Reliability implications of advanced analysis in design of steel frames

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Advanced Analysis

• Non-Linear Structural Analysis
  – e.g. Inelastic materials, P-Δ effects, fiber elements

• System Behavior
  – e.g. Frame stability, moment redistribution

• No member checking, etc., in design

• Analysis code: OpenSees
  (opensees.berkeley.edu)
Reliability & Advanced Analysis (AA)

- Compare Reliability of Steel Frames Designed by the American AISC-LRFD vs. AA
- Compare Limit States Member (AISC-LRFD) vs. System (AA)
- Calculate Resistance Factors for AA
- Loads (D, L) and $F_y$ random in our analysis
Frames for Study

- Steel Frames from Ziemian et al. (1992)
- Designed by both AISC-LRFD (1986) and AA
- AA Design saves ~12% by weight
Member Size Comparison

Design by:
LRFD
AA
Frames for Study

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<td>1</td>
<td>Member Sizes:</td>
<td>LRFD or AA</td>
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<td>2</td>
<td>Yield Strength:</td>
<td>Uncorrelated and Correlated</td>
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<td>Base Fixity</td>
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<td>Geometry</td>
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Total = 32 Frames Analyzed
Overview

Non-Linear Analyses with Random Yield Strength

Advanced Analysis with Nominal Properties

Load Distributions

Strength Distributions

Reliability Estimates

Resistance Factors
Overview

Non-Linear Analyses with Random Yield Strength

Load Distributions

Strength Distributions

Reliability Estimates

Advanced Analysis with Nominal Properties

Resistance Factors
Load-Deflection Behavior

Uncorrelated $F_y$
AA Design

Normalized Total Gravity Load

Lateral Deflection of Top Story (in.)

- 1st Plastic Hinge
- Plastic Collapse
Load-Deflection Behavior

Correlated $F_y$
AA Design

1st Plastic Hinge
Plastic Collapse
Mean Strength at Plastic Collapse

Design by
- LRFD
- AA
Overview

Non-Linear Analyses with Random Yield Strength

Advanced Analysis with Nominal Properties

Load Distributions

Strength Distributions

Reliability Estimates

Resistance Factors
Strength Distributions

Normalized Strength and Load

Relative Frequency

Load

Strength

Plastic Collapse

1st Plastic Hinge

Uncorrelated $F_y$
LRFD Design

10,000 samples

10,000 samples
Reliability at Plastic Collapse

Design by

\[ Pf = \]

\[ 9.90 \times 10^{-10} \]
\[ 2.87 \times 10^{-7} \]
\[ 3.17 \times 10^{-5} \]
\[ 1.35 \times 10^{-3} \]
\[ 0.02 \]
Reliability at 1st Plastic Hinge

Design by
- LRFD
- AA

$P_f = 3.17 \times 10^{-5}$

$P_f = 1.35 \times 10^{-3}$

$P_f = 0.02$

$P_f = 0.16$

$P_f = 0.50$
Member vs. System Limit State

1st PH Strength

Plastic Collapse Strength

Uncorrelated $F_y$
AA Design
Member vs. System Limit State

1st PH Strength

Plastic Collapse Strength

Uncorrelated $F_y$
AA Design

Uncorrelated $F_y$
LRFD Design
Overview

Non-Linear Analyses with Random Yield Strength

Advanced Analysis with Nominal Properties

Load Distributions

Strength Distributions

Reliability Estimates

Resistance Factors
Resistance Factors for AA at PC

\[ \beta_t = 3.00 \]
Resistance Factors for AA at 1st PH

\[ \beta_t = 3.00 \]
Conclusions

• Probabilistic Basis for Advanced Analysis Resistance Factors

• No Simple Transformation from Member Design [Reliability] to System Design [Reliability]

• Increased Probability of 1st PH with Design by AA. Serviceability controls?
Frame Analysis Details

• Analysis code: OpenSees (opensees.berkeley.edu)

• Random Properties

  Yield Strength
  – 50 ksi (345 MPa) Nominal
  – Normal Distribution with COV=0.10
  – Members Uncorrelated and Correlated

Gravity Loads
  – Dead Load ~ Normal Distribution
  – Live Load ~ Extreme Type I Distribution
  – Both COV=0.10

Consistent with LRFD Assumptions
Frame Analysis Details

• OpenSees (http://opensees.berkeley.edu)
  – Geometric Non-Linear
  – Fiber-Element Cross-Section
  – Elastic-Plastic Material
  – Out-of-Plumb Column Imperfection of $H/400$
  – Out-of-Plane Behavior Restrained
  – No Residual Stresses

• Random Yield Strength and Gravity Loads
## Random Properties

- **Yield Strength**
  - 50 ksi (345 MPa) Nominal
  - Normal Distribution with COV=0.10
  - Members Uncorrelated and Correlated

- **Gravity Loads**
  - Dead Load ~ Normal Distribution
  - Live Load ~ Extreme Type I Distribution
  - Both COV=0.10

- **Consistent with LRFD Assumptions**
Frame Simulation Details

• Each Frame of 32
  – 10,000 Samples with Random $F_y$

• Each Sample
  – Load Increased Until Failure
  – Strength Limit States considered
    • 1st Plastic Hinge
    • Plastic Collapse
Future Work

• Serviceability Concerns due to Increased Probability of 1st PH with AA

• Is a Single $\phi$ Appropriate for ALL Steel Frames?

• How to Apply $\phi$ in AA?
  – To System Strength
  – To Member Properties
Reliability and LRFD

\[ Q = \text{Load} \quad R = \text{Strength} \]

\[ \ln \left( \frac{R}{Q} \right) = \beta_{FO} \left( \sqrt{V_R^2 + V_Q^2} \right) \]

First-Order Reliability

\[ \beta_{FO} = \frac{\ln \left( \frac{R_m}{Q_m} \right)}{\sqrt{V_R^2 + V_Q^2}} \]
Reliability by Sampling

Probability of Failure

\[ P_f = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(r,q) f_R(r) f_Q(q) \, dr \, dq \]

\[ I(r,q) \begin{cases} = 1 \text{ for } R \leq Q \\ = 0 \text{ for } R > Q \end{cases} \]

Monte Carlo Estimate

\[ \hat{P}_f = \frac{1}{N} \sum_{i=1}^{N} I(r_i,q_i) \]

\[ r_i \text{ sampled from } f_R \]

\[ q_i \text{ sampled from } f_Q \]

Reliability

\[ \beta = -\Phi^{-1}(\hat{P}_f) \]
Resistance Factors

\[ \phi = \left( \frac{R_m}{R_n} \right) \exp(-0.55 \beta_t V_R) \]

- Mean of “True” Strength
- Nominal Strength
- Target Reliability
- Variation of “True” Strength

![Diagram showing True Strength, \( \phi R_n \), \( R_n \), \( R_m \), and \( V_R \)]
Resistance Factors of LRFD

\[ \phi = \left( \frac{R_m}{R_n} \right) \exp(-0.55 \beta_t V_R) \]

\[ \frac{R_m}{R_n} = P_m M_m F_m = 1.07 \]

\[ V_R = \sqrt{V_P^2 + V_M^2 + V_F^2} = 0.15 \]

LRFD Bias Factors:

\( P \) = Professional

\( M \) = Material

\( F \) = Fabrication
Resistance Factors for AA

\[ \phi = \left( \frac{R_m}{R_n} \right) \exp(-0.55 \beta_t V_R) \]

\[ \frac{R_{m}^{AA}}{R_{n}^{AA}} = B_{m}^{AA} \]

\[ V_{B_{m}^{AA}} = \sqrt{V_{R_{m}^{AA}}^2 + V_{F}^2} \]

AA Strength Distribution

AA Nominal Strength

AA Strength Distribution
Mean Bias Factors
<table>
<thead>
<tr>
<th>Limit State</th>
<th>Plastic Collapse</th>
<th>1st Plastic Hinge</th>
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</thead>
<tbody>
<tr>
<td>$F_y$ Uncorr.</td>
<td>0.86</td>
<td>0.80</td>
</tr>
<tr>
<td>Min</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>Mean</td>
<td>0.89</td>
<td>0.83</td>
</tr>
<tr>
<td>Max</td>
<td>0.91</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Target Reliability, $\beta_t = 3.00$
Reliability by Sampling

Uncorrelated $F_y$
AA Design

Strength vs. Load

Safe
Failure
Reliability by Sampling

Uncorrelated $F_y$
AA Design

![Graph showing $P_f$ vs Number of Samples with 'Direct Monte Carlo' and 'Importance Sampling' lines.](image-url)
Member Size Comparison

Member Sizes by:
LRFD
Advanced Analysis
Reliability at Plastic Collapse

![Graph showing reliability at plastic collapse with symbols for First-Order and Monte Carlo methods.](image)