A key issue in Small Strain Modeling: capturing the dependence of soil response on the direction of loading

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• which non-linearity are we talking about
• experimental evidence
• practical relevance

a few basic ideas which are often overlooked
the (overemphasized!) $G$ vs. $\gamma$ curves

Elastic threshold
$(G/G_0 > 0.99)$

→ increases with mean stress and PI (% of fines)
→ smallest values: $5 \times 10^{-6}$
  (e.g. sand at 20 kPa)
→ highest values: $1 \times 10^{-4}$
  (e.g. kaolinite at 200 kPa)

(after Lancellotta & Calavera, 1999)
stiffness decay with strain

variation of shear modulus with strain level and relevance to in-situ tests (after Mayne & Schneider, 2001)
$G-\gamma$ constitutive framework: non-linear elasticity

- can be traced back to the pioneering work of Kondner & Zelasko (1963) and Duncan & Chang (1970)

- in essence: an isotropic elastic constitutive equation, in which the elastic stiffness parameters are assumed to depend on the strain level

- the stiffness tensor is typically given by general expressions of the form:

\[
D(\sigma, \varepsilon) = K_I(p, \varepsilon_s)1 \otimes 1 + 2G_I(p, \varepsilon_s) \left[ I - \frac{1}{3} 1 \otimes 1 \right]
\]

- the dependence of the tangent bulk and shear moduli on the strain invariants is obtained by curve-fitting the observed stress-strain response in standard loading paths, e.g. drained (or undrained) TXC and IC

- empirical expressions are typically suggested in terms of secant moduli, which are easier to determine directly

\[
K_s(p, \varepsilon_s) = K_{s0}(p)F_K(\varepsilon_s) \quad G_s(p, \varepsilon_s) = G_{s0}(p)F_G(\varepsilon_s)
\]
is the variable-moduli approach OK?

this approach has largely inspired much of the experimental research on pre-failure deformation of soils over the last decade (Shibuya et al. 1994, Jamiolkowski et al. 1999)

major improvements of laboratory measurement techniques (Jardine et al. 1984, Goto et al. 1991) have allowed to obtain very high quality stress-strain data, which in turn have motivated new, more refined expressions for the decay function Fc

these models have become popular in the geotechnical community. The reason for their appeal stems from their relative simplicity and ease of incorporation into standard FE codes

→ However…
The behavior of soils is incrementally non-linear (i.e., inelastic) whereas non-linear elasticity is incrementally linear.

$\sigma$ $\varepsilon$

$A$, $B$, $B'$, $B''$

$\Delta \varepsilon_f = 0$

$\epsilon_{a,1}$, $\epsilon_{a,B}$

$\rightarrow$ non-linear elasticity is “an easy answer to a difficult question” (Whittle, 1999)
problems with the variable-moduli approach (2)

typically, the functions $F_K$ and $F_G$ are derived from single, specific loading path (axisymmetric or isotropic compression), which, although easily accessible in the laboratory, might be quite far from those commonly experienced by the soil in practical applications → the use of them for all possible loading paths represents a potentially dangerous extrapolation

→ soil response depends (among other things) on the direction of loading

how can we visualize this?
incremental strain response envelopes (ISRE)

**SRE** (after Gudehus 1979): image in the strain rate space of the unit sphere in the stress rate space, under the map defined by the constitutive equation

![Diagram of ISRE](image)

in triaxial (axisymmetric) loading, independent stress and strain variables are just two → the ISRE can be represented in the Rendulic plane of strain increments

→ the size of each strain increment vector defining the RE can be directly interpreted as a **directional secant compliance** of the material, for the associated loading direction and stress increment magnitude
how to discriminate among different inelastic theories?

Different inelastic constitutive formulations are available

How can we assess which of them is “the best one”?

1st answer: understanding the very mathematical structure

(what a given model can, and what it cannot, do; why is it so)

2nd answer: check which is the effect of constitutive hypotheses at the level of a given Boundary Value Problem

(qualitative and quantitative standpoint)

3rd answer: check the model performance against experimentally observed behavior
investigating the incremental behavior of soils

the most natural approach: program of stress-probing experiments, in which incremental strains are measured by applying a series of "small" stress increments to a number of "identical" specimens, with a common initial state.


→ to control stress history
→ to minimize the differences from one specimen to another

Beaucaire Marl ID
CL silt (clay content: 30%)
w_L = 38%  PI = 17%  CaCO_3 = 34%  G_S = 2.75

reconstituted specimens from a single, large batch

ε_a = ε_v (ε_r) reliable down to 5 E-4
stress probes: experimental program

drained probes, from virgin stress state A ($p' = 150$ kPa, $q = 0$ kPa)

$\rightarrow$ drained probes, from virgin stress state B ($p' = 150$ kPa, $q = 60$ kPa)
experimental ISRE, anisotropic state B

SRE is not centered about the origin → incremental non-linearity
evolution of the REs with $R_\sigma$

- **stiffest response** for full stress path reversal with respect to the previous loading history

- **largest deformations** for TX compression at constant $p'$ (pointing towards failure locus in TX compression)

**consequence**

marked asymmetry of RE about the origin along both the consolidation direction and the direction of purely deviatoric loading

**strong argument for**

- incrementally non-linear, inelastic behavior already at relatively small strain levels

$\rightarrow$ contour spacing indicative of rate of change of soil secant stiffness
experimental results vs. models prediction

results obtained from the isotropic state have been used for calibrating different models

assessment of models performance has been carried out w.r.t. the data from the tests from the anisotropic state

Masin, Tamagnini, Costanzo, Viggiani (2005)
investigating the directional response of soils by means of stress-probing tests

- important for understanding some **fundamental features** of soil behaviour

- also relevant from the standpoint of **practical applications**

there are a number of important geotechnical engineering problems – notably the analysis of soil-structure interaction in deep excavations and tunnelling – where **large differences** typically exist between stress-paths experienced in different regions of the surrounding soil, both in terms of magnitude and direction

in these cases, the **quality of the engineering prediction** crucially depends on the ability of the constitutive model adopted for the soil to correctly reproduce soil behaviour along a wide range of loading paths, which in turn requires data from **experimental investigations going beyond conventional loading programs**
conclusive remark

the question

what are a few specific steps (up to, say, five) that can be taken to promote the use of advanced numerical methods in geotechnical practice?

my answer

1. Education
2. Education
3. Education
4. Education
5. Education

(mechanics, plasticity theory, numerical analysis, mathematical modeling)
how to discriminate among different inelastic theories?

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conclusions from stress-probing experiments

F.E. prediction of ground and wall movements around an excavation in sand supported by a propped diaphragm wall

(Viggiani & Tamagnini 2000)

horizontal displacements are prevented along the vertical boundaries AB and CD; both horizontal and vertical displacements are zero along the boundary BC

the wall is propped at the crest by a perfectly rigid strut, preventing any horizontal movement

details of the construction process and soil-structure interface behavior are not considered

the excavation process is simulated by sequentially removing four layers of soil elements until the final depth of 10.0 m is reached

looking for the minimum level of complexity needed for engineering purposes
qualitative vs. quantitative differences  
(Viggiani & Tamagnini 2000)

**Stiff wall**

\( K_o = 0.47 \)

**Soft wall**

\( K_o = 0.47 \)