Towards optimization of CFS beam-column industry sections

TWG-RR02-12

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July 2012
This report was prepared independently, but was motivated from, the American Iron and Steel Institute sponsored project: Direct Strength Prediction of Cold-Formed Steel Beam Columns. The project also received supplementary support and funding from the Metal Building Manufacturers Association. Project updates are available at www.ce.jhu.edu/bschafer/dsmbeamcol. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the author and do not necessarily reflect the views of the American Iron and Steel Institute, nor the Metal Building Manufacturers Association.

Acknowledgements:

The authors would like to acknowledge AISI and MBMA for their support throughout the duration of this project. The author would also like to acknowledge Jizahen Leng and Dr. Zhanjie Li for their input on this work.
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1. Introduction

The objective of this research is to develop industry sections optimized for maximum strength subjected to shape and dimensional constraints that can be used in low-cost sustainable mid-rise cold-formed steel housing construction. The optimization of common industry sections that include channel, zee, hat and sigma sections used in framing, truss and metal building cold-formed steel applications (Figure 1) and non-standard sections subjected to combined loading of compression and bending is examined. The effect of the complex stability behavior of thin-walled cross-section when subjected to combined loading is incorporated into the optimization code via yielding, elastic critical buckling and strength envelopes. Preliminary Direct Strength Method approach for combined loading in cold-formed steel members subjected to combined loading is adopted for strength prediction in place of current linear interaction approach. The methods of gradient-based optimization and stochastic optimization via genetic algorithm are explored for optimal solution. Based on strength envelopes of the preliminary Direct Strength Method in the axial-bending space, efficient members beam-column use can be readily selected.

Figure 1 Beam-column industry applications that motivated the research a) Framing b) Truss c) Metal building

2. Optimization problem formulation

The optimization of cold-formed steel industry sections subjected to combined loading is examined. The maximum combined strength is sought given a rectangular-sized sheet of steel. A 1.57 mm thick and 215 mm wide sheet of steel is considered based on the average dimensions of SSMA standard channel sections. Sections selected for
optimization study are in the spirit of the standard sections used in the framing, truss and metal building industry. The cross-sectional dimensions of the various applications are determined based on the average dimension for that particular standard shape. The material properties are Young’s modulus \((E=203000 \text{ MPa})\) and yield strength \((f_y=344.5 \text{ MPa})\). The member length is chosen as 3000mm.

The optimization problem is formulated in the standard form as:

\[
\text{Max } \beta_n = \text{Min } \frac{1}{\beta_n}
\]

Subject to the following dimensional constraints:

**Framing applications**

**Channel section**

\[
h + 2b + 2d \leq w
\]

\[
2d < h
\]

**Sigma section**

\[
1.069h + 2b + 4d \leq w
\]

\[
2d < h
\]

**Truss applications**

**Hat/U section**
**Metal building applications**

Zee section

\[
2.059h + b + 5d + 3.943 \leq w \\
2d < h
\]

**Non-standard S section**

\[
h + 3b + 2d \leq w \\
2d < h
\]

where \(w\) is the width of the sheet of steel and \(\beta_n\) is the predicted combined load capacity of the section:

\[
\beta_n = \min\{\beta_{sw}, \beta_{sl}, \beta_{sd}\}
\]

where
for $\lambda_{n} \leq 0.776$
\[ \beta_{n} = \beta_{ne} \]

for $\lambda_{n} > 0.776$
\[ \beta_{n} = \left(1 - 0.15 \left( \frac{\beta_{cr}}{\beta_{y}} \right)^{0.4} \right) \left( \frac{\beta_{cr}}{\beta_{y}} \right)^{0.4} \beta_{y} \]

where $\lambda_{n} = \sqrt{\beta_{y}/\beta_{cr}}$

$\beta_{cr}$ = Critical elastic local buckling magnitude under combined $P-M$ resultant
$\beta_{y}$ = First yield under combined $P-M$ resultant.

for $\lambda_{d} \leq 0.673c$
\[ \beta_{nd} = \beta_{y} \]

for $\lambda_{d} > 0.673c$
\[ \beta_{nd} = \left(1 - 0.22a \left( \frac{\beta_{crd}}{\beta_{y}} \right)^{0.5b} \right) \left( \frac{\beta_{crd}}{\beta_{y}} \right)^{0.5b} \beta_{y} \]

where $a = (1.136)^{2\gamma/\pi}$ $b = (1.2)^{2\gamma/\pi}$ $c = (0.834)^{2\gamma/\pi}$, and
\[ \lambda_{d} = \sqrt{\beta_{y}/\beta_{crd}} \]
$\beta_{crd}$ = Critical elastic distortional buckling $P-M$ resultant
$\gamma$ = Angular direction (in radians) in the $P-M$ space measured from the positive $x$-axis for the first quadrant and from the negative $x$-axis for the second quadrant.

For a $\gamma_{l} = fixed \ angle \ in \ P - M \ space \ (fixed \ for \ each \ run)$
$\beta_{n}$ is $f(L = 3000, t = 1.57, f_y = 344.5, h, b, d)$
3. Genetic algorithm (GA) approach to beam-column dimensional optimization

3.1 Genetic Algorithm [3, 4, 5]

Steps: Initial population, evaluate initial fitness, use crossover and mutation to generate population, reevaluate the population

\[
\text{Min}(f = \frac{1}{\beta_n} + C \times I^2)
\]

C = penalty constant
\(I\) = inequality constraint function
Advantage: easy to code
Disadvantage: Lots of function evaluations, no guarantee of optimal solution

The constrained optimization problem is first transformed into an unconstrained problem by making use of penalty functions. Among the various penalty methods, the additive form is chosen [3]. Using the general formulation of an exterior penalty function, the new objective function to be optimized is given as

\[
\text{Minimize } f = \frac{1}{Pc_r} + \sum_{i=1}^{q} r_i G_i + \sum_{j=1}^{m} c_j L_j
\]

where \(G_i\) and \(L_j\) are the functions of \(g_i(X)\) and \(h_j(X)\), inequality and equality constraints respectively, and \(r_i\) and \(c_j\) are penalty parameters. General formulas of \(G_i\) and \(L_j\) are,

\[
G_i = \max[0, g_i(X)]^\beta
\]

\[
L_j = \max[0, h_j(X)]^\gamma
\]

where \(\beta\) and \(\gamma\) are commonly 1 or 2. If the inequality holds, \(g_i(X) \leq 0 \text{ and } \max[0, g_i(X)]\) will be zero. Therefore the constraint does not affect the new objective function. If the constraint is violated (\(g_i(X) \text{ or } h_j(X) \neq 0\)), a big term will be added to the new objective function such that the solution is pushed back towards the feasible region. The severity of the penalty depends on the penalty parameters \(r_i\) and \(c_j\). If either the penalty is too large or too small, the problem could be very hard for genetic algorithm. A big penalty prevents it from searching an infeasible region. In this case, the genetic algorithm (GA) will converge to a feasible solution very quickly even if it is far from the optimal. A small penalty will cause it to spend too much time in searching in an infeasible region; thus GA would converge to an infeasible solution.

The basic operations of natural genetics-reproduction, crossover, and mutation-are implemented by use of genetic algorithm. It starts with a set of designs, randomly generated using the allowable values for each design variable. Each design is also assigned a fitness value, using the penalty function for constrained problems. From the
current set of designs, a subset is selected to generate new designs using the selected subset of designs. The size of the set of designs is kept fixed. Since more fit members of the set are used to create new designs, the successive sets of designs have a higher probability of having designs with better fitness values. The process is continued until a stopping criterion is met.

Reproduction is a process in which the individuals are selected based on their fitness values relative to that of the population. In this process, each individual string (design vector) is assigned a probability of being selected as

\[ P_i = \frac{F_i}{\sum_{j=1}^{N_p} F_j} \]

where \( N_p \) is the size of population and \( F_i \) is the fitness value for each design that defines its relative importance in the set, and is given by

\[ F_i = (1 + \varepsilon) f_{\text{max}} - f_i \]

where \( f_i \) is the cost function (penalty function value for constrained problems) for the \( i \)th design, \( f_{\text{max}} \) is the largest recorded cost (penalty) function value, and \( \varepsilon \) is a small value to prevent numerical difficulties when \( F_i \) becomes 0.

After reproduction, the crossover operation is implemented in two steps. First two individual strings (designs) are selected from the mating pool generated by the reproduction operator. Next, a crossover site is selected at random along the string length, and the binary digits (alleles) are swapped between the two strings following the crossover site.

Mutation is the next operation which in terms of a genetic string, corresponds to selecting a few members of the population, determining a location on each string randomly, and switching 0 to 1 or vice versa. The number of members selected for mutation (\( I_m \)), is based on heuristics, and the selection of location on the string for mutation is based on a random process. \( I_m = \text{integer}(P_m \times N_p) \)

where

- \( P_m \)-fraction of population selected for mutation

In each generation, while the number of reproduction operations is always equal to the size of the population, the amount of crossover (\( I_{\text{max}} \)) and mutation (\( I_m \)) can be adjusted to fine-tune the performance of the algorithm. If the improvement for the best cost (penalty) function value is less than \( \varepsilon \) for the last \( I_g \) consecutive iterations, or if the number of iterations exceeds a specified value, then the algorithm terminates.

Genetic algorithm operations

Selection-Roulette wheel (fitness proportionate) selection [ Spall p. 243]
3.2 Parametric study for GA

Example: channel section

Comparison with gradient based approach for channel: GA [174.63] vs Gradient [174.79]
Recommended: Use of GA to zoom in and use gradient methods for optimal solution
4. Gradient-based approach to beam-column dimensional optimization

4.1 Approach

FMINCON: Multi-dimensional constrained nonlinear minimization

Min F(x) subject to
A.x<=B
LB<=x<=UB

Optimization variables
Tolerance=1.e-6
Maxiter=500

Example for a channel section, gammai=pi/4

xo=[100 50 7.5]  \( \beta_{no}=140.91 \times 10^3 \)
xf=[80.6 40.1 27]  \( \beta_{nf}=174.79 \times 10^3 \)

4.2 Parameters in gradient-based

Gammai=pi/4

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<th>iteration</th>
<th>constant</th>
<th>Beta,n</th>
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Results:

• Gradient-based solution for $\gamma = \pi/4$

- Gradient-based optimum results:
  - $[h, b, d] = [70.8, 47.2, 24.9]$
  - $\beta_n = 173.02 \times 10^3$
  - $[168.6, 14.9, 1.2]$
  - $\beta_n = 216.4 \times 10^3$
  - $[80.6, 40.1, 27.0]$
  - $\beta_n = 174.79 \times 10^3$
  - $[70.8, 47.2, 24.9]$
  - $\beta_n = 175.8 \times 10^3$
5. Conclusions

The optimization of a channel cross-section subject to combined loading has been studied using genetic algorithm and gradient-based methods. The effect of complex buckling behavior of thin-walled cross-section when subjected to combined actions has been incorporated into the optimization code by using the finite-strip method and the direct strength method. The study indicated that effective use of genetic algorithm to such optimization problem requires examination of parameters such as the penalty function, mutation rate and sufficiently large number of population and generations to capture the optimal solution. The penalty, cross-over and mutation parameters need to be adjusted for optimum solution. In comparison, the gradient-based methods……. Future research that combines genetic algorithm with gradient-based methods to ensure globally optimal results is an approach that should be explored. In addition, optimization of beam-column sections under combined axial and minor-axis and biaxial bending has the potential to exploit additional strength as predicted under newly developed Direct Strength Method for combined loading.

References
1. Schafer BW. Progress on the direct strength method. 16th International Specialty Conference on Cold-formed Steel Structures, Orlando, FL. 2002.