

## REVIEW: THE DIRECT STRENGTH METHOD OF COLD-FORMED STEEL MEMBER DESIGN

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**Abstract.** *The objective of this paper is to provide a review of the Direct Strength Method for cold-formed steel design. The Direct Strength Method was formally adopted in North American design specifications in 2004 as an alternative to the traditional Effective Width Method. The appendix of this paper provides an excerpt from the North American Specification which provides the Direct Strength Method equations for the design of columns and beams. A brief comparison of the Direct Strength Method with the Effective Width Method is provided. The advantage of methods that integrate computational stability analysis into the design process, such as the Direct Strength Method, is highlighted. The development of the Direct Strength Method for beams and columns, including the reliability of the method is provided. Current and ongoing research to extend the Direct Strength Method is reviewed and references provided.*

### 1 INTRODUCTION

Cold-formed steel members are typically thin-walled, i.e., local plate buckling and cross-section distortion must be treated as an essential part of member design. These complications also provide certain opportunities, as local plate buckling, in particular, has the capacity for beneficial post-buckling reserve that can be drawn upon for increased strength in design. As a result, the ultimate efficiency, e.g. in terms of strength-to-weight ratio, can be quite high for cold-formed steel members. The challenge for any cold-formed steel design method is to incorporate as many of these complicated phenomena, that are largely ignored in conventional design of ‘compact’ sections, into as simple and familiar a design method as possible. Further complicating the creation of simple design methods is the lack of symmetry in many cross-sections, the enhanced possibility of other limit states such as web crippling, and other issues.

### 2 DESIGN METHODS FOR THIN-WALLED MEMBERS

Currently, two basic design methods for cold-formed steel members are formally available in design specifications in North America the traditional Effective Width Method, also known as the unified method or the main Specification method [1], and the Direct Strength Method, also known as the Appendix 1 method [2]. The Effective Width Method is available, in some form, nearly world-wide for formal use in design, while the Direct Strength Method has only been adopted in North America, and Australia/New Zealand. One additional design approach worthy of mention, but not detailed here further, is the Erosion of Critical Bifurcation Load approach championed by Dubina [3,4].

#### 2.1 Effective Width Method and Direct Strength Method

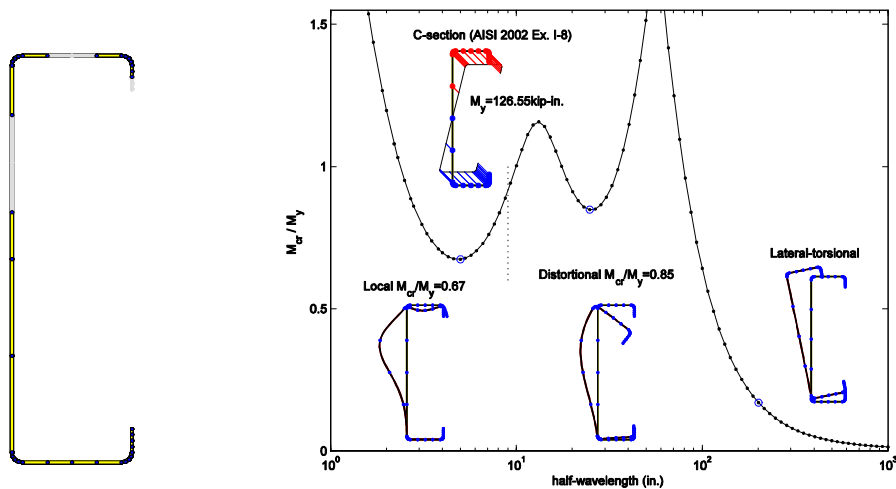
The basis for the Effective Width Method is well explained in textbooks and Specifications, the essential idea is that local plate buckling leads to reductions in the effectiveness of the plates that comprise a cross-section (more formally this can be understood as an idealized version of equilibrium in

an effective plate under a simplified stress distribution vs. the actual plate with a nonlinear stress distribution due to buckling<sup>1</sup>). This reduction from the gross cross-section to the effective cross-section as illustrated in Figure 1(a) is fundamental to the application of the Effective Width Method. The effective cross-section (i) provides a clear model for the locations in the cross-section where material is ineffective in carrying load, (ii) cleanly leads to the notion of neutral axis shift in the section due to local-buckling, (iii) provides an obvious means to incorporate local-global interaction where reduced cross-section properties influence global buckling (although specifications often simplify this interaction somewhat).

However, the Effective Width Method (i) ignores inter-element (e.g., between the flange and the web) equilibrium and compatibility in determining the elastic buckling behavior, (ii) incorporation of competing buckling modes, such as distortional buckling can be awkward, (iii) cumbersome iterations are required to determine even basic member strength, and (iv) determining the effective section becomes increasingly more complicated as attempts to optimize the section are made, e.g., folded-in stiffeners add to the plates which comprise the section and all plates must be investigated as being potentially partially effective. The Effective Width Method is a useful design model, but it is intimately tied to classical plate stability, and in general creates a design methodology that is different enough from conventional (hot-rolled) steel design that it may impede use of the material by some engineers in some situations.

## 2.2 Direct Strength Method

If the effective width (or section) is the fundamental concept behind the Effective Width Method, then accurate member elastic stability, as shown in Figure 1(b) is the fundamental idea behind the Direct Strength Method. The Direct Strength method is predicated upon the idea that if an engineer determines all of the elastic instabilities for the gross section (i.e., local, distortional, and global buckling) and the load (or moment) that causes the section to yield, then the strength can be directly determined.



(a) effective section of a C-section in bending

(b) semi-analytical finite strip solution of a C-section in bending showing local, distortional and lateral-torsional buckling

Figure 1 Fundamental “thinking model” for (a) Effective Width and (b) Direct Strength Methods

<sup>1</sup> As users of shell finite element analysis of thin-walled members know from examining stress contours of buckled plates, the common nonlinear two-dimensional stress distribution that is shown to explain the effective width of a plate is itself an approximation, representing the average of the axial membrane stress along the length of the plate – and ignoring variation in stress through the thickness (flexural) as well as stress variation along the length.

The Direct Strength Method has been detailed in textbooks and review articles [5–8]. The method is essentially an extension of the use of column curves for global buckling, but with application to local and distortional buckling instabilities. The development of, and continued research into, the Direct Strength Method is explored further in this paper.

### 2.3 Long-term goals for design methods of thin-walled members

It is important to recognize in any discussion regarding the Effective Width Method, the Direct Strength Method, or other semi-empirical design methods that none of these design methods are theoretically correct. Rather, a complicated nonlinear problem is simplified in some manner so that engineers may have a working model to design from without resorting to testing every individual member. These models serve us well when backed up by the application of reliability to incorporate uncertainty in their predictive powers.

It is this writer's contention that the long-term goal for thin-walled member design should be a fully nonlinear computational analysis. To this end, the computational member elastic buckling stability analysis that is at the heart of the Direct Strength Method is a useful stepping stone. In particular, the underlying mechanics for the member stability solutions, in e.g., the finite strip method [9] are necessary (but not sufficient) for understanding fully nonlinear analysis. Such a nonlinear analysis will also need to incorporate geometric and material imperfections into a consistent reliability framework so that we may provide engineers with a realistic prediction of strength and sensitivity that can be used in design.

More attempts to understand the inputs to thin-walled member strength such as geometric imperfections and residual stresses [10] as well as modeling assumptions (elements, material modeling) related to the underlying mechanics are needed [11]. Finally, for a full structural simulation the member analysis will need to be wedded to realistic connection and system models. While these remain long-term goals this writer contends that we should try to place as much emphasis on mechanics that we can agree on today (such as member elastic stability) into current design codes and specifications – as we drive towards more robust solutions in the future.

## 3 DEVELOPMENT OF DIRECT STRENGTH METHOD FOR COLUMNS

For columns, the beginning of the Direct Strength Method (though it was not called this at the time) can most clearly be traced to research into distortional buckling of rack post sections at the University of Sydney [12,13]. In particular, Hancock et al. [14] collected the research and demonstrated that for a large variety of cross-sections the measured compressive strength in a distortional failure correlated well with the slenderness in the elastic distortional mode.

Development of the Direct Strength Method beyond distortional buckling was completed using a much wider set of cold-formed steel cross-sections and tests that included failures in local, distortional, and global flexural or flexural-torsional modes [15,16]. In addition, interaction between the modes was systematically studied. Based on the experimental data it was concluded that local-global interaction was strong and must be included, but inclusion of local-distortional and distortional-global interactions lead to overly conservative solutions and were thus not recommended for inclusion. Recent work [17] has questioned whether local-distortional interaction should be included in some specific cases.

The resulting provisions for columns are summarized in the Appendix of this paper and comparison with the test data is provided graphically in Figure 2. Note, that for the local failures the normalization of  $P_{\text{test}}$  is to  $P_{\text{ne}}$ , the maximum strength due to global buckling (thus reflecting local-global interaction), while for distortional buckling the normalization of  $P_{\text{test}}$  is to  $P_y$ , the squash load of the column.

While Figure 2, indicates that the Direct Strength Method is a reasonable predictor of strength over a wide range of slenderness it does not provide a comparison with the Effective Width Method. Such a comparison is provided in [15] and recent work [18] has underscored the importance of sharing these findings more broadly. An example of systematic error in the Effective Width Method (as implemented in [1]) is demonstrated in Figure 3 – where the strength predictions of the Effective Width Method and the Direct Strength Method are compared as a function of the web slenderness of a C-section column. As

web slenderness increases the Effective Width Method solution becomes systematically more unconservative. This behavior is exacerbated by the fact that for C-sections the flange is usually approximately the same width, so high web slenderness is strongly correlated with high web height/flange width ratios (i.e., C-sections which are ‘narrow’). This detrimental behavior is primarily one of local web/flange interaction, not distortional buckling. Since the Effective Width Method uses an element approach, no matter how high the slenderness of the web becomes, it has no effect on the solution for the flange. Such an approach holds an advantage in some cases, in this case, it is systematically unconservative.

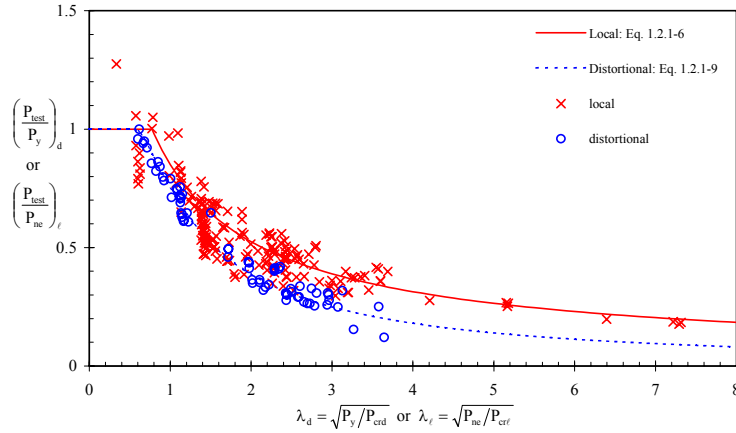


Figure 2. Comparison of the Direct Strength Method predictor curves with test data for columns

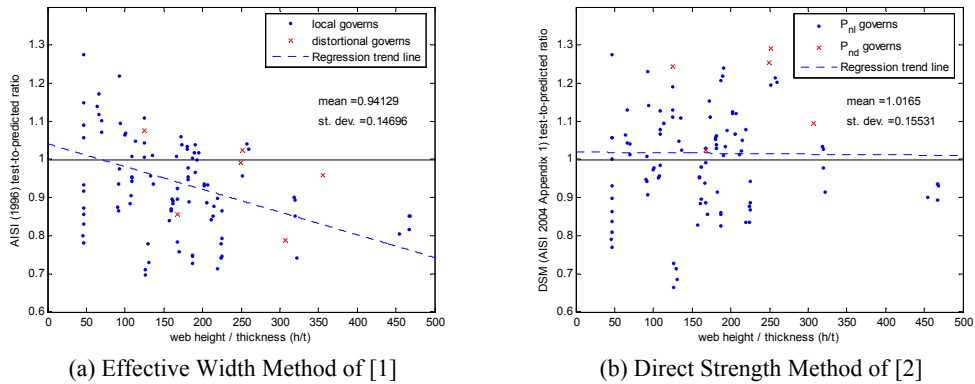


Figure 3. Test-to-predicted ratio for for all lipped channel columns used in the development of Direct Strength Method vs. web slenderness (h/t) of experimental specimens

#### 4 DEVELOPMENT OF DIRECT STRENGTH METHOD FOR BEAMS

The first mention of the Direct Strength Method occurs in [19] and was closely coupled to the development of the method for beams, in particular application of the large database of sections that was collected by the writer to explore two problems: distortional buckling in C- and Z-section beams, and local and distortional buckling in deck sections with multiple longitudinal intermediate stiffeners in the

compression flange. At the same time Hancock and related researchers at the University of Sydney demonstrated that distortional buckling failures for a wide variety of failures were well correlated with the elastic distortional slenderness [14,20]. The form of the presentation of the Direct Strength Method for beams evolved somewhat from [19]. In particular, curve (2) of [19] as discussed in [21] is identical to the distortional buckling expressions developed in [20] and became the distortional buckling Direct Strength curve. For local buckling, curve (3) of [19] was employed. The Appendix of this paper provides the Direct Strength Method expressions for beams, and the performance against experimental data is graphically provided in Figure 4.

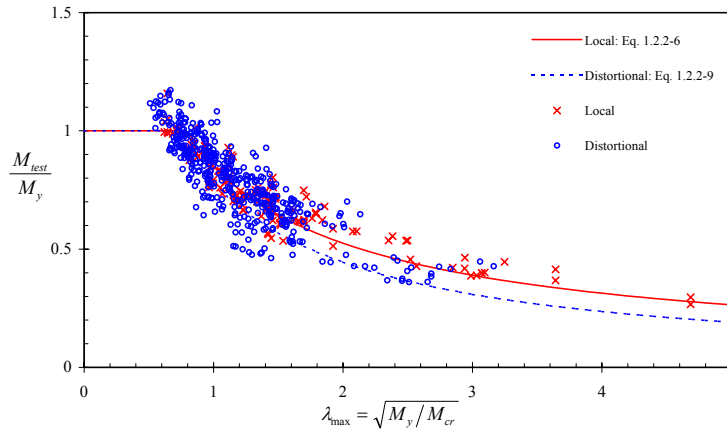


Figure 4 Comparison of the Direct Strength Method predictor curves with test data for beams

Note, for the beam data of Figure 4 all of the  $M_{test}$  values are normalized against the moment at first yield,  $M_y$ . This is due to the fact that all of the test data employed was for laterally braced members. It is worth noting that while local-global interaction was experimentally examined for columns, and the same methodology applied for beams, local-global, distortional-global, and local-distortional interactions have not been experimentally examined in the context of the Direct Strength Method for beams. Based on the findings for columns local-global interaction has been included and local-distortional and distortional-global interactions ignored. The performance of laterally un-braced beams deserves further study, not only in the context of the Direct Strength Method and potential interactions, but also to better understand how warping torsion should be treated. For moderate rotations the influence of the torsional stress on local and distortional buckling modes is real [22] and its potential inclusion in the Direct Strength Method is worthy of further study.

The beam data of Figure 4 shows many more distortional buckling failures than the column data of Figure 2. This is due to two reasons: (i) distortional buckling failures are more common in typical C- and Z-sections where the web is stabilized by the bending stress, and (ii) the database of sections includes a large number of deck/hat sections with longitudinal intermediate stiffener(s) in the compression flange – buckling of those members in which the longitudinal intermediate stiffeners is engaged is defined as distortional buckling failure.

In the development of the Direct Strength Method for C- and Z-section beams separation of local and distortional buckling failure modes was initially somewhat difficult and complicated by the bracing and boundary conditions used in the testing, which typically restrained distortional buckling in part, but not necessarily in full. Nonetheless expressions were arrived at as provided in the Appendix and adopted in [2]. A recent series of flexural tests on a variety of C- and Z-sections in local buckling [23,24] and distortional buckling [24,25] used specific details to isolate the two modes and unequivocally

demonstrated the robustness of the strength predictions for C- and Z-sections definitively failing in the local or distortional mode. A summary of the performance of these sections is provided in Figure 5.

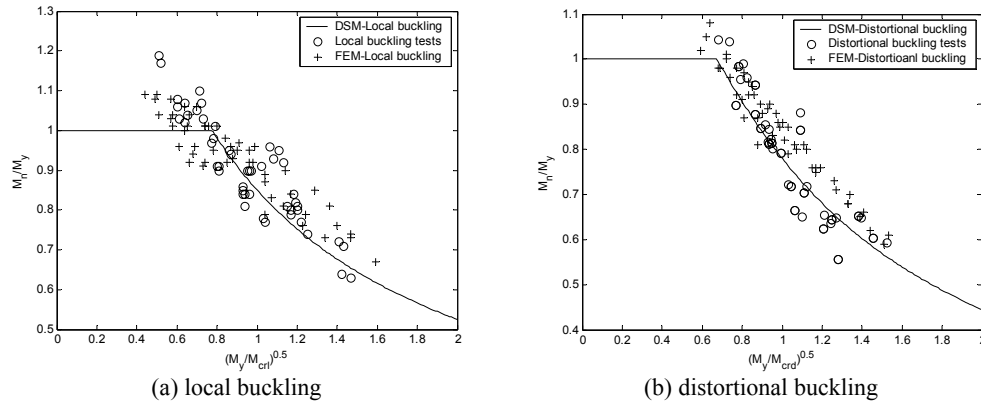


Figure 5 Comparison of DIRECT STRENGTH METHOD to tests and extended FE results

**4.3 Deflection calculation (serviceability)**

To examine serviceability, deflections are typically determined at the service stress level of interest. In the Effective Width Method, to account for reduced stiffness due to cross-section instability, the effective member properties are determined at the service stress. The Direct Strength Method uses a similar philosophy, but since the equations are in terms of strength, the implementation is more awkward. As detailed in [2] the service level moment ( $M$ ) is used as the peak moment (i.e.,  $M$  replaces the yield moment  $M_y$  in the expressions) and the deflection strength  $M_d$  of the cross-section is determined. The ratio of these two moments ( $M_d/M$ ) provides an approximate reduction in the stiffness of the member at the service moment,  $M$ . Results of the calculation for a typical C-section is shown in Figure 6 while the full solution is detailed in [26]. The approach follows the same basic trends as the Effective Width Method for reduced stiffness in a cross-section.

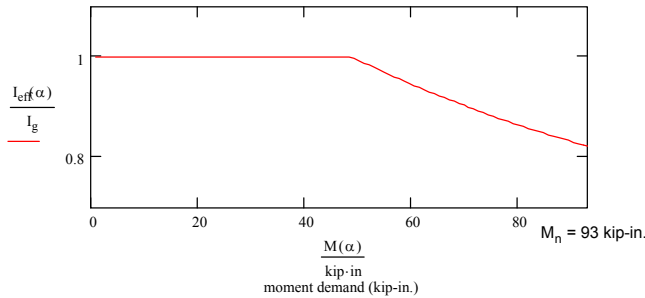


Figure 6 Reduced stiffness as a function of service moment for 9CS2.5x059

**4.1 The good and the bad in deck sections**

The first paper discussing the Direct Strength Method [19] reported a fundamental limitation of the method: when the stability of one part of the cross-section becomes so slender that the member elastic critical buckling stress approaches zero the Direct Strength Method will assume the member strength approaches zero. In contrast, the Effective Width Method presumes only that the element itself

(not the member) will have no strength in such a situation. Deck or hat sections with low yield stress and very slender (wide) compression flanges without intermediate stiffeners tend to fall in this category and thus have unduly conservative predictions by the Direct Strength Method, but quite reasonable predictions via the Effective Width Method. However, ignoring inter-element interaction, as the Effective Width Method traditionally does, is not universally a good idea as illustrated for columns in Figure 3.

For highly optimized deck sections with multiple longitudinal intermediate stiffeners in the web and the flange (see e.g., [27]) the Direct Strength Method is highly desirable over the Effective Width Method – here the benefit is primarily convenience not theoretical. If a computational solution is employed for determining the elastic buckling stresses (moments) a highly optimized deck section is no more complicated than a simple hat for strength determination; but for the Effective Width Method the calculation of effective section properties and accurately handling the effective width of the numerous sub-elements leads to severe complication without increased accuracy, or worse in the case of many specifications (e.g. [1] or [28]) no design approach is even available for such a section using the Effective Width Method. In general, as sections are optimized the Direct Strength Method provides a simpler design methodology with wider applicability than the Effective Width Method.

## 5 RELIABILITY AND PRE-QUALIFIED SECTIONS

The reliability of the Direct Strength Method was established using the limit-states design format in use in the United States: Load and Resistance Factor Design (LRFD). Chapter F of [28] provides the formal expressions for deriving the resistance factors,  $\phi$  where capacity ( $\phi R_n$ ) must be greater than demand ( $\gamma Q$ ) in the LRFD format via  $\phi R_n > \Sigma \gamma_i Q_i$ . A target reliability  $\beta$  of 2.5 was employed. The resulting resistance factors for the Direct Strength Method of [2] and the Effective Width Method of [1] are provided in Table 1.

Table 1 Reliability of Design Methods

	$\phi$ Beams	$\phi$ Columns
<b>AISI (1996) Specification [1]</b>		
Based on DSM data <sup>1,2</sup>	0.77	0.82
Specified	0.90 or 0.95	0.85
<b>Direct Strength Method [2]</b>		
Local ( $M_{n\ell}$ or $P_{n\ell}$ controls)	0.89	0.79
Distortional ( $M_{nd}$ or $P_{nd}$ controls)	0.93	0.90
Combined	0.92	0.85
Specified in [2]	0.90	0.85

(1) Sections which are outside the geometric bounds of [1] or include longitudinal web stiffeners or other features not covered in [1] are excluded from the calculation.

(2) The DSM data includes all the tested sections cited in [2]

During the formal codification of the Direct Strength Method in [2] it was determined that the users of the method should be aware of the cross-sections employed to verify the approach. Further, it was decided that the geometric and material bounds of the cross-sections used in the verification of the Direct Strength approach should be able to use the derived  $\phi$  factors, but new sections falling outside the boundaries of tested sections should use slightly reduced (more conservative)  $\phi$  factors. Thus, the idea of pre-qualified section (or limits) was established, and [2] includes a number of tables that provide the geometric and material bounds for pre-qualified members. Essentially, the pre-qualified sections in [2] represent a summary of the experimental database used in verifying the Direct Strength Method. It is perhaps worthy to note that this experimental database is larger than that used for determining the Effective Width Method approach of [1] or [28].

### 5.1 Development of new and optimal cross-sections

No definitive method has yet been established for extending the limits of pre-qualified section, but in [26] initial guidance is provided. Of particular interest is the potential to use a small number of tests and extend one of the pre-qualified categories – to this end the number of samples ( $n$ ), the average test-to-predicted ratio also known as the professional factor ( $P_m$ ) and the coefficient of variation of the professional factor ( $V_p$ ) are provided in Table 2. These statistics provide further information on how the method performs across different cross-section types.

Table 2 Summary Statistics for Direct Strength Method Development

	$n$	$P_m$	$V_p$
<b>Beams</b>			
C-sections	185	1.10	0.11
C-sections with web stiffeners	42	1.12	0.07
Z-sections	48	1.13	0.13
Hat sections	186	1.10	0.15
Trapezoidal sections	98	1.01	0.13
ALL BEAMS	<b>559</b>	<b>1.09</b>	<b>0.12</b>
<b>Columns</b>			
C-sections	114	1.01	0.15
C-sections with web stiffeners <sup>1</sup>	29	0.88	0.14
Z-sections	85	0.96	0.13
Rack sections	17	1.02	0.05
Hat sections	4	0.98	0.02
ALL COLUMNS	<b>249</b>	<b>0.98</b>	<b>0.14</b>

(1) Thomasson's (1978) tests contribute to the low  $P_m$ , more recent tests by Kwon and Hancock (1992) showed much better agreement. See [2] or [26] for full citations and further details.

### 5.2 Members with complex stiffeners and extension of pre-qualified sections

In 2006, based on the work in [29] and [30] the limits on pre-qualified sections in [2] were extended to cover C- and Z-section beams with complex lip stiffeners. For columns the category of Lipped C-Section and Rack Upright were merged, as a rack upright is a C-section with a complex stiffener. In addition, the complex stiffener limits from the original Rack Upright category were relaxed to match those found for C-section beams with complex stiffeners. Finally, the Effective Width Method of [28], i.e., the main Specification, was restricted to only cover simple lip stiffeners – thus the Direct Strength Method became the preferred approach for these more complicated sections.

## 6 DESIGN AIDS, THE NEW AISI DIRECT STRENGTH METHOD DESIGN GUIDE

As detailed in [31] a Design Guide for the Direct Strength Method [26] has recently been authored to aid engineers in the application of the Direct Strength Method. The Guide covers the following areas: elastic buckling, overcoming difficulties with elastic buckling determination in the finite strip method, beam design, column design, beam-column design, and product development. The Design Guide includes nearly 100 pages of design examples. The Design Guide provides a complete discussion of the details associated with application of the finite strip method, and the difficulties encountered as well. Topics covered include: indistinct local mode, indistinct distortional mode, multiple local or distortional modes (stiffeners), global modes at short unbraced lengths, global modes with different bracing conditions, influence of moment gradient, partially restrained modes, boundary conditions for repeated members, members with holes, boundary conditions at the supports not pinned, and built-up cross-sections. The discussion is directed at aiding engineers who need the finite strip method for more than just cursory use.

The Design Guide provide complete details for development of beam span tables or charts and column height tables or charts using the Direct Strength Method. An example beam chart is provided in

Figure 7. In this example one can readily see how the local buckling strength,  $M_{n\ell}$ , is a reduction below the global buckling strength,  $M_{ne}$ . The point where  $M_{n\ell}$  and  $M_{ne}$  merge (approximately 9 ft) indicates that local buckling no longer provides a reduction in the strength of this beam (in the main Specification [28] this occurs when the stress used to determine the effective section,  $F_n$  is low enough that the section is fully effective at that stress.) Further, the detrimental impact of distortional buckling on intermediate length beams is shown in Figure 7.

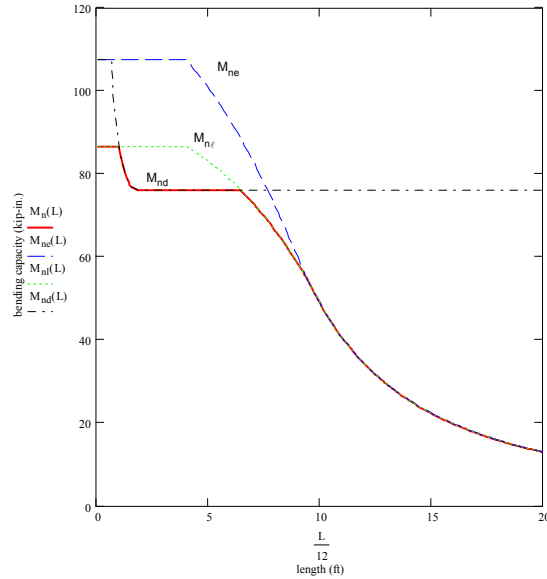


Figure 7 Example beam chart for a Z-section with lips

Additional information on the design of purlins using the Direct Strength Method beyond that in [26] is also offered in [32]. Further, the behavior of purlins as struts was explored in [33], however comparisons to the Direct Strength Method did not incorporate the beneficial influence of rotational restraint to the purlins as discussed in [26] and detailed in [24]. Built-up sections are explored in [26] and in recent research [34]. The work reported in [34] has been corrected since its publication and the authors should be contacted for corrected comparisons to the Direct Strength Method.

As engineers employ the Direct Strength Method on novel cross-sections one important piece of advice from [26] is that when in doubt about whether to define a given buckling mode as local or distortional it is always conservative to assume it is both modes. Such an approach is conservative, but ensures reduced post-buckling strength at intermediate unbraced lengths (i.e., the distortional reduction) as well as inclusion of interaction effects (i.e. local-global interaction).

## 7 ADVANCING THE DIRECT STRENGTH METHOD

### 7.1 Shear

No formal provisions for shear currently exist for the Direct Strength Method. However, it is recommended in [26] that existing provisions [28] could be suitably modified. As a rational analysis extension the existing equations are recast into the Direct Strength format and are suggested for use:

$$\text{for } \lambda_v \leq 0.815 \quad V_n = V_y \quad (1)$$

$$\text{for } 0.815 < \lambda_v \leq 1.231 \quad V_n = 0.815 \sqrt{V_{cr} V_y} \quad (2)$$

$$\text{for } \lambda_v > 1.231 \quad V_n = V_{cr} \quad (3)$$

$$\text{where} \quad \lambda_v = \sqrt{V_y / V_{cr}} \quad (4)$$

$$V_y = A_w 0.60 F_y \quad (5)$$

$V_{cr}$  = critical elastic shear buckling force.

For members with flat webs where  $V_{cr}$  is determined only for the web, these expressions yield the same results as in [28], for more unique cross-sections  $V_{cr}$  can be determined by finite element analysis or other methods. Further research to validate these expressions for unique sections is needed.

### 7.2 Inelastic reserve capacity in beams

A number of the flexural test results reported in Figure 4 and Figure 5 have bending capacity in excess of the moment at first yield,  $M_y$ , i.e. they exhibit inelastic reserve capacity. Current methods to account for inelastic reserve capacity, see e.g. [28] are highly involved. A simplified method using a Direct Strength Method approach is currently being investigated by the writer to account for inelastic reserve capacity. Two approaches are being pursued, one in which the slenderness,  $\lambda$ , is used only to estimate a limiting extreme fiber strain, and a second method where  $\lambda$  combined with the  $M_p/M_y$  ratio is used to directly provide the inelastic bending capacity.

### 7.3 Members with holes

Research is actively underway to extend the Direct Strength Method to members with holes [18,35, 36]. Using general purpose finite element analysis and focusing on isolated holes in columns (primarily C-sections) the research in [35,36] has identified that interactive elastic buckling modes triggered by the holes are possible, and even common, as shown for example in Figure 8 where distortional buckling occurs near the hole but local buckling away from the hole. For the sections studied in [35,36] the existing Direct Strength Method expression, but with  $P_{crf}$ ,  $P_{crd}$ , and  $P_{cre}$  defined as the minimum elastic buckling mode that displays characteristics of local, distortional, and global buckling respectively, provide a reasonable and conservative strength prediction. Research in this area is actively proceeding.

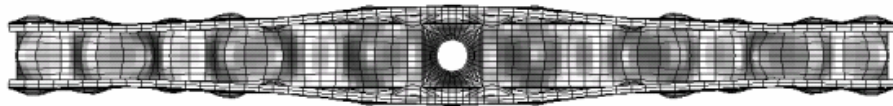


Figure 8 Mixed local and distortional mode that occurs because of a hole in a C-section column

### 7.4 Angles

Although angles are geometrically one of the simplest cold-formed steel members they are not pre-qualified for use in the Direct Strength Method implementation in [2]. Rasmussen in [37] extended his work on angles to include a Direct Strength Method approach. The work explicitly considers eccentricity – thus requiring a beam-column approach even for nominally concentrically loaded angle columns. Consistent with the Direct Strength Method the developed beam-column approach uses the stability of the angle under the applied compression + bending stresses which accurately reflects the fact that some eccentricities (away from the legs) benefit the strength and others (towards the legs) do not. Work performed in [38] examines a Direct Strength Method approach that ignores eccentricity for angle columns, and also further explores the relationship between local-plate buckling and global-torsional buckling of equal leg angle columns; arguing that when one considers the potential for multiple half-

waves along the length local-plate and global-torsional should be treated as unique modes. For now, the Direct Strength Method detailed in [37] is the most consistent and rational extension of current design methodologies, though the work in [38] may eventually provide a simpler approach.

### 7.5 Beam-columns

The design of beam-columns represents an opportunity for the Direct Strength Method to significantly diverge from current practice. Since the stability of the section can be considered directly under the applied loads ( $P$ ) and moments ( $M$ ) the interaction between  $P$  and  $M$  becomes cross-section specific; instead of the invariant interaction equations used in design specifications such as [28]. A basic methodology for the application of the Direct Strength Method for beam columns was proposed in [39, 40] and a complete design example using this methodology provided in [26]. The method is conceptually summarized in Figure 9 – where a cross-section specific interaction diagram is constructed for the sections reported in [41] and discussed in [42]<sup>2</sup>. For any applied combination of  $P$  and  $M$  (which defines the angle  $\theta$  in the interaction diagram) the combination that causes first yield,  $\lambda_y$ , and elastic buckling,  $\lambda_{cr}$ , (typically determined by finite strip analysis) are constructed. Using the same basic Direct Strength Method equations as before, but now replacing, e.g.,  $P_{cr}$  and  $P_y$  with  $\lambda_{cr}$  and  $\lambda_y$  – the nominal capacity,  $\lambda_n$ , may be determined. An example of the resulting Direct Strength Method interaction curve is illustrated in Figure 9. As discussed above, the methodology has been applied to angles in [37]. Comparison to long column data is provided in [43] with further discussion and an example in [44]. Further experimental and analytical research in this area is currently underway.

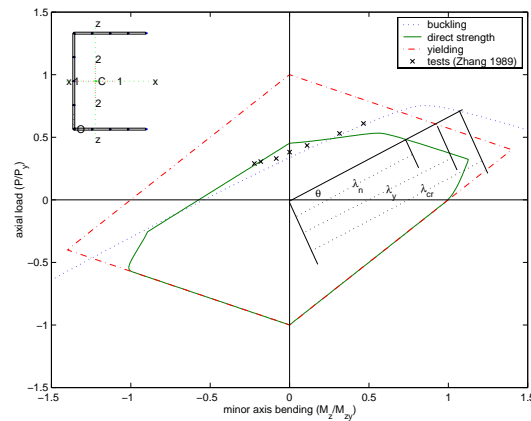


Figure 9 Proposed interaction diagram solutions for distortional buckling of unflipped C

### 7.6 Using pure mode analysis from GBT or cFSM to expedite Direct Strength Method design

Application of the Direct Strength Method is greatly aided by computational elastic buckling analysis. In fact, the development of the Direct Strength Method equations relied on the finite strip method, in particular [45]. However, the finite strip method does not always provide a definitive identification of the modes (i.e., which result is local, distortional, and/or global buckling), see [46] for example. Further, the finite element method (using plate or shell elements to comprise the section) provides no definitive method for identifying the modes. The Direct Strength Method requires that the modes be positively identified so that the equations may be applied. Generalized Beam Theory (GBT)

<sup>2</sup> Please note, the results of Figure 9 differ from those reported in [42], in which it was assumed that a linear interaction diagram could be used for the Direct Strength Method, and no elastic buckling analysis was performed for the eccentric loading.

[47,48], and now the constrained Finite Strip Method (cFSM) [49,50] provide methods for definitively separating the buckling modes from one another. This not only provides the potential for a cleaner and clearer implementation of the Direct Strength Method, but goes much further to opening up the possibility of automating the strength calculation, which enables optimization efforts, such as [51].

One word of caution about the application of the pure mode solutions of GBT (e.g., [52,53]) or cFSM, they are not identically the same as those used in developing the Direct Strength Method. As shown in [54,55] the minima in the finite strip method curve (e.g., Figure 1b) include interaction with the other modes. In the case of local and global buckling this interaction generally is small, but in the case of distortional buckling the minima (i.e.,  $P_{crd}$ ) identified by the conventional finite strip method may be as much as 10% or more lower than that identified by GBT or cFSM when only focused on distortional buckling. While it may be possible to re-calibrate the Direct Strength Method curves to these “pure mode” solutions for now it is recommended that the GBT or cFSM solutions be used only for determining the critical half-wavelength but the “all mode” or conventional finite strip method solution be used for determining the elastic buckling load (or moment).

### **7.7 Other materials: stainless steel, hot-rolled steel, aluminum, plastics**

While not the focus of this review, the application of the Direct Strength Method to other materials where cross-section stability plays an important or dominant role in the strength determination is underway. For example, in stainless steel see [56], for hot-rolled steel see [44], for aluminum see [57,58], and for thermoplastics see [59]. The basic methodology has even proven useful in investigating the stability of more unique cross-sections such as the human femur [60].

## **8 CONCLUSIONS**

The Direct Strength Method is a new design methodology for cold-formed steel members. The method has been formally adopted as an alternative design procedure in Appendix 1 of the North American Specifications for the Design of Cold-Formed Steel Structural Members, as well as in the Australian/New Zealand Standard for cold-formed steel design. The Direct Strength Method employs gross cross-section properties, but requires an accurate calculation of member elastic buckling behavior. Numerical methods, such as the finite strip method or generalized beam theory, are the best choice for the required stability calculations. The reliability of the Direct Strength Method equals or betters the traditional Effective Width Method for a large database of tested beams and columns. Extensive design aids are now available for engineers who want to apply the Direct Strength Method in design. Expansion of the Direct Strength Method to cover, shear, inelastic reserve, and members with holes are all underway. In addition, development of a Direct Strength Method for beam-columns continues and will provide cross-section specific interaction with far greater accuracy than the simple (essentially linear) interaction equations in current use. Much work remains for the continued development of the Direct Strength Method.

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**APPENDIX 1: DIRECT STRENGTH METHOD FOR COLUMNS**

(As excerpted from Appendix 1 of the North American Specification for the Design of Cold-Formed Steel Structural Members, 2004 Supplement to the 2001 Edition)

**1.2.1 Column Design**

The nominal axial strength,  $P_{nr}$  is the minimum of  $P_{ne}$ ,  $P_{nl}$  and  $P_{nd}$  as given below. For columns meeting the geometric and material criteria of Section 1.1.1.1,  $\Omega_c$  and  $\phi_c$  are as follows:

USA and Mexico		Canada
$\Omega_c$ (ASD)	$\phi_c$ (LRFD)	$\phi_c$ (LSD)
1.80	0.85	0.80

For all other columns,  $\Omega$  and  $\phi$  of Section A1.1(b) apply.

**1.2.1.1 Flexural, Torsional, or Torsional-Flexural Buckling**

The nominal axial strength,  $P_{ne}$  for flexural, ... or torsional- flexural buckling is

$$\text{for } \lambda_c \leq 1.5 \quad P_{ne} = (0.658^{\lambda_c^2}) P_y \quad (\text{Eq. 1.2.1-1})$$

$$\text{for } \lambda_c > 1.5 \quad P_{ne} = \left( \frac{0.877}{\lambda_c^2} \right) P_y \quad (\text{Eq. 1.2.1-2})$$

where  $\lambda_c = \sqrt{P_y/P_{cre}} \quad (\text{Eq. 1.2.1-3})$

$$P_y = A_g F_y \quad (\text{Eq. 1.2.1-4})$$

$P_{cre}$  = Minimum of the critical elastic column buckling load in flexural, torsional, or torsional-flexural buckling ...

**1.2.1.2 Local Buckling**

The nominal axial strength,  $P_{nl}$  for local buckling is

$$\text{for } \lambda_\ell \leq 0.776 \quad P_{nl} = P_{ne} \quad (\text{Eq. 1.2.1-5})$$

$$\text{for } \lambda_\ell > 0.776 \quad P_{nl} = \left[ 1 - 0.15 \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} \right] \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} P_{ne} \quad (\text{Eq. 1.2.1-6})$$

where  $\lambda_\ell = \sqrt{P_{ne}/P_{cr\ell}} \quad (\text{Eq. 1.2.1-7})$

$P_{cr\ell}$  = Critical elastic local column buckling load ...

$P_{ne}$  is defined in Section 1.2.1.1.

**1.2.1.3 Distortional Buckling**

The nominal axial strength,  $P_{nd}$  for distortional buckling is

$$\text{for } \lambda_d \leq 0.561 \quad P_{nd} = P_y \quad (\text{Eq. 1.2.1-8})$$

$$\text{for } \lambda_d > 0.561 \quad P_{nd} = \left( 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \left( \frac{P_{crd}}{P_y} \right)^{0.6} P_y \quad (\text{Eq. 1.2.1-9})$$

where  $\lambda_d = \sqrt{P_y/P_{crd}} \quad (\text{Eq. 1.2.1-10})$

$P_{crd}$  = Critical elastic distortional column buckling load ...

$P_y$  is given in Eq. 1.2.1-4.

**APPENDIX 2: DIRECT STRENGTH METHOD FOR BEAMS**

(As excerpted from Appendix 1 of the North American Specification for the Design of Cold-Formed Steel Structural Members, 2004 Supplement to the 2001 Edition)

**1.2.2 Beam Design**

The nominal flexural strength,  $M_n$ , is the minimum of  $M_{ne}$ ,  $M_{n\ell}$  and  $M_{nd}$  as given below. For beams meeting the geometric and material criteria of Section 1.1.1.2,  $\Omega_b$  and  $\phi_b$  are as follows:

USA and Mexico		Canada
$\Omega_c$ (ASD)	$\phi_c$ (LRFD)	$\phi_c$ (LSD)
1.67	0.90	0.85

For all other beams,  $\Omega$  and  $\phi$  of Section A1.1(b) apply.

**1.2.2.1 Lateral-Torsional Buckling**

The nominal flexural strength,  $M_{ne}$ , for lateral-torsional buckling is

for  $M_{cre} < 0.56M_y$   $M_{ne} = M_{cre}$  (Eq. 1.2.2-1)

for  $2.78M_y \geq M_{cre} \geq 0.56M_y$   $M_{ne} = \frac{10}{9} M_y \left( 1 - \frac{10M_y}{36M_{cre}} \right)$  (Eq. 1.2.2-2)

for  $M_{cre} > 2.78M_y$   $M_{ne} = M_y$  (Eq. 1.2.2-3)

where  $M_y = S_f F_y$ , where  $S_f$  is the gross section modulus referenced to the extreme fiber in first yield (Eq. 1.2.2-4)

$M_{cre}$  = Critical elastic lateral-torsional buckling moment...

**1.2.2.2 Local Buckling**

The nominal flexural strength,  $M_{n\ell}$ , for local buckling is

for  $\lambda_\ell \leq 0.776$   $M_{n\ell} = M_{ne}$  (Eq. 1.2.2-5)

for  $\lambda_\ell > 0.776$   $M_{n\ell} = \left( 1 - 0.15 \left( \frac{M_{cr\ell}}{M_{ne}} \right)^{0.4} \right) \left( \frac{M_{cr\ell}}{M_{ne}} \right)^{0.4} M_{ne}$  (Eq. 1.2.2-6)

where  $\lambda_\ell = \sqrt{M_{ne}/M_{cr\ell}}$  (Eq. 1.2.2-7)

$M_{cr\ell}$  = Critical elastic local buckling moment...

$M_{ne}$  is defined in Section 1.2.2.1.

**1.2.2.3 Distortional Buckling**

The nominal flexural strength,  $M_{nd}$ , for distortional buckling is

for  $\lambda_d \leq 0.673$   $M_{nd} = M_y$  (Eq. 1.2.2-8)

for  $\lambda_d > 0.673$   $M_{nd} = \left( 1 - 0.22 \left( \frac{M_{crd}}{M_y} \right)^{0.5} \right) \left( \frac{M_{crd}}{M_y} \right)^{0.5} M_y$  (Eq. 1.2.2-9)

where  $\lambda_d = \sqrt{M_y/M_{crd}}$  (Eq. 1.2.2-10)

$M_{crd}$  = Critical elastic distortional buckling moment...

$M_y$  is given in Eq. 1.2.2-4.