A SIMPLE ELASTO-PLASTIC MODEL
FOR SOILS AND SOFT ROCKS

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1. MODEL HISTORY

The model is the result of the evolution of successive models elaborated in the last thirty years, partly in collaboration with others:

a) sand and clay
Nova (1977) Archiwum Mechaniki Stosowanej
Nova (1988) Cleveland Conference

b) soft rocks
Nova (1986) Computers & Geotechnics

c) soft rocks with mechanical degradation

d) soft rocks with mechanical and chemical degradation

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2. MODEL STRUCTURE FOR UNCEMENTED SOILS

Simple elastic plastic strain-hardening model (as Cam Clay):

a) plastic potential $g$
b) yield function $f$
c) hardening rule

d) elastic law (for unloading-reloading)
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Plastic potential

\[ g = 9(\gamma - 3) \ln \frac{p'}{p_c} - \gamma J_{3\eta} + \frac{9}{4}(\gamma - 1)J_{2\eta} = 0 \]

\[ \gamma = \frac{9 - M^2}{2 \left( M^3 + 3 - M^2 \right)} \]

\[ \dot{\varepsilon}_{ij}^p = \Lambda \frac{\partial g}{\partial \sigma_{ij}}; \quad \eta_{ij} \equiv \frac{s_{ij}}{p'}; \quad J_{2\eta} \equiv \eta_{ij} \eta_{ij}; \quad J_{3\eta} \equiv \eta_{ij} \eta_{ik} \eta_{ki} \]

‘Triaxial’ (axisymmetric) plane

Deviatoric plane

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Loading (or yield) function

\[ f \equiv 3 \beta (\gamma - 3) \ln \frac{p'}{p_c} - \gamma J_{3\eta} + \frac{9}{4} (\gamma - 1) J_{2\eta} \leq 0 \]

If \( \beta = 3 \), normality holds true (plastic potential and loading function coincide). Soil deformability depends linearly on \( \beta \). Soil behaviour is considered to be elastic within the region delimited by \( f = 0 \) (yield locus).

'Biaxial' (axisymmetric) plane

'Deviatoric' plane

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\[ \dot{p}_c = \frac{p_c}{B_p} \left\{ \dot{\varepsilon}_{rs}^{p} \delta_{rs} + \xi (\dot{\varepsilon}_{rs}^{p} \dot{\varepsilon}_{rs}^{p})^{1/2} + \psi (\dot{\varepsilon}_{st}^{p} \dot{\varepsilon}_{tr}^{p})^{1/3} \right\} \]

Hardening rule:

\[ \dot{\varepsilon}_{hk} = \dot{\varepsilon}_{hk} - \frac{1}{3} \dot{\varepsilon}_{rs} \delta_{rs} \]

\(p_c\) is the isotropic preconsolidation pressure, controlling the size of the elastic domain

\(B_p\) is the plastic volumetric logarithmic compliance

\(\xi\) and \(\psi\) control dilatancy in triaxial compression and extension

Hypoelastic relationship:

\[ \dot{\varepsilon}_{ij}^e = B_e \frac{\dot{p}'}{p'} \delta_{ij} + L \dot{\eta}_{ij} \]

\(B_e\) is the elastic volumetric logarithmic compliance

\(L\) is a shear stiffness proportionality constant (\(G\) increasing with \(p'\))

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Summary of constitutive parameters:

- $\gamma$ linked to $M$ (characteristic (or critical) state)
- $\beta$ measure of non-normality linked to soil deformability
- $\xi$ and $\psi$ control dilatancy at failure in compression and extension
- $\gamma$, $\xi$ and $\psi$ control friction angles in compression and extension
- $B_p$ isotropic plastic (logarithmic) compressibility
- $B_e$ isotropic elastic (logarithmic) compressibility
- $L$ shear deformability (proportional to $1/G$ via $p'$)
- $p_{c0}$ size of elastic nucleus

All parameters have a clear physical meaning. They can be determined by performing an isotropic compression and a compression triaxial test (also an extension test is needed if friction angle in extension is assumed to be different from that in compression). They are also linked to traditional soil constants.

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3. MODEL PREDICTIONS FOR SAND AND REMOULDED CLAY

Cleveland Symposium (1987): Hollow cylinder, Hostun sand

\[
\sin 2\beta^* = \frac{2\tau_{rz}}{\sigma_z - \sigma_r}
\]

Test at constant \( \beta^* \)

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Class A predictions - hollow cylinder tests

\[ \beta^* = \text{const} \]
\[ \sigma'_r = \text{const} \]

Dotted lines are calculated results. Parameters predetermined in conventional triaxial tests.

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Cleveland Symposium (1987): True triaxial (or ‘cube’) tests, Hostun sand

\[ b \equiv \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \]
Class A predictions - cube tests

\[ p' = \text{const} \]
\[ b = \text{const} \]

Dotted lines are calculated results. Parameters predetermined in conventional triaxial tests.
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Pontida Clay (silty clay); normally consolidated

Drained

Undrained

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Undrained behaviour of Banding sand

Note: non-normality is a necessary condition to model static liquefaction

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4. MODEL FOR BONDED SOILS AND SOFT ROCKS

\[
\begin{align*}
    p_m &= \alpha p_t \\
    \dot{p_t} &= -\rho p_t |\dot{\varepsilon}_v^p| \\
    p^* &= p' + p_t \\
    \eta_{ij}^* &= \frac{s_{ij}}{p^*}
\end{align*}
\]
Natural calcarenite: isotropic compression
Natural calcarenite: constant p’ tests

Experimental data

Calculated results

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Stevn’s Klint chalk: undrained test after $K_0$ consolidation

Stress-strain

Stress path

Calc.

Exp.

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5. MODEL FOR BONDED SOILS WITH CHEMICAL DEGRADATION

\[ p_t = p_t(\varepsilon_{ij}^p, X_d) \]

Stress path in oedometric test with loading-unloading-reloading, subsequent chemical attack and final unloading
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Axial strains in oedometric test: constant loading, acid seeping through specimen

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Code GeHoMadrid (Fernandez Merodo et al.)

Strains are larger below foundation

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Problems of practical interest can be solved by means of numerical methods with advanced constitutive models. Academicians must convince practitioners that the solutions obtained in this way are better than those one can obtain with ‘simple’ constitutive models.

Select a number of problems, compare the solution one can obtain with an advanced constitutive model and that one can obtain with e.g. Drucker Prager and show which is the qualitative enhancement by using the former.

WORKSHOPS ON B.V.P.? Explore the role of:
- dilatancy (non normality)
- occurrence of plastic strains prior to failure (‘anisotropy’)
- cementation and degradation
- softening, shear banding and other types of instabilities (e.g. static liquefaction)

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Academicians must also show that the number of parameters and their determination is not an obstacle for practical applications.

Academicians should guarantee that their numerical solutions are objective. More research is needed for the numerical treatment of softening, shear banding and other types of instability. Creep effects must be explicitly taken into account.

Progress in science (knowledge) implies that more and more concepts should be studied and understood by ‘students’. Therefore academicians must:

- give a sound theoretical basis even at the undergraduate level
- disseminate *extra moenia* recent ideas
- try to confront themselves with practical problems

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Students and practitioners MUST STUDY

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PANEL DISCUSSION

The static problem is characterised by:

- 3 different materials (loose sand, soft clay, stiff clay)
- Clay consolidation (negative friction on piles)
- Three-dimensional loading conditions (piles)
- Rotation of principal stresses
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• 3 different materials (loose sand, soft clay, stiff clay)

It is convenient to use the same model for all materials (with different parameters). Non-normality must be accounted for to describe loose sand behaviour.

• Clay consolidation (negative friction on piles)

Plasticity prior to failure must be accounted for (oedometric loading). Elastic perfectly plastic models not suitable. It is of vital importance to model correctly the soil-pile interface behaviour. It is necessary to use a finite element code allowing for coupled analyses.
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• Three-dimensional loading conditions (piles)

Drucker-Prager (and any other model with circular cross section in the deviatoric plane) is not suitable for this problem.

• Rotation of principal stresses

This item could be properly addressed only by using a constitutive model with coupled isotropic-kinematic hardening (yield function rotating with plastic strains, see e.g. di Prisco et al. (1995)). Isotropic elasto-plastic models with strain-hardening provide at least a response that depends on the incremental stress direction.

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Results of di Prisco’s model (di Prisco et al. (1993)) in a test in which principal stresses rotate while all stress invariants remain constant.

Experimental data (after Ishihara et al. (19))

Calculated results (after di Prisco et al. (1995))

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