Interim Design Rules for Flexure in Cold-Formed Steel Webs

B.W. Schafer¹, T.W.J. Trestain²

Abstract
The design of cold-formed steel webs in flexure is governed by section B2.3 of the AISI Specification. Harmonization of the AISI (1996) Specification with the Canadian Standard (S136 1994), for the development of the new North American Specification (NAS 2001) has brought to light shortcomings in both the U.S. and Canadian documents and lead to the adoption of an interim design approach in the NAS (2001). The interim approach employs the AISI (1996) rules for one class of members and the S136 (1994) rules for a second class. Assessment of the resulting method with existing bending tests on Cees and Zees reveals significant “scatter” in the prediction of cold-formed steel beams and highlights problems associated with ignoring web/flange interaction, as is done in current methods. Determination of the “classes” in which the two methods are employed is presented, as is the rejection of a specific exclusion for sheathed members which was proposed during the development of the interim method. Finally, the practical implications of the new design rules are explored in a design example with the step discontinuity in strength between the "classes" highlighted.

Introduction
The new North American Specification is a joint standard that harmonizes the cold-formed steel design practices in Canada, Mexico and the United States. To facilitate harmonization, a North American Specification (NAS) Committee was created with a mandate to resolve any differences among the three countries. In the absence of a Mexican Standard for cold-formed steel design, the focus was on harmonization between the CSA S136 (1994) and the AISI Specification (1996). While the AISI and S136 have much in common, there are significant differences between some of the predictor equations. It was agreed to resolve these differences by choosing the equations that best fit the available data and if both approaches fit the data equally well, then the simpler of the two design expressions would be selected. Generally, there was insufficient time to undertake new research.

Since the effective width equations for webs elements in flexure are significantly different between S136 (1994) and the AISI Specification (1996), some resolution was required. However, when reviewing which approach was the better predictor, there was no clear winner. The AISI Specification was a better predictor for one class of members and the S136 for another. The first author undertook some analytical work and determined that the relevant variable for distinguishing between the classes was the overall depth to width ratio, h₀/b₀. Sheathing attached

¹. Assistant Professor, Johns Hopkins University, 203 Latrobe Hall, Baltimore, MD 21218, USA (schafer@jhu.edu)
². Professional Engineer, T.W.J Trestain Structural Engineering, 573 Durie Street, Toronto, Ontario, M6S 3H2, Canada (tomtrestain@rogers.com)
to the compression flange was also explored as a relevant variable but the impact on local buckling in the web was not significant enough for inclusion in the NAS. After an evaluation of the available data and given the time constraints, it was agreed to adopt an interim solution with the AISI (1996) approach to be used when $h_0/b_0 \leq 4$ and the S136 (1994) approach when $h_0/b_0 > 4$. The justification for this decision is presented here.

**Development of AISI (1996) and S136 (1994)**

As explained in the commentary (see Appendix B) the AISI (1996) method is the original implementation of the unified effective width approach. However, this approach contains discontinuities that were felt to be undesirable and the S136 Committee adopted changes suggested by the author’s of the AISI (1986) provisions, in anticipation of similar adoption by AISI. The AISI, however, did not follow suit. Despite discontinuities in the AISI provisions and based on good agreement with available experimental data at the time, the AISI COS decided to keep the original 1986 equations.


The AISI (1996) method for the effective width of webs has been recognized as having a number of peculiarities, discontinuities, and inconsistencies for quite some time (most recently discussed in Schafer and Peköz 1999). Thus, in the process of harmonization of the AISI (1996) and Canadian S136 (1994) for the North American Specification it was anticipated that difficulties may arise. Conceptually the methods are the same, but the implementation for the AISI (1996) method, as presented in equations B2.3-3, -4 and -5 and the S136 (1994) method as presented in B2.3-6 and –7 differs. The following example serves to demonstrate the primary difference.

Consider defining

$$\rho^* = \frac{b_1 + b_2}{b_{comp}},$$

thus $\rho^*$ is the ratio of effective portion of the element in compression. For the case of $\xi=2$ ($\psi=-1$), i.e. pure bending, then the S136 method calculates $b_1$ and $b_2$ as follows:

$$b_1 = \frac{b_e}{4}, \quad b_2 = \frac{b_e}{2} - b_1 = \frac{b_e}{4}, \quad \text{where} \quad b_e = \rho w .$$

Therefore, for the S136 method:

$$\rho^* = \frac{b_1 + b_2}{b_{comp}} = \frac{\frac{b_e}{4} + \frac{b_e}{4}}{\frac{b_{comp}}{w}} = \frac{\rho w}{2} = \rho .$$

Now, for the same example consider the AISI (1996) approach to $b_1$ and $b_2$:

$$b_1 = \frac{b_e}{4}, \quad b_2 = \frac{b_e}{2}, \quad \text{and therefore,}$$

$$\rho^* = \frac{b_1 + b_2}{b_{comp}} = \frac{\frac{b_e}{4} + \frac{b_e}{2}}{\frac{b_{comp}}{w}} = \frac{3\rho w}{4} = \frac{3}{2} \rho !$$

Thus, the effective width expressions for the web using current AISI expressions result in a 50% greater capacity for the web alone. In essence, the effective width expression for an element in pure bending by AISI is:
\[
\rho_{\text{AISI}}^* = \frac{3}{2} \left( 1 - \frac{0.22}{\lambda} \right) \frac{1}{\lambda}
\]
which for \( \rho^* = 1.0 \) implies a limiting \( \lambda = 1.23 \).

So, how can the AISI expression be ok? At least two mitigating factors must be considered: (1) little of the bending strength is derived from the web in typical members, (2) for most Cee and Zee members with stable flanges, the flange can provide rotational restraint to the web, and thus elevate ‘k’ far above the simply supported value used in design.

Other rational methods for calculating \( b_1 \) and \( b_2 \) have been derived (see Schafer and Peköz 1999) but for our purposes here just the AISI (1996) and S136 (1994) method will be considered.

**North American Specification**
The details of the North American Specification (2001) procedure are given in Appendix A.

**Evaluation with Experimental Data**
A large amount of existing experimental data was collected in Schafer and Peköz (1999) primarily for the purposes of evaluating new provisions for distortional buckling of Cees and Zees. A complete set of references, is given in the North American Specification and is provided in the appendix to this paper.

The tests primarily covered paired specimens of statically loaded Cees and Zees with 1/4 point loading. A few of the tests are done in vacuum boxes, but not the majority.

The range of studied dimensions is given in Table 1. The dimensions include \( h_0 = \) web depth, \( b_0 = \) flange width, \( D = \) lip length, \( t = \) thickness.

**Table 1 Geometry of members**

<table>
<thead>
<tr>
<th></th>
<th>( h_0/t )</th>
<th>( b_0/t )</th>
<th>( D/t )</th>
<th>( h_0/b_0 )</th>
<th>( D/b_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>Cohen (1987)</td>
<td>78</td>
<td>128</td>
<td>33</td>
<td>55</td>
<td>9</td>
</tr>
<tr>
<td>Ellifritt et al. (1997)</td>
<td>113</td>
<td>139</td>
<td>31</td>
<td>48</td>
<td>11</td>
</tr>
<tr>
<td>Laboube and Yu (1978)</td>
<td>77</td>
<td>269</td>
<td>28</td>
<td>75</td>
<td>11</td>
</tr>
<tr>
<td>Moreyra (1993)</td>
<td>120</td>
<td>124</td>
<td>34</td>
<td>36</td>
<td>12</td>
</tr>
<tr>
<td>Rogers (1995)</td>
<td>53</td>
<td>228</td>
<td>15</td>
<td>61</td>
<td>3</td>
</tr>
<tr>
<td>Schardt and Schrade (1982)</td>
<td>178</td>
<td>183</td>
<td>45</td>
<td>71</td>
<td>10</td>
</tr>
<tr>
<td>Schuster (1992)</td>
<td>166</td>
<td>168</td>
<td>33</td>
<td>34</td>
<td>10</td>
</tr>
<tr>
<td>Shan et al. (1994)</td>
<td>43</td>
<td>256</td>
<td>19</td>
<td>58</td>
<td>6</td>
</tr>
<tr>
<td>Willis and Wallce (1990)</td>
<td>126</td>
<td>131</td>
<td>38</td>
<td>40</td>
<td>14</td>
</tr>
<tr>
<td>Grand Total</td>
<td>43</td>
<td>269</td>
<td>15</td>
<td>75</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2 provides definitive numerical evidence that systematic problems exist with the AISI (1996) provisions for Cee and Zee members in bending with \( h_0/b_0 \) ratios greater than 4. Further, the method indicates that the S136 (1994) method unfairly penalizes members with \( h_0/b_0 \) ratios less than 4. A combination of the two approaches, while inelegant, does provide better agreement with the mean tested strength.
Table 2 Test-to-predicted ratio of members

<table>
<thead>
<tr>
<th>h/b</th>
<th>average test-to-predicted</th>
<th>st. dev. of test-to-predicted</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt;4</td>
<td>&gt;4</td>
<td>&lt;4</td>
</tr>
<tr>
<td>Cohen (1987)</td>
<td>1.08</td>
<td>1.13</td>
<td>0.06</td>
</tr>
<tr>
<td>Ellifritt et al. (1997)</td>
<td>0.78</td>
<td>0.85</td>
<td>0.10</td>
</tr>
<tr>
<td>Laboube and Yu (1978)</td>
<td>1.04</td>
<td>0.94</td>
<td>1.10</td>
</tr>
<tr>
<td>Moreyra (1993)</td>
<td>0.86</td>
<td>0.94</td>
<td>0.08</td>
</tr>
<tr>
<td>Rogers (1995)</td>
<td>1.08</td>
<td>0.92</td>
<td>1.09</td>
</tr>
<tr>
<td>Schardt and Schrade (1982)</td>
<td>1.06</td>
<td>0.94</td>
<td>1.11</td>
</tr>
<tr>
<td>Schuster (1992)</td>
<td>0.82</td>
<td>0.89</td>
<td>0.04</td>
</tr>
<tr>
<td>Shan et al. (1994)</td>
<td>1.01</td>
<td>0.94</td>
<td>1.06</td>
</tr>
<tr>
<td>Willis and Wallace (1990)</td>
<td>1.02</td>
<td>1.13</td>
<td>0.08</td>
</tr>
<tr>
<td>Totals</td>
<td>1.03</td>
<td>0.92</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Although Table 2 presents the standard deviation of the data it does not provide a direct sense of the large scatter that exists in these test results. For an individual experimenter the deviations are not too large, but taken as a whole, as in Figure 1, the large scatter is clear.

Figure 1 presents the test-to-predicted ratio for the North American Specification as a function of the web height to flange width ratio. The average test-to-predicted ratio is also shown as a moving window average on $h_0/b_0$. The step change at $h_0/b_0 = 4$ is indicated by the switch in the solid line from the lower line (AISI) to the upper line (S136). The plot reinforces that (1) the scatter is large, and (2) a trend on $h_0/b_0$ exists in the data.
In the absence of research to bring a complete model forth for local web/flange interaction for either the web or the flange it was proposed to adopt the more conservative S136 expressions for webs of members with \( h_0/b_0 > 4 \). This provides a partial fix on this problem, as on average the S136 expressions result in approximately 7 to 8 % lower strength predictions for members than AISI (1996), (the difference in the two expressions can be smaller or greater for a given member and the inclusion of cold work of forming issues further separates the two prediction methods), but this fix does not remove the systematic error. For tested members with \( h_0/b_0 > 5 \) even the S136 method has test to predicted ratios on average less than 1.0, and for \( h_0/b_0 > 8 \) the test to predicted ratio is on average greater than 5% unconservative with the more extreme test points as much as 20% unconservative.

Web/Flange Interaction
This “quick fix” came about because experimental data shows that for \( h_0/b_0 \) in excess of 4 the predicted bending strength by AISI (1996) method becomes progressively unconservative. The reason for this is the fact that AISI (1996) ignores local web/flange interaction. (Note that the S136 method also ignores web/flange interaction but the method does not become unconservative until \( h_0/b_0 \) reaches values substantially higher than 4.) For members with high \( h_0/b_0 \), i.e. with deep webs and narrow flanges, the local buckling of the web drives the local buckling of the flange and causes the \( k \) for the flange to be significantly lower than that used in AISI (1996) For example, as demonstrated in Figure 2, in pure bending at \( h_0/b_0 \) of 4, \( k_{flange} \sim 1.75 \), at \( h_0/b_0 \) of 6 \( k_{flange} \sim 1.0 \) versus the generally assumed \( k_{flange} = 4 \) for an adequately stiffened flange.

Using the finite strip analysis results as a guide, and comparing to current practice in the AISI (1996) Specification we may make some interesting observations:

- \( k \) for the web may be overly conservative for many common members; however this is apparently offset by effective width equations which increase \( \rho \) to 1.5\( \rho \),
- \( k \) for the flange may be unconservative for common members, however, in some cases the AISI Spec. still arrives at approximately the correct value, by implementing a reduction on \( k \) as a function of \( L/L_a \) when actually the reduction is a flange/web interaction issue that can better be expressed through the \( h_0/b_0 \) ratio.

Since current methods do not separate between local and distortional buckling of members, it is difficult to distinguish all the ramifications of ignoring local flange/web interaction. An experimental project now underway at Johns Hopkins under the direction of the first author is investigating these issues further.
Sheathing Exclusion
The North American Specification’s adoption of the S136 (1994) expressions for $h_o/b_o > 4$ result in an average strength reduction of 7 to 8% compared with AISI (1996) for Cees and Zees with this geometry. Members of the AISI Committee on Specifications wondered if such a reduction in strength was warranted for members with rigid sheathing attached to the compression flange. In effect, posing the following question: can a sheathed member provide an average strength increase of 7 to 8% for members with deep webs and narrow flanges versus an unsheathed member?

To answer this question several ideal sheathing situations were considered. First, consider rigid sheathing such that when the compression flange buckles against the sheathing it is resisted by contact and can only buckle inward. This provides a maximum boost in the elastic buckling of the flange of 33%. At best this provides an increase in the effective width of the flange of slightly less than 10%. For a typical member of concern, e.g., an 8 in. deep member with 2 in. flanges even a 10% increase in the flange effective width only results in a 2-3% increase in the moment of inertia (and the effective section modulus) a far cry from the 7 to 8% change that would be put in effect for a “sheathed member”.

What if one considers that the fasteners themselves effectively fix the flange and thus increase the capacity by directly limiting its rotation ability at the fastener location? A study comparing a flat plate with simple supports on the unloaded edges and with fixed supports at the loaded edges (i.e. the fasteners) versus pinned supports at the loaded edges sheds some light on this notion. The benefit is a function of the length – e.g., if the ideal fasteners are spaced at 3 times the buckling half-wavelength then the boost in elastic buckling is only 10% - if the ideal fasteners
are spaced equal to the buckling half-wavelength the boost in elastic buckling is as much as 54%. The buckling half-wavelength is 70% of the depth of the web (i.e. a 10 in. deep member has a buckling half-wavelength of approximately 7 in. in local buckling). Thus effective fastener spacing has to be as low as .7h – which is impractical for essentially all members. That aside, even significant increases in the flange buckling capacity will not reach the 7 to 8% boost put in effect by using the AISI (1996) equations.

However, it is important to remember quality (i.e., rigid) sheathing fastened at 12 in. o.c. is highly beneficial for performance. The benefit is far more significant in restricting distortional buckling and lateral-torsional buckling, than it is in increasing the capacity in local buckling failures, which is our interest in Chapter B of the Specification. For members with deep webs and narrow flanges in local buckling failures, even a rigid sheathing with ideal fasteners cannot be expected to provide a 7 to 8% boost in capacity unless the fastener spacing is tight enough to strongly disturb the local buckling wave in the web, i.e. fastener spacing less than 70% of the web depth. Based on these observations it was concluded that no exclusion should be allowed for the presence of sheathing in the North American Specification provisions.

A word about reliability
The North American Specification includes factors for LRFD (U.S. and Mexico) and LSD (Canada). The resistance factors in the Specification for bending members are \( \phi_b = 0.95 \) for the U.S. and \( \phi_b = 0.90 \) for Canada. For member design, Canada uses a target reliability (\( \beta \)) of 3.0 as opposed to 2.5 in the U.S., hence Canada systematically employs more conservative \( \phi \) factors.

Resistance factors were calculated in line with Chapter F of the NAS (2001), but with \( C_p \) set to 1, as was generally done in the original derivations. For determination of the variation of the prediction method a standard deviation weighted by the number of samples from each researcher was employed. (The ensemble standard deviation reported in Table 2 was not used, because it is higher and reflects variation across samples (researchers), as well as within samples.) The resulting \( \phi \) factors, for \( \beta = 2.5 \) are given in Table 3.

| Table 3 Calculated Resistance Factors for members of Table 1, \( \beta = 2.5 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | \(<4\) \>4\)    | \(<4\) \>4\)    | \(<4\) \>4\)    | all data       |
| \( \phi_b \)   | 0.90 0.80 0.87  | 0.95 0.86 0.92  | 0.89            |

These results suggest that the current AISI and S136 \( \phi \) values are too high. However, the current \( \phi \) factors are based on long historical practice. Further, it is argued that the data set should not be accepted in its entirety for a number of reasons:

- some of the test members were not adequately braced,
- many of the tests were based on 1/4 point loading which is more severe than the typical field applications with attached sheathing,
- some of the tests reflect other modes of failure, such a distortional buckling, and
- the virgin yield strength is not known for many of the tests.

Recent testing on Cees and Zees (Yu and Schafer 2002) indicates that when care is taken to insure local buckling is the failure mode, and the yield stress is determined for each member, the NAS (2001) method is generally in good agreement.
Design Example
A design example using the new NAS is provided in Appendix B. A typical cold-formed steel framing member (2" x 8" x 0.0451") with $h_0/b_0 = 4$ has been selected. This member lies on the dividing line between the two classes and the example calculates the section properties twice—once using the web design rules for $h_0/b_0 \leq 4$ and again using the rules for $h_0/b_0 > 4$. The results are summarized in Table 4.

<table>
<thead>
<tr>
<th>$h_0/b_0$</th>
<th>$b_1$ (in.)</th>
<th>$b_2$ (in.)</th>
<th>$M_n$ (in.kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 4$</td>
<td>1.301</td>
<td>2.590</td>
<td>42.7</td>
</tr>
<tr>
<td>$&gt; 4$</td>
<td>1.273</td>
<td>1.367</td>
<td>38.0</td>
</tr>
</tbody>
</table>

As expected, the most significant difference is in the effective width $b_2$ with the resulting drop in nominal moment capacity across the $h_0/b_0 = 4$ boundary of 11%.

Conclusions
The newly adopted North American Specification method for effective width of webs in flexure does not solve all issues with regard to local web/flange interaction nor any issues with regard to distortional buckling, but it does provide improved strength prediction ability for both users of the AISI and S136 Specifications; as S136 is overly conservative for members with low $h_0/b_0$ and AISI is unconservative for members with high $h_0/b_0$.

This is an interim solution only. Work is ongoing to eliminate discontinuities, in particular, the step discontinuity in strength across the $h_0/b_0 = 4$ boundary.

Acknowledgments
This work was partially sponsored by a research grant from the American Iron and Steel Institute and the Metal Building Manufacturers Association.

References


S136 (1994) “Cold-Formed Steel Structural Members” S136-94. Canadian Standards Association. Rexdale, Ontario, Canada


Yu, C., Schafer, B.W.. (2002). “Local Buckling Tests on Cold-Formed Steel Beams.” 16th International Specialty Conference on Cold-Formed Steel Structures October, Orlando, FL.
B2.3 Webs and other Stiffened Elements under Stress Gradient

The following notation is used in this section:

- $b_1$ = Effective width, dimension defined in Figure B2.3-1
- $b_2$ = Effective width, dimension defined in Figure B2.3-1
- $b_e$ = Effective width $b$ determined in accordance with Section B2.1 with $f$ substituted for $f$ and with $k$ determined as given in this section
- $b_0$ = Out-to-out width of the compression flange as defined in Figure B2.3-2
- $f_1$, $f_2$ = Stresses shown in Figure B2.3-1 calculated on the basis of effective section. Where $f_1$ and $f_2$ are both compression, $f_1 \geq f_2$
- $h_o$ = Out-to-out width of the web as defined in Figure B2.3-2
- $k$ = Plate buckling coefficient
- $\psi = \frac{|f_2|}{f_1}$ (absolute value) \hspace{1cm} (Eq. B2.3-1)

(a) Strength Determination

(i) For webs under stress gradient (both stresses are compression as shown in Figure B2.3-1)

$k = 4 + \psi (3 \psi) + 2 (1 - \psi)$ \hspace{1cm} (Eq. B2.3-2)

For $h_o/bo \leq 4$

$b_1 = \frac{b}{3 + \psi}$ \hspace{1cm} (Eq. B2.3-3)

$b_2 = \frac{b}{2}$ when $\psi > 0.236$ \hspace{1cm} (Eq. B2.3-4)

$b_2 = b - \frac{b}{2}$ when $\psi \leq 0.236$ \hspace{1cm} (Eq. B2.3-5)

In addition, $b_1 + b_2$ shall not exceed the compression portion of the web calculated on the basis of effective section.

For $h_o/bo > 4$

$b_1 = \frac{b}{3 - \psi}$ \hspace{1cm} (Eq. B2.3-6)

$b_2 = \frac{b}{1 - \psi} - b_1$ \hspace{1cm} (Eq. B2.3-7)

(ii) For other stiffened elements under stress gradient (both stresses are compression as shown in Figure B2.3-1)

$k = 4 + \psi (3 \psi) - 2 (1 - \psi)$ \hspace{1cm} (Eq. B2.3-8)

$b_1 = \frac{b}{3 \psi}$ \hspace{1cm} (Eq. B2.3-9)

$b_2 = \frac{b}{1 - \psi}$ \hspace{1cm} (Eq. B2.3-10)

(b) Serviceability Determination

The effective widths used in determining serviceability shall be calculated in accordance with Section B2.3(a) and based on $f$ and $\psi$ on the effective section at the load for which serviceability is determined.
When a beam is subjected to bending moment, the compression portion of the web may buckle due to the compressive stress caused by bending. The theoretical critical buckling stress for a flat rectangular plate under pure bending can be determined by Equation C-B2-1, except that the depth-to-thickness ratio, h/t, is substituted for the width-to-thickness ratio, w/t, and the plate buckling coefficient, k, is equal to 23.9 for simple supports as listed in Table C-B2-1.

Prior to 1986, the design of cold-formed steel beam webs was based on the full web depth with the allowable bending stress specified in the AISI Specification for cold-formed steel construction. In order to unify the design methods for web elements and compression flanges, the "effective design depth" approach was adopted in the 1986 edition of the AISI Specification on the basis of the studies made by Pekoz (1986b), Cohen and Pekoz (1987). This is a different approach as compared with the past practice of using a full area of the web element in conjunction with a reduced stress to account for local buckling and postbuckling strength (LaBoube and Yu, 1982b; Yu, 1985).

Prior to 2001, the expressions used in the Specification for the effective width of webs (Equations B2.3-3 through B2.3-5) implicitly...
assumed that the flange provided beneficial restraint to the web. Collected data (Cohen and Peköz (1987), Elhouar and Murray (1985), Ellifritt et al (1997), Hancock et al (1996), LaBoube and Yu (1978), Moreyra and Peköz (1993), Rogers and Schuster (1995), Schardt and Schrade (1982), Schuster (1992), Shan et al (1994), and Willis and Wallace (1990) as summarized in Schafer and Peköz (1999)) on flexural tests of C's and Z's indicate that Specification equations B2.3-3 through B2.3-5 can be unconservative if the overall web width \( h_o \) to overall flange width \( b_o \) ratio exceeds 4. Consequently, in 2001, in the absence of a comprehensive method for handling local web and flange interaction, the North American Specification adopted a two-part approach for the effective width of webs: an additional set of alternative expressions (Eqs B2.3-6 and B2.3-7), originally developed by Cohen and Pekoz (1987) were adopted for \( h_o > 4 \); while the expressions adopted in the 1986 edition of the AISI Specification (Eqs B2.3-3 through B2.3-5) remain for \( h_o < 4 \). For flexural members with local buckling in the web, the effect of these changes is that the strengths [resistances] will be somewhat lower compared with the 1996 Specification (1996). When compared with the CSA S136 (CSA, 1994) there are only minor changes for members with \( h_o / b_o > 4 \), but an increase in strength [resistance] will be experienced when \( h_o / b_o \leq 4 \).

It should be noted that in the North American Specification, the stress ratio \( \psi \) is defined as an absolute value. As a result, some signs for \( \psi \) have been changed in Specification Equations B2.3-2, B2.3-3, B2.3-6 and B2.3-7 as compared with the 1996 edition of the Specification (1996).

REFERENCES:

Appendix B: Design Example

Section properties are calculated for a typical channel geometry with \( h_0/b_0 = 4 \) and \( F_y = 33 \) ksi. The effective section properties for strength are calculated twice – once using the web expressions for \( h_0/b_0 \leq 4 \) and again for \( h_0/b_0 > 4 \).

The geometry of the cross section is illustrated in Figure B-1.

![Diagram of channel section](image)

**FIGURE B-1**

Properties are calculated using the linear method. See Table B-1 for the fully effective (unreduced for local buckling) properties.

For effective properties, the neutral axis is below the mid-depth of the section and the outer fiber compressive stress is therefore at \( F_y = 33 \) ksi.

Effective Web Calculations for \( h_0/b_0 \leq 4 \)

For the last iteration, \( Y_{cg} = 4.038 \) in.

By similar triangles, \( f_1 = 32.050 \) ksi and \( f_2 = 31.427 \) ksi and \( \psi = |f_2/f_1| = 0.9806 \)

\[
k = 4 + 2(1 + \psi)^3 + 2(1 + \psi) = 23.499
\]

\[
w = 7.767 \text{ in.}
\]

\[
E = 29,500 \text{ ksi}
\]
\( \mu = 0.3 \)

\[ F_{cr} = \frac{k\pi^2E}{12(1-\mu^2)} \left( \frac{t}{w} \right)^2 = 21.123 \text{ ksi} \]

\[ \lambda = \frac{f}{\sqrt{F_{cr}}} = \sqrt[3]{\frac{32.050}{21.123}} = 1.2318 \]

\[ \rho = \frac{(1-0.22/\lambda)}{\lambda} = 0.66684 \]

\[ b_c = \rho w = 0.66684(7.767) = 5.180 \text{ in.} \]

\[ b_1 = b_c/(3 + \psi) = 1.301 \text{ in.} \text{ and } b_2 = b_c/2 = 2.590 \text{ in.} \]

Check that \( b_1 + b_2 \) is less than the compression portion of the web:

\[ \text{Web compression portion} = Y_{cg} - t - R_i = 4.038 - 0.0451 - 0.0712 = 3.922 \text{ in.} \]

\[ b_1 + b_2 = 3.891 < 3.922 \text{ in.} \text{ OK} \]

Represent the ineffective portion of the web as an element with negative length

\[ b_{neg} = 3.922 - 3.891 = 0.031 \text{ in.} \]

\[ Y_{neg} = t + R_i + b_1 + b_{neg}/2 = 1.433 \text{ in.} \]

\[ I_{neg} = (1/12) b_{neg}^3 = 0.00000243 \text{ in}^4 \]

These values along with the effective properties for the flange and lip (elements 1 & 3 – detailed calculations not shown here) are given in Table B-2.

**Effective Web Calculations for \( h_0/b_0 > 4 \)**

For the last iteration, \( Y_{cg} = 4.287 \text{ in.} \)

By similar triangles, \( f_1 = 32.105 \text{ ksi} \) and \( f_2 = 27.692 \text{ ksi} \) and \( \psi = |f_2/f_1| = 0.8626 \)

\[ k = 4 + 2(1 + \psi)^3 + 2(1 + \psi) = 20.648 \]

\[ w = 7.767 \text{ in.} \]

\[ E = 29,500 \text{ ksi} \]

\[ \mu = 0.3 \]

\[ F_{cr} = \frac{k\pi^2E}{12(1-\mu^2)} \left( \frac{t}{w} \right)^2 = 18.560 \text{ ksi} \]

\[ \lambda = \frac{f}{\sqrt{F_{cr}}} = \sqrt[3]{\frac{32.105}{18.560}} = 1.3152 \]
\[ \rho = (1-0.22/\lambda)/\lambda = 0.63315 \]

\[ b_c = \rho w = 0.63315(7.767) = 4.918 \text{ in.} \]

\[ b_1 = b_c/(3 + \psi) = 1.273 \text{ in. and } b_2 = b_c/(1 + \psi) - b_1 = 1.367 \text{ in.} \]

For this case, it is not necessary to check that \( b_1 + b_2 \) is less than the compression portion of the web.

Represent the ineffective portion of the web as an element with negative length

\[ b_{neg} = Y_{cg} - t - R_i - (b_1 + b_2) = 1.530 \text{ in.} \]

\[ y_{neg} = t + R_i + b_1 + b_{neg}/2 = 2.154 \text{ in.} \]

\[ I_{neg} = (1/12) b_{neg}^3 = 0.298 \text{ in}^4 \]

These values along with the effective properties for the flange and lip (elements 1 & 3 – detailed calculations not shown here) are given in Table B-3.

**TABLE B-1**

<table>
<thead>
<tr>
<th>Element</th>
<th>L</th>
<th>Y</th>
<th>LY</th>
<th>LY²</th>
<th>I₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.509</td>
<td>0.371</td>
<td>0.189</td>
<td>0.070</td>
<td>0.011</td>
</tr>
<tr>
<td>2</td>
<td>0.147</td>
<td>0.057</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>1.767</td>
<td>0.023</td>
<td>0.040</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.147</td>
<td>0.057</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>7.767</td>
<td>4.000</td>
<td>31.070</td>
<td>124.278</td>
<td>39.052</td>
</tr>
<tr>
<td>6</td>
<td>0.147</td>
<td>7.943</td>
<td>1.170</td>
<td>9.292</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>1.767</td>
<td>7.977</td>
<td>14.099</td>
<td>112.477</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.147</td>
<td>7.943</td>
<td>1.170</td>
<td>9.292</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.509</td>
<td>7.629</td>
<td>3.881</td>
<td>29.610</td>
<td>0.011</td>
</tr>
<tr>
<td>Σ</td>
<td>12.909</td>
<td>51.635</td>
<td>285.021</td>
<td>39.075</td>
<td></td>
</tr>
</tbody>
</table>

\[ Y_{cg} = \Sigma LY/\Sigma L = 51.635/12.909 = 4.000 \text{ in.} \]

\[ I_{cg} = [\Sigma LY^2 + \Sigma I_0 - \Sigma LY_{cg}^2] t \]
\[ = [285.021 + 39.075 - 12.909(4.000)^2][0.0451] \]
\[ = 5.302 \text{ in}^4 \]
TABLE B-2
Effective Section Properties about X-X Axis - Linear Method
\( h_0/b_0 \leq 4 \)

<table>
<thead>
<tr>
<th>Element</th>
<th>L</th>
<th>Y</th>
<th>LY</th>
<th>LY²</th>
<th>I₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.496</td>
<td>0.364</td>
<td>0.180</td>
<td>0.066</td>
<td>0.010</td>
</tr>
<tr>
<td>2</td>
<td>0.147</td>
<td>0.057</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>1.676</td>
<td>0.023</td>
<td>0.038</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.147</td>
<td>0.057</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>7.767</td>
<td>4.000</td>
<td>31.070</td>
<td>124.278</td>
<td>39.052</td>
</tr>
<tr>
<td>-ve web ele.</td>
<td>-0.031</td>
<td>1.433</td>
<td>-0.044</td>
<td>-0.063</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.147</td>
<td>7.943</td>
<td>1.170</td>
<td>9.292</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>1.767</td>
<td>7.977</td>
<td>14.099</td>
<td>112.477</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.147</td>
<td>7.943</td>
<td>1.170</td>
<td>9.292</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.509</td>
<td>7.629</td>
<td>3.881</td>
<td>29.610</td>
<td>0.011</td>
</tr>
<tr>
<td>Σ</td>
<td>12.773</td>
<td>51.580</td>
<td>284.953</td>
<td>39.074</td>
<td></td>
</tr>
</tbody>
</table>

\( Y_{cg} = \Sigma LY/\Sigma L = 51.580/12.773 = 4.038 \text{ in.} \)

\[ I_{cg} = [\Sigma LY^2 + \Sigma I_0 - \Sigma LY_{cg}^2 ] t \]
\[ = [284.953 + 39.074 - (12.773)(4.038)^2][0.0451] = 5.220 \text{ in}^4 \]

\( M_n = I_{cg}F_y/Y_{cg} = (5.220)(33)/4.038 = 42.66 \text{ in.kips} \)
### TABLE B-3
Effective Section Properties about X-X Axis - Linear Method
\( h_0/b_0 > 4 \)

<table>
<thead>
<tr>
<th>Element</th>
<th>L</th>
<th>Y</th>
<th>LY</th>
<th>LY²</th>
<th>I/t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.496</td>
<td>0.364</td>
<td>0.180</td>
<td>0.066</td>
<td>0.010</td>
</tr>
<tr>
<td>2</td>
<td>0.147</td>
<td>0.057</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>1.676</td>
<td>0.023</td>
<td>0.038</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.147</td>
<td>0.057</td>
<td>0.008</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>7.767</td>
<td>4.000</td>
<td>31.070</td>
<td>124.278</td>
<td>39.052</td>
</tr>
<tr>
<td>-ve web ele.</td>
<td>-1.530</td>
<td>2.154</td>
<td>-3.296</td>
<td>-7.101</td>
<td>-0.298</td>
</tr>
<tr>
<td>6</td>
<td>0.147</td>
<td>7.943</td>
<td>1.170</td>
<td>9.292</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>1.767</td>
<td>7.977</td>
<td>14.099</td>
<td>112.477</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.147</td>
<td>7.943</td>
<td>1.170</td>
<td>9.292</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.509</td>
<td>7.629</td>
<td>3.881</td>
<td>29.610</td>
<td>0.011</td>
</tr>
<tr>
<td>Σ</td>
<td>11.274</td>
<td>48.328</td>
<td>277.915</td>
<td>38.775</td>
<td></td>
</tr>
</tbody>
</table>

\[
Y_{cg} = \frac{\Sigma LY}{\Sigma L} = \frac{48.328}{11.274} = 4.287 \text{ in.}
\]

\[
I_{cg} = \frac{[\Sigma LY^2 + \Sigma I_0 - \Sigma LY_{cg}^2]}{\Sigma L} t \\
= \left[277.915 + 38.775 - (11.274)(4.287)^2\right][0.0451] = 4.940 \text{ in}^4
\]

\[
M_n = I_{cg}F_y/Y_{cg} = (4.940)(33)/4.287 = 38.03 \text{ in.kips}
\]