18-19. The elevator car $E$ has a mass of 1.80 Mg and the counterweight $C$ has a mass of 2.30 Mg. If a motor turns the driving sheave $A$ with a torque of $M = (0.069^2 + 7.5) \text{ N.m}$, where $\theta$ is in radians, determine the speed of the elevator when it has ascended 12 m starting from rest. Each sheave $A$ and $B$ has a mass of 150 kg and a radius of gyration of $k = 0.2 \text{ m}$ about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.

\[
\theta = \frac{12}{0.33} = 36.39 \text{ rad} \\
T_i + \Sigma F_{1-2} = 0 \\
0 = 2300(9.81)(12) - 1800(9.81)(12) \int_0^{1.05} (0.069^2 + 7.5) \, d\theta \\
= \frac{1}{2}(1800)(v)^2 + \frac{1}{2}(2300)(v)^2 + (3)\left(\frac{1}{2}(150)(0.2)^2\right)\left(\frac{v}{0.33}\right)^2 \\
5880 + (0.025^2 + 7.5)\left[1.05^{1.05} - 1\right] = 2098.58J \\
v = 5.34 \text{ m/s}
\]

18-31. The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at $A$, which has a stiffness of $k = 80 \text{ N.m/rad}$, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0^\circ$ after being opened at $\theta = 90^\circ$ and released from rest. Hint: For a torsional spring $M = k\theta$, when $k$ is the stiffness and $\theta$ is the angle of twist.

\[
T_i + \Sigma F_{1-2} = T_3 \\
0 + \int_{\theta_0}^{\theta + \frac{\pi}{2}} 809 \, d\theta = \frac{1}{2} \left[ \frac{1}{3} (20)(0.8)^2 \right] (12)^2 \\
40 \left[ (\theta_0 + \frac{\pi}{2}) - 60^\circ \right] = 307.2 \\
\theta_0 = 1.66 \text{ rad}
\]
18.51. A uniform ladder having a weight of 30 lb is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle \( \theta \) at which the bottom end \( A \) starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at \( A \).

**Potential Energy:** Since is set at point \( A \). When the ladder is at its initial and final position, its center of gravity is located 5 ft and \( (5 \cos \theta) \) ft above the floor. Its initial and final gravitational potential energy are \( 30(5) = 150 \text{ ft-lb} \) and \( 30(5 \cos \theta) = 150 \cos \theta \text{ ft-lb} \) respectively. Thus, the initial and final potential energy are:

\[
V_i = 150 \text{ ft-lb} \quad V_f = 150 \cos \theta \text{ ft-lb}
\]

**Kinetic Energy:** The mass moment inertia of the ladder about point \( A \) is \( I_A \):

\[
I_A = \frac{1}{12} (32.2) (30) = 31.66 \text{ slug-ft}^2
\]

Since the ladder is initially at rest, the initial kinetic energy is \( T_i = 0 \). The final kinetic energy is given by

\[
T_f = \frac{1}{2} I_A \omega^2 = \left( \frac{1}{2} \right) (31.66) \omega^2 = 15.33 \omega^2
\]

**Conservation of Energy:** Applying Eq. 18–18, we have

\[
T_f + V_f = T_i + V_i
\]

\[
0 + 150 \cos \theta = 150 \text{ ft-lb}
\]

\[
\omega^2 = 9.66 (1 - \cos \theta)
\]

**Equation of Motion:** The mass moment inertia of the ladder about its mass center is \( I_c = \frac{1}{12} (32.2) (30) = 7.764 \text{ slug-ft}^2 \). Applying Eq. 17–16, we have

\[
+ 7.764 \omega^2 = m (\dot{\omega}) \quad - 3 \sin \theta \frac{\partial (\sin \theta)}{\partial \theta} \omega = 7.764 \frac{30}{32.2} \sin \theta
\]

\[
\omega = 4.83 \sin \theta
\]

\[
+ 7.764 \omega^2 = m (\dot{\omega}) \quad \dot{\omega} = - \frac{30}{32.2} [4.83 \sin \theta (5)] \sin \theta
\]

\[
\dot{\omega} = 30 - \frac{30}{32.2} (48.3 \sin \theta - 48.3 \cos \theta + 24.15 \sin \theta \cos \theta)
\]

\[
= 30 - 4.5 \cos \theta + 4.5 \cos \theta - 22.5 \sin \theta
\]

\[
= 30 - 4.5 \cos \theta + 4.5 \cos \theta - 22 \cos (1 - \cos \theta)
\]

\[
= 7.50 (3 \cos \theta - \sin \theta + 1)
\]

\[
= 7.50 (3 \cos \theta - 1)^2
\]

If the ladder lifts off the ground, then \( \dot{\omega} = 0 \). Thus,

\[
7.50 (3 \cos \theta - 1)^2 = 0
\]

\[
\theta = 70.5^\circ \quad \text{Ans}
\]
19-4. The space capsule has a mass of 1200 kg and a moment of inertia \( I_G = 900 \text{ kg} \cdot \text{m}^2 \) about an axis passing through \( G \) and directed perpendicular to the page. If it is traveling forward with a speed \( v_0 = 800 \text{ m/s} \) and executes a turn by means of two jets, which provide a constant thrust of 400 N for 0.3 s, determine the capsule's angular velocity just after the jets are turned off.

\[
\begin{align*}
T &= 400 \text{ N} \\
\theta &= 15^\circ \\
v_0 &= 800 \text{ m/s} \\
\end{align*}
\]

\[
(1) \quad (I_G)_1 + 2 \int M_{gd} dt = (I_G)_2
\]

\[
(2) \quad \omega = \frac{2(400 \cos 15^\circ)(0.3)(1.5)}{900} = 9.0 \text{ rad/s} \\
\omega &= 0.386 \text{ rad/s} \quad \text{Ans}
\]

19-17. The drum has a mass of 70 kg, a radius of 300 mm, and radius of gyration \( k_G = 125 \text{ mm} \). If the coefficients of static and kinetic friction at \( A \) are \( \mu_s = 0.4 \) and \( \mu_k = 0.3 \), respectively, determine the drum's angular velocity 2 s after it is released from rest. Take \( \theta = 30^\circ \).

\[
\begin{align*}
T &= 400 \text{ N} \\
\theta &= 15^\circ \\
\end{align*}
\]

\[
\begin{align*}
(1) \quad (I_G)_1 + 2 \int M_{gd} dt = (I_G)_2 \\
(2) \quad \omega = 27.863 \text{ rad/s} = 27.9 \text{ rad/s} \quad \text{Ans}
\end{align*}
\]

\[
\begin{align*}
\omega &= \frac{70(9.81) \sin 30^\circ (2) - 70(27.863)(0.3)}{50.79} \\
F &= 50.79 \text{ N} \\
N &= 594.7 \text{ N}
\end{align*}
\]

\[
\begin{align*}
F_{\text{max}} &= 0.4(594.7) = 237.87 \text{ N} > 50.79 \text{ N} \quad \text{OK}
\end{align*}
\]