Effective Force Testing Using A Robust Loop Shaping Controller

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Abstract

Effective force testing (EFT) is one of the force-based experimental methods used for performance evaluation of structures that incorporate dynamic force control using hydraulic actuators. While previous studies have shown successful implementations of force control, controllable frequency ranges are limited to low frequencies (10 Hz). This study presents the EFT method using a robust loop shaping force feedback controller that can extend the frequency range up to 25 Hz or even higher. Unlike the conventional PID controllers, loop shaping controllers can provide robustness for a high level of force measurement noise. This study investigates the dynamic properties of hydraulic actuators and the design of a loop shaping controller that compensates for control-structure interaction as well as suppresses the effect of oil-column resonance. The designed loop shaping controller was successfully implemented into an EFT setup at the Johns Hopkins University. An experimental investigation of the loop shaping controller was performed under step, random and earthquake force loadings. Experimental results showed that the loop shaping controller provided excellent force tracking performance as well as robustness for dynamic force loadings. It was also shown that the loop shaping controller had the gain margin of 9.54 dB at the frequency of 28 Hz.

Keywords

Effective force testing; dynamic force control; hydraulic actuators; experimental methods; loop shaping; robust control;
INTRODUCTION

Experimental methods in earthquake engineering have come a long way since the initial development of a hand-powered shake table in the 19th century in Japan (Severn 2011). Significant developments have been made along with upgrades of equipment (e.g., hydraulic actuators) and advancements of digital computers (e.g., high-performance computing, real-time operating systems, etc.). Evolution was also not possible without innovative ideas to expand the experimental capabilities such as pseudo dynamic test by Hakuno et al. (1969), substructure technique by Dermitzakis and Mahin (1985), real-time hybrid testing by Nakashima et al. (1992), and so forth. In the last ten years, developments and applications of experimental methods have been further accelerated with recently constructed large, unique, and versatile facilities such as those in the George E. Brown, Jr. Network for Earthquake Engineering Simulation in the United States, E-Defense shake table in Japan, and European Laboratory for Structural Assessment (ELSA) in JRC. While progresses made in this area are promising, most of the efforts have been limited to displacement-based experimental methods such as shake table tests and hybrid simulation techniques; while combined displacement, velocity, and acceleration controls (Tagawa and Kajiwara 2007) and acceleration feed-forward control (Nakata 2010) for shake tables are reported, most shake tables are primarily controlled in displacement. Potential methods that have not been thoroughly explored in earthquake engineering are force-based experimental methods.

One of the force-based experimental methods that has advantages over shake table tests and hybrid simulation is effective force testing (EFT). Idea of the EFT method was first introduced by Thewalt and Mahin (1987), and it is as follows. Dynamic performance of a structure can be evaluated by directly imposing a controlled force that is equivalent to the impact of an earthquake loading (i.e., $f$ in the following equation of motion).

$$m\ddot{x} + c\dot{x} + kx = f$$

(1)

where $m$, $c$ and $k$ are the mass, damping, and stiffness of the test structure; and $x$ is the displacement. Unlike shake table tests, EFT does not require shaking of the whole test structure. Therefore, EFT can be performed with smaller actuators than those required for shake table tests. Unlike hybrid simulation, the EFT method does not need to solve equations of motion; which is preferred since solution algorithms introduce numerical errors in simulation. A more detailed description of the EFT method can be found in Dimig et al. (1999).

While the EFT method is conceptually straightforward, implementation is challenging. Due to the influence of the actuator piston velocity on the flow of oil in actuator chambers (i.e., natural velocity
feedback), actuators do not have abilities to apply force at the natural frequencies of the test structure (Dyke et al. 1995). This phenomenon is called control-structure interaction, and it largely limits capabilities of the EFT method without proper compensations. Furthermore, because force measurement generally has a high level of noise, stability and robustness also become critical in force feedback control.

Several researchers have investigated dynamic force control through studies of the EFT method and force tracking problems. Dimig et al. (1999) verified the effect of control-structure interaction using a linear-elastic single-degree-of-freedom system. To compensate the control-structure interaction, an additional velocity feedback was suggested. Subsequently, Zhao et al. (2005) investigated nonlinear velocity feedback compensation for the control-structure interaction using the same test setup as Dimig, and experimentally verified their effectiveness. While these studies showed successful implementations of force control along with velocity feedback compensation, the bandwidth of the controlled force was limited to 10 Hz. An explanation of the limited bandwidth in force control using hydraulic actuators can be found in Alleyne et al. (1998). Using the root locus of a proportional controller, they showed that the conventional PID controllers severely limit the bandwidth. This conclusion is consistent with the limitations of the bandwidth found in Dimig and Zhao where a conventional proportional controller was used. Alleyne and Liu (1999) proposed a Lyapunov-based nonlinear controller for force tracking problems using hydraulic actuators. While their algorithm has shown to have high bandwidth in simulation, it is significantly simplified in an experimental validation. As the result, the controllable frequency range was 10 Hz (Alleyne and Liu 2000).

As briefly reviewed here, the previous studies in dynamic force control have limitations in controllable bandwidth. Furthermore, they have not addressed the robustness of controllers that is important in force feedback control where a high level of measurement noise exists. Stability and robustness are critical particularly due to the existence of oil-column resonance in hydraulic actuators. To improve force control capabilities in EFT as well as to provide enabling force control techniques for the other force-based experimental methods such as force-based real-time hybrid simulation (Nakata and Stehman 2011), dynamic force control using hydraulic actuators has to be further investigated.

It should be mentioned that a number of researchers suggested indirect approaches to dynamic force control by introducing conversions from a reference force to an equivalent reference displacement (Sivaselvan et al. 2008, Wu et al. 2007, etc.). While overall goals of the indirect approaches are similar to the EFT method, their implementations are totally different from a control point of view; indirect approaches use displacement feedback control for actuators, not force feedback control. Furthermore, such indirect approaches face different challenges including an estimation of stiffness for the
displacement conversion that is not required in the EFT method. Therefore, indirect approaches are not discussed further in this study.

This paper presents effective force testing using a robust loop shaping force feedback controller. Loop shaping is a frequency domain technique for feedback control systems that can provide high performance and robustness as well as large bandwidth. Firstly, a theoretical background of actuator dynamics is presented to facilitate controller designs. Following, a description of the experimental set up for EFT at Johns Hopkins University, controller designs including velocity feedback, proportional controller, and loop shaping, are discussed based on performance, stability, and robustness. A loop shaping controller is designed to compensate the control-structure interaction and suppress the oil-column resonance of the actuator. Experimental investigation of the loop shaping force feedback controller is performed under step, random, and earthquake force loadings. Experimental results and observations are also presented in this paper.

**DYNAMICS OF HYDRAULIC ACTUATORS**

Hydraulic actuators consist of electro-magnetic, mechanical, and hydraulic components that have unique dynamics. As a result of series, parallel and feedback connections of those components, the dynamics of hydraulic actuators become higher-order and even nonlinear under severe loading cases. Open-loop actuator dynamics govern not only performance of the controlled system but also its stability and robustness. To facilitate the development of controller designs for EFT, this section presents the open-loop dynamics of hydraulic actuators derived from component dynamics and governing equations.

**Transfer Functions of Actuator Components**

The input signal to a hydraulic actuator is a valve command that is generated by a controller. The valve command is usually a DC voltage, and when a voltage-driven valve that has onboard integrated electronics (e.g., Parker D1FH series) is used, the valve command can be directly sent to the valve. However, when a current-driven servo valve (e.g., Moog G761 series) is used, the valve command needs to be converted to a proportional current by a voltage-to-current converter (e.g., Axiomatic USC-CVB225). The first-order approximation of the transfer function of the converter from voltage $u$ to current $i$ can be expressed as (Erickson 1997):

$$H_{iu}(s) = \frac{n_{iu}(s)}{d_{iu}(s)} = \frac{i_{\text{max}}}{u_{\text{max}}} \frac{1}{1 + \tau c s}$$ (2)
where \( n_{iu} \) and \( d_{iu} \) are the numerator and denominator polynomials of the transfer function \( H_{iu} \), respectively; \( i_{\text{max}} \) and \( u_{\text{max}} \) are the maximum rated current and voltage of the converter, respectively; and \( \tau_c \) is the time constant of the converter; and \( s \) is the Laplace variable.

The valve current alters a magnetic field of the coil in the servo valve which in turn changes the inclination of a nozzle flapper. This inclination of the flapper changes the position of a spool. The dynamics of the spool opening \( d \) from the valve current \( i \) can be described by a first-order differential equation and the transfer function is expressed as (Dyke et al. 1995; and Conte and Trombetti 2000):

\[
H_{di}(s) = \frac{n_{di}(s)}{d_{di}(s)} = \frac{d_{\text{max}}}{i_{\text{max}}} \frac{1}{1 + \tau_v s}
\]  

where \( n_{di} \) and \( d_{di} \) are the numerator and denominator polynomials of the transfer function \( H_{di} \), respectively; \( d_{\text{max}} \) is the maximum opening of the spool; and \( \tau_v \) is the time delay of the servo valve.

The spool opening regulates the oil flow in the actuator chambers. Under severe loading cases, a pressure drop across actuator chambers also affects the oil flow. Nonlinear relationship between the spool opening, pressure drop and the oil flow is experimentally obtained and reported by Merritt (1967) as:

\[
q = k_q d \sqrt{1 - \frac{d}{d_{s}} \frac{p}{p_{s}}} \]  

where \( q \) is the oil flow in the actuator chambers; \( p \) is the pressure drop across the actuator chambers; \( p_{s} \) is the supply pressure; and \( k_q \) is the flow gain of the servo valve. A linearized relationship around the neutral position of the spool opening and a zero pressure drop is expressed as:

\[
q = k_q d
\]  

The transfer function from the spool opening to the oil flow yields

\[
H_{qd}(s) = \frac{n_{qd}(s)}{d_{qd}(s)} = k_q
\]  

where \( n_{qd} \) and \( d_{qd} \) are the numerator and denominator polynomials of the transfer function \( H_{qd} \), respectively.
The oil flow can be also expressed from the flow continuity equation of the actuator piston as:

\[ q = A v + k_e f + \frac{V}{4\beta A} \dot{f} \]  

(7)

where \( v \) is the actuator piston velocity; \( f \) is the actuator force; \( A \) is the cross sectional area of the actuator piston; \( k_e \) is the flow-force coefficient; \( V \) is the volume of the chambers; and \( \beta \) is the bulk modulus of the hydraulic fluid. The transfer function from the oil flow to the actuator force can be written as (Conte and Trombetti 2000):

\[ H_{fq}(s) = \frac{n_{fq}(s)}{d_{fq}(s)} = \frac{1}{AsH_{sf} + k_e + k_s} = \frac{d_{sf}}{Asn_{sf} + (k_e + k_s)d_{sf}} \]  

(8)

where \( n_{fq} \) and \( d_{fq} \) are the numerator and denominator polynomials of the transfer function \( H_{fq} \), respectively; \( H_{sf} \) is the transfer function from the force \( f \) to the actuator piston displacement \( x \); \( n_{sf} \) and \( d_{sf} \) are the numerator and denominator polynomials of the transfer function \( H_{sf} \), respectively; and \( k_s = V/4\beta A \). If the test structure is a single-degree-of-freedom system with a mass \( m \), damping coefficient \( c \), and stiffness \( k \), the transfer function \( H_{sf} \) can be expressed as:

\[ H_{sf}(s) = \frac{n_{sf}(s)}{d_{sf}(s)} = \frac{1}{ms^2 + cs + k} \]  

(9)

Finally, dynamic properties of interest such as force and velocity are measured using transducers and processed through signal conditioners. The transducers and signal conditioners themselves are dynamic processes, and their transfer functions are expressed as:

\[ S_f(s) = \frac{n_f(s)}{d_f(s)}, \quad S_v(s) = \frac{n_v(s)}{d_v(s)}, \quad \text{and} \quad S_x(s) = \frac{n_x(s)}{d_x(s)} \]  

(10)

where \( n_f \) and \( d_f \) are the numerator and denominator polynomials of the transfer function \( S_f \) for the load cell; \( n_v \) and \( d_v \) are the numerator and denominator polynomials of the transfer function \( S_v \) for the velocity sensor; and \( n_x \) and \( d_x \) are the numerator and denominator polynomials of the transfer function \( S_x \) for the displacement sensor.
Open-Loop Transfer Functions for Input Valve Command

The overall open-loop transfer functions from the valve command $u$ to the actuator force $f$, the actuator velocity $v$, and the actuator displacement $x$, are obtained from products of the above transfer functions as:

$$H_{fu}(s) = S_{f}(s) H_{fv}(s) H_{qd}(s) H_{d}(s) H_{iu}(s)$$

$$= \frac{n_f n_{qd} n_{di} n_{iu}}{d_f d_{qd} d_{di} d_{iu} Asn_{sf} + (k_e + k_i s)d_{sf}}$$  \hspace{1cm} (11)

$$H_{uu}(s) = S_{v}(s) sH_{xf}(s) H_{jq}(s) H_{qd}(s) H_{d}(s) H_{iu}(s)$$

$$= \frac{n_v n_{qd} n_{di} n_{iu}}{d_v d_{qd} d_{di} d_{iu} Asn_{sf} + (k_e + k_i s)d_{sf}}$$  \hspace{1cm} (12)

$$H_{ux}(s) = S_{x}(s) H_{xf}(s) H_{jq}(s) H_{qd}(s) H_{d}(s) H_{iu}(s)$$

$$= \frac{n_x n_{qd} n_{di} n_{iu}}{d_x d_{qd} d_{di} d_{iu} Asn_{sf} + (k_e + k_i s)d_{sf}}$$  \hspace{1cm} (13)

Equation (11) reveals two important dynamic characteristics of the actuator force. First, the denominator polynomial of the test structure $d_{sf}$ appears in the numerator of the force transfer function. This means that poles of the structural system (i.e., roots of $d_{sf}$) are the zeros of the force transfer function, indicating inability to apply force at the natural frequencies of the structure. This phenomenon is caused by the effect of the actuator velocity on oil flow equation (see Equation (7)) and often referred to as control-structure interaction due to natural velocity feedback (Dyke et al. 1995). The natural velocity feedback can be verified from a block diagram of the open-loop actuator dynamics shown in Figure 1.

Secondly, the force transfer function has complex conjugate poles from the roots of $Asn_{sf} + (k_e + k_i s)d_{sf}$. This vibration mode is due to the compressibility of the hydraulic fluid and is called oil-column resonance. The oil-column resonances of hydraulic actuators are generally high frequency (around 100 Hz) and tend to cause vibration and stability issues.
EXPERIMENTAL SETUP AND SYSTEM IDENTIFICATION

An experimental study of effective force testing is conducted in the Smart Structures and Hybrid Testing Laboratory at Johns Hopkins University. Following the description of the experimental setup and system components for EFT at JHU, transfer functions and system parameters that are experimentally obtained through system identification tests are presented in this section.

Mass-Spring-Actuator System

The experimental setup consists of a hydraulic actuator and a mass-spring model shown in Figure 2. The actuator is a fatigue rated actuator manufactured by Shore Western, Inc. (Model number: 911D) and has a total stroke of 152 mm and a maximum dynamic loading capability of 24.5 kN. An MTS 252 series servo valve is used to drive the 911D actuator. The 911D actuator has an embedded DC-operated linear variable differential transducer (LVDT) for the measurement of actuator piston displacement. For the measurement of actuator force, acceleration, and velocity, a 22.2 kN load cell from Interface, Inc., 4g general purpose accelerometer from Omega, Inc., and a 100mm-stroke electro-magnetic velocity transducer from Trans-Tek, Inc., are employed in the test system.

The mass is a 52.7 kg steel block that is rigidly connected to the actuator. Two linear bearings and guides are placed under the mass to support gravity of the mass as well as to provide smooth motion of the mass with low friction. The spring is made of steel and has a total stroke of 25.4 mm. In this test setup, the spring is not attached to the mass due to the connection difficulty under tension force. Therefore, zero force point is defined in the middle stroke of the spring and difference from the middle reference point is regarded as the force measurement to have reaction force in both positive and negative directions. Because the spring here is employed as a linear elastic structural member, this approach does not cause loss of generality.

A National Instruments PXI Express system is employed for an integrated control and data acquisition process. An embedded real-time controller, PXI-8031, allows analog-to-digital and digital-to-analog signal conversions at a sampling rate of 4 kHz. LabVIEW Real-Time is used as a software platform for the implementation of controller designs in this study. For more details of hardware and software systems at the Smart Structures and Hybrid Testing Laboratory at JHU, refer to Nakata (2011).

System Identification of Structure and Control Parameters

Structural and actuator parameters for the test setup are experimentally identified using system identification techniques. For this preliminary system identification test, the actuator servo loop is closed
with the displacement measurement (i.e., displacement feedback) and band-limited white noise with a frequency range from 0.1 to 125 Hz is used as an excitation to the actuator. Figure 3 shows transfer functions from the servo valve command to the actuator force, from the servo valve command to the actuator velocity, and from actuator force to the actuator displacement. These transfer functions are obtained using spectral methods. It should be noted that these transfer functions are open-loop transfer functions that are directly linked to the dynamics of the actuator and the structure, and thus influence of the displacement controller is not present.

Figure 3 (a) and (c) show that the natural frequency of the mass-spring system is around 10 Hz and it appears in the transfer function from the valve command to the actuator force as a zero. This observation is consistent with control-structure interaction in Equation (11). It can be also seen from Figure 3 (a) and (b) that the oil-column resonance of the hydraulic actuator appears around 90 Hz. Analytical models are developed using the least square curve fitting technique. Complete analytical transfer functions are listed in Table 1. These analytical models include transducer dynamics and have the same order in Equations (11-13). It can be seen that each analytical transfer function accurately captures the dynamic characteristics of experimental data both in magnitude and phase.

Structural and actuator parameters are identified from coefficients of the analytical transfer functions in Table 1 and Equations (2-13). These parameters as well as dynamic models of the transducers and signal conditioners are shown in Table 2. As shown in the Table, the transducers have a first order or a series of first order dynamics. Although the transducer dynamics do not influence the magnitude of the outputs in the frequency range in this study (50Hz), they introduce a phase lag in the order of 2 milli seconds. This phase lag is slightly small compared to the period of vibration considered in EFT test, but it may have an impact on the stability if it is used for feedback control. Therefore, the dynamics of the transducer and signal conditioners are explicitly included in the following studies. It should be noted that the parameters listed in Table 2 provide the complete dynamics of the linearized mass-spring-actuator system at JHU. The parameters will be used for the development of controller designs for the EFT method in the next section.
CONTROLLER DESIGN FOR EFFECTIVE FORCE TESTING

As previously stated, controller designs for EFT have to provide compensation for the control-structure interaction as well as stability for the oil-column resonance. This section explores performance and robustness of previously studied approaches in the EFT method including velocity feedback (Zhao et al., 2005) as well as a loop shaping controller for EFT at JHU. Note that this study investigates only direct force feedback controller designs and indirect controller designs that adopt displacement feedback for the actuator servo inner-loop (Sivaselvan et al. 2008) are not considered here.

Figure 4 shows a block diagram for the generalized actuator servo inner-loop with velocity and force feedback as well as input noise, force measurement noise and velocity measurement noise. Feed-forward approaches that are often applied to the reference command are not included here; such approaches are referred to as command shaping (Daley et al. 2004) and they do not alter performance and stability of the closed-loop control system.

The valve command \( u \) consists of velocity and force feedback terms as:

\[
U = U_f + U_v
\]

where \( U_f \) and \( U_v \) are the Laplace transforms of command signals from velocity term \( u_v \) and force term \( u_f \), respectively. Each component of the valve commands can be expressed as:

\[
U_f = C_f E_f = C_f \left( F_r - F_m \right)
\]

\[
U_v = C_v V_m
\]

where \( C_f \) and \( C_v \) are the transfer functions of force feedback controller and velocity feedback controller, respectively; \( E_f, F_r, F_m, \) and \( V_m \) are the Laplace transforms of force error \( e_f \), reference force \( f_r \), measured force \( f_m \), and measured velocity \( v_m \), respectively.

Measured force and velocity of the actuator are subjected to the influence of input noise \( w_u \) and include force measurement noise \( w_f \) and velocity measurement noise \( w_v \) as:

\[
F_m = H_{fu} \left( U + W_u \right) + W_f
\]
where $W_u$, $W_f$ and $W_v$ are the Laplace transforms of input noise $w_u$, force measurement noise $w_f$, and velocity measurement noise $w_v$, respectively. Using Equations (13-17), transfer functions from the reference force $f_r$, input noise $w_u$, force measurement noise $w_f$, velocity measurement noise $w_v$, to measured force $f_m$ and valve command $u$ can be obtained and summarized in a matrix form as:

$$
\begin{align*}
F_m & = H_v \left( U + W_u \right) + W_v \\
\begin{bmatrix}
F_m \\
U \\
W_u \\
W_f \\
W_v
\end{bmatrix} & = \frac{1}{1+C_f H_f u - C_v H_v u} \begin{bmatrix}
C_f H_f u & H_f u & 1 & C_v H_f u \\
C_f & -C_f H_f u & -C_f & C_v
\end{bmatrix} \begin{bmatrix}
F_r \\
W_u \\
W_f \\
W_v
\end{bmatrix}
\end{align*}
$$

Performance and robustness of the closed-loop control system can be evaluated from the above transfer functions. Controller designs are investigated based on the criteria for compensation of the control-structure interaction and stabilization of the oil-column resonance.

**Consideration of Velocity Feedback for Compensation of Control-Structure Interaction**

Velocity feedback has been applied to compensate control-structure interaction (Zhao *et al.* 2005). Their approach aims to cancel out the natural velocity feedback by feeding velocity measurement into the actuator servo loop. It is equivalent to have a pole-zero cancellation for the roots of $d_{sf}$ with a proper design of velocity feedback controller, $C_v$. Substituting Equations (11-12) into Equation (19), the force transfer function yields:

$$
H_f = \frac{F_m}{F_r} = \frac{C_f n_f n_{qu} n_{sf}}{d_{sf} + Asn_{sf} d_f d_{qu} - sC_v n_{qu} n_{sy} d_f}
$$

where $n_{qu} = n_{qd} n_{di} n_{iu}$ and $d_{qu} = d_{qd} d_{di} d_{iu}$ for the simplification of the equation. To have a pole-zero cancellation at the roots of $d_{sf}$, following conditions have to be met:

$$
Asn_{sf} d_f d_{qu} - sC_v n_{qu} n_{sy} d_f = \alpha d_{sf}
$$

where $\alpha$ can be an arbitrary number. Because the above equation is not generally satisfied if $\alpha$ is not zero, the velocity feedback controller has to satisfy the following.
Equation (22) suggests that the velocity feedback controller has to be the inverse of the dynamics of the velocity transducer and the dynamics from valve command to the oil flow. For the EFT setup,

\[
C_v = A \frac{d^*d_{qu}}{n_vn_{qu}} = A \left( S_vH_{qu} \right)^{-1}
\]  

(22)

Because the inverse of the above model is not strictly proper, it is not possible to cancel out the control-structure interaction from the velocity feedback. While phase-lead compensators can be applied (Zhao et al. 2005), the controllable frequency range in EFT is significantly lowered due to phase characteristics of the phase-lead compensators. Therefore, the velocity feedback technique is not used in this study. The compensation of the control-structure interaction is addressed in the force feedback controller.

**Loop Shaping Force Feedback Controller**

Loop shaping is a frequency domain technique for design of feedback control systems (Astrom and Murray 2006). The method is based on the loop transfer function, \( C_f H_{fu} \), and the Nyquist stability criterion. Controllers are designed by changing the gain and adding poles and zeros until the loop transfer function has the desired shape. Good performance can be achieved with a high gain and a steep slope of the gain curve for the loop transfer function at low frequencies (i.e., below crossover frequency). Good stability can be obtained with a low gain at the frequencies higher than the crossover frequency.

For EFT in this study, a loop shaping controller is selected to compensate the control-structure interaction as:

\[
C^{LS}_f = \frac{\gamma}{d_{sf}}
\]  

(24)

where \( \gamma \) is the gain for the loop shaping controller. The form in Eq (24) is the lowest order controller that compensates for the control-structure interaction. The gain is set to have a crossover frequency at 12 Hz. The mathematical form of the loop shaping controller is expressed as:

\[
C^{LS}_f = \frac{37}{s^2 + 17.14s + 4002}
\]  

(25)
Figure 5 shows the gain curves of the loop shaping force feedback controller. For comparison, a proportional force feedback controller that has been used in the previous studies (Dimig et al. 1999 and Zhao et al. 2005) is included in the figure. Gain for the proportional controller is chosen to be close to the stability limit, $C_f^{PR} = 0.0006$, so that the best performance with the proportional controller is considered herein.

It can be seen from the gain curve of the controller in Figure 5 (a) that the loop shaping controller has poles at the natural frequency of the mass-spring model to compensate the control-structure interaction. The magnitude continues to decrease as the frequency increases whereas the proportional gain is constant regardless of the frequency. The gain curves of the loop transfer function in Figure 5 (b) shows a desirable shape with the loop shaping controller (i.e., high gain at lower frequency and low gain at higher frequency). It should be noted that the oil-column resonance is successfully suppressed with the loop shaping controller. On the other hand, the loop transfer function from the proportional controller shows undesirable features; lower gain at lower frequency and high gain at high frequency. This result indicates that even with the largest gain, the proportional force feedback controller neither compensates for the control-structure interaction nor suppresses the oil-column resonance. It is also not surprising to see from the Nyquist plot in Figure 5 (c) that the loop shaping controller provides a larger stability margin than the proportional controller; while the proportional controller is at the edge of the stability limit, the loop shaping controller maintains the gain margin of 12.0 dB and the phase margin of 64 degrees.

Closed-loop sensitivity functions of the loop shaping and proportional controllers are shown in Figure 6. Complementary sensitivity function, $C_f /\left(1 + C_f H_{fu}\right)$, in Figure 6 (a) indicates good tracking capability of dynamic force from the loop shaping controller up to 25 Hz while the proportional controller shows poor performance. Noise sensitivity functions, $C_f /\left(1 + C_f H_{fu}\right)$, in Figure 6 (b) shows the influence of the reference force to the valve command. As expected, the loop shaping controller has a higher amplification of the valve command at the natural frequency of the mass-spring system. Load sensitivity functions, $H_{fu} /\left(1 + C_f H_{fu}\right)$, and sensitivity function, $1 /\left(1 + C_f H_{fu}\right)$, in Figure 6 (c) and (d) show the impact of input noise and force measurement noise on the measured force. As seen in these plots, the loop shaping controller provides excellent robustness over the entire frequency range including the oil-column resonance at 90 Hz. Unlike the loop shaping controller, the proportional controller is very sensitive to the input and measured noises. It is clear from the above results and observation, the loop
shaping controller provides better performance as well as robustness when compared to the proportional controller for EFT.

EXPERIMENTAL VERIFICATION OF LOOP SHAPING FOR EFT

The loop shaping force feedback controller designed in the previous section is experimentally investigated using the EFT setup. The loop shaping controller is successfully implemented into a control system developed by Nakata (2011) that allows for an implementation of sophisticated actuator inner-loop servo control algorithms beyond the conventional PID controllers. EFT in this study considered three types of loadings: step, random and earthquake loadings. Experimental results in the test series are presented in this section.

Step Response and Comparison with Displacement Control

Firstly, performance of the force control with the loop shaping controller for a step input is investigated. Figure 7 shows force and displacement time histories for the step force input of 222 N. For comparison, force and displacement time histories under displacement control for a step displacement input with a similar amplitude are plotted in the figure.

Rise time, settling time, and overshoot in the force control are 0.0015 sec, 0.045 sec and 80%, respectively. While some disturbances follow after the target force is reached, they can be considered due to unsettlement of actuator that can be observed in displacement time history. It is interesting to see that the actuator displacement is oscillating to maintain the constant force for the step force input. On the other hand, the displacement control shows a large oscillation of force with an approximately 600% force overshoot while excellent performance in displacement is achieved. This oscillation is due to the oil-column resonance and takes longer than 0.09 sec for the force to settle. The results here demonstrate that force control with the loop shaping force feedback controller has better dynamic performance than the displacement control in tracking force with a smaller overshoot. It can be also confirmed that the loop shaping controller successfully suppresses the oil-column resonance that is present in displacement control.

Random Excitation and Frequency Response Curves

Next, performance of force control with the loop shaping controller for a random excitation is evaluated. Band-limited white noise with a frequency range from 0.1 to 50 Hz is used as a reference force input to the actuator. Figure 8(a) shows the force time history during the random excitation. It can be seen that reasonable force tracking is achieved with force control. Figure 8 (b-d) show magnitude, phase, and
coherence characteristics of the frequency response curve from the reference force to the measured force. Spectral methods are used to develop these curves. The magnitude plot shows excellent force control performance over a wide frequency range, particularly up to 25 Hz. While a small dip can be found at the natural frequency of the mass-spring system in the magnitude plot, the control-structure interaction is well compensated by the application of the loop shaping. This imperfection in the compensation is due to an estimation error of the natural frequency of the system. From the phase plot, the time lag in force tracking and the phase crossover frequency are estimated 0.014 sec and 28 Hz, respectively. The gain margin is approximately 9.54 dB, which is close to the design value of 12.0 dB. The coherence plot exhibits high correlation between the reference and the measurement force up to 20 Hz. In summary, these results prove that the force control with the loop shaping controller provides excellent performance as well as robustness.

**Earthquake Loading Tests**

Finally, performance of the force control with the loop shaping controller is evaluated for earthquake loadings. Figure 9 shows time histories and Fourier spectrum of the reference and measured forces for the 1995 Kobe earthquake KJMA record. The maximum input force is scaled to 1500 N. As shown in the time history, the measured force exhibits excellent force tracking to the reference force for the entire duration of input. While a high level of noise in force measurement is inevitable, the force control is not influenced by the noise, demonstrating the robustness of the loop shaping. The Fourier spectrum also shows very good agreement of the measurement force spectrum with the reference up to 20Hz. This result is consistent with the characteristics found in the frequency response curve discussed in the previous section. Discrepancy in the spectrum over 20 Hz is due to the noise in the force measurement.

To demonstrate the versatility of the force control, further studies are conducted using a series of different earthquake loadings at different force levels. Figure 10 shows time histories and Fourier power spectrum for the 1989 Loma Prieta earthquake recorded at the Oakland station as a sample. In the same manner, excellent performance of force tracking in the time domain and good agreement in the Fourier spectrum are obtained in force control with the loop shaping controller. The test results here demonstrate superior performance and robustness of force control with the loop shaping controller for effective force testing.
CONCLUSIONS

This paper presented an investigation of effective force testing using a loop shaping controller design. The controller was designed in the frequency domain based on the thorough theoretical and experimental investigation of the actuator dynamics. Then, the controller was successfully implemented in the EFT setup at JHU and a series of force control tests were performed. Not only did the controller developed here compensate for the control-structure interaction but also suppressed the oil-column resonance which makes direct force feedback control feasible. The test results showed that force control with the loop shaping controller had excellent force tracking as well as provided robustness for a high level of force measurement noise.

The robust force feedback control capability demonstrated through the EFT method in this paper has a lot of potential to enable new experimental methods that are not possible without force control. They include, but are not limited to, effective force testing with nonlinear test structures (Nakata and Krug 2012), force-based real-time hybrid simulation (Nakata and Stehman 2011), substructure effective force testing, multi-degrees-of freedom effective force testing, force-based control of shake tables, etc. The author believes that high-fidelity robust force control will bring a new dimension to the experimental earthquake engineering that has utilized the conventional displacement control in almost all current tests.

ACKNOWLEDGEMENT

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REFERENCES


Table 1. Analytical transfer functions for the EFT setup at JHU.

\[
H_{vf}(s) = \frac{7.881 \times 10^{13} s^2 + 3.154 \times 10^{17}}{s^6 + 1591 s^5 + 1.201 \times 10^6 s^4 + 6.687 \times 10^5 s^3 + 2.491 \times 10^{11} s^2 + 3.94 \times 10^{13} s + 4.331 \times 10^{17}}
\]

\[
H_{nv}(s) = \frac{1.109 \times 10^{15} s}{s^7 + 2441 s^6 + 2.593 \times 10^6 s^5 + 1.736 \times 10^7 s^4 + 8.45 \times 10^9 s^3 + 2.655 \times 10^{14} s^2 + 3.699 \times 10^{16} s + 4.066 \times 10^{19}}
\]

\[
H_{xf}(s) = \frac{6.752 s + 3039}{s^4 + 817.1 s^3 + 1.777 \times 10^6 s^2 + 5.944 \times 10^9 s + 6.404 \times 10^8}
\]
<table>
<thead>
<tr>
<th>Properties</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>m Mass of the structure</td>
<td>52.7 kg</td>
</tr>
<tr>
<td>c Damping of the system</td>
<td>902.5 N sec/m</td>
</tr>
<tr>
<td>k Stiffness of the spring</td>
<td>$2.11 \times 10^5$ N/m</td>
</tr>
<tr>
<td>$i_{max}$ Maximum current to the servo valve</td>
<td>20 mA</td>
</tr>
<tr>
<td>$u_{max}$ Maximum voltage from the controller</td>
<td>10 V</td>
</tr>
<tr>
<td>$d_{max}$ Maximum opening of the spool</td>
<td>100 %</td>
</tr>
<tr>
<td>A Piston area</td>
<td>$1303 \text{ mm}^2$</td>
</tr>
<tr>
<td>$\beta$ Bulk modulus of the fluid</td>
<td>$1.5 \times 10^9$ N/m$^2$</td>
</tr>
<tr>
<td>$V$ Volume of the actuator chambers</td>
<td>$6.17 \times 10^5 \text{ mm}^3$</td>
</tr>
<tr>
<td>$k_q$ Flow gain of the servo valve</td>
<td>$5.03 \times 10^{-6}$ m$^3$/sec/%</td>
</tr>
<tr>
<td>$k_e$ Flow-force coefficient</td>
<td>$3.24 \text{ mm}^3$/sec/N</td>
</tr>
<tr>
<td>$k_t$ $= V/4\beta A$</td>
<td>$7.89 \times 10^{-2}$ mm$^3$/N</td>
</tr>
<tr>
<td>$\tau_c$ Time delay of the converter</td>
<td>0.0018 sec</td>
</tr>
<tr>
<td>$\tau_v$ Time delay of the spool</td>
<td>0.0020 sec</td>
</tr>
<tr>
<td>$S_f$ Load cell and signal conditioner model</td>
<td>$1/(1+0.0022s)$</td>
</tr>
<tr>
<td>$S_v$ Velocity transducer and signal conditioner model</td>
<td>$1/(1+0.0015s)^2$</td>
</tr>
<tr>
<td>$S_x$ LVDT and signal conditioner model</td>
<td>$1/(1+0.0025s)^2$</td>
</tr>
</tbody>
</table>
Figure 1. A block diagram of the open-loop actuator dynamics.
Figure 2. A mass-spring-actuator system for EFT at JHU.
Figure 3. Open-loop transfer functions: (a) and (d) magnitude and phase from the valve command to the actuator force, respectively; (b) and (e) magnitude and phase from the valve command to the actuator velocity, respectively; (c) and (f) magnitude and phase from the actuator force to the actuator displacement, respectively.
Figure 4. A block diagram for closed-loop control system incorporating force and velocity feedback.
Figure 5. Gain curves of the loop shaping force feedback controller: (a) controller gain; (b) loop transfer function gain; and (c) Nyquist plot.
Figure 6. Closed-loop transfer functions of the loop shaping force feedback controller: (a) complementary sensitivity function; (b) noise sensitivity function; (c) load sensitivity function; and (d) sensitivity function.
Figure 7. Comparison between force control for a step force input and displacement control for a step displacement input.
Figure 8. Response for a random excitation: (a) force time histories; (b), (c), and (d) magnitude, phase, and coherence of the frequency response curve from the reference force to the measured force, respectively.
Figure 9. Performance of force control with a loop shaping controller for Kobe earthquake loading: (a) force time histories and (b) Fourier spectrum.
Figure 10. Performance of force control with a loop shaping controller for Loma Prieta earthquake loading: (a) force time histories and (b) Fourier spectrum.