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Modeling Electricity Demand: A Neural Network Approach

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Support

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“Implications of Climate Change for Regional Air Pollution and Health Effects and Energy Consumption Behavior”

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Outline

I. Context of the Research
II. Introduction to ANN Modeling
III. Basics of Electricity Demand
IV. Developing the Demand Model
V. Results

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I. Context of the Research

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The modeling efforts of the STAR grant are
1. Electricity load modeling and forecasting
2. …
3. …
4. …
Context

The modeling efforts of the STAR grant are
1. Electricity load modeling and forecasting
2. Electricity generation and dispatch modeling
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The modeling efforts of the STAR grant are
1. Electricity load modeling and forecasting
2. Electricity generation and dispatch modeling
3. Regional air pollution modeling
4. …
Context

The modeling efforts of the STAR grant are
1. Electricity load modeling and forecasting
2. Electricity generation and dispatch modeling
3. Regional air pollution modeling
4. Health effects characterization
II. Intro to ANN Modeling
ANN Model Architecture

Independent Variables

ANN

Dependent Variables

“In this figure, the ‘Independent Variables’ serve as inputs to the ‘ANN’ model (also known as an Artificial Neural Network). The ‘Dependent Variables’ are the outputs generated by the ANN. This diagram illustrates a basic feedforward neural network architecture, where the connection from input to output is unidirectional.”

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ANN Model Architecture

Input Layer

Output Layer

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ANN Model Architecture

- **Input Layer** (Hidden Layer)
  - Input neuron #1
  - Input neuron #2

- **Output Layer**
  - Output neuron #1

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ANN Model Architecture

Elements of a neuron

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Elements of a neuron

• Multiplicative Weight ($w$)
ANN Model Architecture

Elements of a neuron

• Multiplicative Weight ($w$)
• Additive Bias ($b$)
ANN Model Architecture

Elements of a neuron

- Multiplicative Weight ($w$)
- Additive Bias ($b$)
- Transfer Function ($f$)

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ANN Model Architecture

Elements of a neuron

- Multiplicative Weight \((w)\)
- Additive Bias \((b)\)
- Transfer Function \((f)\)

The Neuron uses these elements in its operation, when it receives an input \((p)\) and produces a result \((a)\)
ANN Model Architecture

Once a neuron receives an input \((p)\), the neuron’s operation consists of two parts:
ANN Model Architecture

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- Linear Transformation
  
  using the weight \((w)\) and bias \((b)\)
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- Linear Transformation
  
  using the weight \((w)\) and bias \((b)\)

- Application of the Transfer Function \((f)\)
ANN Model Architecture

Once a neuron receives an input \((p)\), the neuron’s operation consists of two parts:

- Linear Transformation using the weight \((w)\) and bias \((b)\)
- Application of the Transfer Function \((f)\)

The neuron then passes the result \((a)\) on to the next part of the ANN.
ANN Model Architecture

Linear Transformation

• Multiply the input \((p)\) by a weight \((w)\) and add the bias \((b)\) to get the intermediate result \((n)\):
ANN Model Architecture

Linear Transformation

- Multiply the input \((p)\) by a weight \((w)\) and add the bias \((b)\) to get the intermediate result \((n)\):

\[
    n = w \times p + b
\]
ANN Model Architecture

Linear Transformation

- Multiply the input \( p \) by a weight \( w \) and add the bias \( b \) to get the intermediate result \( n \):

\[
    n = w \times p + b
\]

- Pass the intermediate result \( n \) to the transfer function
ANN Model Architecture

Transfer Function

• Apply the transfer function \((f)\) to the intermediate result \((n)\) to create the neuron’s result \((a)\)
ANN Model Architecture

Transfer Function

• Apply the transfer function \( f \) to the intermediate result \( n \) to create the neuron’s result \( a \)

• For neurons in the input layer, the transfer function is typically a hard-limiting switch at some threshold, say \( n = 0 \):

\[
f(n) = (0 \text{ for } n \leq 0; \ 1 \text{ for } n > 0)
\]
ANN Model Architecture

Transfer Function

• Apply the transfer function \( f \) to the intermediate result \( n \) to create the neuron’s result \( a \)

• For neurons in the input layer, the transfer function is typically a hard-limiting switch at some threshold, say \( n = 0 \):
  \[
  f(n) = (0 \text{ for } n \leq 0; 1 \text{ for } n > 0)
  \]

• For neurons in the output layer, the “pure linear” transfer function typically has no effect:
  \[
  f(n) = n
  \]
ANN Model Architecture
ANN Model Architecture

\[ \sum w \cdot p + b = n \]

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ANN Model Architecture

\[ \sum \overrightarrow{w} \cdot \overrightarrow{p} + b = n \]

- **weights**
- **bias**

Linear Transformation

Transfer Function

\[ f(n) = a \]

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\[ \sum \bar{w} \cdot \bar{p} + b = n \]

\[ f(n) = a \]

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ANN Model Architecture

Example: A Neuron’s Operation

• Say a neuron has a weight of 10, a bias of 100, and a hard-limiting transfer function with a threshold of $n = 0$:

  $w = 10; \ b = 100$

  $f(n) = (0 \text{ for } n \leq 0; \ 1 \text{ for } n > 0)$
ANN Model Architecture

Example: A Neuron’s Operation

• Say a neuron has a weight of 10, a bias of 100, and a hard-limiting transfer function with a threshold of $n = 0$:
  \[ w = 10; \quad b = 100 \]
  \[ f(n) = (0 \text{ for } n \leq 0; \quad 1 \text{ for } n > 0) \]

• Say the neuron receives an input of 5:
  \[ p = 5 \]
ANN Model Architecture

Example: A Neuron’s Operation

• Say a neuron has a weight of 10, a bias of 100, and a hard-limiting transfer function with a threshold of \( n = 0 \):
  \[
  w = 10; \quad b = 100
  \]
  \[
  f(n) = (0 \text{ for } n \leq 0; \; 1 \text{ for } n > 0)
  \]

• Say the neuron receives an input of 5:
  \[
  p = 5
  \]

• The neuron’s linear transformation produces an intermediate result of 150:
  \[
  n = 10 \times 5 + 100 = 150
  \]
ANN Model Architecture

Example: A Neuron’s Operation

• Say a neuron has a weight of 10, a bias of 100, and a hard-limiting transfer function with a threshold of \( n = 0 \):
  \[
  w = 10; \quad b = 100
  \]
  \[
  f(n) = (0 \text{ for } n \leq 0; \ 1 \text{ for } n > 0)
  \]

• Say the neuron receives an input of 5:
  \[
  p = 5
  \]

• The neuron’s linear transformation produces an intermediate result of 150:
  \[
  n = 10 \times 5 + 100 = 150
  \]

• The neuron’s transfer function produces the neuron’s result of 1:
  \[
  a = f(n) = 1
  \]

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ANN Model Selection

- The weights and biases are determined by the ANN during training

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ANN Model Selection

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• Training involves presenting the ANN with input-output examples
ANN Model Selection

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- Training involves presenting the ANN with input-output examples.
- Training is finding the weights and biases that minimize the performance function.
ANN Model Selection

• The weights and biases are determined by the ANN during **training**
• Training involves presenting the ANN with input-output **examples**
• Training is finding the weights and biases that minimize the **performance function**
• The performance function is typically some measure of model error, such as MSE
ANN Model Selection

To build an ANN, the researcher specifies

- the number of *layers*
  - Input
  - Output
  - Hidden (if any)
ANN Model Selection

To build an ANN, the researcher specifies

- the number of *layers*
  - Input
  - Output
  - Hidden (if any)

- the number of *neurons* in each layer
  - typically 1-5 input neurons
  - typically 1 output neuron
ANN Model Selection

To build an ANN, the researcher specifies

➢ the number of *layers*
  • Input
  • Output
  • Hidden (if any)

➢ the number of *neurons* in each layer
  • input layer: typically 1-5 neurons
  • output layer: typically 1 neuron

➢ the type of *transfer function* for each neuron
  • input layer: typically hard-limiting (0-1 switch)
  • output layer: typically pure-linear (no effect)
ANN Model Selection

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- the number of *layers*
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- the type of *transfer function* for each neuron
  - input layer: typically hard-limiting (0-1 switch)
  - output layer: typically pure-linear (no effect)

- the *performance function* for the network (MSE)
III. Basics of Electricity Demand
Electricity Demand

- Utilities are interested in forecasting peak demand
Electricity Demand

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• Peak demand determines how many generators the utility must bring on line
Electricity Demand

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• Overestimating demand is costly
Electricity Demand

- Utilities are interested in forecasting peak demand
- Peak demand determines how many generators the utility must bring on line
- Overestimating demand is costly, wasteful
- Underestimating peak leads to electricity shortfalls

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Electricity Demand

Hourly peak electricity demand depends on
• Weather
• ...
• ...
• ...
• ...

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Electricity Demand

Hourly peak electricity demand depends on
• Weather
• Time of the Year
• ...
• ...
• ...

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Electricity Demand

Hourly peak electricity demand depends on

- Weather
- Time of the Year
- Day of the Week
- ...
Electricity Demand

Hourly peak electricity demand depends on

- Weather
- Time of the Year
- Day of the Week
- Holidays
- …
Electricity Demand

Hourly peak electricity demand depends on

- Weather
- Time of the Year
- Day of the Week
- Holidays
- Year
- Recent Electricity Demand
IV. Developing the Demand Model
General Model

*Independent Variable* (Peak Electric Demand in MWh):

\[ Load^h_d \]
General Model

*Independent Variable* (Peak Electric Demand in MWh):

\[
Load^h_d
\]

\( h = 1 \ldots 24 \)

is the hour being modeled

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General Model

Independent Variable (Peak Electric Demand in MWh):

\[ \text{Load}^h_d \]

\( h = 1 \ldots 24 \)

is the hour being modeled

\( d = \text{January 1, 1995} \ldots \text{September 30, 2003} \)

is the date of the observation
General Model

Dependent Variables

1. Weather (Temperature in Fahrenheit degrees):

\[ Temp_d^h \]

\[ h = 1 \ldots 24 \]

is the hour being modeled

\[ d = \text{January 1, 1995} \ldots \text{September 30, 2003} \]

is the date of the observation
General Model

**Dependent Variables**

2. **Time of the Year:**

   $Month_i$

   $i = 1 \ldots 12$

   is the month of the observation

   $(Month_1 = 1$ for January, 0 otherwise,
   $Month_2 = 1$ for February, 0 otherwise, etc.)

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General Model

*Dependent Variables*

3. Day of the Week:

\[ \text{Day}_j \]

\[ j = 1 \ldots 7 \]

is the weekday of the observation

\( \text{Day}_1 = 1 \) for Monday, 0 otherwise,  
\( \text{Day}_2 = 1 \) for Tuesday, 0 otherwise, etc.)
General Model

Dependent Variables

4. Holidays:

\[ \text{Holiday} \]

dummy for recording days with different demand profiles due to businesses closing

\((\text{Holiday} = 1 \text{ for New Years Day, Independence Day etc.}, \text{ and } 0 \text{ otherwise})\)
General Model

Dependent Variables

5. Year of observation:

Year

records the year of the observation to account for trends
General Model

**Dependent Variables**

6. Recent Electricity Demand:

\[ \text{Load}_d^{h-1}, \text{Load}_d^{h-2}, \text{Load}_d^{h-3} \]

electricity demand for the previous three hours
General Model

Dependent Variables

5. Recent Electricity Demand:

\[ \text{Load}_{d}^{h-1}, \ \text{Load}_{d}^{h-2}, \ \text{Load}_{d}^{h-3} \]

electricity demand for the previous three hours

\[ \text{Load}_{d-1}^{h} \]

electricity demand for the previous day at this hour
General Model

\[ Load_d^h = f \left( \begin{array}{c}
Temp_d^h, \\
Month_i, Day_j, Holiday, Year \\
Load_d^{h-1}, Load_d^{h-2}, Load_d^{h-3}, Load_{d-1}^h
\end{array} \right) \]
V. Model Selection

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Data

• Electricity demand data were obtained from PJM, a large Independent System Operator (ISO) in the mid-Atlantic U.S.
Data

• Electricity demand data were obtained from PJM, a large Independent System Operator (ISO) in the mid-Atlantic U.S.

• Temperature data were obtained from the US National Climatic Data Center (NCDC) for the dates and areas of interest
Model Selection

I chose to build an ANN with two layers

- Input Layer with 1-20 hard-limiting neurons
- Output Layer with 1 pure-linear neuron
Model Selection

- I chose to build an ANN with two layers
  - Input Layer with 1-20 hard-limiting neurons
  - Output Layer with 1 pure-linear neuron
- I trained the ANN using the MSE as the performance function
Model Selection

I chose to build an ANN with two layers

- Input Layer with 1-20 hard-limiting neurons
- Output Layer with 1 pure-linear neuron

I trained the ANN using the MSE as the performance function

The optimal number of input neurons was to be determined by evaluating their effect on the ANN

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Model Selection

• Adding neurons allows a model to be more flexible
Model Selection

• Adding neurons allows a model to be more flexible
• But each additional neuron requires estimating several more parameters (weight and bias vectors)
Model Selection

• Adding neurons allows a model to be more flexible
• But each additional neuron requires estimating several more parameters (weight and bias vectors)
• McMenamin and Monforte (1998) suggest observing the Schwartz Information Criterion (SIC) as additional neurons are added to an ANN
Model Selection

- The SIC is a measure of model performance, based on the MSE, penalized for degrees of freedom. (NB the statistic reported is often the natural log of the SIC)

\[
SIC \equiv e^{\frac{2k}{T}} \left[ \frac{\sum_{t=1}^{T} e_t^2}{T} \right]
\]

\[
\begin{align*}
e_t & \text{ is the model error} \\
T & \text{ is the total number of observations} \\
k & \text{ is the number of estimated parameters} \\
e & \text{ is the base of the natural logarithm}
\end{align*}
\]
Model Selection

• The SIC is a measure of model performance, based on the MSE, penalized for degrees of freedom. (NB the statistic reported is often the natural log of the SIC)

$$SIC \equiv e^{\frac{2k}{T} \left[ \sum_{t=1}^{T} e_t^2 \right] / T}$$

• If a neuron’s benefit outweighs its cost, SIC falls
Model Selection

- The SIC is a measure of model performance, based on the MSE, penalized for degrees of freedom.
  (NB the statistic reported is often the natural log of the SIC)

\[
SIC \equiv T^T \left[ \sum_{t=1}^{T} \frac{e_t^2}{T} \right]^{k \frac{1}{T}}
\]

- If a neuron’s benefit outweighs its cost, SIC falls
- When the SIC starts to rise, the optimal number of neurons has likely been surpassed

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Model Selection

SIC as a Function of Neurons in the Input Layer
Hour 13 – Hour 18

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Model Selection
SIC as a Function of Neurons in the Input Layer
Average over all 24 Hours

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V. Model Results
Model Results

For hourly models estimated for the Summer months:

- four models use 1 input neurons
- fifteen models use 2 input neurons
- four models use 3 input neurons
- one model uses 4 input neurons
Model Results

ANN models typically perform well with abundant data from non-linear relationships. The hourly models account for about 99% of the variation in the test data, with MAPE around 1.2%.

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Model Results

• The models appear to be biased towards low values in the test period.
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• As the test period is composed of the most recent data, this bias may be due to a growth trend (data were not de-trended)
Model Results

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• As the test period is composed of the most recent data, this bias may be due to a growth trend (data were not de-trended)
• Bias could also be due to high demand in the test period, or a structural shift
Model Results

Typical Performer – Hour 12

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Model Results
Poor Performer – Hour 14

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