Professor Cheney, volunteered the information that in many of the tests cited by Professor Johnston, development of slight waves and kinks was noticed at loads below the ultimate, which is in agreement with the findings of this investigation.

The writer wishes to thank the American Rolling Mill Company for releasing for publication the test data given in Messrs. Brown and Wolford's contribution. These data furnish important, supplementary evidence, since the b/l -ratios of these specimens, 242 to 429, are far beyond the range of those discussed in the paper. The test results given in Table 6, as well as conclusions by Messrs. Brown and Wolford, confirming the validity of the proposed method in this high range of b/l , speak for themselves. It is seen that the determinations made on the basis of Eq. 6, if compared with test results, are slightly on the conservative side, both for deflection and for strength. The deviations are small and within the range of discrepancy observed in most structural testing, but they point to one factor that should be emphasized in closing.

The behavior of thin compression flanges is naturally influenced by the amount of rotational edge fixity provided by adjoining elements, such as the webs in Fig. 3. A reasonably simple design method cannot be expected to take explicit account of this involved factor, particularly since, in the great number of tests, its numerical effect was found to be small. The amount of restraint