BWS
3.23.01

Alternative L_r expression

$$X_1 = \frac{\pi}{S_x} \cdot \sqrt{\frac{E \cdot G \cdot J \cdot A}{2}}$$ AISC F1-8

$$X_2 = 4 \cdot \frac{C_w}{I_Y} \cdot \left(\frac{S_x}{G \cdot J}\right)^2$$ AISC F1-9

L_r is the length at which $M_r = S_x(F_y - F_r) = S_x F_L = M_{cr}$ (note C_b=1.0 always for L_r)

$$L_r = \frac{r_y \cdot X_1}{F_L} \cdot \sqrt{1 + \sqrt{1 + X_2 \cdot F_L^2}}$$ AISC F1-6

X1 and X2 are clumsy, the direct expression for L_r (found by substitution)

$$L_r = \frac{\pi}{\sqrt{2} \cdot S_x \cdot F_L} \cdot \sqrt{E \cdot I_Y \cdot G \cdot J} \cdot \sqrt{1 + \sqrt{1 + 4 \cdot \frac{C_w \cdot (S_x \cdot F_L)^2}{I_Y \cdot (G \cdot J)^2}}}$$

A numerical check on the derived formula for Lr (units are kips and in.):

$E := 29000 \cdot ksi$

$I_Y := 184 \cdot in^4$

$G := \frac{E}{2 \cdot (1 + 0.3)}$

$J := 11.2 \cdot in^4$

$C_w := 32500 \cdot in^6$

$S_x := 345 \cdot in^3$

$F_L := 40\ ksi$

$A := 37.8 \cdot in^2$

$r_y := \sqrt{\frac{I_Y}{A}}$ $\quad X_1 := \frac{\pi}{S_x} \cdot \sqrt{\frac{E \cdot G \cdot J \cdot A}{2}}$ $\quad X_2 := 4 \cdot \frac{C_w}{I_Y} \cdot \left(\frac{S_x}{G \cdot J}\right)^2$

$r_y = 0.056 \cdot m$ $\quad X_1 = 1.643 \cdot 10^{10} \cdot kg \cdot m^{-}$ $\quad X_2 = 0 \cdot kg^{-2} \cdot m^2 \cdot s^4$

$$L_r := \frac{r_y \cdot X_1}{F_L} \cdot \sqrt{1 + \sqrt{1 + X_2 \cdot F_L^2}}$$ $L_r = 266.181 \cdot in$

$$L_r := \frac{\pi}{\sqrt{2} \cdot S_x \cdot F_L} \cdot \sqrt{E \cdot I_Y \cdot G \cdot J} \cdot \sqrt{1 + \sqrt{1 + 4 \cdot \frac{C_w \cdot (S_x \cdot F_L)^2}{I_Y \cdot (G \cdot J)^2}}}$$ $L_r = 266.181 \cdot in$