

### 13 THIN-WALLED METAL CONSTRUCTION

Thin-walled metallic members are used in a significant number of structural applications: buildings, bridges, storage tanks, cars, ships, aircraft, etc. Here thin-walled metal construction refers primarily to civil engineering applications, with particular focus on building structures. Thin-walled metallic members are employed as the primary framing system in low-rise and mid-rise construction, and as the secondary framing system in high-rise or long span construction, such members are also commonly used in specialty structures such as storage racks, greenhouses, and others.

Cold-formed steel, stainless steel, and aluminum members, either by the use of thin sheet material or the inherent nature of the stress-strain response of the base material, all qualify as thin-walled metallic members. Example cross-sections include tubular members (Fig. 4.1b), I-sections (Fig. 4.1h), channels, Z-sections, hat sections (Fig. 4.1i), T-sections (Fig. 4.1m), and panels (Fig. 4.1o, p, q). The depth of such members generally ranges from 1 to ~12 in. (25 to 305mm), and the thickness of material ranges from about 0.003 to 0.5 in. (0.076 to 12.7mm) or thicker (particularly for aluminum).

In thin-walled metal construction understanding stability behavior and accounting for, or mitigating, this behavior in design plays a dominant role in successful engineering. In a thin-walled member local plate buckling and cross-section distortion must be treated as an essential part of member design. These complications also provide certain opportunities, as local plate buckling, in particular, has the capacity for beneficial post-buckling reserve that can be drawn upon for increased strength in design. As a result, the ultimate efficiency, e.g. in terms of strength-to-weight ratio, can be quite high for thin-walled metallic members. The challenge for any design method is to incorporate as many of these complicated phenomena, that are largely ignored in conventional design of ‘compact’ metal sections, into as simple and familiar a design method as possible. Further complicating the creation of simple design methods for thin-walled members is the lack of symmetry in many cross-sections, the enhanced possibility of limit states related directly to the use of thin sheet such as web crippling, and other unique characteristics of their manufacture and application.

Thin-walled cold-formed steel enjoys a wide and growing base of application in civil structures. For a number of years cold-formed steel members have been a mainstay of metal building systems serving as purlins, girts, and the building skin. Also, in high-rise construction cold-formed steel panels are widely used as floor decking. Today high-rise construction also uses a significant amount of cold-formed steel for curtain walls and partition walls. In addition, load bearing cold-formed steel for low-rise and mid-rise buildings has seen significant growth in the last two decades, where cold-formed steel members frame the walls, floors, and roof (including trusses). Along with this increase in applications has come research to support the new applications and new challenges to overcome.

The emphasis in this Chapter is on design and stability related to cold-formed steel construction. Stainless steel and aluminum members are covered in the last two sections of this Chapter. The first requirement for any thin-walled metallic member is to determine the elastic stability modes of the member: local, distortional, and global – this is covered in Section 13.1. Two design methods currently exist for cold-formed steel the classical Effective Width Method and the newly developed Direct Strength Method. These important design approximations are the focus of Sections 13.2 and 13.3 respectively. Other design approaches such as Reduced Stress, Effective Thickness, the Q-(or form-)factor approach, and more recently the Erosion of Critical Bifurcation Load (Ungureanu and Dubina 2004) are not detailed here. As the focus of

the presented design methods (Sections 13.2 and 13.3) is columns and beams, section 13.4 provides the additional stability and strength limit states that must also be considered for a successful cold-formed steel member. Much of the current research focuses on cold-formed steel systems, in Section 13.5 the stability and strength of such assemblies is discussed.

## 13.1 MEMBER STABILITY MODES (ELASTIC)

A distinguishing feature of thin-walled members is that cross-section stability must be considered in their design, as it often contributes to, or dominates, the observed behavior under load. This section presents and discusses the elastic stability modes of thin-walled members, namely: local, distortional, and global buckling. Historically, closed-form expressions have been employed by engineers in design, and today this trend continues, though some relief using computational methods is typically allowed. Given the historical importance of the closed-form expression for the stability modes Sections 13.1.2, 13.1.3, and 13.1.4 cover the analytical expressions in use for local, distortional, and global buckling, respectively. Finally, in Section 13.1.5 computational tools for thin-walled member stability, with particular emphasis on the finite strip method, are discussed.

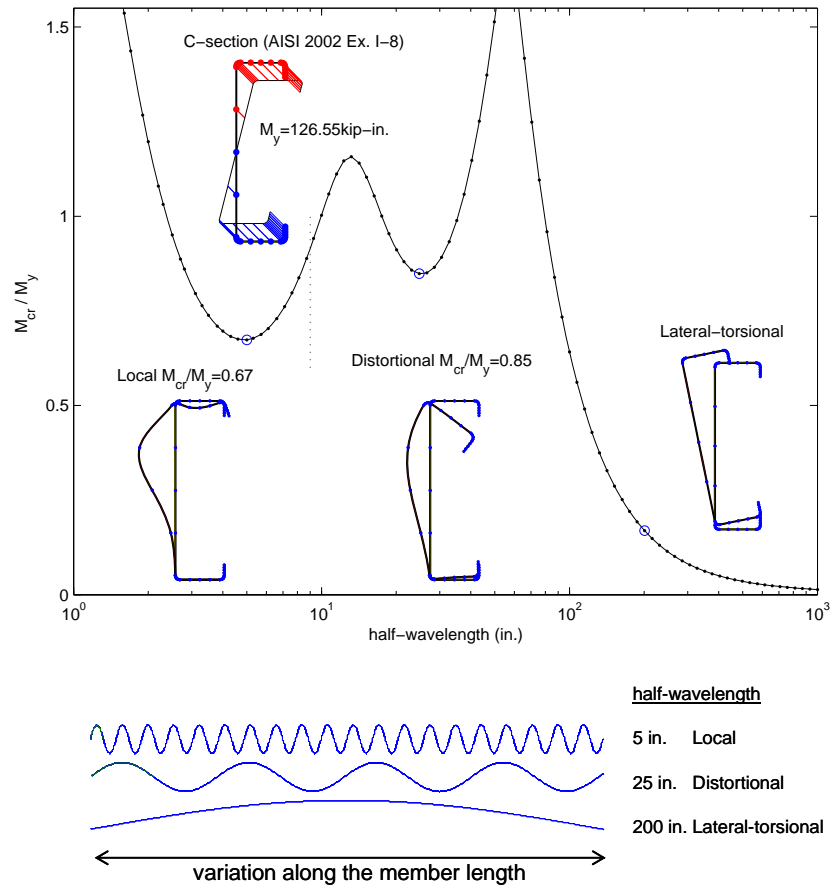
### 13.1.1 Local, distortional, and global buckling

Thin-walled members typically have at least three stability modes that are of interest in design: local, distortional, and global buckling. The AISI Specification (2007) provides definitions for the three buckling modes of a flexural member as follows: *Local Buckling*: buckling of elements only within a section, where the line junctions between elements remain straight and angles between elements do not change; *Distortional Buckling*: a mode of buckling involving change in cross-sectional shape, excluding local buckling, and *Lateral-torsional Buckling*: Buckling mode of a flexural member involving deflection out of the plane of bending occurring simultaneously with twist about the shear center of the cross-section..

As an example consider the three stability modes for a cold-formed steel lipped channel in bending as provided in Fig. 13.1. In the example, the local buckling moment is 67% of the yield moment ( $M_y$ ), and the buckling mode shape has a half-wavelength of 5 in., the distortional modes is 65% of  $M_y$  with a 25 in. half-wavelength, and the global mode is lateral-torsional buckling with only one half-wave along the length relevant. Given the magnitude of the buckling moments (significantly less than  $M_y$ ) all three buckling modes, and potentially their interactions, may be involved in the design of this member. This is typical in thin-walled member design.

Examination of the buckled shapes provided in Fig. 13.1 provides support for the AISI definitions; however particularly for distortional buckling, the provided definitions are limited in their application. From a practical standpoint the modes may often be identified by the characteristics of the buckled shape and their appearance at a given half-wavelength. Getting beyond heuristics, mechanics-based definitions of the three modes have been proposed and implemented in the context of generalized beam theory (Silvestre and Camotim 2002a; Silvestre and Camotim 2002b) and the finite strip method.(Adany and Schafer 2006a; Adany and Schafer 2006b). These methods provide the potential for automatic identification of the three primary buckling modes.

The “signature curve” of a thin-walled cross-section as given in Fig. 13.1 was pioneered and popularized by Hancock (see e.g. (Hancock 1978) and the figures in Chapter 4, Section 4.6). This “signature curve” of a cross-section has become an organizing principle for understanding the behavior of thin-walled sections. This curve was generated using the finite strip method and is discussed further in Section 13.1.5 below. Further, the implications of, and ease with which, such information can now be obtained was the motivating tool for a new design method: the Direct Strength Method, discussed in Section 13.3 below.



**Fig. 13.1 Normalized buckling moment vs. buckling mode half-wavelength for a lipped channel**

### 13.1.2 Local buckling via plate stability

As discussed at length in Chapter 4, the classical method for determining local stability of thin-walled cross-sections is to break the section into a series of plates. For a lipped channel in compression the procedure is illustrated in Fig. 13.2. The web (part A) is a stiffened element and may be idealized as a plate simply supported on all four sides. The plate buckling stress is:

$$f_{crl} = k \frac{\pi^2 E}{12(1-\nu^2)} \left( \frac{t}{w} \right)^2 \quad (13.1)$$

where  $k = 4.0$ , via case 1 of Fig. 4.2, and  $E$  is the Young's modulus,  $\nu$  the Poisson's ratio,  $t$  the thickness, and  $w$  the plate width. The lip (part C) is an unstiffened element and may be idealized as a simply supported plate with one longitudinal edge free,  $k = 0.425$ , via case 4 of Fig. 4.2.

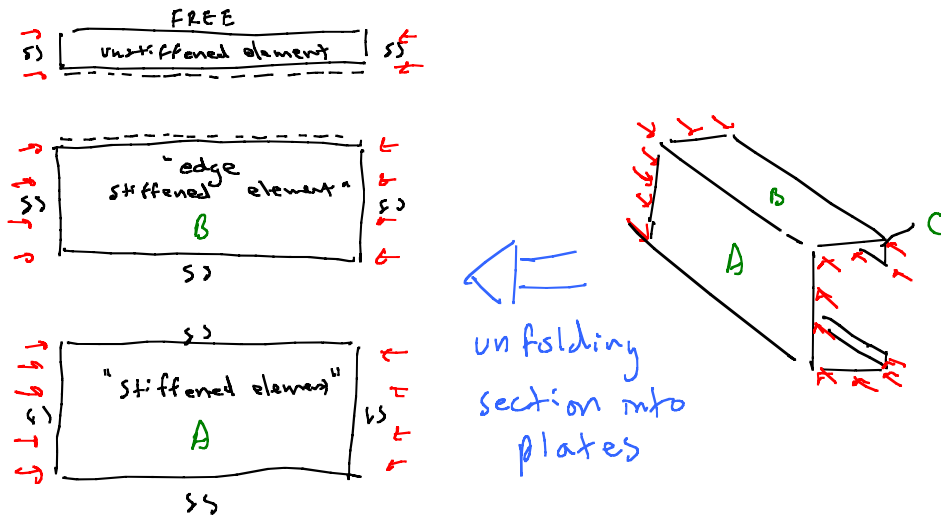


Fig. 13.2 Breakdown of a thin-walled cross-section into elements for plate buckling evaluation

The flange (part B) is an edge stiffened element and represents a potentially more complicated condition. For pure local buckling the buckling mode shape (see inset of Fig. 13.1) of the flange is similar to the web – and may be approximated as a plate simply supported on four sides with a  $k$  of 4.0. However, if the lip is short, then the edge stiffened element may behave more like an unstiffened element; hence the reason the flange is referred to as an “edge stiffened” element instead of simply a “stiffened” element. Treatment of this mode of buckling has varied significantly over the years, but today is known as distortional buckling (again, see inset buckled shapes of Fig. 13.1) and will be discussed in detail in the next Section.

If the lipped channel of Fig. 13.2 is under bending instead of compression then the plate buckling coefficients ( $k$ 's) must be suitably modified. Stiffened elements under stress gradients and unstiffened elements under stress gradients are handled by the expressions reported in Section 4.2.2 of Chapter 4 (Bambach and Rasmussen 2004; Peköz 1987). In addition, the influence of moment gradient on the local plate buckling may also be considered as reported in Section 4.2.7 of Chapter 4 (Yu and Schafer 2007a).

### Interaction of elements in local buckling

As first discussed in Section 4.6 of Chapter 4 the classical approach to local buckling using isolated plates violates equilibrium and compatibility of the cross-section. Consider for instance the cross-section of Fig. 13.1, but now under the simpler case of pure compression. The centerline dimensions (ignoring corner radii) are  $h = 8.94$  in. (227.1 mm),  $b = 2.44$  in. (62.00 mm),  $d = 0.744$  in. (18.88 mm), and  $t = 0.059$  in. (1.499 mm), the critical buckling stress,  $f_{cr}$  of each element using the classical plate stability approach are as follows:

$$\begin{aligned} \text{lip: } k &= 0.43, & f_{crl\text{-lip}} &= 0.43[\pi^2 E / (12(1-\mu^2))] (t/d)^2 = 72.1 \text{ ksi (497 MPa)} \\ \text{flange: } k &= 4, & f_{crl\text{-flange}} &= 4.0[\pi^2 E / (12(1-\mu^2))] (t/b)^2 = 62.4 \text{ ksi (430 MPa)} \\ \text{web: } k &= 4, & f_{crl\text{-web}} &= 4.0[\pi^2 E / (12(1-\mu^2))] (t/h)^2 = 4.6 \text{ ksi (32.0 MPa)} \end{aligned}$$

Each element predicts a different buckling stress, even though the member is a connected group. The high flange and lip buckling stresses have little relevance given the low web buckling stress. A finite strip analysis (see Section 13.1.5 for more on computational solutions), which includes the interaction amongst the elements, shows that the flange aids the web significantly in local

buckling, increasing the web buckling stress from 4.6 ksi (32.0 MPa) to 6.6 ksi (45.4 MPa), but the buckling stress in the flange and lip are much reduced due to the same interaction.

It is possible to approximate the local buckling stress as the minimum of the element buckling stresses, but this method is typically overly conservative as demonstrated in the preceding example and discussed further in Section 4.6 of Chapter 4. The nomographs and references of Section 4.6 of Chapter 4 provide a means to determine the local buckling stress including the interaction for many simple shapes. Analytical expressions for flange-web and flange-lip local buckling interaction do exist for beams (Schafer and Peköz 1999) and columns (Schafer 2002). However, by far the simplest method for including the interaction of the connected elements (plates) in local buckling is to use one of the computational methods discussed in Section 13.1.5.

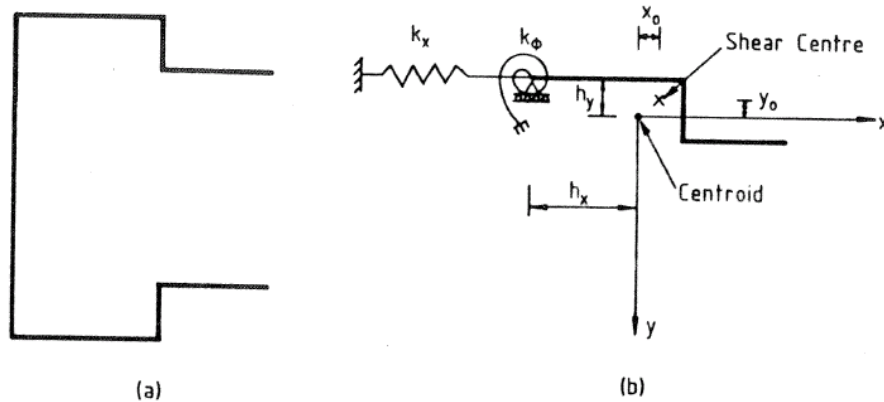
### ***13.1.3 Distortional buckling expressions***

#### ***Edge stiffeners***

Analytical models for the elastic critical distortional buckling stress (see Fig. 13.1) have proven to be relatively complicated. The deformations involved include both membrane deformations, primarily in the flange and lip, and bending deformations, primarily in the web. Desmond et al. (1981a) provided expressions for the plate buckling coefficient ( $k$ ) of the flange that are a function of the flange width and lip stiffener moment of inertia, and these are provided in AISI (2007). However, they have been shown to be poor predictor of the elastic distortional buckling stress (Schafer and Peköz 1999) and are only intended to be used in conjunction with specific effective width expressions provided (that is to say the  $k$ 's provided by Desmond et al. 1981 are not actually elastic buckling  $k$ 's, but empirically modified).

An alternative approach has been to account for the distortional mode of buckling as a compressed strut on an elastic foundation where the elastic foundation is represented by a spring that depends upon the bending stiffness of adjacent parts of plane elements and on the boundary conditions of the element. This procedure has been adopted in Eurocode 3, Part 1.3 (Eurocode 2004). The method accounts for the elastic restraint of all elements in the section, including the web by incorporation of their flexibility in the elastic spring restraint. A detailed discussion of this method applied to channel sections is given in Buhagiar et al. (1992). Another interesting alternative approach is the analytical application of Generalized Beam Theory to determining closed-formed expressions for distortional buckling (Silvestre and Camotim 2004a; Silvestre and Camotim 2004b; Silvestre and Camotim 2004c).

The analytical model in widest use is Lau and Hancock's (1987a) and is based primarily on the assumption that the flange acts as an isolated column undergoing flexural-torsional buckling, while the web provides elastic restraint to the flange, as shown in Fig. 13.3. This model was first considered by aluminum researchers (Sharp 1966) and subsequently improved to include more consistent treatment of the web by Lau and Hancock (1987a). Schafer and Peköz (1999) further improved this model to allow for the impact of applied stresses on the web's rotational stiffness; thus allowing for the case when distortional buckling is triggered by instability of the web as opposed to the flange. Teng et al. (2003) examined the method's application to beam-columns. Lau and Hancock's treatment is used in the Australian cold-formed steel standard (AS/NZS:4600 2005) and Schafer and Peköz's in the AISI (2007) Specification.



**Fig. 13.3 Lau and Hancock's model for distortional buckling: Flange elastically restrained along flange/web junction (a) original column section (b) isolated flange (Lau and Hancock 1987b)**

Treatment of the flange as a column requires that separate section properties ( $I$ ,  $J$ ,  $C_w$ , etc.) for the flange-lip components be calculated. These section properties are then used in the torsional-flexural buckling problem, which itself requires the solution to a quadratic equation (see Eq. 13.3). The involved nature of these calculations make computational solutions far more attractive, and indeed the AISI (2007) Specification makes it explicitly clear that rational elastic buckling analysis is allowed for in determining the elastic distortional buckling stress.

#### ***Intermediate stiffeners***

To a certain extent the elastic distortional buckling treatment of intermediate stiffeners has developed in a manner similar to edge stiffeners. Desmond provided a method for predicting  $k$  as a function of the plate width and stiffener moment of inertia (Desmond et al. 1981b) that was employed from the 1986-1996 versions of the AISI Specification. Eurocode employs the model of a compressed strut on an elastic foundation (Eurocode 2004). Based on the classical expressions for stiffened plates (See Section 4.4 of Chapter 4) AISI (2001) adopted new expressions for plates with intermediate stiffeners (Schafer and Peköz 1998a) that are provided in Section 4.4.1 of Chapter 4. The primary difference in expressions for elastic distortional buckling of intermediate stiffeners, as opposed to edge stiffeners, is that the interaction of the elements (e.g., web-flange) is typically ignored and the focus remains only on the element with the intermediate stiffeners (the classic stiffened plate).

#### ***Edge and intermediate stiffeners***

Analytical expressions for distortional buckling of sections with both intermediate and edge stiffeners are essentially too involved to be practical. The only potential exception to this is the case of single mode generalized beam theory solution – if the generalized beam theory cross-section parameters are known then the resulting analytical expressions are tractable (Silvestre and Camotim 2004c). However, even the calculation of the cross-section properties is essentially a computational method. Computational solutions for sections with edge and intermediate stiffeners pose no particularly unique problem for computational solutions and are discussed further in 13.1.5, and recommended for the design of such sections.

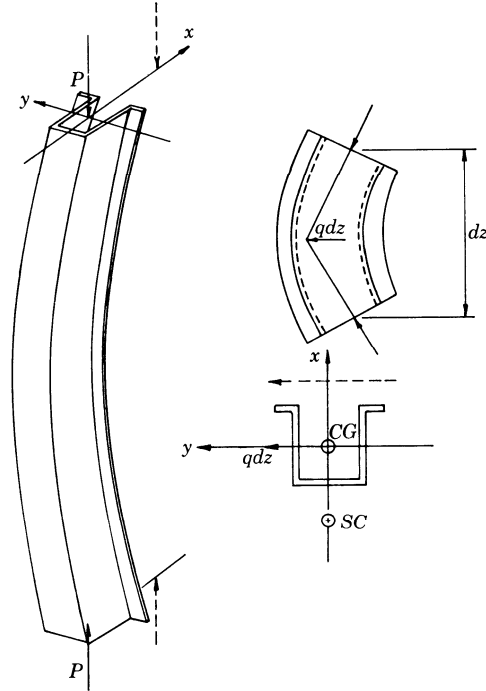
### ***13.1.4 Global/Flexural–Torsional buckling***

#### ***Columns***

Concentrically loaded columns can buckle by (1) flexure about one of the principal axes, (2) twisting about the shear center (torsional buckling), or (3) a combination of both flexure and twisting, called flexural–torsional buckling. Torsional buckling is a possible failure mode for point symmetric sections. Flexural–torsional buckling must be checked for open sections that are singly symmetric and for sections that have no symmetry. Open sections that are doubly symmetric or point symmetric are not subject to flexural–torsional buckling because their shear center and centroid coincide. Closed sections also are immune to flexural–torsional buckling. Flexural-torsional buckling is common in thin-walled construction and is thus the focus of the discussion here, see Chapter 3 for further discussion on flexural column buckling.

One can explain the nature of flexural–torsional buckling with the aid of Fig. 13.4. At buckling, the axial load can be visualized to have a lateral component ( $qdz$ ) as a consequence of the column deflection. The torsional moment of this lateral component about the shear center of the open section shown in Fig. 13.4 causes twisting of the column. The degree of interaction between the torsional and flexural deformations determines the amount of reduction of the buckling load in comparison to the flexural buckling load. Therefore, as the distance between the shear center and the point of application of the axial load increases, the twisting tendency increases and therefore the flexural–torsional buckling load decreases. Flexural–torsional buckling can be a critical mode of failure for thin-walled open sections because of their low torsional rigidity. The theory of elastic flexural–torsional instability is well developed (Goodier, 1942; Vlasov, 1959; Timoshenko and Gere, 1961; Galambos, 1968). Flexural–torsional buckling of singly symmetric thin-walled open sections under concentric and eccentric loading also has been studied in detail, and design aids have been devised (Klöppel and Schardt, 1958; Pfluger, 1961; Chajes and Winter, 1965; Chilver, 1967; Peköz, 1969; Peköz and Winter, 1969). The AISI specification since 1980 has contained flexural–torsional buckling provisions based on the work of Chajes et al. (1966), Peköz (1969), and Peköz and Winter (1969).





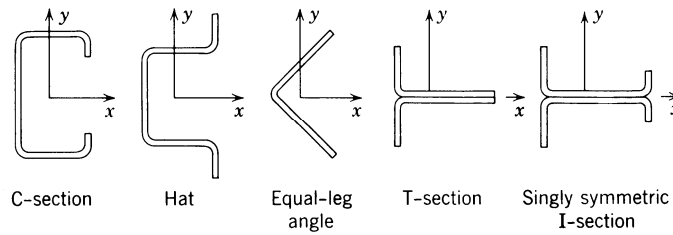
**Fig. 13.4 Forces causing flexural-torsional buckling.**

Differential equations of equilibrium for the general case of biaxial eccentricities have been solved by Thurlimann (1953), Vlasov (1959), Dabrowski (1961), Prawel and Lee (1964), Culver (1966), and Peköz and Winter (1969) using different procedures of solution. If the section is singly symmetric, such as the sections shown in Fig. 13.5, and is acted on by an axial load not in the plane of symmetry; or if the section is not symmetric, the solution of the differential equations indicates that as the axial load increases the member continuously twists and deflects biaxially. The principal axes, twist angle  $\phi$  and deflections  $u$  and  $v$  are shown in Fig. 13.6. Analogous to small deflection flexural beam-column theory, infinite deflections and rotation are predicted for a certain value of the axial load.

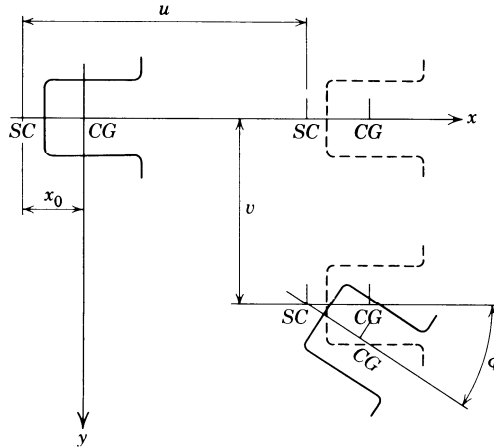
However, if the section is singly symmetric and the axial load is applied through the centroid, the behavior of the member is described by three homogeneous differential equations, two of which are coupled. If the member is assumed to be hinged at both ends, namely,  $u'' = v'' = \phi' = 0$ , the solution of the one uncoupled equation gives the critical load for buckling in the direction of the symmetry axis (taken here as the  $x$ -axis):

$$P_{ye} = K_{11} EI_y \frac{\pi^2}{L^2} \quad (13.2)$$

where  $I_y$  is the moment of inertia about the  $y$ -axis and  $L$  is the length of the column.  $K_{11}$  and other  $K$  values determined by the Galerkin method for various boundary conditions are given by Peköz (1969). The discussion here will be limited to hinged ends.



**Fig. 13.5 Some singly symmetric sections and coordinate axis orientation.**



**Fig. 13.6 Principal axes and deflection components.**

The two coupled equations describing deformations  $v$  and  $\phi$  result in a single buckling load  $P_{TF}$  for the flexural–torsional mode. The same buckling mode also occurs in the more general case of the load acting eccentrically in the plane of symmetry. Then the member continuously deflects as a beam–column in the plane of symmetry ( $x$ -direction), but is subject to flexural–torsional buckling out of this plane under load  $P_{TF}$  given in this case by Eq. 13.3. (The solution for a concentric load is obtained by setting  $e_s = 0$  in this equation.)

$$P_{TF} = \frac{(P'_{\phi e} + \alpha P_{xe}) \pm \sqrt{(P'_{\phi e} + \alpha P_{xe})^2 - 4\gamma P_{xe} P'_{\phi e}}}{2\gamma} \quad (13.3)$$

where

$$\alpha = 1 + e_x \beta_2 \frac{A}{I_0} \quad (13.4)$$

$$\gamma = \frac{A}{I_0} (x_0 - e_x)^2 + e_x \beta_2 \frac{A}{I_0} + 1 \quad (13.5)$$

$$P'_{\phi e} = P_{\phi e} \alpha \quad (13.6)$$

$$P_{\phi e} = \frac{A}{I_0} \left( EC_w \frac{\pi^2}{L^2} + GJ \right) \quad (13.7)$$

$$P_{xe} = EI_x \frac{\pi^2}{L^2} \quad (13.8)$$

$$\beta_2 = \frac{1}{I_y} \left( \int_A x^3 dA + \int_A xy^3 dA \right) - 2x_0 \quad (13.9)$$

and where

$e_x$  = eccentricity with respect to the center of gravity

$X_0$  =  $x$ -coordinate of the shear center

$I_x$  = moment of inertia about the  $x$ -axis

$I_0$  = polar moment of inertia about the shear center

$A$  = area of the cross section

$C_w, J$  = warping and St.-Venant torsional constants for the cross section, respectively.

The parameter  $P_{\phi e}$  has the physical meaning that it is the concentric torsional buckling load if the displacements  $u$  and  $v$  are prevented,  $P'_{\phi e}$  is the corresponding value for eccentric loading, and  $P_{xe}$  designates the load for buckling in the direction of the  $y$ -axis if displacements  $\phi$  and  $u$  are prevented. A simplified expression for  $P_{TF}$  is employed in the AISI Specification.

### Beams

For doubly- and mono-symmetric sections global lateral-torsional buckling is discussed in Chapter 5. For thin-walled metal construction, the critical stress for lateral buckling of an I-beam having unequal flanges can be determined by the following formula (Winter, 1943, 1970):

$$\sigma_c = \frac{\pi^2 ED}{2S_{xc} L^2} \left( I_{yc} - I_{yt} + I_y \sqrt{1 + \frac{4GJL^2}{\pi^2 EI_y d^2}} \right) \quad (13.10)$$

where

$S_{sc}$  = compressive section modulus of the entire section about the major axis

$I_{yc}, I_{yt}$  = moments of inertia of the compression and tension portion, respectively, of a section about its centroidal axis parallel to the web

$E$  = modulus of elasticity

$G$  = shear modulus

$J$  = torsional constant of the section

$d$  = depth of the section

$L$  = unbraced length

For thin-walled steel sections, the first term under the radical in Eq. 13.10 usually exceeds the second term by a considerable margin (Winter, 1947). If the second term is omitted and considering that  $I_y = I_{yc} + I_{yt}$ , the following equation can be obtained for determination of critical stress for lateral buckling in the elastic range:

$$\sigma_c = \pi^2 EC_b \left( \frac{dI_{yc}}{L^2 S_{sc}} \right) \quad (13.11)$$

where  $C_b$  is a bending coefficient to account for moment gradient (see Chapter 5 or the AISI Specification). In the AISI Specification the lateral buckling strength for I-section beams in the elastic range is based on Eq. 13.11. For Z-section beams, Eq. 13.11 is divided by 2, a conservative approximation to the actual lateral-torsional buckling stress of a Z.

For singly-symmetric sections (Fig. 13.5) the torsional-flexural buckling solution of the previous section can be extended. When there is no axial load, the Galerkin method solution (Peköz, 1969) of the general differential equations of equilibrium gives the following expression for the critical moments,  $M_{CR}$ :

$$M_{CR} = -\frac{P_e \beta_2}{2} \left( 1 \pm \sqrt{1 + \frac{4I_0 R}{\beta_2^2 A}} \right) \quad (13.12)$$

where  $R = P_{\phi e}/P_{xe}$ . For singly and doubly symmetric section bending about the symmetry axis perpendicular to the web and introducing the notation used in the AISI Specification (as opposed to Peköz 1969) including the moment gradient  $C_b$  factor, then the elastic critical lateral-torsional buckling moment may be expressed as:

$$M_{cre} = C_b r_0 A \sqrt{\sigma_{ey} \sigma_t} \quad (13.13)$$

where

$$r_0 = \sqrt{r_x^2 + r_y^2 + x_0^2} \quad (13.14)$$

$$\sigma_{ey} = \frac{\pi^2 E}{(K_y L_y / r_y)^2} \quad (13.15)$$

$$\sigma_t = \frac{GJ + \pi^2 EC_w / (K_t L_t)^2}{Ar_0^2} \quad (13.16)$$

and where

$r_x$  and  $r_y$  = radii of gyration of the cross section about the centroidal principal axis;  
 $x_0$  = distance from shear center to centroid along principal  $x$ -axis, taken as negative;  
 $A$  = the full cross-sectional area.

$K_y, K_t$  = effective length factors for bending about the  $y$ -axis and for twisting

$L_y, L_t$  = unbraced length for bending about the  $y$ -axis and for twisting

For laterally unbraced hat sections bent about the  $x$ -axis, no stress reduction is necessary if  $I_y > I_x$ , because there is no tendency to buckle. When  $I_y < I_x$  a conservative estimate of the critical elastic stress may be determined by regarding the compression portion of the section as an independent strut, which gives

$$\sigma_e = \frac{\pi^2 E}{(L/r_y)^2} \quad (13.17)$$

where  $r_y$  is the radius of gyration about the vertical axis of that portion of the hat section which is in compression. A more accurate analysis for such hat-shaped sections and for any other singly symmetric section is to use the equations given in Chapter 5. This is the approach required in the AISI specification for the design of singly symmetric section. The AISI Design Manual (2002) and Yu (2000) provide design aids for calculation of the necessary section properties for

common thin-walled shapes. However, due to the involved nature of the preceding expressions it is not uncommon for computational tools to be employed to (a) determine the necessary section properties and (b) implement and provide solutions for  $P_{TF}$  and  $M_{CR}$ .

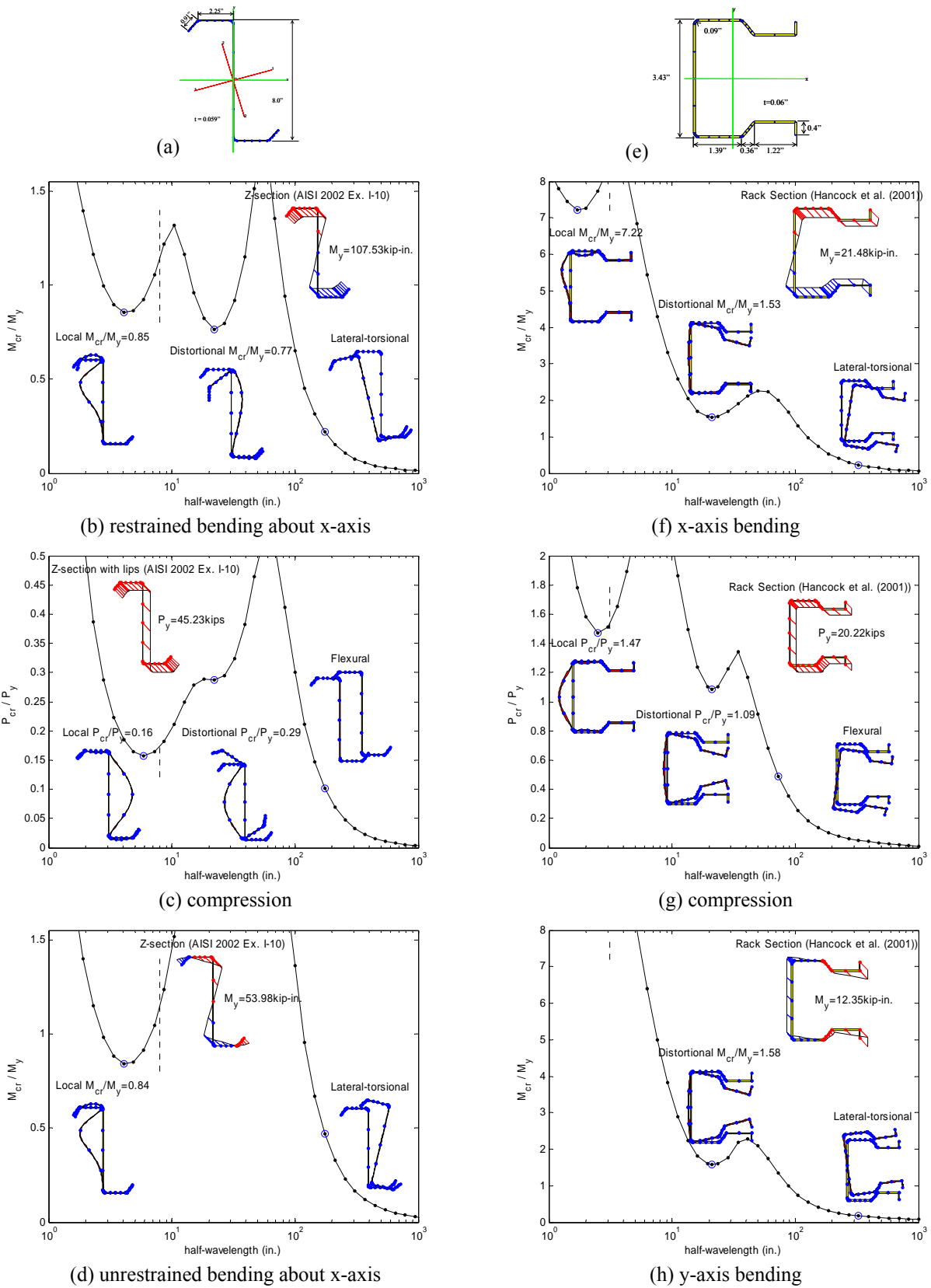
### ***13.1.5 Computational elastic buckling solutions***

Complete analytical expressions for local, distortional, and global buckling exist, as detailed in the previous sections, but even though the underlying mechanics is relatively straight forward (this is after all just elastic bifurcation buckling, no large deformations or inelasticity) the resulting expressions are significantly involved. Today, computational solutions offer a powerful alternative – particularly for elastic buckling where the solution sensitivity is not so great. A variety of numerical methods: finite element, finite differences, boundary element, generalized beam theory, finite strip analysis, and others, provide accurate elastic buckling solutions for thin-walled beams and columns. (See also Chapter 21 for additional discussion.)

Traditional finite element analysis using thin plate or shell elements may be used for elastic buckling prediction. Due to the common practice of using polynomial shape functions, the number of elements required for reasonable accuracy can be significant. Finite element analysis books such as Cook et al. (1989) and Zienkiewicz and Taylor (1989, 1991) explain the basic theory; while a number of commercial implementations can provide accurate elastic buckling answers if implemented with a modicum of care. Finite difference solutions for plate stability are implemented by Harik et al. (1991) and others. The boundary element method may also be used for elastic stability (Elzein, 1991). Generalized beam theory, developed originally by Schardt (1989, 1994) with contributions from Davies et al. (1994) and Davies and Jiang (1996, 1998) and significant extensions in recent years by Silvestre and Camotim (2002a, 2002b) who have also recently provided a free user-friendly software implementation (Bebiano et al. 2008).

Finite strip analyses, such as presented in Fig. 13.1 and Fig. 13.7, are a specialized variant of finite element analysis. For elastic stability of thin-walled sections, finite strip is one of the most efficient and popular methods. The specific version of the finite strip method which can account for both plate flexural buckling and membrane buckling in thin-walled members was developed by Plank and Wittrick (1974). Cheung and Tham (1998) explain the basic theory. Hancock and his collaborators (see Hancock et al., 2001 for full references and descriptions) pioneered the use of finite strip analysis for stability of cold-formed steel members and convincingly demonstrated the important potential of finite strip analysis in both cold-formed steel design and behavior. Consider, for example the results of Fig. 13.7: local, distortional, and global buckling loads (or moments) along with the corresponding mode shapes are identified in the figure. All of the instabilities that need to be considered in basic thin-walled member design of these members are presented in one compact result.

Finite strip analysis is a general tool that provides accurate elastic buckling solutions with a minimum of effort and time. Finite strip analysis, as implemented in conventional programs, does have limitations, the two most important ones are: the model assumes the ends of the member are simply supported, and the cross-section may not vary along its length. These are not limitations of the method per se, but rather the implementations which are commonly available (Papangelis and Hancock 1995; Schafer and Adany 2006). Despite these limitations the tool is useful, and a major advance over plate buckling solutions and plate buckling coefficients ( $k$ 's) that only partially account for the important stability behavior of thin-walled members.



**Fig. 13.7 Example finite strip analysis results for local, distortional, and global buckling of (a)-(d) Z-section under different loading conditions and (e)-(h) rack post section under different loading conditions**

## **13.2 EFFECTIVE WIDTH MEMBER DESIGN**

Since the first cold-formed steel specification the concept of effective width reductions to account for local buckling and post-buckling strength has been central to capacity determination of members. Until the 1986 Specification in addition to effective width, reduced stress, and form factor (or Q-factor) approaches were also in use. In 1986 reduced stress methods were replaced with effective width methods for webs and unstiffened elements, and the Q-factor approach for local-global interaction was replaced with a novel extension to the effective width approach, under what is now known as the unified Effective Width Method (Peköz 1987).

Today the AISI Specification (2007) provides two alternative procedures for strength determination of cold-formed steel members: the unified Effective Width Method in the main body of the Specification, and the Direct Strength Method in Appendix 1 of the Specification. There is significant overlap between the two approaches. Further, not all material in the main body of the Specification directly uses the concept of effective width. This Section provides an examination of beams and columns by the Effective Width Method, while the following Section examines the Direct Strength Method.

The focus of this section is to provide the Effective Width Method solution to the strength of columns and beams which are subject to the member elastic stability modes addressed in the Section 13.1. Namely, how does one determine the strength in local, distortional, and global buckling – as well as the potential interactions amongst these modes? Local buckling, which is the focus of the effective width method is treated first, followed by global buckling, local-global interaction, and finally distortional buckling. Comparison between the Effective Width Method and the Direct Strength Method are provided in Section 13.3

### ***13.2.1 Local buckling strength***

#### ***Columns***

The role of local buckling in cold-formed steel columns has been studied since the 1940's with Winter (1949) summarizing U.S. contributions (e.g., Winter 1940, Winter 1943) and Chilver (1951, 1953) and Harvey (1953) work in the U.K. After sixty years of progress, modern column research is still similar to Chilver's work: elastic stability solutions for local plate buckling and "effective width" for the ultimate strength. The elastic plate buckling solution of Chilver and Harvey was based on Lundquist and Stowell (1943) who extended the work of Timoshenko and Gere (1936) by providing practical methods for calculating the stability of connected plates. The "effective width" solution was based on von Kármán et al. (1932) and the experimental corrections of Winter (1947). Notably, both Chilver and Harvey properly included the interaction of elements in determining the local buckling stress.

The basic premise of the effective width method for local buckling is to reduce each plate comprising a cross-section to an effective plate (Fig. 13.8). The effective plate is an approximation of the longitudinal normal stress distribution in the real plate, where the effective plate can carry the full applied stresses, but only in the effective portions. This method has long been used for flat plates in compression and is fully detailed in Section 4.3.3.

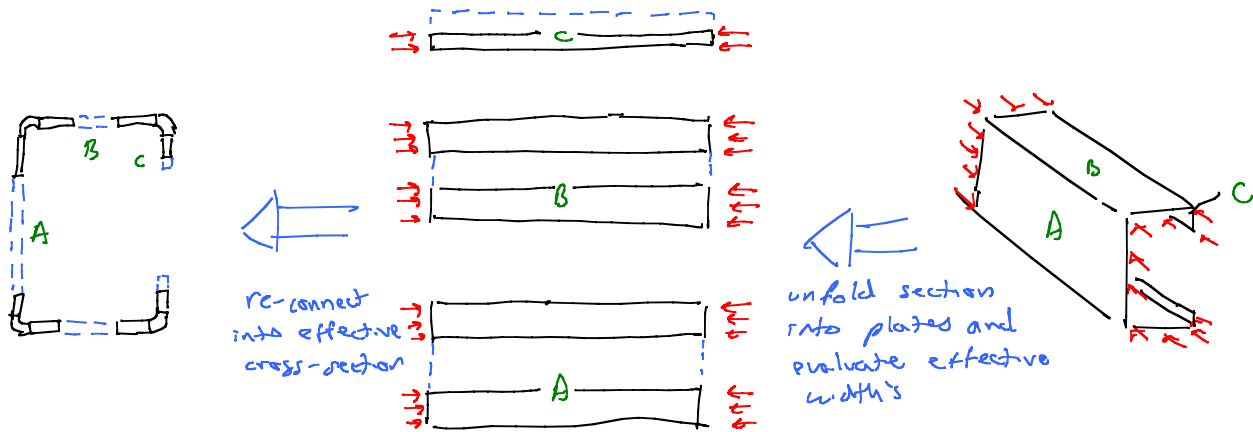


Fig. 13.8 Lipped channel : effective section, effective width of component plates, gross section

The actual expressions used to determine the effective width,  $b$ , of a given element with gross width,  $w$ , (i.e. see Fig. 13.2 or Fig. 13.8 for typical elements of a cross-section) are given, for instance, by AISI (2007) as:

$$b = \begin{cases} w & \text{when } \lambda \leq 0.673 \\ \rho w & \text{when } \lambda > 0.673 \end{cases} \quad (13.18)$$

where

$$\rho = (1 - 0.22 / \lambda) / \lambda \quad (13.19)$$

$$\lambda = \sqrt{f / f_{cr}} \quad (13.20)$$

$$f_{cr} = k \frac{\pi^2 E}{12(1 - \nu^2)} \left( \frac{t}{w} \right)^2 \quad (13.21)$$

and  $f$  is the uniform stress in the effective portions ( $f = f_y$  for the maximum plate or section capacity),  $k$  is the plate buckling coefficient,  $E$  is the modulus of elasticity,  $\nu$  is Poisson's ratio, and  $t$  is the plate thickness. The expression for  $\rho$  is identical to Eq. 4.30. The key step in the implementation of this effective width is the determination of,  $k$ , which is detailed in Section 4.2. The 5<sup>th</sup> Ed. of this Guide provides further information on minor changes in formatting with regard to the presentation of the effective width expression over the years in the AISI Specification, and Section 4.3.3 provides the complete history of the development of this expression.

The summation of the effective plates also creates an effective cross-section, as shown in Fig. 13.8. This effective cross-section provides an initial means to understand why local-global interaction is so important in thin-walled members: if the moment of inertia of the section is effectively reduced, then so must the global flexural buckling stress, more on this in Section 13.2.3. If the applied stress on the elements,  $f$ , is set to the yield stress,  $f_y$ , then the effective cross-section is an estimate of the stiffness at collapse. If the applied stress on the elements,  $f$ , is set to the stresses on the section under service loads, then the effective cross-section is an estimate of the stiffness for serviceability considerations.

### Beams

Use of effective width expressions for beams actually came significantly later than that for columns and was initiated by an extensive experimental program (LaBoube and Yu 1982). The traditional approach, prior to the advent of the Unified Effective Width Method (Peköz 1987)



was to use effective width for compression flanges and average stress for webs. The unified effective width method provided expressions for the effective width of webs (i.e., elements under a stress gradient ) based on Cohen and Peköz (1987) that were adopted by AISI. During development of the North American version of the cold-formed steel specification (AISI 2001) it was determined that significant differences existed between the methods adopted from Cohen and Peköz (1987) – the U.S. (AISI 1996) adopted a more liberal set of effective width expressions than Canada’s (CSA:S136 1994) though both were from the same source document. A comprehensive study comparing strength predictions with test data demonstrated unconservative predictions for AISI (1996) with members with tall webs and narrow flanges (Schafer and Trestain 2002) and as a result the AISI (1996) method was adopted for  $h/b$  less than 4 and the CSA:S136 (1994) method for  $h/b > 4$ .

A complication of the effective width method applied to beams is the necessity for iteration. The gross neutral axis is unlikely to be at the same location as the effective neutral axis once effective width’s have been introduced into all the plate elements. Since the effective width of the web is a function of the neutral axis location, iteration is required. Complete design examples are provided in texts (Yu 2000, Hancock et al. 2001) and the AISI Design Manual (AISI 2002).

### **13.2.2 Global buckling strength**

The global buckling strength of cold-formed steel columns and beams is generally treated in two steps. First, the strength without consideration of local or distortional buckling is determined. Then, the strength considering interactions is examined. Since cold-formed steel sections often offer torsionally weak, open profiles, and singly-, point-, and un-symmetric sections are common, even the basic (compact cross-section) global buckling strength can be relatively involved. The basic global buckling strength is not dependent on the choice of Effective Width Method or the Direct Strength Method. Nonetheless, since global buckling strength is found in the main body of the AISI Specification (as is the Effective Width Method) the material is presented in this Section.

#### **Columns**

The global buckling strength of thin-walled cold-formed steel columns (via AISI) uses the same simplification of SSRC curve 2P (Chapter 3) for all columns as AISC (2005). That is the global buckling strength (inelastic or elastic) is expressed at stress  $F_n$  as follows:

$$\text{For } \lambda_c \leq 1.5 \quad F_n = 0.658^{\lambda_c^2} F_y \quad (13.22)$$

$$\text{For } \lambda_c > 1.5 \quad F_n = (0.877 / \lambda_c^2) F_y = 0.877 F_e \quad (13.23)$$

where

$$\lambda_c = \sqrt{F_y / F_e} \quad (13.24)$$

and  $F_y$  is the yield stress and  $F_e$  is the least of the elastic flexural, torsional and flexural-torsional buckling stress of the section. Eq’s 13.22 and 23 were shown to provide adequate strength predictions when originally adopted (Peköz 1987) as well as in more recent studies (Schafer 2002). This is somewhat surprising given the vastly different state of initial imperfections and residual stresses in cold-formed steel members (Moen et al. 2008; Schafer and Peköz 1998b; Weng and Pekoz 1990) when compared with hot-rolled steel members as well as the prevalence of flexural-torsional limit states in cold-formed steel members as opposed to flexural limit states in hot-rolled steel members. However, as Fig. 3.22 illustrates a single column curve is by

definition a crude instrument when compared against real data, so the choice of the same column curve for hot-rolled (AISC) and cold-formed (AISI) steel is as much a matter of convenience on the part of AISI as demonstration of theoretical agreement.

Suggesting that global buckling strength begins and ends with the selection of an empirical column curve is to miss the real complexities that underlie the behavior of such sections; particularly for the common singly symmetric sections undergoing flexural-torsional buckling. For the inelastic domain, approximate approaches have been developed and adequately substantiated by tests (Chajes et al., 1966). The AISI specification (since 1980) contains flexural–torsional buckling provisions based on the work of Chajes and Winter (1965), Peköz (1969), and Peköz and Winter (1969).

It is assumed that for the thin-walled sections in question, the attainment of the yield stresses represents the limit of load-carrying capacity; that is, the plastic reserve capacity, if any, is negligible in torsional-flexural buckling. This point has been verified experimentally (Peköz 1969; Peköz and Winter, 1969). Therefore, elastic flexural torsional buckling is a possible mode of failure only if the axial load  $P_{yd}$  that causes incipient yielding (e.g., as predicted by the secant formula) is larger than  $P_{TF}$  (Eq. 13.3). Extensive numerical studies were carried out on a variety of singly symmetric open sections and are reported by Peköz (1969). Fig. 13.9 is a typical sample of the plots given in that reference that illustrates the complex behavior of such compression members. For positive eccentricities, numerical studies indicate that both yielding and instability need to be considered. The following expression is shown to give very satisfactory results:

$$\frac{1}{P_{TFO}} + \frac{e_x}{M_{CR+}} = \frac{1}{P_{TF}} \quad (13.25)$$

in which  $P_{TF}$  is the flexural–torsional buckling load for an eccentricity  $e_x$ ;  $P_{TFO}$  the flexural–torsional buckling load for concentric loading, regardless of whether it is the governing mode; and  $M_{CR+}$  the positive critical moment when there is no axial load, regardless of whether it is the governing mode (see Eq. 13.12). With the aid of charts given by Peköz (1969) for computing  $P_{TFO}$  and  $M_{CR+}$ , this equation is much more convenient to use than Eq. 13.3.

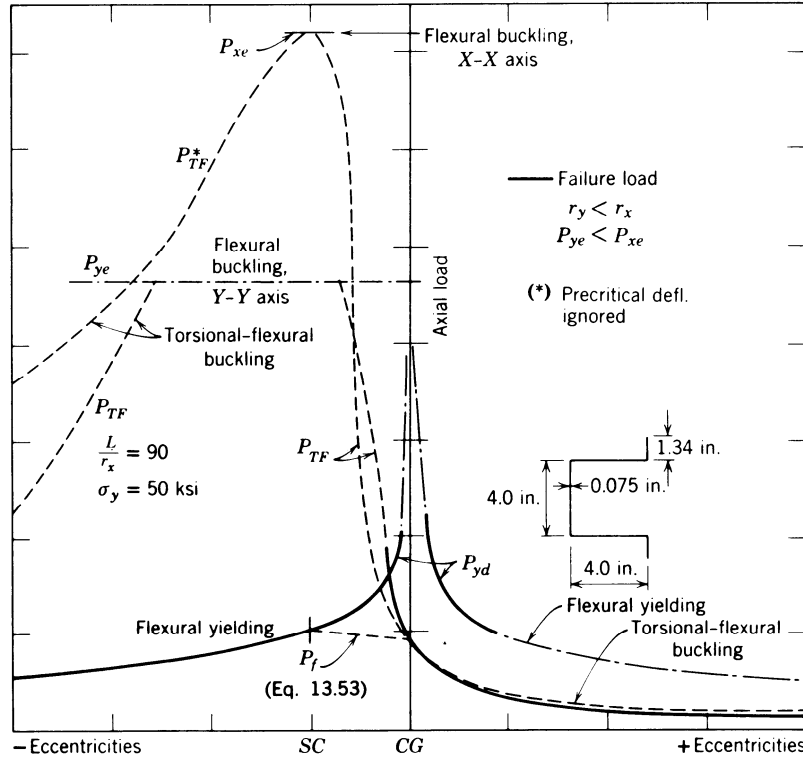


Fig. 13.9 Failure modes and loads (Peköz, 1969).

For negative eccentricities greater than  $x_0$ —that is, if the point of application of the axial load is on the side of the shear center opposite from the center of gravity—numerical studies on hat, channel, lipped channel, angle, and lipped angle sections of typical dimensions and yield stresses below 50 ksi (345 MPa) indicate that flexural–torsional buckling is not a governing mode of failure. For such eccentricities, these members fail by yielding after deflecting in the direction of the symmetry axis as a beam-column. However, for singly symmetric I-sections both yielding and flexural–torsional buckling need to be investigated. For these sections, the following interaction equation may be used:

$$\frac{1}{P_{xe}} + \frac{e_x - x_0}{M_{CR-}} = \frac{1}{P_{TF}} \quad (13.26)$$

in which  $M_{CR-}$  is the negative critical moment when there is no axial load, regardless of whether it is the governing mode (see Eq. 13.12).

If a section when concentrically loaded can fail in flexural–torsional buckling, then flexural–torsional buckling is also a possible mode of failure for some range of eccentricities between the centroid and the shear center. It is seen from Fig. 13.9 that in this region, between shear center and centroid, the two branches of the failure curve (yielding on the left and flexural–torsional buckling on the right) show a definite and sharp peak. This means that small changes or inaccuracies in eccentricity can produce large reductions in load capacities. For this reason it seems reasonable and conservative, in design, to disregard the uncertain high carrying capacity in the region of the peak, and to base design values on the dashed straight cutoff shown in Fig. 13.9. In this range of eccentricities, the following linear interpolation formula between the axial load,  $P_S$ , applied at the shear center, which causes yielding or buckling and the concentric

flexural–torsional buckling load,  $P_{TFO}$ , gives a realistic and conservative flexural–torsional buckling load,  $P_F$ .

$$P_F = P_{TFO} + \frac{e_x}{x_0} (P_S - P_{TFO}) \quad (13.27)$$

For singly symmetric I-sections,  $P_S$  is the smaller of the yield load  $P_{yd}$  or the buckling load  $P_{xe}$ , whereas for the other open sections, only yielding need be considered, as explained previously. In addition to the points discussed in the preceding section, the following need to be considered in the design of thin-walled members to resist flexural–torsional buckling.

First is the inelastic stability behavior for members of relatively low slenderness ratios. Chajes et al, (1966) studied this problem and reported that an expression similar to SSRC curve 2P is satisfactory for concentric flexural–torsional buckling. The AISI specifications reflect this approach for both concentric and eccentric loading.

Second is the frequent case of unequal eccentricities at opposite ends of the member. Peköz (1969) presents the results of an extensive study of this subject and makes the conclusion that application of a modification factor,  $C_{TF}$ , to the second term of Eq. 13.25 is quite accurate. The value  $C_{TF}$  is the same as  $C_m$  discussed in Chapter 8, except that it does not have 0.4 as its lower limit.

Third, the influence of pre-critical beam-column deflections on the flexural–torsional buckling load is an important consideration. Again, on the basis of an analytical and experimental treatment of the subject, Peköz (1969) recommends the use of an amplification factor  $1/(1 - P/P_{ye})$  for the moments.

Fourth, is the wandering centroid problem where centrally loaded, singly symmetric columns become beam-columns upon local buckling and the shifting of the neutral axis. In an extensive statistical study, Peköz (1987) established good correlation with test results if a concentrically loaded column is defined as a member loaded through its effective centroid. The effective centroid is calculated at the reduced column stress  $F_n$  from Eq. 13.22-23. This approach is used in the AISI Specification.

Fifth, is the behavior of biaxially loaded beam-columns. Prior to 1986 the AISI specification did not permit the calculation of singly symmetric beam-columns bending about the symmetry axis. The designer had to resort to tests. Based primarily on the work of Loh (1985), Mulligan and Peköz (1983), and Peköz (1987), the AISI Specification determines the capacity of biaxially loaded open sections using an interaction equation with eccentricities measured from the effective centroid. (The method is similar to procedures adopted previously by the RMI (1979) standard and followed in the CSA (1989) standard.) Lipped channel sections used in the endwalls of metal buildings and the columns in industrial storage racks are a few examples of members subjected to such loads.

### **Beams**

The lateral-torsional buckling strength uses an empirical expression, similar in spirit to the column curve: consisting of yield plateau, inelastic buckling regime, and elastic buckling regime: the strength is expressed as an extreme fiber stress,  $F_n$ , that may be carried in the three regimes:

$$\text{For } F_e \geq 2.78F_y \quad F_n = F_y \quad (13.28)$$

$$\text{For } 0.56F_y < F_e < 2.78F_y \quad F_n = \frac{10}{9} F_y \left( 1 - \frac{10F_y}{36F_e} \right) \quad (13.29)$$

$$\text{For } F_e \leq 0.56F_y \quad F_n = F_e \quad (13.30)$$

where  $F_y$  is the yield stress and  $F_e$  is elastic lateral-torsional buckling stress. The stress  $F_n$  may be expressed in terms of a moment by multiplying by the gross section modulus. The solid curve in Fig. 13.10 shows the variation of this moment,  $M (S_g F_n)$ , with the unbraced length. The three regimes: yielding (Eq. 13.28), inelastic buckling (Eq. 13.29), and elastic buckling (Eq. 13.30) are clearly delineated.

For a given  $L/\sqrt{dI_{yc}}$  ratio, a Z-section (or any point-symmetric section) will buckle laterally at a lower stress than will an I-beam (doubly-) or a channel (singly-symmetric) section. A conservative design approach has been used in the AISI specification since (1986), in which the critical moments for Z-sections in the elastic range are one-half of those permitted for I-beams or channels at the same  $L/\sqrt{dI_{yc}}$  ratio. The lateral buckling curve for Z-shaped beams is shown as the dashed line in Fig. 13.10. Functionally, this is enabled in the AISI Specification by using  $\frac{1}{2}F_e$  for the lateral-torsional buckling stress of point-symmetric sections.

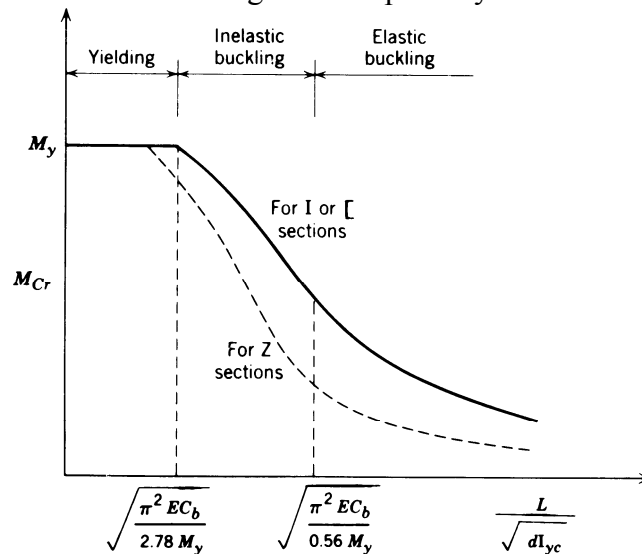


Fig. 13.10 Allowable compressive stress for lateral buckling of beams (Winter, 1970)

Fig. 13.10 shows a yield plateau at the yield moment,  $M_y$ , as opposed to the fully plastic moment  $M_p$  (common in locally stable cross-sections). This is not a completely accurate representation of inelastic reserve capacity in thin-walled sections. If the cross-section can sustain the larger strains associated with capacities greater than  $M_y$  then larger capacities are possible. Inelastic reserve ( $M > M_y$ ) is treated as a strength reserve in local buckling, and provisions are provided in the AISI Specification on an element basis for stiffened elements (Yener and Pekoz 1985) and unstiffened elements (Bambach and Rasmussen 2004).

### 13.2.3 Local-global interactive strength

#### Columns

Although local buckling may have significant post-buckling reserve the deformations and re-distribution of stresses that are associated with that reserve change the global buckling response of the member (column or beam). Study of this local-global interaction has included experimental testing and nonlinear finite element analysis using shell element models.

Recognizing and accounting for local-global interaction in the design of thin-walled members was one of the fundamental steps towards making thin-walled construction practical.

Results of early studies on interaction between local and global buckling were presented by Bijlaard and Fisher (1952a,b). Across the world, column research in the 1970's focused on the interaction between local and overall (i.e, global – flexural, torsional, flexural-torsional) buckling modes (DeWolf 1974, Klöppel and Bilstien 1976, Rhodes and Harvey 1977, Peköz 1977, Loughlan 1979). In the 1980's Hancock (1981) and Sridharan and Benito (1984) investigated the interaction problem using the finite strip method. Muliigan (1983) specifically examined local buckling interaction. In Europe, researchers continued to provide strong evidence for interaction of local and overall column buckling (Batista et al. 1987, Rhodes and Loughlan 1980, Zaras and Rhodes 1987). More recently Rasmussen and Hancock (1991) showed the importance of different end fixity on the post-buckling behavior and Young (1997) experimentally demonstrated that fixed ended columns do not suffer the same interaction problems as pin ended columns.

The first widely used design method, and the one still employed in the AISC (2005) Specification, is the form factor, or Q-factor, approach. The 3<sup>rd</sup> Ed. of this Guide provides a detailed summary of this method. The basic idea is to increase the long column (global) slenderness used in the column strength curve ( $\lambda_c$  in Eq. 13.24) as a function of the local slenderness of the stiffened and unstiffened elements. The procedure is simple and as implemented typically conservative; the difficulties primarily arise with unstiffened elements and the nature of the reductions. Comparisons with cold-formed steel columns demonstrated that the method could be excessively conservative (particularly for sections with slender unstiffened elements), but could also be unconservative and the approach was abandoned in favor of an effective width based method (Peköz 1987) in the AISI Specification.

The original adaptation of the effective width method for local-global interaction envisioned using effective section properties, primarily the effective moment of inertia, in the calculation of the column buckling stress; and then using that buckling stress in the typical column strength curve (e.g., DeWolf et al. 1974, 1976, Kalyanaraman et al. 1977). Such a procedure requires iteration because the effective cross-section properties are a function of the applied stresses. What is now known as the unified Effective Width Method was begun by Springfield and Trestain, who developed a column design method for the 1984 Canadian Standards Association (CSA) Cold-Formed Steel Standard (CAN3-S136-M84). For design office use, the iterative approach utilizing effective section properties was deemed unsuitable. The key features of the new method were the use of gross section properties to determine slenderness and buckling stress, and the determination of effective area at this buckling stress rather than at  $F_y$  as used previously. Another unique feature was the use of effective width for both stiffened and unstiffened elements. The column curve developed by Lind for the 1974 CSA standard was retained; this curve is geometrically similar to the AISC column curve. The method predicted the DeWolf et al. (1974) test results with remarkable accuracy (Trestain 1982).

The 1984 CSA method was incorporated into the unified approach proposed by Peköz (1987) for the 1986 AISI specification. The 1986, and later, AISI specification follows this method using an effective cross-sectional area, as follows:

$$P_n = A_e F_n \quad (13.31)$$

where  $A_e$  is the effective area (summation of the effective width of the elements times the thickness) at stress  $F_n$ , and  $F_n$  is the ultimate stress determined from Eq. 13.22-23. Further evaluation by Peköz led to further refinement: in particular, the use of the effective centroid

rather than the gross centroid as the origin for determination of eccentricity of load. The singly symmetric column which is not fully effective is a unique and difficult problem. Not only are the effective section properties reduced by local buckling, but the effective centroid shifts along the axis of symmetry. Thus an initially concentrically loaded column becomes a beam column. Testing such a column which is truly concentrically loaded throughout its loading history appears difficult if not impossible. Furthermore, the centrally loaded singly symmetric column appears to exist in practice only if it is fully effective and is loaded at its ends uniformly around the periphery. In practice, it may be difficult to be assured that such conditions will exist. Consequently, many columns that have no obvious moment applied to their ends may be, in actual fact, beam columns.

The method (Eq. 13.31) works well for local-global interaction but has been shown to be a poor predictor when distortional buckling is involved, see section 13.2.4 and 13.3.1 for further information on this point.

### ***Beams***

Based on the unified method (Peköz 1987) the effective width approach to local-global interaction in beams is essentially the same as for columns. The effective section modulus (as opposed to effective area for columns) is determined at the design stress  $F_n$  which is based on a beam strength curve that is a function of the global beam slenderness as calculated from gross properties:

$$M_n = S_e F_n \quad (13.32)$$

where  $S_e$  is the effective section modulus (appropriate summation of the effective width of all of the elements) at stress  $F_n$ , and  $F_n$  is the ultimate stress determined from the beam curve, Eq.'s 13.28 - 30. The method generally works well for local-global interaction but has been shown to be a poor predictor when distortional buckling is involved, see section 13.2.4 and 13.3.1.

### ***13.2.4 Distortional buckling strength***

Extension of the effective width method for distortional buckling has proven difficult in many situations. The same complications with predicting the elastic buckling stress are exacerbated when determining the strength: the deformations involved include both membrane deformations, primarily in the flange and lip, and bending deformations, primarily in the web. Effective width expressions for edge stiffeners and intermediate stiffeners were developed in the 1980's and in use in the AISI Specification until 2001 (Desmond et al. 1981a; Desmond et al. 1981b). However, the expressions were found to be in poor agreement with columns and beams failing in pure distortional buckling (Hancock et al. 1994; Kwon and Hancock 1992; Rogers and Schuster 1997; Schafer 2001; Schafer and Peköz 1998a; Schafer and Peköz 1999; Yu and Schafer 2006; Yu and Schafer 2007b). New distortional buckling provisions, adopted in 2007 for the AISI Specification, follow a methodology that is consistent with the Direct Strength Method and are thus detailed in that section (13.3.1). The Eurocode method uses a combination of effective width and reduced thickness methods to determine the distortional buckling strength, they extend their beam on elastic foundation model and assume the stiffeners follow a basic column curve for strength (thus, no post-buckling capacity is allowed).

### ***Distortional-global, local-distortional interaction***

If distortional buckling is treated with effective width's, as was completed for a number of years in the AISI Specification, one advantage is automatic inclusion of the potential for distortional-global interaction, at least in some approximate form. The distortional buckling effective width's

are calculated at the long column, or long beam stress ( $F_n$ ), and thus the potential for this interaction is allowed.

Local-distortional interaction is more difficult to include in the effective width method if an effective width for local buckling and an effective width for distortional buckling are to both be determined (for the same element!). Eurocode combines a reduced thickness approach with traditional effective width local buckling reduction to account for this potential phenomenon.



### 13.3 DIRECT STRENGTH MEMBER DESIGN

Instead of focusing on the elements which comprise a cross-section, as is done in the Effective Width Method, in the Direct Strength Method the key to the solution is an accurate elastic stability analysis (Fig. 13.1, Fig. 13.7), including local buckling with interaction, distortional, and global buckling. The premise is, that if an engineer determines all of the elastic instabilities for the gross section, e.g. for a column: local ( $P_{cr\ell}$ ), distortional ( $P_{crd}$ ), and global buckling ( $P_{cre}$ ), and also determines the load that causes the section to yield ( $P_y$ ), then the strength can be directly determined with simple functions, i.e.:

$$P_n = f(P_{cr\ell}, P_{crd}, P_{cre}, P_y). \quad (13.33)$$

where  $f$  designates the unknown functions that are the “Direct Strength” prediction equations. For example in global column buckling if we multiply the expressions of Eq.’s 13.22-23 by the gross area,  $A_g$ , then they provide the column strength,  $P_n$ , as a function of  $P_{cre}=A_gF_{cre}$  and  $P_y=A_gF_y$ . In this example Eq.’s 22-23 are the “Direct Strength” prediction equation for global buckling. Thus, the Direct Strength Method may be understood as an extension of the use of column curves for global buckling, but now new expressions (new  $f$ ’s for Eq. 13.33) are employed for local and distortional buckling instabilities with appropriate consideration of post-buckling reserve and interaction in these modes. In 2004 the AISI Specification adopted the Direct Strength Method as an alternative design method, see Appendix 1 of AISI (2004, 2007).

It is important to recognize in any discussion regarding the Effective Width Method, the Direct Strength Method, or other semi-empirical design methods for thin-walled construction that none of these design methods are theoretically correct. Rather, a complicated nonlinear problem is simplified in some manner so that engineers may have a working model to design from without resorting to testing every individual member. These models serve us well when backed up by the application of reliability to incorporate uncertainty in their predictive powers. A full discussion of reliability, development of the Direct Strength Method, and summary of current related research is proved in Schafer (2008).

Distortional buckling is covered first in this Section because it is of special significance in the Direct Strength Method. The design prediction equations which were proposed by Hancock for strength in distortional buckling modes may be viewed as the genesis for the Direct Strength Method. Distortional buckling is followed by global buckling in Section 13.3.2, and local and local-global interaction in Section 13.3.3. Since the Direct Strength Method is a new procedure it is contrasted directly with the Effective Width Method (Section 13.3.3). Finally, this Section on the Direct Strength Method closes with a look at current research, including the consideration of interaction amongst the buckling modes.

#### ***13.3.1 Distortional buckling strength***

In any explanation of the Direct Strength Method distortional buckling is of prominent importance since much of the Direct Strength Method development centered around finding adequate design solutions to the complicated problem of elastic buckling and strength determination in open singly- and point-symmetric cold-formed steel cross-sections undergoing distortional buckling.

#### ***Columns***

Today the distortional buckling strength ( $P_{nd}$ ) may be predicted with a simple expression:

$$\text{for } \lambda_d \leq 0.561 \quad P_{nd} = P_y \quad (13.34)$$

$$\text{for } \lambda_d > 0.561 \quad P_{nd} = \left( 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \left( \frac{P_{crd}}{P_y} \right)^{0.6} P_y \quad (13.35)$$

$$\text{where } \lambda_d = \sqrt{P_y / P_{crd}} \quad (13.36)$$

$P_{crd}$  = Critical elastic distortional column buckling load

$P_y$  = is the squash load for the column,  $A_g f_y$ .

Seemingly, the only complication of significance in Eq.'s 13.34-36 is the determination of the critical elastic distortional column buckling load,  $P_{crd}$ . However, the research and design decisions required to arrive at this point are far more significant than the equations reveal. Using the preceding approach (which has been adopted in AISI 2007 in both the main Specification as an additional check on local buckling, and in Appendix 1 the Direct Strength Method) assumes the following: (a) distortional buckling may be treated on the whole section as opposed to its component elements, (b) post-buckling in distortional buckling is specific to the mode (and not the same as local or global buckling), (c) interaction of distortional buckling with other modes need not be considered. The research to arrive at these conclusions includes a number of significant contributions as summarized in the following, but the third point (c) is still actively under study and discussed further in Section 13.3.4 below.

Distortional buckling has been long recognized as a potential problem: Chilver (1951, 1953) stated that the reinforcing “lip” in a lipped channel should be sufficiently stiff to insure local buckling (and thus avoid distortional buckling), but gave no criteria for achieving this. Desmond et al. (1981a; 1981b) studied the distortional mode in detail, referring to the mode as “stiffener” buckling. Desmond et al. recognized that elastic buckling criteria, i.e, ensuring that “stiffener” buckling is a higher critical stress than local buckling, does not meet Chilver’s criteria. Instead, using experimental data Desmond et al. empirically formulated rules for an “adequate” stiffener and the plate buckling coefficient,  $k$ , when the stiffener is only partially effective. As a result, distortional buckling was incorporated into the AISI Specification (1986-2004) as a local mode, implicitly assuming post-buckling reserve in distortional buckling was the same as in local buckling. Additionally, Desmond et al.’s experimental work employed members with back-to-back specimens, this avoided web local buckling, but artificially elevated the distortional buckling stress.

Distortional buckling has long been observed in testing, but often removed before given further consideration. In Sweden, Thomasson (1978) performed experiments on lipped channels with slender webs. To increase the local buckling stress of the webs small intermediate (groove) stiffeners were folded in. This eliminated local buckling, but created what Thomasson called a “local-torsional” problem, i.e, distortional buckling. Thomasson considered this “local-torsional” mode undesirable and thus put closely spaced braces from lip to lip, insuring that distortional buckling did not occur and therefore making the local mode again dominant. Mulligan (1983) encountered the same “local-torsional” mode in testing, and observed that Desmond et al.’s adequate stiffener criteria did not appear to restrain distortional buckling in many cases. Subsequently, Mulligan followed Thomasson’s experimental modification and provided braces which restricted distortional buckling.

Another attempt at understanding distortional buckling focused on isolating the flange from the rest of the section (as is traditionally done in effective width design). Isolated edge stiffeners were studied experimentally and analytically by physically replacing the web/flange juncture with a simple support, thus providing known boundary conditions (Kloppel and Unger

1970, Lim 1985, Lim and Rhodes 1986). The “torsional” mode (distortional buckling) for these flanges may be accurately predicted due to the special boundary conditions.

Eurocode has taken another approach to distortional buckling calculations. Eurocode 3 Part 1.3 (1996, 2004) provides a method for predicting the distortional buckling of simple lipped sections such as channels accounting for the elastic restraint provided by the web and the flange to the lip buckling as a strut. This method approximately accounts for the distortional deformations of the web and flange, but used a global column curve for the failure of the lip, thus assuming that there was no post-buckling reserve in the distortional mode.

At the University of Sydney distortional buckling was directly studied, motivated greatly by the prevalence of distortional buckling in high strength cold-formed steel storage racks (e.g., Hancock 1985, Lau 1988). This work led to the refinement of the finite strip method in elastic distortional buckling (Lau and Hancock, 1990) the development of unique analytical methods (Lau and Hancock, 1987) and strength curves for distortional buckling (Hancock et al. 1994). Based on Hancock et al.’s strength curves the Australian Standard for Steel Storage Racking (1993) and the Australian/New Zealand Standard for Cold-Formed Steel Structures (AS/NZS:4600 1996, 2005) first adopted explicit design rules for distortional buckling in compression. Eq. 13.35 adopted in the Direct Strength Method (AISI 2004, 2007) is identical to Eq. 4b as presented in Hancock et al. (1994).

In Kwon and Hancock’s experiments on lipped channels, with and without groove stiffeners in the web, the distortional mode was unrestricted and the tests showed that interaction of distortional buckling with other modes was weak. Later testing at Sydney, Young (1997) also experimentally observed that the interaction of distortional buckling with other modes is weak. Using Generalized Beam Theory, Davies and Jiang (1996) argued that distortional buckling has weak interactions with other modes and endorsed the strength curves of Hancock et al. (1994) for ultimate strength prediction, of which Eq. 13.35 identical to. Further examination against a wider body of test data also showed interaction of distortional with other modes to be weak (Schafer 2002).

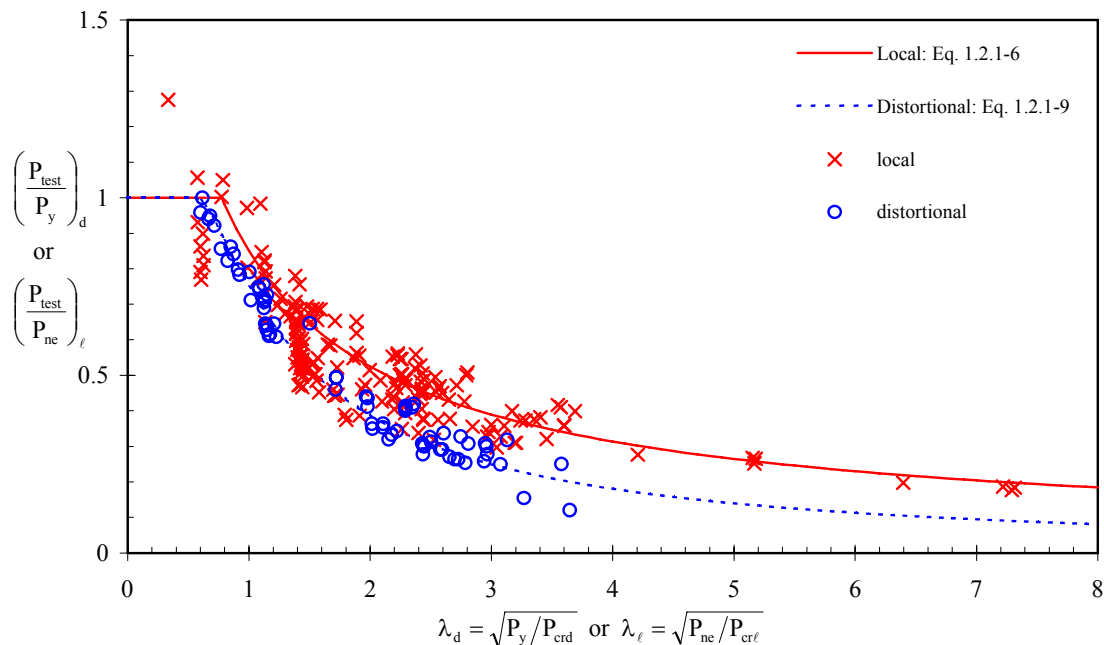
In the University of Sydney tests, distortional buckling was experimentally observed to (a) have post-buckling capacity (thus leading to the rejection of the Eurocode strength methodology), but (b) to a lesser extent than local buckling (essentially a rejection of the prevailing AISI method of the time based on Desmond et al.’s work). The presence of post-buckling capacity indicates that the distortional mode should not be treated as a global mode for strength, while the limitations to the available post-buckling capacity indicate that inclusion in a local buckling based effective width approach must be done with care, as use of the standard expressions will result in over-prediction of the post-buckling reserve. One explanation for the limited post-buckling is that the membrane stress at the flange tips of edge-stiffening lips increase dramatically after distortional buckling (Sridharan’s 1982). Using finite strip and nonlinear finite element analysis, reduced post-buckling capacity and increased imperfection sensitivity for distortional buckling failures, when compared to local buckling, was observed both for edge stiffened and intermediately stiffened beams and columns (Schafer 2002; Schafer and Peköz 1999).

Complementary to the extensive work at University of Sydney were studies at the University of Strathclyde on “torsional” buckling, i.e., distortional buckling (Seah 1989, Seah et al. 1991, Seah and Rhodes 1993, Chou et al. 1996). Hats and channels with compound lips were investigated experimentally, hand methods for the prediction of distortional buckling were provided, ultimate strength in the distortional mode was treated via the effective width approach

(rather than the column curve approach proposed by Sydney researchers). Also complementary to the work at University of Sydney were tests in Finland (Salmi and Talja, 1993), work in Japan on more complicated polygonal cross-sections (Hikosaka, Takami and Maruyama, 1987, Takahashi 1988), and tests in the U.S. on Z-sections columns in distortional buckling (Polyzois and Sudharampal 1990, Purnadi et al. 1990, Polyzois and Charvarnichborikarn 1993).

In 2002 an attempt was made to investigate distortional buckling using the entirety of the available existing experimental data (Schafer 2002). Hancock et al. (1994) had demonstrated that when a section was known to fail in distortional buckling, that for the variety of cross-sections tested at University of Sydney, the measured compressive strength correlated well with the slenderness in the elastic distortional mode. Data in Schafer (2002) provided additional experimental agreement for Hancock's design expression (Fig. 13.11). In addition, the failure mode for specimens collected in Schafer (2002) was generally unknown, thus a variety of methods were examined for strength calculation, considering local, distortional, global buckling and their potential interactions. The result was a method that both identified the failure mode and predicted the strength. The extension to prediction of all failure modes by appropriate expressions correlated to elastic slenderness in the various modes (and combinations) was the essential step in the development of the Direct Strength Method as a general approach.

Hancock et al.'s work provided the essential curve for distortional buckling by the Direct Strength Method. As is often the case with attempts to determine an origin, one can go back even further as Hancock attributes his methodology to Trahair's work on the strength prediction of columns undergoing flexural-torsional buckling. In this regard it becomes clear that the Direct Strength Method is not a new idea, but rather the extension of an old one to new instability limit states.



**Fig. 13.11 Comparison of the Direct Strength Method with test data for columns (Equation numbers in the legend refer to Appendix 1 of AISI 2007)**

## Beams

The first mention of the Direct Strength Method occurs in Schafer and Peköz (1998) and was closely coupled to the development of the method for beams, in particular, application of the large database of sections that was collected to explore two problems: distortional buckling in C- and Z-section beams, and local and distortional buckling in deck sections with multiple longitudinal intermediate stiffeners in the compression flange. At the same time Hancock and related researchers at the University of Sydney demonstrated that distortional buckling failures for a wide variety of failures were well correlated with the elastic distortional slenderness (Hancock et al. 1994; Hancock et al. 1996).

The form of the presentation of the Direct Strength Method for beams evolved somewhat from Schafer and Peköz (1998,1999). In particular, curve (2) is identical to the distortional buckling expression of Equation 4b provided in Hancock et al. (1994), and became the distortional buckling Direct Strength curve, where finally in AISI (2004, 2007) Appendix 1 the nominal flexural strength,  $M_{nd}$ , for distortional buckling via the Direct Strength Method is:

$$\text{for } \lambda_d \leq 0.673 \quad M_{nd} = M_y \quad (13.37)$$

$$\text{for } \lambda_d > 0.673 \quad M_{nd} = \left( 1 - 0.22 \left( \frac{M_{crd}}{M_y} \right)^{0.5} \right) \left( \frac{M_{crd}}{M_y} \right)^{0.5} M_y \quad (13.38)$$

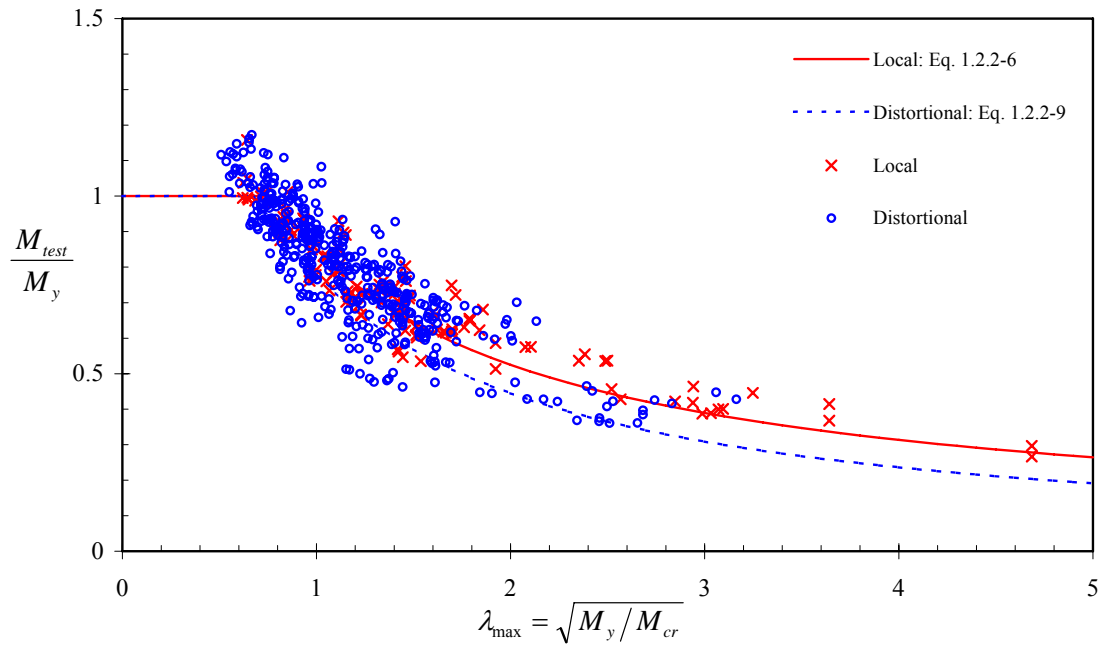
$$\text{where } \lambda_d = \sqrt{M_y / M_{crd}} \quad (13.39)$$

$M_{crd}$  = Critical elastic distortional buckling moment...

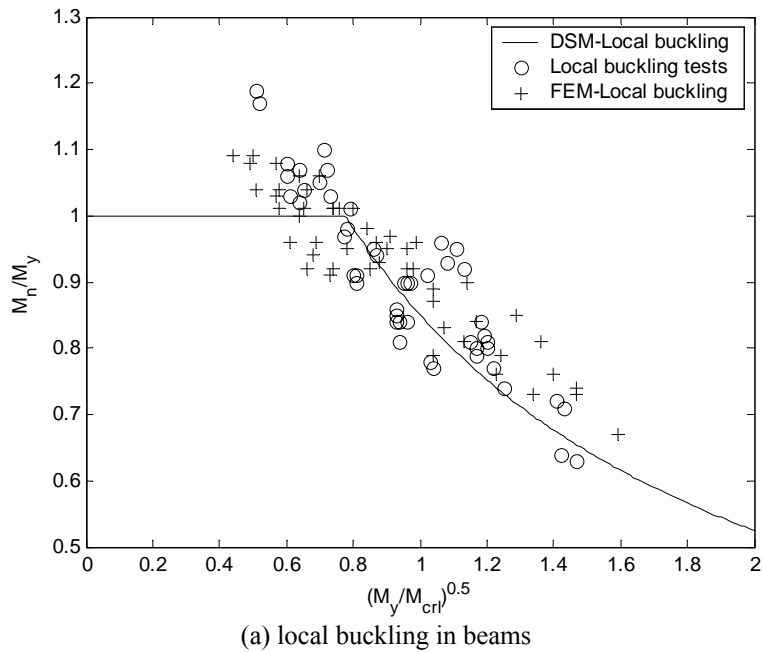
$M_y = S_g f_y$  and  $S_g$  is the gross section modulus referenced to the fiber at first yield.

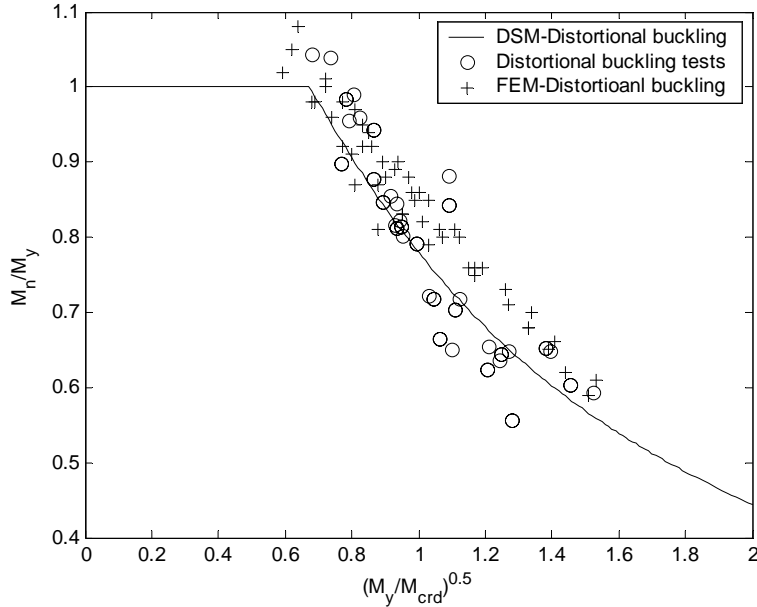
Similar to the work on columns in distortional buckling, the test database was expanded beyond the original University of Sydney results reported in Hancock et al. (1994) and a method for local buckling was also developed, as reported in Schafer and Peköz (1999). The expressions are compared with the expanded test database in Fig. 13.12. The end result is that for an arbitrary cross-section an engineer can not only determine the strength in distortional buckling, but distinguish between the limit states of local and distortional buckling.

In the development of the Direct Strength Method for C- and Z-section beams separation of local and distortional buckling failure modes was initially somewhat difficult and complicated by the bracing and boundary conditions used in the testing, which typically restrained distortional buckling in part, but not necessarily in full. Nonetheless, expressions were arrived at and adopted in AISI (2004, 2007). More recently, a recent series of flexural tests and complementary finite element analysis on a variety of C- and Z-sections in local buckling (Yu and Schafer 2003, 2007, Yu 2005) and distortional buckling (Yu 2005, Yu and Schafer 2006, 2007) used specific details to isolate the two modes and unequivocally demonstrated the robustness of the Direct Strength Method predictions for C- and Z-sections failing in either the local or distortional mode. A summary of the performance of these sections is provided in Fig. 13.13.



**Fig. 13.12 Comparison of the Direct Strength Method with tests data for laterally braced beams (Equation numbers in the legend refer to Appendix 1 of AISI 2004, 2007)**





(b) distortional buckling in beams

**Fig. 13.13 Comparison of Direct Strength Method for beams to tests and nonlinear FE results for C and Z sections in (a) local and (b) distortional buckling as reported in (Yu and Schafer 2007b)**

### 13.3.2 Global buckling strength

The global buckling strength of columns and beams in the Direct Strength Method is the same as in the main body of the AISI Specification (i.e. Eq.'s 13.22-24 and 13.28-30), with two exceptions. First, the provisions are provided in terms of load and moment, instead of stress. Second, no method is prescribed for the elastic global buckling load or moment, instead in the Direct Strength Method (Appendix 1, AISI 2007) rational analysis is allowed and expected for these quantities. Consider for example the main body of the AISI Specification's approach to global buckling of Z-sections, it merely takes ½ the buckling stress of I-sections. This is excessively conservative, particularly given the ease with which either (a) the classical analytical expressions can be solved numerically, or (b) finite strip or finite element solutions may be used to find the elastic buckling load or moment with great accuracy. In the DSM Design Guide (Schafer 2006) a variety of approaches are demonstrated for global buckling determination. The design expressions for columns and beams follow.

#### Columns

##### Flexural, Torsional, or Torsional-Flexural Buckling

The nominal axial strength,  $P_{ne}$ , for flexural, torsional, or torsional-flexural buckling is

$$\text{for } \lambda_c \leq 1.5 \quad P_{ne} = (0.658^{\lambda_c^2}) P_y \quad (13.40)$$

$$\text{for } \lambda_c > 1.5 \quad P_{ne} = \left( \frac{0.877}{\lambda_c^2} \right) P_y \quad (13.41)$$

$$\text{where } \lambda_c = \sqrt{P_y / P_{cre}} \quad (13.42)$$

$$P_y = A_g F_y \quad (13.43)$$

$P_{cre}$  = Minimum of the critical elastic column buckling load in flexural, torsional, or torsional-flexural buckling ...

## Beams

### Lateral-Torsional Buckling

The nominal flexural strength,  $M_{ne}$ , for lateral-torsional buckling is

$$\text{for } M_{cre} < 0.56M_y \quad M_{ne} = M_{cre} \quad (13.44)$$

$$\text{for } 2.78M_y \geq M_{cre} \geq 0.56M_y \quad M_{ne} = \frac{10}{9} M_y \left(1 - \frac{10M_y}{36M_{cre}}\right) \quad (13.45)$$

$$\text{for } M_{cre} > 2.78M_y \quad M_{ne} = M_y \quad (13.46)$$

$$\text{where } M_y = S_f F_y, \text{ where } S_f \text{ is the gross section modulus referenced to} \quad (13.47)$$

the extreme fiber in first yield

$M_{cre}$  = Critical elastic lateral-torsional buckling moment...

### 13.3.3 Local and Local-global interactive strength

#### Columns

The basis of the Direct Strength Method for local buckling can be understood through an examination of Winter's effective width expression for local buckling of plates. If the slenderness parameter,  $\lambda$ , of Eq. 13.20 is substituted into Eq. 13.19 and the resulting expression for  $\rho$  into Eq. 13.18 the effective width,  $b$ , may be expressed as a function of the gross width,  $w$ , with the following expression:

$$b = \left(1 - 0.22 \sqrt{\frac{f_{cr}}{f_y}}\right) \sqrt{\frac{f_{cr}}{f_y}} w \text{ for } \sqrt{\frac{f_y}{f_{cr}}} > 0.673 \quad (13.48)$$

This shows that the effective width (the strength in local buckling) is a function of  $f_{cr}$  and  $f_y$ ; which is the reasoning behind the basic premise of the Direct Strength Method: strength as a function of elastic slenderness for all modes. Consider now that the local plate buckling critical buckling stress,  $f_{cr}$ , in Eq. 13.48 is replaced with the cross-section local buckling stress,  $f_{cr\ell}$  (e.g. from Fig. 13.7) and the effective width and gross width are replaced by effective area and gross area:

$$A_e = \left(1 - 0.22 \sqrt{\frac{f_{cr\ell}}{f_y}}\right) \sqrt{\frac{f_{cr\ell}}{f_y}} A_g \text{ for } \sqrt{\frac{f_y}{f_{cr\ell}}} > 0.673 \quad (13.49)$$

multiplying both sides by  $f_y$  ( $A_e f_y = P_{n\ell}$ ,  $A_g f_y = P_y$ ), and multiplying numerator and denominator under the radical by  $A_g$  ( $A_g f_{cr\ell} = P_{cr\ell}$ ,  $A_g f_y = P_y$ ) and the result is that the strength,  $P_{n\ell}$ , is predicted by the slenderness, as a function of force, i.e., a Direct Strength prediction is provided:

$$P_{n\ell} = \left(1 - 0.22 \sqrt{\frac{P_{cr\ell}}{P_y}}\right) \sqrt{\frac{P_{cr\ell}}{P_y}} P_y \text{ for } \sqrt{\frac{P_y}{P_{cr\ell}}} > 0.673 \quad (13.50)$$

Calibration with experimental data (Schafer 2002) shows this expression to be slightly conservative and the final expression employed in the Direct Strength Method is:

$$P_{n\ell} = \left(1 - 0.15 \left(\frac{P_{cr\ell}}{P_y}\right)^{0.4}\right) \left(\frac{P_{cr\ell}}{P_y}\right)^{0.4} P_y \text{ for } \sqrt{\frac{P_y}{P_{cr\ell}}} > 0.776 \quad (13.51)$$

Local-global interaction is fundamental to thin-walled members, in the effective width approach, as described in Section 13.2.3,  $f_y$ , of Eq. 13.49 is replaced with  $f_n$  from the column curve of Eq. 13.22-23, i.e.



$$b = \left( 1 - 0.22 \sqrt{\frac{f_{cr}}{f_n}} \right) \sqrt{\frac{f_{cr}}{f_n}} \quad \text{for } \sqrt{\frac{f_n}{f_{cr}}} > 0.673 \quad (13.52)$$

Using the same substitutions as before, and making the same final calibration (Schafer 2002), Eq. 13.52 in Direct Strength Method format is expressed as:

$$P_{n\ell} = \left( 1 - 0.15 \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} \right) \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} P_y \quad \text{for } \sqrt{\frac{P_y}{P_{ne}}} > 0.776 \quad (13.53)$$

where  $P_{ne} = A_g f_n$ , and  $f_n$  is from Eq. 13.22-23, or equivalent  $P_{ne}$  is from Eq. 13.40-41. Through this simple substitution local-global interaction is included. Where before the maximum strength in local buckling was  $P_y$ , now the global buckling caps the strength at  $P_{ne}$  and further reductions are made down to  $P_{n\ell}$  as a function of the local slenderness. The final format in the Specification (AISI 2004, 2007) appears as follows, where the nominal axial strength,  $P_{n\ell}$ , for local buckling is

$$\text{for } \lambda_\ell \leq 0.776 \quad P_{n\ell} = P_{ne} \quad (13.54)$$

$$\text{for } \lambda_\ell > 0.776 \quad P_{n\ell} = \left[ 1 - 0.15 \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} \right] \left( \frac{P_{cr\ell}}{P_{ne}} \right)^{0.4} P_{ne} \quad (13.55)$$

$$\text{where } \lambda_\ell = \sqrt{P_{ne}/P_{cr\ell}} \quad (13.56)$$

$P_{cr\ell}$  = Critical elastic local column buckling load ...

$P_{ne}$  is defined in Section 1.2.1.1.

Agreement of Eq. 13.56 with available test data is provided in Fig. 13.11 (Schafer 2002). Significant scatter is prevalent (though not more than in effective width implementations) yet the overall trends are clear. The choice of the coefficients and exponents of Eq. 13.56 were influenced by the solution for the Direct Strength Method for the local buckling strength of beams, which preceded that for columns and is discussed further in the following.

### Beams

Development of the initial Direct Strength prediction equations for local buckling of flexural members followed the same basic progression as described for compression members. Beginning with Winter's basic effective width expression in the form of Eq. 13.48,  $f_{cr}$  is replaced with the cross-section local buckling stress,  $f_{cr\ell}$  (referenced to the compression flange). The effective width and gross width are replaced with the effective section modulus,  $S_e$ , and gross section modulus  $S_g$  and both sides are multiplied times the yield stress:

$$S_e f_y = \left( 1 - 0.22 \sqrt{\frac{f_{cr\ell}}{f_y}} \right) \sqrt{\frac{f_{cr\ell}}{f_y}} S_g f_y \quad \text{for } \sqrt{\frac{f_y}{f_{cr\ell}}} > 0.673 \quad (13.57)$$

Stresses under the radical are multiplied times  $S_g$  while  $S_e f_y$  is defined as the nominal capacity in local buckling,  $M_{n\ell}$ , and  $S_g f_y$  as the yield moment,  $M_y$ , thus resulting in Winter's effective width expression for the bending strength:

$$M_{n\ell} = \left( 1 - 0.22 \sqrt{\frac{M_{cr\ell}}{M_y}} \right) \sqrt{\frac{M_{cr\ell}}{M_y}} M_y \quad \text{for } \sqrt{\frac{M_y}{M_{cr\ell}}} > 0.673 \quad (13.58)$$

In Schafer and Peköz (1998) Eq. 13.58 is compared to available data, the basic trend is clear but the expression is overly conservative. As a result curve (3) of Schafer and Peköz (1998) was

employed and the nominal flexural strength,  $M_{n\ell}$ , for local buckling in the Direct Strength Method was found to be:

$$\text{for } \lambda_\ell \leq 0.776 \quad M_{n\ell} = M_{ne} \quad (13.59)$$

$$\text{for } \lambda_\ell > 0.776 \quad M_{n\ell} = \left( 1 - 0.15 \left( \frac{M_{cr\ell}}{M_{ne}} \right)^{0.4} \right) \left( \frac{M_{cr\ell}}{M_{ne}} \right)^{0.4} M_{ne} \quad (13.60)$$

$$\text{where } \lambda_\ell = \sqrt{M_{ne}/M_{cr\ell}} \quad (13.61)$$

$M_{cr\ell}$  = Critical elastic local buckling moment...

$M_{ne}$  is defined in Section 1.2.2.1.

Agreement of Eq. 13.59-60 with experimental data is provided in Fig. 13.12 and Fig. 13.13(a). Note, for the beam data (as opposed to the column data) all of the  $M_{test}$  values are normalized against the moment at first yield,  $M_y$ . This is due to the fact that all of the test data employed was for laterally braced members. It is worth noting that while local-global interaction was experimentally examined for columns, and the same methodology applied for beams, local-global, distortional-global, and local-distortional interactions have not been experimentally examined in the context of the Direct Strength Method for beams. Based on the findings for columns local-global interaction has been included and local-distortional and distortional-global interactions ignored. See Section 13.3.4 for further discussion on modal interaction. The performance of laterally un-braced beams deserves further study, not only in the context of the Direct Strength Method and potential interactions, but also to better understand how warping torsion should be treated. For moderate rotations the influence of the torsional stress on local and distortional buckling modes is real (Gotluru et al. 2000) and its potential inclusion in the Direct Strength Method (as well as in the main body of the AISI Specification) is worthy of further study.

Finally, it is worth noting that the testing on C- and Z- section beams has focused on strong-axis bending and associated buckling, extension to weak-axis bending has been assumed. This assumption is justified in part by the inclusion of hats and decks in the experimental database, these sections are bent about their weak-axis, and are similar in their behavior to a C-section in weak axis bending. Further, the major-axis bending modes are considered more critical since the primary effect of weak-axis bending in comparison to strong-axis bending is the elimination of global lateral-torsional buckling modes.

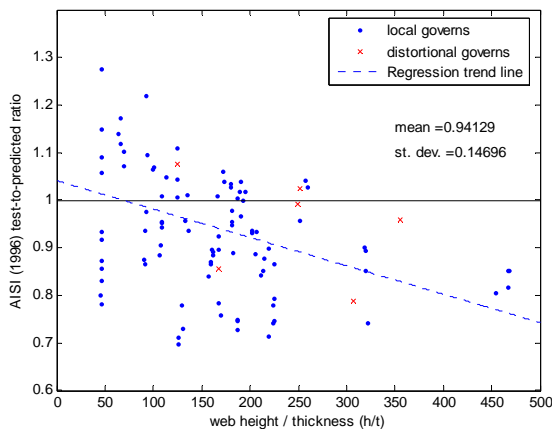
### ***Element Interaction***

A significant difference between the Effective Width Method and the Direct Strength Method is the replacement of element or plate buckling,  $f_{cr}$  with cross-section local buckling,  $f_{cr\ell}$ . In the elastic regime the use of cross-section  $f_{cr\ell}$  insure that equilibrium and compatibility around the cross-section are maintained. Though both design methods have reasonable levels of overall reliability (Schafer 2008) they arrive at that reliability in different ways. In Fig. 13.14 the strength predictions of the Effective Width Method and the Direct Strength Method are compared as a function of the web slenderness of a C-section column. As web slenderness increases the Effective Width Method solution becomes systematically unconservative. This behavior is exacerbated by the fact that for typically available C-sections as the web becomes deeper the flange width remains at approximately the same width, so high web slenderness is strongly correlated with high web-to-flange width ratios (i.e., C-sections which are 'narrow'). This detrimental behavior is primarily one of local web/flange interaction, not distortional

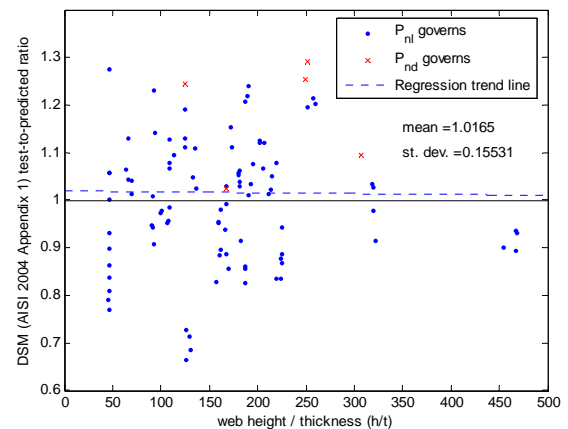
buckling. Since the Effective Width Method uses an element approach, no matter how high the slenderness of the web becomes, it has no effect on the solution for the flange. In contrast the Direct Strength Method, which includes element interaction in local buckling (i.e., interaction between the flange and the web), performs accurately over the full range of web slenderness. Proper inclusion of element interaction is necessary for accurate strength prediction of these columns.

Taken to extremes, inclusion of elastic element interaction can also work against the Direct Strength Method, making the method overly conservative. This fundamental limitation of the Direct Strength Method was reported in the first paper to propose the approach (Schafer and Peköz 1998). When one part (element) of the cross-section becomes extraordinarily slender that element will drive the member elastic critical buckling stress to approach zero. The Direct Strength Method will assume the member strength, like the member elastic critical buckling stress, will also approach zero. In contrast, the Effective Width Method presumes only that the element itself (not the member) will have no strength in such a situation. Deck or hat sections in bending with low yield stress and very slender (wide) compression flanges without intermediate stiffeners tend to fall in this category and thus have unduly conservative predictions by the Direct Strength Method, but quite reasonable predictions via the Effective Width Method. However, ignoring inter-element interaction, as the Effective Width Method traditionally does, is not universally a good idea as illustrated for the C-section columns of Fig. 13.14.

For optimized deck sections with multiple longitudinal intermediate stiffeners in the web and the flange (see e.g., Höglund 1980) the Direct Strength Method is highly desirable over the Effective Width Method – here the benefit is primarily convenience not theoretical. If a computational solution is employed for determining the elastic buckling stresses (moments) an optimized deck section is no more complicated than a simple hat for strength determination; but for the Effective Width Method the calculation of effective section properties and accurately handling the effective width of the numerous sub-elements leads to severe complication without increased accuracy, or worse in the case of many specifications (e.g. AISI 1996, 2001) no design approach is even available for such a section using the Effective Width Method. In general, as sections are optimized the Direct Strength Method provides a simpler design methodology with wider applicability than the Effective Width Method.



(a) Effective Width Method of AISI (1996)



(b) Direct Strength Method of AISI (2004) App. 1

**Fig. 13.14 Test-to-predicted ratio for (a) the Effective Width Method of AISI (1996) and (b) the Direct Strength Method of AISI (2004) App. 1 for all lipped channel columns used in the development of Direct Strength Method predictor equations plotted as a function of web slenderness ( $h/t$ )**

### ***13.3.4 Modal interactions***

If thin-walled cross-sections can generally be characterized as having local, distortional, and global buckling modes then there exists a potential for any of these modes to interact. For example, local-global interaction is known to occur and be of significance, and is thus accounted for specifically in design (e.g., Eq. 13.31 or 13.54-56). Interaction of the other modes: local-distortional, distortional-global, and local-distortional-global have been the subject of research.

No definitive consensus exists on what it means for a mode to interact, nor when such interactions will occur. Definitions depend on whether one is considering a mathematical interaction, or even the coupling of multiple modes, or one is considering observed behavior where one buckling mode influences the deformations and strength in a second mode. When such interaction will occur is complicated by the varied degree of post-buckling in the modes, wavelength of the modes, and the dependence of post-buckling on material properties (e.g., yield stress) as buckling modes trigger plastic mechanisms and ultimately failure.

The Effective Width Method (as implemented in AISI 2007) explicitly includes local-global interaction. Local-distortional, distortional-global, and local-distortional-global interaction are assumed to not occur or be irrelevant for design in the Effective Width Method<sup>1</sup>. The Direct Strength Method (as implemented in AISI 2007) includes only local-global interaction, and ignores local-distortional, distortional-global, and local-distortional-global interaction. These conclusions were drawn from conflicting data, which nonetheless largely point out that if interactions are included for all modes the resulting capacities are not consistent with observations.

#### ***Local-Distortional/Distortional-Global interaction***

As summarized in Section 13.3.1 initial analytical and experimental investigations of distortional buckling largely indicated that interaction of distortional buckling with other buckling modes (local, global) is generally weak. The tests by Kwon and Hancock (1992) on lipped channels with and without groove stiffeners in the web were designed to determine whether adverse interaction occurred if local and distortional buckling were simultaneous or nearly simultaneous. No adverse interaction was found between local and distortional buckling for the channel sections tested. However tests of trapezoidal decks by Bernard et al, (1992a,b, 1993a,b) included sections that underwent local buckling before and after distortional buckling. Sections that underwent local buckling well before distortional buckling needed to account for local-distortional interaction, while in sections where distortional buckling occurred first no similar reduction for local buckling was required. Later testing at Sydney, Young (1997) experimentally observed that the interaction of distortional buckling with other modes is weak. Using Generalized Beam Theory, Davies and Jiang (1996) argued that distortional buckling has weak interactions with other modes.

Further examination against a wider body of test data also showed interaction of distortional with other modes to be weak (Schafer 2002). Interaction of the buckling modes was systematically studied for local-global, distortional-global, and local-distortional buckling of the columns. Based on overall test-to-predicted ratios, and when available the failure modes noted by the researchers in their testing, it was determined that local-global interaction should be

---

<sup>1</sup> This statement is subject to some interpretation, as the empirical expressions of Desmond et al. which are still used for the effective width of edge stiffened elements in AISI (2007) includes some amount of local-distortional interaction in their development; however it may be shown that the distortional buckling strength using Eq. 13.34-36 or Eq. 13.37-39 results in lower capacities than Desmond's expressions; therefore the local-distortional interaction is not meaningfully included in AISI (2007).

included, but not distortional-global, or local-distortional interaction. For example the local-distortional interactive strength was formulated by replacing  $P_{ne}$  of Eq. 13.54-56 with  $P_{nd}$  of Eq. 13.34-36. Such a strength check results in overly conservative predictions: 169 of the 187 studied tests would be identified to fail in local-distortional interaction and the average test-to-predicted ratio would be 1.35 (Schafer 2000, 2002). Neither the failure mode or strength prediction is consistent with the observations from the tests when local-distortional interaction is included for all columns. As a result, it was recommended to only include local-global interaction in the Direct Strength Method.

Recent experimental and analytical work has left this conclusion somewhat in question. Hancock and his students have continued to study the distortional buckling mode for high strength steel sections, including lipped channels with intermediate stiffeners (Yang and Hancock 2004; Yap and Hancock 2008b), and more unique cross-shaped open sections with multiple distortional buckling modes (Yap and Hancock 2008a). They find for these sections that the strength does not follow the expected distortional strength curve, i.e. that of the Direct Strength Method Eq. 13.34-36 or Hancock et al. (1994) Eq. 4(b), but rather is somewhat reduced. (Post-buckling strength is still observed). It is unclear if the reduction should be attributed to local-distortional or distortional-global interaction. The best numerical (strength prediction) agreement is found with ignoring local-distortional interaction in the calculation and including distortional-global interaction by replacing  $P_y$  in Eq. 13.34-36 with  $P_{ne}$ . However, local-distortional interaction is visually observed in the testing.

In addition to studying the mechanics of post-buckling in local and distortional buckling Silvestre et al. (2006) have also been studying local-distortional interaction in lipped channels. Their approach has been to examine cross-sections where  $P_{cr_l}$  and  $P_{cr_d}$  are at the same or nearly the same elastic critical buckling load (moment) and then examine the ultimate strength through the use of nonlinear finite element analysis. Comparisons with the Direct Strength Method show a decrease from the expected post-buckling strength. Based on their analyses an experimental study is now underway to confirm the exact nature of the reductions and when they should be applied in design.

## 13.4 ADDITIONAL DESIGN CONSIDERATIONS

The two previous sections cover the design of thin-walled axial and flexural members by either the Effective Width Method or the Direct Strength Method. Regardless of the method selected a number of additional design considerations must be considered in construction using thin-walled members; including: shear, inelastic reserve capacity, web crippling and interactions between bending and web crippling, bending and shear, and bending and axial load.

### 13.4.1 Shear

The design expressions for cold-formed steel members in shear closely parallel those of the AISC Specification and are based primarily on the experimental work of LaBoube and Yu (1978). The 5<sup>th</sup> Ed. of this Guide provides complete details of the expressions used prior to 2001. In 2001 the shear design expressions were modified slightly so that a uniform resistance factor could be employed through the three regimes: yield, inelastic buckling, and elastic buckling (Craig and Schuster 2000). Development of a Direct Strength Method for shear has sparked more recent work (Pham and Hancock 2008; Schafer 2008)

### 13.4.2 Bending: Inelastic Reserve

It is possible, and indeed relatively common, for thin-walled cold-formed steel beams to have bending capacities in excess of the moment at first yield. One method for determining the capacity in such a situation is to determine the strains that the compressed elements can sustain, and then based on that strain determine the capacity in excess of  $M_y$ . The AISI Specification provides the strain capacity of stiffened elements (Yener and Pekoz 1985) and unstiffened elements (Bambach and Rasmussen 2004) that may be utilized for this calculation. In addition, work has begun on a Direct Strength Method approach to inelastic reserve (Shifferaw and Schafer 2007). Additional inelastic reserve capacity due to the redistribution of moments in statically indeterminate beams and profiled decks was studied by Unger (1973, Yener and Peköz (1980), Yu (1981), and Bryan and Leach (1984). The post-local-buckling behavior of continuous beams is discussed by Wang and Yeh (1974).

### 13.4.3 Web Crippling

The necessity to check web crippling is an important distinction for thin-walled members. The thin webs of beams may cripple due to the high local stresses caused by concentrated loads or reactions. Fig. 13.15 shows the types of deformation that occur due to crippling of unrestrained single webs and restrained double webs.

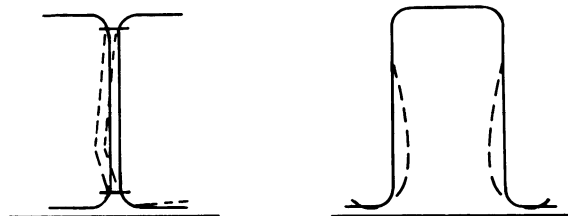


Fig. 13.15 Web crippling of beams.

Early experimental work (Winter and Pian, 1946; Rockey et al., 1972; Hetrakul and Yu, 1978) indicated that the web crippling strength of thin-walled beams depends on  $N/t$ ,  $h/t$ ,  $R/t$ , and  $F_y$ , where  $t$  is the web thickness,  $N$  the bearing length,  $h$  the flat width of the web,  $F_y$  the yield stress

of the steel, and  $R$  the inside bend radius. In 2001 all of the web crippling design expressions were brought together into one format in the AISI Specification (Prabakaran and Schuster 1998):

$$P_n = Ct^2 F_y \sin \theta \left( 1 - C_R \sqrt{\frac{R}{t}} \right) \left( 1 - C_N \sqrt{\frac{N}{t}} \right) \left( 1 - C_h \sqrt{\frac{h}{t}} \right) \quad (13.62)$$

where  $P_n$  is the web crippling capacity,  $\theta$  the web angle and  $C$ ,  $C_R$ ,  $C_N$ , and  $C_h$  are empirical coefficients that are cross-section and loading dependent.

The empirical coefficients,  $C$ 's, are based on the extensive experimental testing that has been completed on web crippling, including significant recent testing (Beshara and Schuster 2000; Gerges and Schuster 1998; Holesapple and LaBoube 2003; Prabakaran and Schuster 1998; Santaputra et al. 1989; Wallace and Schuster 2005; Young and Hancock 2001; Young and Hancock 2004) as well as earlier testing as reported in the AISI Specification. Eq. 13.62 has been shown to provide reliable predictions across the broad data set (Beshara and Schuster 2002).

Experimental work is now moving away from testing individual cross-sections and to testing structural systems, this trend is particularly evident in lightweight steel framing systems where testing as been completed on web crippling in trusses (Ibrahim et al. 1998), headers (Stephens and LaBoube 2000), stud-to-track connections (Fox and Schuster 2002), and joist-to-rim connections (Serrette 2002) and framing members with holes (LaBoube et al. 1999; LaBoube et al. 1997; Langan et al. 1994).

Though it is powerful and simple, Eq. 13.62 is not without its drawbacks. The empirical coefficients, vary significantly over the 46 different categories provided in the Specification, and not always in a rational fashion. Alternatives to testing include nonlinear finite element analysis (Fox and Brodland 2004; Ren et al. 2006b; Sivakumaran 1989) as well as yield-line theory (Bakker and Stark 1994). To date these alternatives are too involved for regular design use.

#### **13.4.4 Bending and Web Crippling**

While web crippling is generally associated with shear loads, the presence of bending demands will also decrease the web crippling capacity. The AISI Specification (2007) provides a series of interaction equations for checking bending and web crippling, the equations in current use are summarized in Wallace et al. (2002). In addition to the experimental work summarized in Wallace et al. (2002) recent work on isolated channels (Young and Hancock 2002) and headers in light steel framing (Stephens and LaBoube 2002; Stephens and LaBoube 2003) have investigated bending and web crippling interaction. Work continues on analytical models to understand web crippling and its interaction with bending (Hofmeyer et al. 2001) as well as fully computational solutions (Ren et al. 2006a).

#### **13.4.5 Bending and Shear**

When high bending stresses and high shear stresses act simultaneously, as in cantilever beams and at supports of continuous beams, the webs of beams will buckle at a lower stress than if only one stress were present. For a combination of bending and shear, Eq. 4.8 can be used to predict buckling.

$$\left( \frac{\sigma_{cb}}{\sigma_{cb}^*} \right)^2 + \left( \frac{\tau_c}{\tau_c^*} \right)^2 \leq 1 \quad (13.63)$$

where  $\sigma_{cb}$  is the actual compressive stress at the junction of flange and web,  $\tau_c$  is the actual average shear stress, and  $\sigma_{cb}^*$  and  $\tau_c^*$  are the critical stresses for bending and shear, respectively. Eq. 13.64 has been adapted for design by replacing the critical stresses for bending and shear with permissible design strength values. The equation has also been presented in a load format by replacing the shear stress with shear force and bending stress with bending moment:

$$\left(\frac{M}{M_n}\right)^2 + \left(\frac{V}{V_n}\right)^2 \leq 1 \quad (13.64)$$

where  $M$  and  $V$  are the applied moments and shears, and  $M_n$  and  $V_n$  are the nominal strengths. Webs with transverse stiffeners can develop strength in excess of Eq. 13.64 (LaBoube and Yu 1978) and a slightly more liberal interaction equation is provided for that case in AISI (2007).

### 13.4.6 Bending and Axial load

The design of beam-columns, or members under bending and axial load, has seen much study in recent years, see Chapter 8 and Chapter 16 of this Guide for example. Similar to hot-rolled steel construction, cold-formed steel now provides a traditional  $K$ -factor approach in the main body of the Specification (AISI 2007) and a 2<sup>nd</sup> order analysis approach in the Appendix (AISI 2007, App.2).

The main Specification approach uses a simple linear interaction equation with classical moment amplification for P- $\delta$  or P- $\Delta$  moments:

$$\frac{P}{\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} + \frac{M_y}{\phi_b M_{ny}} \leq 1.0 \quad (13.65)$$

where  $P$ ,  $M_x$ ,  $M_y$  are the 2<sup>nd</sup> order demands, and  $P_n$ ,  $M_{nx}$ ,  $M_{ny}$  are the capacities, and  $\phi$  the resistance factor. Demands  $M_x$  and  $M_y$  reflect P- $\delta$  or P- $\Delta$  contributions through amplification of the 1<sup>st</sup> order moments (e.g.  $M_{x0}$ ) by the traditional approach:

$$M_x = C_m \frac{1}{1 - P/P_{cr}} M_{x0} \quad (13.66)$$

where  $C_m$  accounts for the location of  $M_{x0}$  in the amplification and  $P_{cr}$  is the buckling load determined through knowledge of the effective length(s),  $KL$ (s). Specific provisions are not provided for determining  $K$ , reference is given to the AISC commentary and earlier editions of this SSRC Guide.

New to the 2007 Specification is Appendix 2 which provides guidelines for the application of 2<sup>nd</sup> order analysis in the determination of  $M_x$  and  $M_y$  as opposed to Eq. 13.66 for cold-formed steel structures (Sarawit and Pekoz 2006). Initial imperfections, or equivalent notional loads, are added to a frame model of the structure and analysis conducted to determine the structural demands in the deformed geometry. Since member  $\delta$  imperfections are inherently assumed in the  $P_n$  capacity of Eq. 13.22-23 or 13.40-41, such imperfections are not required to be explicitly modeled in the 2<sup>nd</sup> order analysis. In addition, following the procedure in AISC, the elastic stiffness,  $E$ , in the models are reduced (reducing  $E$  is simply a convenient way to reduce bending rigidity  $EI$  and axial rigidity  $EA$ ) in an attempt to account for member reliability and reduced stiffness due to partial yielding under large axial loads. Further details are provided in the AISI commentary, the AISC commentary, and Chapter 16 of this Guide.



## 13.5 STRUCTURAL ASSEMBLIES

### 13.5.1 Built-up Sections

Built-up sections are common in light steel framing, but to date the AISI Specification provisions remain somewhat rudimentary. For built-up columns, to account for reduced shear rigidity in built-up sections with discrete fastening, AISI (2007) prescribes a modified slenderness  $(KL/r)_m$  for use in flexural and flexural-torsional buckling calculations:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \quad (13.67)$$

where  $(KL/r)_o$  is the overall slenderness ratio of the entire section about the built-up member axis,  $a$  = fastener spacing, and  $r_i$  is the minimum radius of gyration of an individual shape in the built-up member. In addition the fasteners must be designed for  $2.5\%P_n$ , the faster spacing is limited, and end details are prescribed. The design of built-up cold-formed steel members is derived largely from the extension of similar research for built-up hot-rolled steel members and further work in this area is needed.

### 13.5.2 Bracing

As discussed in Chapter 12 structural bracing may be divided into two general types, according to its function: (1) bracing provided to resist secondary loads on structures, such as wind bracing, and (2) bracing provided to increase the strength of individual structural members by preventing them from deforming in their weakest direction (Winter, 1958). In the latter instance, there are again two different cases: (1) bracing applied to prevent buckling and thereby increase the unstable strength of the member, and (2) bracing applied to counteract stable but detrimental types of deformation. As examples of the latter, C- and Z-shaped beams loaded in the plane of the web twist or deflect laterally, with consequent loss of strength unless they are properly braced (Murray and Elhouar, 1985). For C-sections the eccentricity between the shear center and the plane of loading creates a torsion demand (a tendency to roll), while for Z-sections the difference between the principal (inclined) axes and the geometric axis creates a similar torsion demand, in either case bracing maybe used to restrict the movement and appropriate expressions for the developed forces are provided in AISI (2007).

Bracing may be *continuous*, such as that provided by wall panels, roof decking, or floor systems, or it may be noncontinuous or *discrete*, such as cross-bracing. For discrete bracing the spacing of the braced points also is important. Finally, bracing may also be distinguished according to its behavior: (1) that which provides restraint through resistance to axial deformation, as does cross-bracing, and (2) that which provides restraint through resistance to shear deformation, as do diaphragms (Fig. 13.16).

For bracing against buckling to be effective in an actual situation, it must possess not only the requisite strength but also a definite minimum rigidity. However, the required strength cannot be computed uniquely, except on the basis of assumed imperfections of shape and/or loading of the member to be braced (Winter, 1958). Recognizing this fact lead to the development of recent AISC provisions (as discussed in Chapter 12). For cold-formed steel explicit adoption of this design philosophy is still underway. In many cases the stiffness is assumed and the brace force is assumed to be 2% of the axial load. However, for axial loaded members undergoing flexural buckling, research by Green et al. (2006) have lead to recent

adoption of strength and stiffness bracing provisions in AISI (2007). The provisions require an axial brace capacity of 1% of the member axial capacity, and a stiffness based on the discrete brace spacing developed directly from Winter's work. Diaphragm bracing and continuous bracing can be quite application specific in thin-walled steel construction and are thus covered in that way, as opposed to a general treatment, in AISI (2007).

### ***13.5.3 Light-frame Construction***

Since the last edition of this Guide the practice of light-frame cold-formed steel construction has evolved significantly, particularly with respect to Standards. A significant amount of research has gone into the development of a series of new standards for cold-formed steel framing, these North American Standards for Cold-Formed Steel Framing include: General Provisions (AISI S200-07), Product Data (AISI S201-07), Floor and Roof System Design (AISI S210-07), Wall Stud Design (AISI S211-07), Header Design (AISI S212-07), Lateral Design (AISI S213-07), and Truss Design (AISI S214-07). These standards attempt, when possible, to treat the system as opposed to the individual members.

General Provisions (AISI S200-07) and Product Data (AISI S201-07) are essentially self-explanatory. Floor and Roof System Design (AISI S210-07) covers the design of floors and roofs by either the discretely braced design or continuously braced design philosophy. For continuously braced design prescriptive sheathing requirements are provided (insuring a level of rigidity for the brace) along with the forces required to counteract rolling of the joists. The Floor and Roof System Design standard also provides a simple means to design clip angle bearing stiffeners, based on recent research (Fox 2006).

The Wall Stud Design standard (AISI S211-07) is similar to the floor and roof provisions and covers the design of wall studs by either the all steel design or sheathing braced design philosophy. Prescriptive load limits are provided for gypsum sheathed designs based on experimental testing (Miller and Pekoz 1994). Sheathing braced design does not imply diaphragm-based design methods, which were essentially abandoned for sheathed walls, based on the observation that local fastener deformations, not sheathing in shear, dominates the response. Also covered in this Standard are stud-to-track connection strength including web crippling (Fox and Schuster 2000) and deflection track strength.

As first discussed in Section 13.5.1, the general built-up section provisions in AISI (2007) are rudimentary, but significant research has been conducted on built-up headers used in light frame construction (Elhajj and LaBoube 2000; Stephens and LaBoube 2000; Stephens and LaBoube 2003) This research has lead to provisions for box headers, double L headers, and single L headers covering web crippling, bending and web crippling, and simplified moment calculations in the Header Design standard (AISI S212-07).

The Lateral Design standard (AISI S213-07) has had a significant impact on practice as this standard provides a means to determine the lateral strength of cold-formed steel systems used in wind and seismic demands. The standard provides compiled test results for cold-formed steel shear walls and diaphragms with a variety of sheathing, fastener spacing, stud spacing, etc. Specific seismic detailing provisions are provided, for example for strap-braced shear walls (Al-Kharat and Rogers 2007). In addition to strength, expressions are provided for deflection calculations (Serrette and Chau 2006).

The Truss Design standard (AISI S214-07) provides specific guidance on beam-column design for chord and web members of cold formed steel trusses. In addition due to the presence of concentrated loads (Ibrahim et al. 1998) in locations with compression and bending a unique interaction equation check for compression, bending, and web crippling is provided. Specific

guidance is also provided for gusset plate design (Lutz and Laboube 2005) and methods for testing trusses.

#### ***13.5.4 Diaphragm Construction (Metal Roof and Wall Systems)***

Thin-walled metal panels are often used as wall cladding, roof decking, and floor decking, where their primary structural function is to carry loads acting normal to their surface. Properly designed and interconnected metal roof, wall, and floor systems are also capable of resisting shear forces in their own planes, referred to as diaphragm action. Thus, primary components of the lateral force resisting system for wind or seismic may be a properly detailed floor and/or roof diaphragm composed of fastened and/or welded thin-walled metal panels. Such a system can be highly advantageous, for example, procedures have been developed that recognize the ability of diaphragms assembled from such panels to transfer load from a heavily loaded frame to less heavily loaded adjacent frames in a single-story structure, thus reducing the required maximum frame size (Luttrell, 1967; Bryan and Davies, 1981). The shear strength and rigidity of thin-walled panels can be utilized in folded plate structures (Nilson, 1960), hyperbolic paraboloids (Gergely et al., 1971), and other shell roof structures (Bresler et al., 1968). In addition, theory and test results both have shown that the shear strength and rigidity of properly connected diaphragms can be effective as bracing for individual beams and columns.

#### ***Diaphragm-Braced Columns***

In the elastic range the predicted weak-axis buckling load of an ideal axially loaded I-section column or wall stud with directly attached symmetrical diaphragm bracing (Larson, 1960; Pincus and Fisher, 1966) is determined as

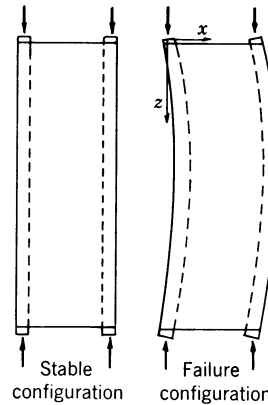
$$P = P_{yy} + Q \quad (13.68)$$

where  $P_{yy}$  is the weak-axis buckling load of the unbraced column and  $Q$  is the shear rigidity of the diaphragm contributing to the support of the column. The shear rigidity can be expressed as

$$Q = A_d G_{\text{eff}} \quad (13.69)$$

where  $A_d$  is the cross-sectional area of the diaphragm normal to the column axis and contributing to the support of the member and  $G_{\text{eff}}$  is the effective shear modulus of the diaphragm. Similar expressions are also possible for columns undergoing flexural-torsional buckling, or having sheeting on only one side, or dissimilar sheeting (Simaan and Pekoz 1976). Of course, Eq. 13.68 only predicts the increased buckling load for ideal members, initial imperfections and a full treatment of the fastener and diaphragm stiffness are still required.

Due to the complexity of predicting the strength of diaphragm-braced columns the AISI Specification (2007) provides empirical methods for compression members with one flange attached to metal deck (Glaser et al. 1994), or standing seam roof systems (Stolarczyk et al. 2002). Strict prescriptive limits define the system and details for the application of these expressions. The more general treatment of Simaan and Peköz (1976) was employed in the AISI Specification for wall studs (with different sheathing types) from 1981 to 2004, but has been replaced by rational engineering analysis in the Wall Stud Standard (AISI S211-07). A key requirement for diaphragm action to occur is that the fasteners must be stiff enough to engage the diaphragm, in many traditional sheathing materials (as opposed to thin-walled metal panels), this does not appear readily possible with conventional detailing (Miller and Pekoz 1993; Miller and Pekoz 1994) and it may therefore be difficult to get full diaphragm action engaged.



**Fig. 13.16 Members with diaphragm bracing.**

### ***Diaphragm-Braced Beams***

The same type of diaphragm action is also useful in counteracting lateral-torsional buckling of beams. For ideal I-section beams braced directly by diaphragms on the compression flange, the critical lateral-buckling moment can be estimated as (Errera et al., 1967)

$$M_{cr} = M_0 + 2Qe \quad (13.70)$$

where  $M_0$  is the lateral-buckling moment of the unbraced beam,  $e$  the distance between the center of gravity of the beam cross section and the plane of the diaphragm, and  $Q$  the shear rigidity of the diaphragm contributing to the support of the member, as defined previously. Again, it is emphasized that Eq. 13.70 predicts the buckling load of an ideal member. For real members the initial imperfections and the strength of the bracing must be taken into consideration.

As discussed in Section 13.5.2 above C- and Z-shaped beams have a tendency to twist under lateral load applied through the web. When both flanges of such beams are connected to deck or sheathing material in such a manner as to restrain lateral deflection effectively, no further bracing is required. However, when only one flange is connected the problem can be complex. For purlins and girts with one flange connected to sheathing as commonly found in metal buildings, design may proceed by either test methods or through empirical procedures (valid only within prescriptive limits) as provided by the AISI Specification (2007) both for conventional metal sheeting and standing seam roof systems.

Proper performance of a diaphragm requires adequate anchorage to the structure. Determination of the anchorage forces for sloped diaphragms common in metal building systems has seen significant investigation in recent years. The key to the recent improvements is the consideration of the stiffness of the full system: diaphragm, purlin, and anchorage in determination of the forces. Considering anchorage stiffness (as opposed to ideally rigid) reduces the forces the anchorage must be designed for, and allows for adequate performance so long as a minimum stiffness is provided. Development of the design method included experiments and shell finite element analysis (Seek and Murray 2005) as well as creation of a complete rational engineering analysis method capable of handling essentially all situations encountered in practice (Seek and Murray 2006; Seek and Murray 2007; Seek and Murray 2008). Ultimately this general method was simplified somewhat (Sears and Murray 2007) to that provided in the AISI (2007).

### 13.6 STAINLESS STEEL STRUCTURAL MEMBERS

Cold-formed stainless steel sections have been widely used architecturally in buildings because of their superior corrosion resistance, ease of maintenance, and pleasing appearance. Typical applications include column covers, curtain-wall panels, mullions, door and window framing, roofing and siding, stairs, elevators and escalators, flagpoles, signs, and many others. Since 1968, their use for structural load-carrying purposes has been increased due to the availability of the AISI and ASCE design specifications (AISI, 1974; ASCE, 1990, 2002).

The main reason for having a different specification for stainless steel structural members is because stainless steel has the following differing characteristics compared with carbon steel:

1. Anisotropy
2. Nonlinear stress-strain relationship
3. Low proportional limit
4. Pronounced response to cold work

Fig. 13.17 shows the stress-strain curves of annealed, half-hard, and full-hard stainless steels. Because of the differences in mechanical properties and structural uses between stainless steel and carbon steel, the ASCE specification for stainless steel design contains modified design provisions for (1) local buckling of flat elements, (2)  $w/t$  limitations, (3) deflection calculations, (4) service stress limitations, (5) lateral buckling of beams, (6) column buckling, and (7) connections. In general, the factors of safety used for the allowable stress design of stainless steels are somewhat larger than those used for carbon steel. Due consideration has been given to the development of the load and resistance factor design criteria (Lin et al. 1992).

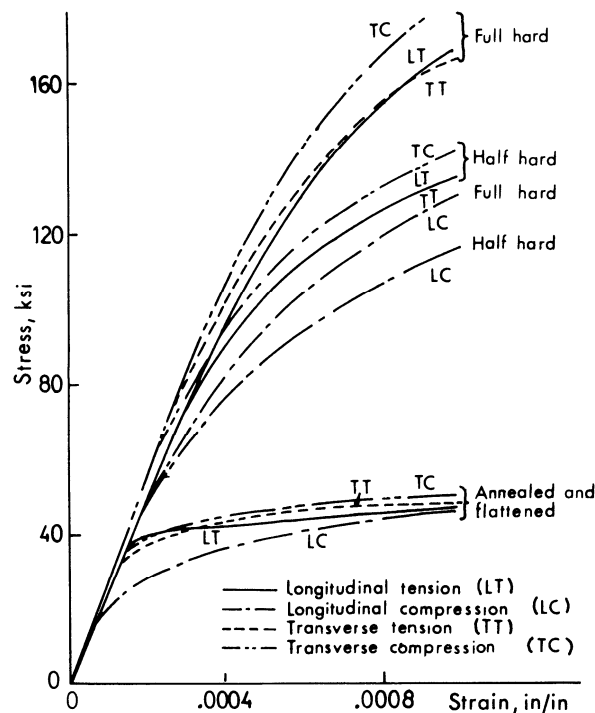


Fig. 13.17 Stress-strain curves of annealed half-hard, and full-hard stainless steels (Wang, 1969; Johnson and Kelsen, 1969).

The first edition of the *Specification for the Design of Cold-Formed Stainless Steel Structural Members* was issued by American Iron and Steel Institute in 1968 on the basis of the research conducted by Johnson and Winter (1966) at Cornell University. This specification was revised in 1974 to reflect the results of additional research (Wang and Errera, 1971) and the improved knowledge of material properties and structural applications. This 1974 edition of the AISI specification contained design information on annealed and cold-rolled grades of sheet and strip stainless steels, types 201, 202, 301, 304, and 316. In 1990, a new ASCE standard, *Specification for the Design of Cold-Formed Stainless Steel Structural Members* (ASCE, 1990; Lin et al., 1992; Yu and Lin, 1992), was published by the American Society of Civil Engineers to supersede the AISI specification. This new ASCE specification is based on both limit-states design and allowable stress design and is applicable to the use of four types of austenitic stainless steels (types 201, 301, 304, and 316) and three types of ferritic stainless steels (annealed types 409, 430, and 439). In 2002 the ASCE Standard was updated, largely to keep the standards in-line with the AISI cold-formed carbon steel standards wherever appropriate. Austenitic UNS Designation S20400 (annealed and 1/4 hard) stainless steels were added to the 2002 Specification. Fully developed stainless steel standards also exist in Europe, South Africa, and Australia/New Zealand, just to name a few.

Research on stainless steel members remains active. Fundamental experimental work on beam and column strength of stainless steel tubular and hollow sections including high-strength sections has provided a wealth of new reliable data for better understanding the section strength (Gardner and Nethercot 2004a; Gardner and Nethercot 2004b; Rasmussen 2000; Rasmussen and Hancock 1993a; Rasmussen and Hancock 1993b; Van den Berg 1998; Van Den Berg 2000; Young 2008; Young and Hartono 2002; Young and Liu 2003; Young and Lui 2005; Young and Lui 2006; Zhou and Young 2005). Analytical methods for material modeling including treatment of the highly worked corner regions (Ashraf et al. 2005; Rasmussen 2003) and complete summaries of imperfections and residual stresses (Cruise and Gardner 2006) are leading to improved modeling capabilities. Successful shell finite element models are being used by a wide variety of researchers to extend parametric studies and more closely examine the impact of differences between stainless steel and carbon steel on the behavior and strength (Ashraf et al. 2007; Ellobody and Young 2005; Gardner and Nethercot 2004c; Rasmussen et al. 2003; Young and Ellobody 2006). Web crippling in stainless steel sections has seen recent study (Zhou and Young 2006; Zhou and Young 2007a; Zhou and Young 2007b) as has the performance of stainless steel sections at elevated temperatures, i.e., fire conditions (Chen and Young 2006; Gardner and Ng 2006; Ng and Gardner 2007; To and Young 2008).

Development of design methods for stainless steel also remains active, as researchers look for robust methods that can accommodate the unique properties of stainless steel, but still provide simple solutions for use in conventional design. For example, alternatives to ASCE's iterative column design methods are available with similar reliability (Rasmussen and Rondal 1997). Improvements to the Effective Width Method to better handle gradual yielding behavior of stainless steel sections have been proposed (Rasmussen et al. 2004). Other design method improvements have been proposed such as basing the resistance on deformation capacity and the use of continuous instead of discrete cross-section classification methods (used in Eurocode) (Ashraf et al. 2008; Gardner and Nethercot 2004d; Gardner et al. 2006; Gardner and Theofanous 2008). Finally, the initial development of the Direct Strength Method for stainless steel has also been completed (Becque et al. 2008).

## 13.7 ALUMINUM STRUCTURAL MEMBERS

Renumber equations to start with Eq. 13.71

Consider adding to Alum section

Kim, Y., Peköz, T. "Flexural strength of aluminum extrusions of arbitrary cross-section." *Proceedings of the Annual Stability Conference*, Structural Stability Research Council, 455-474.

Mennink, J. (2002). Cross-sectional stability of aluminium extrusions. Ph.D. Thesis, TU-Eindhoven.

Zhu, J., Young, B. (2006). "Aluminum alloy tubular columns-Part II: Parametric study and design using direct strength method." *Thin-walled Structures*, 44 (9) 969-985

### 13.7.1 Effective Widths

Postbuckling strengths of thin aluminum plate elements are generally based on the von Kármán concept that the width,  $b_e$ , for which the elastic buckling stress (Eq. 4.1)

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)(b/t)^2} = \frac{\pi^2 E}{(mb/t)^2} \quad (13.49)$$

is equal to the yield stress gives a limiting capacity which remains constant for all other widths. Thus, from Eq. 13.49,

$$b_e = \frac{\pi t}{m} \sqrt{\frac{E}{\sigma_y}} \quad (13.50)$$

For a simply supported plate the buckling coefficient  $k = 4$ , so  $m = 1.63$  and  $b_e = 1.93t \sqrt{E/\sigma_y}$ .

This is not entirely consistent with the treatment for buckling of thin walls, which for the elastic-plastic range uses the equivalent slenderness ratio,  $mb/t$ , in an expression of the type

$$\sigma_c = \left( B_p - \frac{D_p mb}{t} \right) \quad (13.51)$$

This expression represents an approximation of the true limiting stress (Jombock and Clark, 1968), and the maximum load carried by a plate element in the elastic-plastic range is

$$p = \sigma_c bt = \left( B_p - \frac{D_p mb}{t} \right) bt \quad (13.52)$$

The effective width, defined as that width which when multiplied by the yield stress and the thickness, gives the failure load for the element, then becomes

$$b_e = \frac{b}{\sigma_y} \left( B_p - \frac{D_p mb}{t} \right) \quad (13.53)$$

As  $b$  increases the highest load that can be carried occurs when

$$\frac{b}{t} = \frac{B_p}{2D_p m} \quad (13.54)$$

giving

$$P = \frac{B_p^2 t^2}{4D_p m} \quad (13.55)$$

This load capacity remains reasonably constant for all higher values of  $b/t$ . Jombock and Clark (1968) provide values for  $B_p$  and  $D_p$  which can be represented as

$$B_p = \sigma_y \alpha^2 \quad (13.56)$$

$$D_p = \frac{\sigma_y k \alpha}{g} \quad (13.57)$$

where

$$g = (E / \sigma_y)^{1/2}$$

$$\alpha = (1 + 2/g^{2/3})^{1/2} \text{ for fully heat-treated alloy}$$

$$\alpha = (1 + 3/g^{2/3})^{1/2} \text{ for other alloys}$$

$$k = 0.1 \text{ for fully heat-treated alloys}$$

$$k = 0.12 \text{ for other alloys}$$

The effective width is then

$$b_e = b \alpha (\alpha - kmb / t) \quad (13.58)$$

with a maximum value of

$$b_e = \alpha^3 gt / 4mk \quad (13.59)$$

Jombock and Clark (1968), on the basis of limiting strain, provided a theoretical foundation for the model above, which has been adopted in North American codes (AA, 1994; CSA, 1984). This model gives a continuous transition from compact to thin-walled elements without discontinuities or changes in the form of the design expression.

Postbuckling behavior of elements supported on the long, unloaded edges, to which the model above applies, differs from that of outstanding, flange-type elements, in that, for the latter, initial elastic buckling precipitates the collapse of single unsymmetrical elements, while there may be some reserve capacity in symmetrical sections which can be represented by the treatment for edge-supported elements using an appropriate value of the coefficient  $m$ , usually 5.

The limiting stress on unstiffened elements of an unsymmetrical section such as a channel or Z, is obtained using the expression

$$\frac{mb}{t} < C \quad \sigma = B - D \left( \frac{mb}{t} \right) \quad (13.60)$$

$$\frac{mb}{t} < C \quad \sigma = \pi^2 E / \left( \frac{mb}{t} \right) \quad (13.61)$$

where the value of  $m$  lies between 3 and 5, depending on the degree of edge restraint, and  $C$  is the slenderness parameter separating elastic and inelastic buckling.

In symmetrical sections, such as I- and double-channel shapes the effective width will be given by Eqs. 13.58 and 13.59 using  $m = 5$ , unless local buckling can precipitate overall flexural or lateral-torsional buckling, which will occur when the critical stresses for the different buckling modes are close in value.

### 13.7.2 Effective Section at Service Loads

A margin of over 1.5 between service loads and the ultimate load is usual; thus the extent of any local buckling in service will be small and confined to zones of maximum moment. For this reason, the influence of local buckling on deflections under service loads has been neglected in some codes. If a calculation is to be made, the difficulty of computing deflections with a varying and initially unknown effective moment of inertia is usually resolved by assuming that the effective section at the point of highest moment applies throughout. The effective width  $b_e$  of the elements comprising the effective section are obtained using the actual width  $b$  and the ratio of



the critical stress  $\sigma_c$  to the applied stress  $\sigma$  computed on the basis of a fully effective section, thus:

$$b_e = b \left( \frac{\sigma_c}{\sigma} \right)^{1/2} \quad (13.62)$$

### **13.7.3 Torsional Buckling**

#### **13.7.3.1 Angles.**

A single angle may fail by flexure or torsional buckling; only by a special proportion of heavy root bulbs and very thin legs can local buckling be made to occur. To optimize shapes, bulbs and root fillets are added to increase the torsional rigidity such that the equivalent slenderness ratio for torsional buckling is around 60. Because of the interaction of torsion with flexure about the stronger axis, is it not effective to design sections of equal inertia about the two principal axes, and the optimum is an equal-leg right-angle section for both plain and bulb shapes. This is also true of double angles, designed to balance torsional and flexural buckling, in which case equal-leg angles are again very close to the optimum.

#### **13.7.3.2 Eccentrically Loaded Columns.**

Unsymmetrical open sections loaded axially fail in combined torsion and flexure. Should they be loaded through the shear center, the modes are uncoupled and torsional buckling can be eliminated. Use has been made of this in T-sections for diagonals which, when bolted through the flanges, are loaded through the shear center. This allows much thinner sections to be used with a considerable increase in efficiency despite the moment due to the eccentricity. The optimum form is a lipped shape to control local buckling of the flanges.

Single angles loaded through one leg fail by lateral-torsional buckling in the manner of a beam-column (Marsh, 1969) and the design procedure adopted by CSA (1983) and ASCE (1972) treat this interaction by using an effective slenderness ratio:

$$\left( \frac{KL}{r} \right)_{\text{eff}} = \left[ \left( \frac{5b}{t} \right)^2 + \left( \frac{KL}{r_v} \right)^2 \right]^{1/2} \quad (13.63)$$

where  $5b/t$  is the slenderness ratio for torsional buckling of angles and  $KL/r_v$  is the slenderness ratio for flexural buckling.

#### **13.7.3.3 Postbuckling Strength.**

In general, the critical stress for a column failing by torsional buckling represents the maximum capacity of the member. This is always true of pin-ended unsymmetrical sections, as the application of the load through the centroid requires a uniform stress in the section for equilibrium. Should the column be loaded by fixed platens, the axis of load application can shift as the member twists, causing an increase in stress toward the shear center (Smith, 1955). Symmetrical sections, such as a cruciform, even when pin-ended, can accept a higher stress at the center as the member twists, giving a higher critical load than that obtained for a uniform stress.

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