Sliding/Rolling Constitutive Theory and Its Generalization

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• **Promotion of the use of Advanced Methods in Practice**

• **Sliding/Rolling Constitutive Theory**
Promotion of the use of Advanced Methods in Practice

• Use in situ data to estimate input parameters as much as possible
• Develop computationally-efficient methods for handling three-dimensional problems
• Develop physical meaning for parameters
Use in situ data to estimate input parameters as much as possible

- Back-calculate parameters from in situ tests
- Establish correlations between in situ data and needed parameters
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Back-calculation of parameters for the anisotropic bounding surface clay model from pressuremeter data


Figure 4. Axi-symmetric finite element mesh for analysis of pressuremeter problem.
Back-calculation of parameters for the anisotropic bounding surface clay model from pile-load test

0.15G_{\text{max}}
Use in situ data to estimate input parameters as much as possible

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Fig. 3.2 The relationship between the slope of the isotropic consolidation line, $\lambda$, and the magnitude of dielectric dispersion, $\Delta \varepsilon_0$. 
Promotion of the use of Advanced Methods in Practice

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Earthquake Response of a 2-Storey Building

Theory Versus Experiment
Complex Soil-Structure System

Domains to be analyzed
Maximum Horizontal Displacement Contour @ 10% of Pile-Head
Maximum Displacement
Sliding/Rolling Constitutive Theory and Its Generalization
VELACS Predictions: Anandarajah (1992)

Data from Arulmoli et al. (1992)

Fig. 1. Comparison of the Model Behavior with Experimental Data of Cyclic Triaxial Behavior of Nevada Sand of Dr=40 percent at p_0=40 kPa.
Fig. 6. Long Term Pore Pressure Histories.
Fig. 7. Ground Settlement History.
Sliding/Rolling Constitutive Theory and Its Generalization
Link

2D Assembly

Mechanisms Based on Interparticle Sliding

\[ \frac{F_2}{F_1} = \tan(\beta - \alpha) \quad \beta_{\text{max}} = \tan^{-1}(S_v) + \phi_\mu \]

Sliding:
\[ \alpha \equiv \phi_\mu \]
\[ u_1^p = 4r \theta \sin \beta_{\text{max}}^{vt} \]
\[ u_2^p = -4r \theta \cos \beta_{\text{max}}^{vt} \]
\[ \Delta \varepsilon_1^p = \frac{(2N \theta r) \cos(\beta_{\text{max}}^1 - \phi_{\mu})}{(\beta_{\text{max}}^1 - \beta_{\text{min}}^1) \sigma_1} \langle L_1 \rangle \sin \beta_{\text{max}}^1 \]

\[ \Delta \varepsilon_3^p = \frac{(2N \theta r) \cos(\beta_{\text{max}}^1 - \phi_{\mu})}{(\beta_{\text{max}}^1 - \beta_{\text{min}}^1) \sigma_1} \langle L_1 \rangle (-\cos \beta_{\text{max}}^1 / 2) \]

\[ \beta_{\text{max}} = \tan^{-1}(S_v) + \phi_{\mu} \]

\[ \sigma_2 \sigma_1 = \tan(\beta_{\text{max}} - \phi_{\mu}) \]

\[ \beta_{\text{min}}^1 = 50^0 \quad \beta_{\text{min}}^1 = 10^0 \]
1, 2: Sliding
3, 4: Rolling
1, 3: Failure in Dir. 1
2, 4: Failure in Dir. 3
2D TO TRIAXIAL:
Discrete Element Method
Examples

Some Aspects of Anisotropy
Anandarajah and Kuganenthira, 1995, *Geotechnique*

Fig. 5. Assembly of 625 rods at different stages of loading: (a) initial ($\eta = 0$); (b) point A ($\eta = 0$); (c) point F ($\eta = 0.5$); (d) point J ($\eta = 0.5$)
Two-Dimensional Analysis

2914 Element Assembly
Consider fcc:

Brauns (1968)
Hendron (1963)
\[ S_v = \frac{\sigma_3}{\sigma_1} = \tan(\beta - \phi_\mu) \]
\[ S_v = \frac{\sigma_3}{\sigma_1} = \frac{1}{2} \tan(\beta - \phi_\mu) \]
\[ r_{ij} = \sigma_{ij} + \alpha n_{ij} \]
Conventional

\[ r_{ij} = S_{ij} + d\delta_{ij} \]

Sliding/Rolling

\[ r_{ij} = \sigma_{ij} + \alpha n_{ij} \]
Generalization of Sliding/Rolling Theory to 3D Loading Using the Bounding Surface Concept

**Loading**

\[ \beta_{\text{max}} = 46.5^0 \]

\[ \beta_{\text{max}} = 65^0 \]

\[ \beta_{\text{max}} = \tan^{-1}(s) + \phi_\mu \]

**Reverse Loading**

\[ \beta_{\text{max}} = 83.4^0 \]

\[ \beta_{\text{max}} = 65^0 \]

\[ \beta_{\text{max}} = 46.5^0 \]

\[ \beta_{\text{max}} = \tan^{-1}\left(\frac{1}{s}\right) + \phi_\mu \]

\[ K_p \propto [\beta_{\text{max}} - \beta_{\text{min}}] \]

\[ r_{ij} = \sigma_{ij} + \alpha n_{ij} \]

\[ \delta = \frac{n_{ij} s_{ij}}{\sqrt{2} \left| n \right| I} \]

\[ \beta_{\text{max}} = \text{fun}(\delta) \]
$$\dot{\alpha}_{ij}^A = \lambda^* \dot{\lambda} r_{ij}^{ad}$$

$$\lambda^* = C_1 \exp\left[-C_2 (1 + \rho) \delta_f^a \right]$$

$$\beta_{\text{min}} = \beta_{\text{fail}} + \left[ \beta_{\text{min}} - \beta_{\text{fail}} \right] \exp\left(C_{\beta} | \varepsilon_v^p | \right)$$
Triaxial Undrained Monotonic Loading

\[ \beta_{init} = 41.7^0 \]

\[ \beta_{init} = 42.8^0 \]
Triaxial Undrained Cyclic Simulation with $\beta_{\text{init}} = 41.7^0$
Modeling Cyclic Behavior

Experimental Data for Nevada Sand (VELAS Project, Arulmoli, et al., 1992)
Concluding Remarks

- Calibrate constitutive laws using in situ data as much as possible
- Develop simpler finite element models for three-dimensional problems
- Develop physical meaning for parameters