CFS-NEES

Advancing Cold-Formed Steel Earthquake Engineering

RESEARCH REPORT

Moment-Rotation Characterization of Cold-Formed Steel Beams

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SUMMARY

The objective of this study is to provide a prediction method for characterizing the complete moment-rotation (M-θ) response of cold-formed steel (CFS) members in bending. The work is an ancillary effort related to the National Science Foundation funded Network for Earthquake Engineering Simulation (NEES) project: CFS-NEES (www.ce.jhu.edu/bschafer/cfsnees). The goal of CFS-NEES is to enable performance-based seismic design for cold-formed steel framed buildings. A basic building block of performance-based seismic design is nonlinear structural analysis. For cold-formed steel members, which suffer from local and distortional buckling, existing codes provide peak strength and approximations for stiffness loss prior to peak strength, but no estimation of post-peak M-θ behavior. Complete M-θ response is necessary for nonlinear structural analysis of CFS framed buildings. In this research, existing data, obtained by experiments and finite element analysis, are processed to examine the complete M-θ response in cold-formed steel beams. Using a modification of the simplified model introduced in ASCE 41 for pushover analysis, the M-θ response is parameterized into a simple multi-linear curve. The parameters include the initial stiffness, fully effective limit, reduced pre-peak stiffness, peak moment, post-peak plateau, and post-peak rotation at 50% of the peak moment. It is shown herein that the parameters of this multi-linear M-θ curve may themselves be readily predicted as a function of either the local slenderness or distortional slenderness of the cross-section, as appropriate. Accuracy of the proposed M-θ approximation is assessed. The impact of utilizing the full M-θ response in a single and multi-span CFS beam is demonstrated. The proposed prediction method for M-θ provides a necessary step in the development of nonlinear structural analysis of CFS systems.

Table of Contents

List of Figures

List of Tables

1 Introduction

Cold-formed steel (CFS) enjoys a wide and growing base of application in civil structures. Although design codes provide full guidance for strength prediction and partial guidance for stiffness of CFS members, member ductility - specifically the full moment-rotation (M-θ) response of members is not addressed.

Collapse analysis of a CFS building system (i.e. a building comprised of load bearing cold-formed steel framing), whether for static loads, wind loads, progressive collapse, or seismic design is predicated on knowledge of the nonlinear response of the components and connections that make up a building. Simple determination of the force or moment redistribution in a CFS building system after one member fails may not be accurately completed with current knowledge, requiring current design to ignore system effects and instead concentrate on first member failure. Given that CFS cross-sections are typically locally slender, they have a more complicated and less forgiving moment-rotation response than compact hot-rolled steel beams. Therefore, simple elastic-perfectly plastic response as commonly used in conventional steel analysis is generally not appropriate for CFS members.

Further, since much of the nonlinear response in CFS building systems is related to the shear walls, CFS member response has not been pursued in much detail. Regardless, this lack of understanding has consequences. For example, in CFS seismic design, buildings are detailed with the goal of concentrating all nonlinear response in pre-tested shear walls. The capacity of other members (or connections) to absorb any of the deformation (energy) is ignored – as is the potential for redistribution of forces – leading to model predictions divorced from reality and structural systems that do not achieve full economy.

For modeling collapse, particularly under dynamic (seismic) loads, no current method provides guidance on member ductility of CFS members. Without fundamental information on CFS member ductility, system modeling for CFS structures to collapse or under dynamic loads, is impossible. This report attempts to take the initial steps toward providing this needed information for CFS beams.

2 ASCE41 M-θ Definitions

The latest in a series of documents developed to assist engineers with the seismic assessment and rehabilitation of existing buildings (FEMA 273, 1997; FEMA 356, 2000) is ASCE/SEI 41 (2007). These documents provide a comparison of deformation and force demands for different seismic hazards against deformation and force capacities for various performance levels to provide a performance-based seismic engineering framework.

The ductile performance of steel structures is highly dependent on the ability of its members to dissipate energy by means of hysteretic behavior. The amount of dissipated energy is usually correlated with the area under the force-deformation/moment-rotation curve. ASCE/SEI 41 (2007) provides three basic types of component force-deformation curves (Fig. 1, where Q=P or M and Δ=Δ or θ, all parameters are define in ASCE41). The acceptance criteria for each type of moment-rotation curve is defined depending on the performance level.

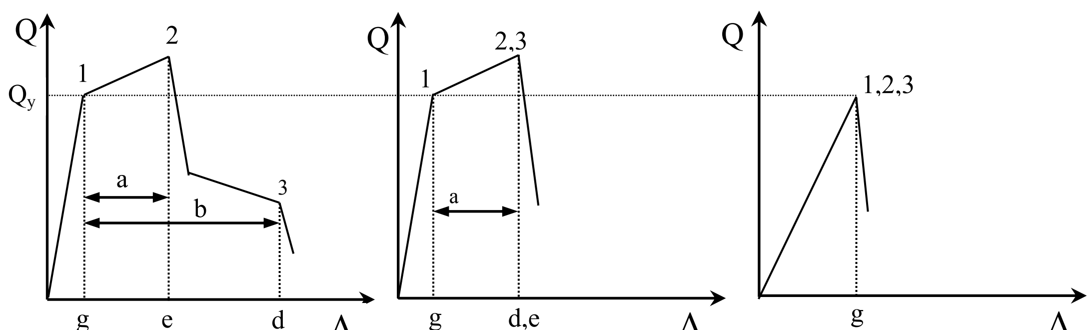


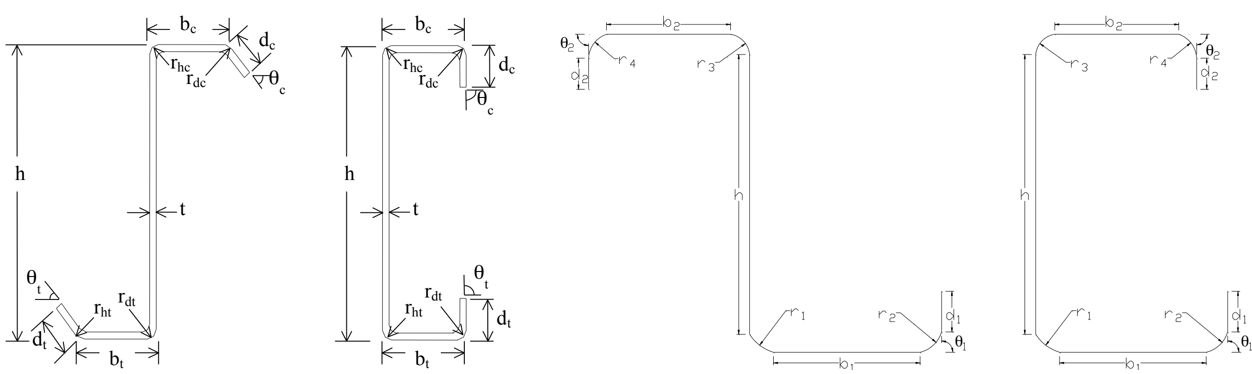
Figure 1: Component force-deformations curves of ASCE 41 (2007)

ASCE41 does not include explicit predictions for CFS members; therefore, here ASCE 41 backbone 'curve fitting' exercises are realized for CFS members. The ASCE41 Type 1 curves assume an elastic range followed by a plastic range including strain hardening, then a post-peak strength degraded range. This is modified for CFS members, which instead have a pre-peak fully effective (elastic) range. The pre-peak partially effective range is followed by a peak (moment), that is typically less than the yield moment of the beam, and then followed by a post-peak strength-degraded range.

3 CFS M-θ Data for Local and Distortional Buckling

Cross-sections studied

The experiments of Yu and Schafer (2003, 2006, and 2007) and finite element (FE) analysis results of Shifferaw and Schafer (2010), on local and distortional buckling of CFS beams, are utilized herein as the available moment-rotation response of CFS beams. The out-to-out dimensions of the cross-sections (Fig. 2a) for twenty-four local and twenty-two distortional buckling tests of Yu and Schafer (2003, 2006) are listed separately in Table 1 and Table 2. The centerline dimensions (Fig. 2b) of seventeen cross-sections from Yu and Schafer (2003, 2006) (tests having $M_{test}>0.95M_y$) are used in the FE analysis study of Shifferaw and Schafer (2011) as listed in Table 3. From these centerline dimensions the thickness (Table 4) was varied from 0.0538 in. to 0.1345 in. resulting in 187 different models in the Shifferaw and Schafer (2010) study.



(a) Yu and Schafer (2003, 2006) (b) Shifferaw and Schafer (2010)

Figure 2: Definitions of specimen dimensions for C and Z-section

Table 1: Measured geometry of specimens for local buckling tests of Yu and Schafer (2003, 2006)

Test label	Specimen	h (in.)	b_c (in.)	d_c (in.)	θ_c (deg)	b_t (in.)	d_t (in.)	θ_t (deg)	r_{hc} (in.)	r_{dc} (in.)	r_{ht} (in.)	r_{dt} (in.)	t (in.)
8.5Z120-3E2W	8.5Z120-3	8.44	2.58	0.96	47.2	2.46	0.99	48.9	0.36	0.36	0.35	0.35	0.1183
	8.5Z120-2	8.47	2.59	0.96	47.8	2.46	1.00	48.9	0.36	0.36	0.34	0.34	0.1180
8.5Z105-2E1W	8.5Z105-2	8.48	2.66	0.95	50.5	2.36	0.95	48.7	0.32	0.32	0.34	0.34	0.1040
	8.5Z105-1	8.42	2.69	0.97	50.7	2.36	0.91	48.7	0.31	0.31	0.34	0.34	0.1050
8.5Z092-4E2W	8.5Z092-4	8.41	2.61	0.93	53.0	2.41	0.96	50.8	0.29	0.29	0.31	0.31	0.0900
	8.5Z092-2	8.43	2.61	0.92	51.8	2.40	0.95	50.4	0.28	0.28	0.31	0.31	0.0887
8.5Z082-1E2W	8.5Z082-1	8.46	2.50	0.95	49.0	2.36	0.97	50.3	0.28	0.28	0.30	0.30	0.0801
	8.5Z082-2	8.45	2.51	0.95	47.9	2.40	0.95	52.4	0.28	0.28	0.30	0.30	0.0804
8.5Z073-4E3W	8.5Z073-4	8.51	2.53	0.93	49.6	2.41	0.92	50.3	0.28	0.28	0.29	0.29	0.0715
	8.5Z073-3	8.50	2.53	0.91	50.1	2.38	0.96	51.0	0.28	0.28	0.30	0.30	0.0720
8.5Z073-1E2W	8.5Z073-2	8.50	2.54	0.93	50.2	2.41	0.92	51.0	0.28	0.28	0.30	0.30	0.0715
	8.5Z073-1	8.49	2.50	0.92	48.4	2.41	0.95	51.2	0.28	0.28	0.30	0.30	0.0720
8.5Z065-3E1W	8.5Z065-3	8.47	2.42	0.83	47.3	2.43	0.79	47.3	0.27	0.27	0.28	0.28	0.0640
	8.5Z065-1	8.47	2.44	0.76	47.4	2.43	0.84	47.1	0.28	0.28	0.27	0.27	0.0640
8.5Z059-4E3W	8.5Z059-4	8.50	2.50	0.77	50.9	2.35	0.72	48.9	0.28	0.28	0.28	0.28	0.0590
	8.5Z059-3	8.50	2.44	0.78	50.2	2.22	0.69	50.4	0.28	0.28	0.28	0.28	0.0595
8.5Z059-2E1W	8.5Z059-2	8.49	2.51	0.78	50.6	2.33	0.70	50.2	0.28	0.28	0.28	0.28	0.0590
	8.5Z059-1	8.50	2.51	0.78	51.2	2.33	0.71	49.4	0.28	0.28	0.28	0.28	0.0590
8C097-2E3W	8C097-2	8.04	2.12	0.57	85.6	2.08	0.52	85.7	0.30	0.28	0.28	0.30	0.0980
	8C097-3	8.03	2.09	0.56	84.0	2.08	0.54	88.2	0.30	0.28	0.28	0.29	0.0940
8C068-4E5W	8C068-5	8.03	2.03	0.52	83.2	2.04	0.53	87.0	0.28	0.25	0.24	0.24	0.0750
	8C068-4	8.01	2.05	0.52	84.0	2.04	0.54	87.6	0.27	0.26	0.24	0.27	0.0770
8C068-1E2W	8C068-2	8.02	2.04	0.52	83.4	2.04	0.53	87.6	0.28	0.25	0.24	0.26	0.0758
	8C068-1	8.03	2.03	0.53	83.1	2.05	0.53	88.1	0.30	0.26	0.25	0.26	0.0754
8C054-1E8W	8C054-1	8.00	2.04	0.52	88.9	2.07	0.50	84.7	0.22	0.23	0.23	0.23	0.0550
	8C054-8	8.08	2.02	0.58	88.1	1.96	0.48	82.3	0.22	0.20	0.22	0.23	0.0540
8C043-5E6W	8C043-5	8.04	2.02	0.53	88.8	1.98	0.53	87.3	0.18	0.20	0.21	0.20	0.0496
	8C043-6	8.06	2.01	0.53	88.9	2.00	0.46	87.0	0.19	0.20	0.22	0.20	0.0490
8C043-3E1W	8C043-3	8.04	2.02	0.54	89.3	2.01	0.53	87.5	0.19	0.19	0.19	0.19	0.0474
	8C043-1	8.03	2.02	0.54	89.0	1.98	0.54	85.8	0.19	0.19	0.29	0.19	0.0476
12C068-9E5W	12C068-9	12.02	1.92	0.53	82.0	2.00	0.55	85.3	0.28	0.27	0.30	0.28	0.0652
	12C068-5	12.00	1.79	0.55	85.9	2.06	0.53	94.8	0.27	0.27	0.22	0.27	0.0654
12C068-3E4W	12C068-3	11.97	1.96	0.59	82.5	1.99	0.56	77.4	0.26	0.27	0.27	0.27	0.0671
	12C068-4	12.02	2.01	0.52	80.6	2.00	0.52	83.3	0.26	0.27	0.26	0.27	0.0670
10C068-2E1W	10C068-2	10.08	1.93	0.50	83.2	1.98	0.52	83.3	0.27	0.25	0.27	0.25	0.0572
	10C068-1	10.03	2.04	0.55	80.7	1.97	0.54	81.9	0.27	0.26	0.28	0.25	0.0573
6C054-2E1W	6C054-2	6.04	2.00	0.56	85.7	2.00	0.52	90.0	0.21	0.24	0.26	0.25	0.0616
	6C054-1	6.03	2.01	0.56	86.5	2.05	0.52	90.5	0.22	0.25	0.25	0.24	0.0616
4C054-1E2W	4C054-1	3.95	1.99	0.55	79.2	2.02	0.55	77.4	0.24	0.24	0.23	0.24	0.0551
	4C054-2	3.96	1.95	0.50	74.2	1.96	0.55	74.8	0.22	0.27	0.25	0.25	0.0561
3.62C054-1E2W	3.62C054-1	3.65	1.97	0.49	77.1	2.00	0.42	88.1	0.23	0.26	0.26	0.25	0.0555
	3.62C054-2	3.67	1.99	0.51	79.8	1.97	0.44	79.8	0.24	0.25	0.26	0.26	0.0554
11.5Z092-1E2W	11.5Z092-1	11.41	3.33	0.96	50.1	3.51	0.96	49.5	0.25	0.27	0.27	0.27	0.1027
	11.5Z092-2	11.34	3.33	0.98	48.3	3.54	0.89	48.1	0.28	0.27	0.28	0.28	0.1033
11.5Z082-2E1W	11.5Z082-2	11.45	3.50	0.88	50.3	3.45	0.87	52.2	0.31	0.31	0.35	0.35	0.0837
	11.5Z082-1	11.47	3.49	0.90	50.6	3.43	0.88	51.0	0.32	0.32	0.35	0.35	0.0839
11.5Z073-2E1W	11.5Z073-2	11.39	3.51	0.87	46.0	3.35	0.83	44.8	0.27	0.28	0.27	0.28	0.0709
	11.5Z073-1	11.35	3.52	0.95	45.4	3.40	0.90	44.2	0.27	0.11	0.27	0.07	0.0695

Table 2: Measured geometry of specimens for distortional buckling tests of Yu and Schafer (2003, 2006)

Test label	Specimen	h (in.)	b_c (in.)	d_c (in.)	θ_c (deg)	b_t (in.)	d_t (in.)	θ_t (deg)	r_{hc} (in.)	r_{dc} (in.)	r_{ht} (in.)	r_{dt} (in.)	t (in.)
D8.5Z120-4E1W	D8.5Z120-4	8.44	2.63	0.93	54.20	2.47	1.00	50.20	0.34	0.34	0.34	0.34	0.1181
	D8.5Z120-1	8.43	2.65	0.94	48.10	2.52	0.99	52.10	0.36	0.36	0.35	0.35	0.1181
D8.5Z115-1E2W	D8.5Z115-2	8.54	2.56	0.91	49.00	2.40	0.89	48.30	0.35	0.35	0.37	0.37	0.1171
	D8.5Z115-1	8.50	2.66	0.82	48.33	2.47	0.87	48.30	0.37	0.37	0.39	0.39	0.1166
D8.5Z092-3E1W	D8.5Z092-3	8.40	2.58	0.95	51.90	2.41	0.94	51.60	0.29	0.29	0.31	0.31	0.0893
	D8.5Z092-1	8.42	2.59	0.93	52.40	2.39	0.95	50.90	0.28	0.28	0.31	0.31	0.0897
D8.5Z082-4E3W	D8.5Z082-4	8.48	2.52	0.94	48.50	2.39	0.97	51.30	0.28	0.28	0.30	0.30	0.0810
	D8.5Z082-3	8.50	2.53	0.94	49.90	2.37	0.96	49.50	0.28	0.28	0.30	0.30	0.0810
D8.5Z065-7E6W	D8.5Z065-7	8.48	2.47	0.83	50.00	2.47	0.82	49.33	0.32	0.32	0.33	0.33	0.0642
	D8.5Z065-6	8.52	2.48	0.87	53.00	2.43	0.83	48.33	0.32	0.32	0.34	0.34	0.0645
D8.5Z065-4E5W	D8.5Z065-5	8.50	2.36	0.67	51.33	2.52	0.90	47.17	0.27	0.27	0.28	0.28	0.0645
	D8.5Z065-4	8.40	2.40	0.81	47.33	2.25	0.65	51.17	0.30	0.30	0.27	0.27	0.0619
D8.5Z059-6E5W	D8.5Z059-6	8.44	2.42	0.77	50.40	2.39	0.86	48.00	0.32	0.32	0.30	0.30	0.0618
	D8.5Z059-5	8.50	2.42	0.80	48.30	2.40	0.76	48.33	0.30	0.30	0.32	0.32	0.0615
D11.5Z092-3E4W	D11.5Z092-4	11.23	3.47	0.94	48.70	3.40	0.91	49.60	0.33	0.33	0.31	0.31	0.0887
	D11.5Z092-3	11.25	3.43	0.89	49.29	3.46	0.87	49.50	0.33	0.33	0.32	0.32	0.0889
D11.5Z082-3E4W	D11.5Z082-4	11.40	3.41	0.88	48.40	3.40	0.86	49.90	0.30	0.30	0.32	0.32	0.0812
	D11.5Z082-3	11.33	3.41	0.94	50.20	3.42	0.93	50.97	0.31	0.31	0.31	0.31	0.0818
D8C097-7E6W	D8C097-7	8.13	2.15	0.65	80.75	2.13	0.62	80.00	0.27	0.29	0.27	0.30	0.1001
	D8C097-6	8.15	2.09	0.64	81.00	2.09	0.61	80.00	0.27	0.29	0.27	0.30	0.1005
D8C097-5E4W	D8C097-5	8.06	2.00	0.66	86.70	1.99	0.67	83.00	0.28	0.30	0.28	0.28	0.0998
	D8C097-4	8.06	2.03	0.67	83.00	2.00	0.68	83.00	0.27	0.28	0.27	0.28	0.0998
D8C085-2E1W	D8C085-2	8.06	1.98	0.63	86.00	1.96	0.68	86.60	0.22	0.22	0.23	0.22	0.0825
	D8C085-1	8.06	1.98	0.62	88.60	1.96	0.68	89.00	0.22	0.19	0.23	0.19	0.0848
D8C068-6E7W	D8C068-6	7.94	1.91	0.66	80.00	1.97	0.64	77.80	0.16	0.16	0.16	0.16	0.0708
	D8C068-7	7.94	1.97	0.64	76.50	1.95	0.67	77.50	0.16	0.16	0.16	0.16	0.0708
D8C054-7E6W	D8C054-7	8.01	2.04	0.53	83.40	2.03	0.57	88.70	0.24	0.23	0.21	0.23	0.0528
	D8C054-6	8.00	2.05	0.59	89.40	2.04	0.56	83.30	0.22	0.23	0.23	0.24	0.0520
D8C045-1E2W	D8C045-1	8.18	1.95	0.67	89.00	1.92	0.66	87.60	0.28	0.19	0.22	0.20	0.0348
	D8C045-2	8.14	1.94	0.69	88.80	1.92	0.69	88.30	0.28	0.20	0.23	0.20	0.0348
D8C043-4E2W	D8C043-4	8.02	2.01	0.53	87.30	2.01	0.53	88.80	0.17	0.18	0.17	0.20	0.0459
	D8C043-2	8.03	1.99	0.52	88.93	1.98	0.54	87.70	0.18	0.19	0.20	0.19	0.0472
D8C033-1E2W	D8C033-2	8.15	1.99	0.68	87.10	1.91	0.63	85.80	0.17	0.30	0.20	0.30	0.0337
	D8C033-1	8.08	2.00	0.61	86.00	1.96	0.77	88.00	0.21	0.26	0.18	0.28	0.0339
D12C068-10E11W	D12C068-11	12.03	2.03	0.51	81.97	2.00	0.53	85.33	0.22	0.22	0.24	0.23	0.0645
	D12C068-10	12.05	2.02	0.54	85.87	1.98	0.51	94.80	0.24	0.24	0.27	0.23	0.0648
D12C068-1E2W	D12C068-2	11.92	2.05	0.52	82.47	2.03	0.59	77.37	0.26	0.24	0.25	0.24	0.0664
	D12C068-1	11.97	2.12	0.52	80.60	2.00	0.56	83.30	0.25	0.25	0.26	0.26	0.0668
D10C068-4E3W	D10C068-4	10.08	2.00	0.48	83.23	2.08	0.53	83.30	0.26	0.21	0.23	0.23	0.0626
	D10C068-3	10.10	2.07	0.53	80.70	2.08	0.52	81.85	0.24	0.23	0.23	0.22	0.0634
D10C056-3E4W	D10C056-3	9.99	1.97	0.66	88.00	1.95	0.63	89.00	0.13	0.16	0.13	0.13	0.0569
	D10C056-4	10.00	1.94	0.72	88.60	1.92	0.66	87.70	0.13	0.16	0.13	0.18	0.0569
D10C048-1E2W	D10C048-1	9.94	2.06	0.62	86.10	1.94	0.63	79.60	0.20	0.19	0.20	0.19	0.0478
	D10C048-2	9.94	2.02	0.63	85.70	1.95	0.63	83.70	0.18	0.19	0.19	0.20	0.0486
D6C063-2E1W	D6C063-2	5.99	1.99	0.63	88.74	1.97	0.63	87.30	0.19	0.17	0.19	0.22	0.0578
	D6C063-1	5.99	1.99	0.62	87.03	1.97	0.63	86.13	0.22	0.17	0.22	0.17	0.0559
D3.62C054-3E4W	D3.62C054-4	3.73	1.88	0.41	87.00	1.87	0.43	89.00	0.26	0.24	0.27	0.27	0.0555
	D3.62C054-3	3.72	1.89	0.35	88.00	1.86	0.36	88.00	0.24	0.28	0.26	0.26	0.0556

Table 3: Centerline dimensions of cross-sections used in the parametric study of Shifferaw and Schafer (2010)

Specimen	h	b_1	b_2	d_1	d_2	θ_1	θ_2	r_1	r_2	r_3	r_4
8Z2.25x100	7.52	1.95	1.95	0.79	0.79	50.00	50.00	0.19	0.19	0.19	0.19
8.5Z2.5x70	8.00	2.13	2.13	0.78	0.78	50.00	50.00	0.22	0.22	0.22	0.22
8C068	7.58	1.71	1.61	0.47	0.57	77.80	80.00	0.13	0.13	0.13	0.13
8.5Z092	7.81	1.94	2.18	0.85	0.76	50.40	51.80	0.27	0.27	0.24	0.24
8.5Z120	7.80	2.01	2.08	0.85	0.84	48.90	47.80	0.28	0.28	0.30	0.30
8.5Z082	7.88	1.92	2.09	0.83	0.82	50.30	49.00	0.26	0.26	0.24	0.24
8C097	7.47	1.52	1.55	0.27	0.32	86.30	85.10	0.24	0.23	0.23	0.23
8.5Z120-2	7.77	1.97	2.07	0.84	0.80	48.90	47.20	0.28	0.28	0.30	0.30
8C097-3	7.45	1.52	1.54	0.26	0.31	88.20	84.00	0.23	0.24	0.25	0.23
8C068-5	7.51	1.57	1.53	0.30	0.30	87.00	83.20	0.20	0.20	0.24	0.21
6C054-2	5.57	1.49	1.57	0.27	0.34	90.00	85.70	0.23	0.22	0.18	0.21
4C054-2	3.49	1.52	1.53	0.36	0.30	74.80	74.20	0.22	0.22	0.19	0.24
3.62C054-2	3.17	1.49	1.54	0.22	0.30	79.80	79.80	0.23	0.23	0.21	0.22
D8.5Z120-4	7.76	1.97	2.12	0.84	0.76	50.20	54.20	0.28	0.28	0.28	0.28
D8C085-2	7.61	1.52	1.55	0.47	0.42	86.60	86.00	0.19	0.18	0.18	0.18
D10C068-4	9.59	1.65	1.55	0.33	0.29	83.30	83.23	0.20	0.20	0.23	0.18
D3.62C054-3	3.22	1.35	1.38	0.11	0.08	88.00	88.00	0.23	0.23	0.21	0.25

Table 4: Thickness variation for FE models

	1	2	3	4	5	6	7	8	9	10	11
Thickness (in)	0.0538	0.0566	0.0598	0.0673	0.0713	0.0747	0.0897	0.1017	0.1046	0.1196	0.1345

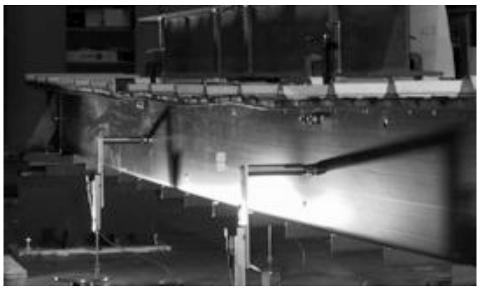
Four point bending tests

The experimental and computational work of Yu and Schafer (2003, 2006, 2007), which examined the strength of C- and Z-section beams failing in local and distortional buckling, is used as the basis for the study conducted herein. The test setup, typical failure modes, strength, and response of the tested specimens are summarized in Fig. 3. The tests consisted of paired CFS beams tested in 4 point bending.

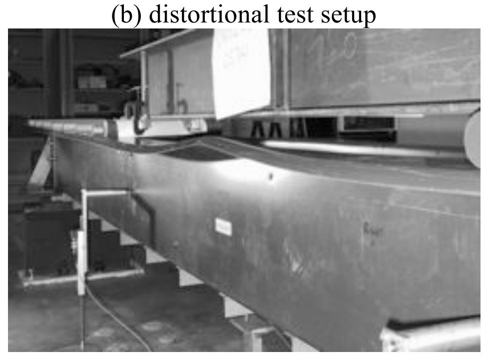
In the local buckling tests a corrugated metal panel was attached to the compression flange in the moment span to insure distortional buckling was restricted (Fig. 3a,c). In the distortional buckling tests the panel remained in the shear spans only and no restraint was provided to the compression flange of the specimens (Fig. 3b, d). Tests were carried out on industry standard lipped channel and lipped zee specimens (Tables 1 and 2).

(a) local test setup

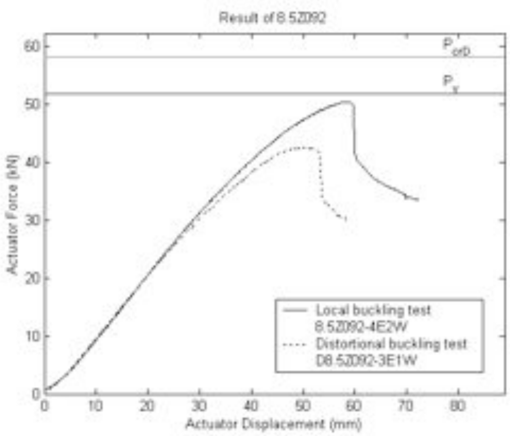
(b) distortional test setup



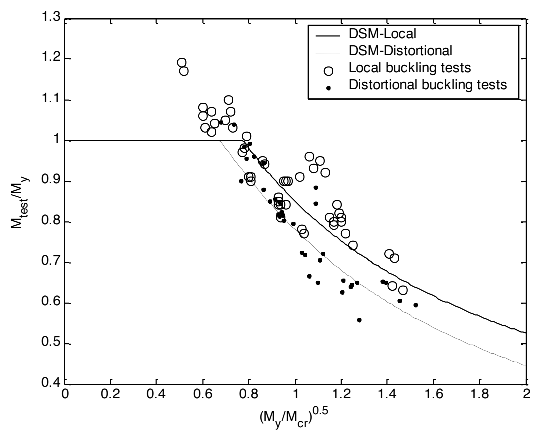
(c) local test at failure



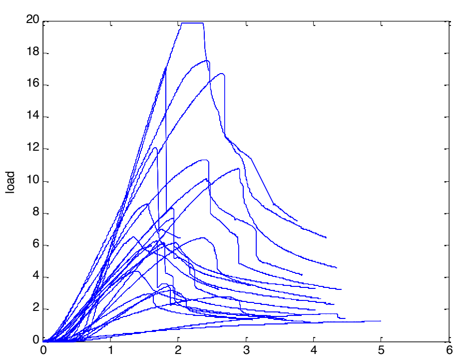
(d) distortional test at failure



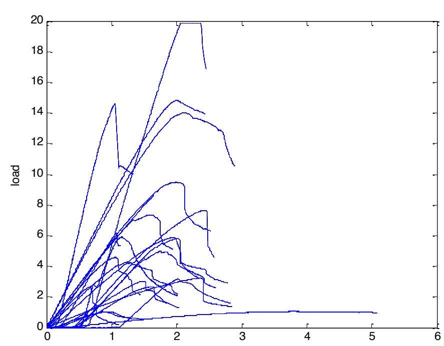
(e) local vs. distortional response for identical specimens



(f) correlation of strength to cross-section slenderness



(g) load-displacement response of local tests



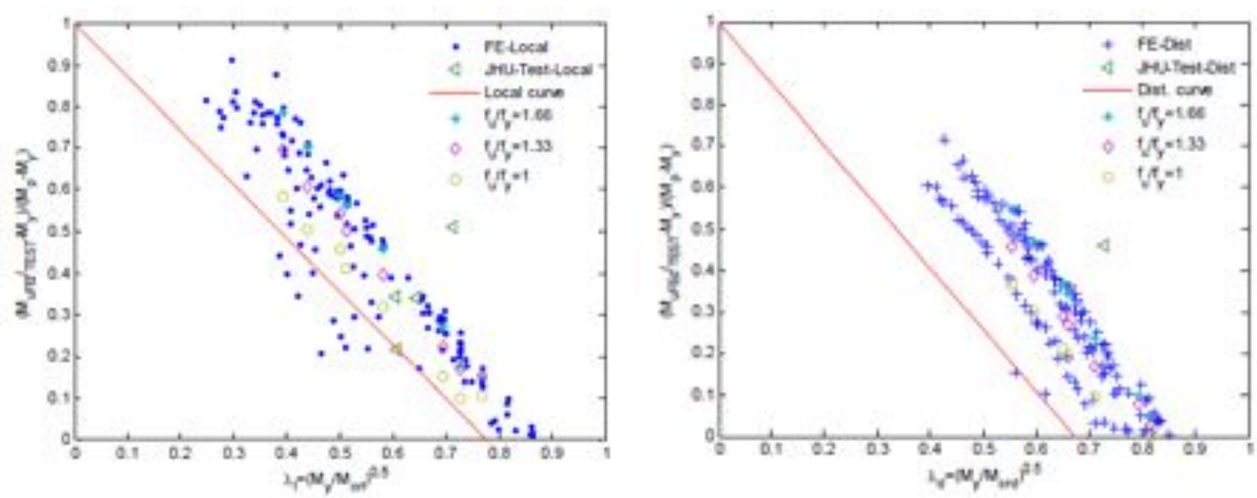
(h) load-displacement response of distortional tests

Figure 3: Local and distortional buckling tests of Yu and Schafer (Note, (a), (b) Yu and Schafer (2007), (c)-(f) Yu and Schafer 2006, (g),(h) original to this paper)

Members failing in distortional buckling typically exhibited lower capacities and a slight decrease in stiffness prior to the peak strength, as shown in Fig. 3e. The peak strength observed in the tests correlated well with cross-section slenderness and the independently determined Direct Strength Method design expressions for local and distortional buckling (AISI-S100-07 Appendix 1) as shown in Fig. 3f.

Four point bending simulations (FE models)

Shifferaw and Schafer (2011) used the experiments of Yu and Schafer (2003, 2006) to develop and validate an ABAQUS nonlinear collapse shell finite element (FE) model focusing on local and distortional buckling limit states in typical lipped channel and lipped zee CFS sections. The goal of these analyses was not to recreate the tests but rather to provide an idealized model that could consistently provide local and distortional buckling failure modes in a computationally efficient manner. The selected model includes only the central 1.63 m (64 in.) constant moment region from the tests and employs special boundary conditions at the ends and along the flanges. Centerline dimensions from seventeen cross-sections from Yu and Schafer (2003, 2006) were selected (Tables 3 and 4). The modeling focused on CFS sections that can develop inelastic reserve; i.e., sections with a peak bending capacity greater than the moment at first yield, see Fig. 4.

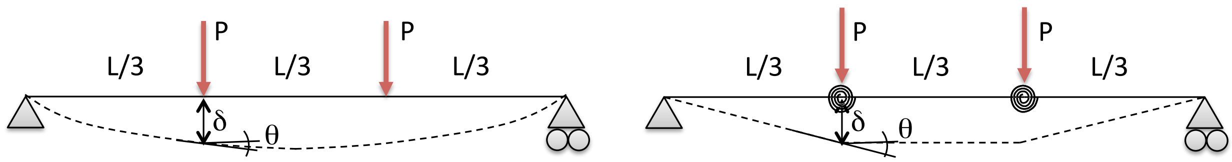


(a) local buckling (b) distortional buckling

Figure 4: Correlation of strength to cross-section slenderness for all available tests

Conversion of data to M-θ

Working from the raw data, twenty-four of the local buckling tests (Fig. 3g) and twenty-two of the distortional buckling tests (Fig. 3h) from Yu and Schafer (2003, 2006) were employed in this study. Force measurements are recorded in the load cell with the actuator (see center of the spreader beam Fig. 3b) and displacement measurements from LVDTs at the load points (e.g., see Fig. 3c). Note, specimens are made up of two cold-formed steel beams in parallel, thus the moment of inertia for the beam in Fig. 5a is equal to the summation of the moment of inertia for the two beams.



(a) variable definitions and continuous model (b) two parameter lumped rotational spring model

Figure 5: Conversion of measured data in 4 point bending test

Conversion of the measured test data to stiffness as well as moment-rotation is as follows:

test: δ = average of LVDTs positioned under two loading points
P = ½ of force measured from load cell
$k = P/\delta$
$\theta = \delta/(L/3)$
$M = P(L/3)$
$k_\theta = M/\theta$

The rotation determined from the test data is approximate, and is consistent with the lumped parameter model of Fig. 5b. For comparison, linear elastic analysis using beam theory (ignoring cross-section deformation), provides the following solution:

elastic: $\delta_e = 5PL^3/(162EI)$
$k_e = 162EI/(5L^3)$
$\theta_e = PL^2/(18EI)$
$k_\theta = 6EI/L$
also note:
$\delta_e/(L/3) = 15PL^2/(162EI)$

The preceding are used to compare observed displacements to expected (analytical) displacements and provide information towards development of a lumped stiffness model (Fig. 5b) that may eventually be used in nonlinear collapse analysis.

In addition to the experimental results, 187 finite element models analyzed by Shifferaw and Schafer for both local and distortional buckling modes were employed in this study.

The raw data (of tests or FE models) was down-sampled to 10 pre-peak points, each one in increments of 10% of the displacement at peak strength, as shown in Fig. 6 and 7. Based on the force levels corresponding to 10% pre-peak displacement increments, post-peak data was determined. Due to the low density of available data a 3rd order polynomial was fit to the response immediately after the peak strength for the experimental results (Fig. 6) – note, for the finite element analysis results the full curve (Fig. 7) was utilized.

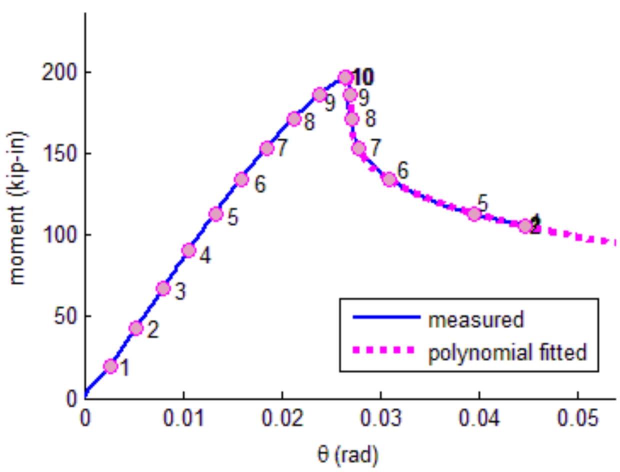


Figure 6: Digitized points (1-10) shown for test 8C068-4E5W

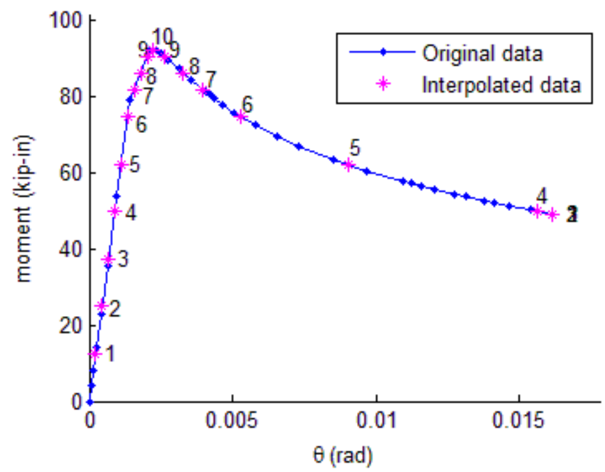


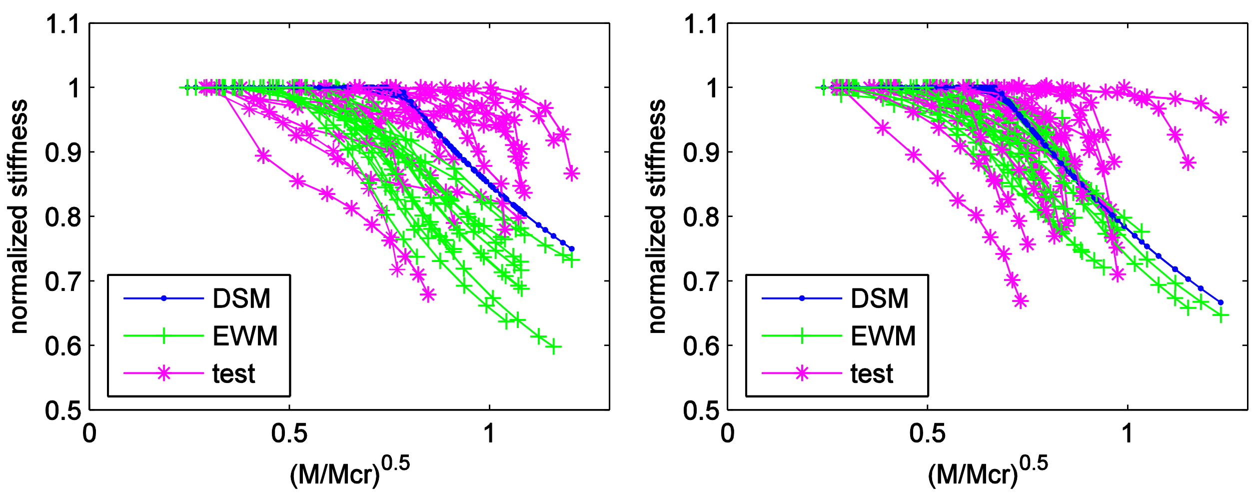
Figure 7: Digitized points (1-10) shown for FE model 8C0685lt11

Down-sampled moment-rotation curves for the finite element models are provided in Appendix 1. The down-sampled data is used to examine pre-peak stiffness. However, the comparisons for ductility predictions are realized with the actual data; pre-peak and post-peak energies (the area under the moment-rotation curve) are calculated according to actual data.

Examination of Pre-Peak Stiffness by Available Data

The secant stiffness for all available experimental data is calculated and reported in previous studies (Ayhan and Schafer 2011). Secant stiffness values were obtained for the Effective Width Method (EWM) (Yu, 2000) and Direct Strength Method (DSM) (Schafer 2006, 2008) and compared against the measured values in Fig. 8. In this figure the horizontal axis is the cross-section slenderness (either local or distortional). As the moment increases the cross-section slenderness increases and the predictive methods proceed from fully effective to partially effective and the stiffness reduces. Neither the EWM nor the DSM method for reducing the stiffness (I_e) follows the same "shape" as the test data as the section stiffness reduces.

The EWM provides cross-section specific predictions of the reduced stiffness. The reductions initiate earlier and are more severe than the observed stiffness reductions. The DSM method provides a singular prediction as a function of cross-section slenderness – so all sections reduce stiffness in the same manner. The predicted DSM reductions follow the mean of observed stiffness, but much scatter remains. The EWM reductions generally follow the same shape as the DSM reductions. The DSM reductions provide an upperbound to the EWM reductions.



(a) local buckling (b) distortional buckling

Figure 8: Comparison of DSM and EWM I_{eff} results with (a) local and (b) distortional tests

A statistical summary comparing EWM and DSM I_{eff} to the measured data is provided in Table 5. The test-to-predicted ratio for the reduced pre-peak moment of inertia is compared at the ten load levels explored. Focusing on predicting the secant stiffness at peak strength, DSM provides a mean test-to-predicted stiffness ratio of 0.97 for both the local and distortional buckling tests and a coefficient of variation of 15% for local buckling and 21% for distortional buckling; while EWM provides a mean test-to-predicted stiffness ratio of 1.13 in local buckling, 1.03 in distortional buckling and coefficients of variation of 18 and 20% respectively. Neither method provides highly accurate stiffness predictions, but DSM is superior in terms of mean and variance; therefore, it seems reasonable to conclude that either method may be used and DSM's simplicity may make it more advantageous in many situations. The conservativeness and variance of existing predictions for stiffness shows the need for this work, even for pre-peak stiffness determination.

Table 5: Summary of Test-to-Predicted Ratios for I_{eff} by EWM and DSM

		$k_{secant\text{-}test}/k_{secant\text{-}predicted}$ at									
		δ_{peak}	$0.9\delta_{pe}$ ak	$0.8\delta_{pe}$ ak	$0.7d_{pe}$ ak	$0.6\delta_{pe}$ ak	$0.5\delta_{pe}$ ak	$0.4\delta_{pe}$ ak	$0.3\delta_{pe}$ ak	$0.2\delta_{pe}$ ak	$0.1\delta_{pe}$ ak
LOCAL BUCKLING TESTS											
	n	24	24	24	24	23	21	18	13	9	7
DSM	μ	0.97	1.01	1.02	1.02	1.01	0.99	0.98	0.98	0.99	1.00
	CV	0.15	0.13	0.12	0.09	0.07	0.03	0.02	0.01	0.01	0.00
	min	0.70	0.75	0.80	0.86	0.90	0.93	0.95	0.96	0.98	1.00
	max	1.19	1.23	1.25	1.23	1.18	1.06	1.00	1.00	1.00	1.00
EWM	μ	1.13	1.17	1.16	1.15	1.11	1.07	1.03	1.00	0.99	1.00
	CV	0.18	0.15	0.13	0.11	0.09	0.07	0.05	0.03	0.01	0.00
	min	0.77	0.83	0.88	0.92	0.96	0.98	0.96	0.96	0.98	1.00
	max	1.54	1.55	1.52	1.46	1.38	1.27	1.15	1.07	1.00	1.00
DISTORTIONAL BUCKLING TESTS											
	n	22	22	21	21	20	20	18	14	9	7
DSM	μ	0.97	1.00	1.01	1.01	0.99	0.98	0.97	0.96	0.97	1.00
	CV	0.21	0.19	0.18	0.16	0.14	0.11	0.08	0.06	0.04	0.00
	min	0.43	0.46	0.50	0.53	0.57	0.62	0.68	0.76	0.88	1.00
	max	1.43	1.42	1.37	1.31	1.25	1.18	1.10	1.01	1.01	1.00
EWM	μ	1.03	1.06	1.07	1.07	1.04	1.02	0.99	0.99	0.98	1.00
	CV	0.20	0.19	0.17	0.15	0.13	0.11	0.08	0.06	0.04	0.00
	min	0.46	0.50	0.54	0.59	0.63	0.68	0.73	0.79	0.89	1.00
	max	1.48	1.46	1.41	1.36	1.29	1.20	1.10	1.04	1.02	1.01

Note: n=number of tests used, μ=average, CV=coefficient of variation

4 Characterization of CFS M-θ with ASCE41-like models

Ductile behavior is defined by a member energy dissipation ability, which for beams is found from the area under the moment-rotation curve. Therefore, equating the area under the experimental (or FE) curve to the modelled (simplified / ASCE41-like) curve is the first aim for the characterization of the multi-linear moment-rotation models (Ayhan and Schafer, 2011b). The shape of the moment-rotation curve is the other important point for characterizing the ASCE41-like M-θ models. In the following, the ASCE41 (2007) moment-rotation definition is applied to CFS beams. The Type 1 (Fig. 1) curve was selected as best able to represent the behavior of CFS beams with its ability to capture post-peak moment loss. Type 1, M-θ includes two key features: pre-peak stiffness loss, and post-peak moment degradation. Accordingly, Model 1, Model 2, and Model 1a (Fig. 9-11) variants of Type 1 are generated to examine the available data. The test data of Yu and Schafer (2003, 2006) and the FE results of Shifferaw and Schafer (2011) were down-sampled and converted from load-displacement to moment-rotation and then ASCE41-like models were "fit" to the data.

Minimization Procedure

The parameters which are needed to characterize CFS moment-rotation response via the Type 1 curve were varied such that the area under the experimental (or FE) M-θ curve was equal to the area under the modeled curve. This was completed in two pieces, pre-peak energy and post-peak energy; so that over/undershooting pre-peak energy is not over/under compensated for in the post peak range. The error considered was calculated as the sum of squares of the difference of pre-peak area under the curves and difference of post-peak area under the curves. This optimization problem is solved with MATLAB routines which are provided in Appendix 2. Error residuals are generally less than $1x10^{-10}$ and the 'fitting' exercises were successful. The key point in selecting from the three moment-rotation models obtained, are the curve shape and its ability to properly represent CFS behavior.

Multi-linear M-θ models for characterization

The notation used in this study is not the same as in ASCE41 (2007), but the shapes of the M-θ curves aimed are similar to the ASCE41 Type 1 curve. The notation below is utilized in the MATLAB programs for fitting to the data.

The selected model parameters are defined in row vector p as follows (see Fig. 9 – 11):

$$p = [M_1\ k_1\ M_2\ k_2\ \Delta\theta\ \Delta M\ \theta_4]$$

where M_1 is the elastic moment, k_1 is the elastic stiffness, M_2 is the peak moment, k_2 is the second stiffness between elastic and the peak point, $\Delta\theta$ is the rotation step after the peak point, ΔM is the moment drop after the peak point, and θ_4 is the maximum rotation where the M-θ curve terminates.

The rotations are defined by the selected model parameters as follows:

$$\theta_1 = \frac{M_1}{k_1}$$

$$\theta_2 = \theta_1 + \frac{M_2 - M_1}{k_2}$$

$$\theta_3 = \theta_2 + \Delta\theta$$

The parameters are constrained in the error minimization as follows:

$$\theta_1 > 0 \text{ and } \theta_1 < \theta_2 < \theta_3 < \theta_4$$

$M_1>0$ and $M_2>M_1$ and $M_3<M_2$ and $M_3>0$

θ violations are scaled by the magnitude of the violation multiplied times 100 and added to the error residual, M violations are scaled by the magnitude of the violation multiplied times 10 and added to the error residual. This heuristic approach to the constrained optimum proved successful.

The 'fit' is sensitive to initial conditions. In addition, in some models as noted below, certain initial conditions such as the initial stiffness, peak moment, and final rotation are treated as constraints. In the most general case the initial conditions are as follows:

$M_{1i}=0.9\max(M_t)$ (where M_t is the test moment)
$k_{1i}=k_t$ (evaluated at $50\%M_{t\text{-prepeak}}$)
$M_{2i}=\max(M_t)$;
$k_{2i}=(M_{2i}-M_{1i})/(\theta_{2i}-M_{1i}/k_{1i})$;

note, $\theta_{2i}=\theta_t$ (evaluated at $\max(M_t)$)

$\Delta\theta_i=\max$ of (θ_t(at $0.8M_{t\text{-postpeak}}$)-θ_t(at $\max(M_t)$) , and 0)
$\Delta M_i=\max(M_t)-\min(M_{t\text{-postpeak}})$ or $0.5\max(M_t)$
$\theta_{4i}=\max(\theta_t)$ or θ_t(at $0.5M_{t\text{-postpeak}}$)

where the subscript "t" denotes 'test' (physical test or nonlinear FE model) and "i" an 'initial' guess in the optimization.

The additional (fitted) model parameters are established by minimizing the error such that the pre-peak, and post-peak energy are equal to that of the tests or FE models.

Model 1: post-peak plateau and strength drop

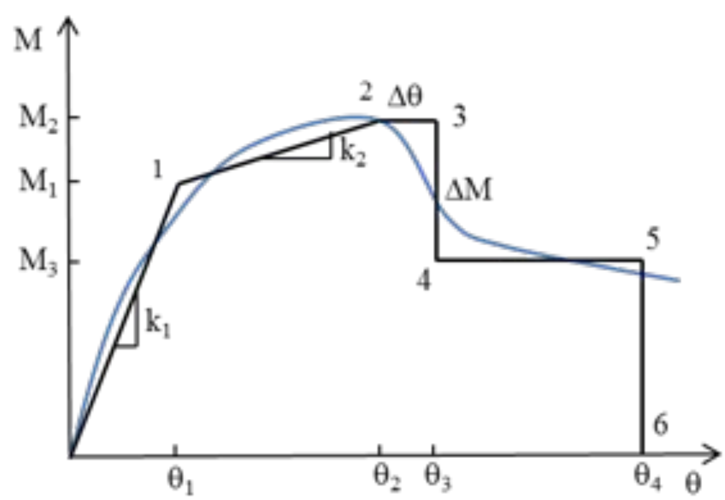


Figure 9: Model 1 backbone curve

Model 1 includes pre-peak stiffness loss and a post-peak moment degradation which is described as a combination of post-peak plateau and strength drop (Fig. 9). This shape is defined with 6 points, see Table 6. Parameters which are necessary to characterize this model were selected and used to solve the optimization problem.

Table 6: Variables defining M-θ curve of Model 1

point no	rotation	moment	stiffness	parameters selected
1	θ_1	M_1	k_1	M_1, k_1
2	θ_2	M_2	k_2	M_2, k_2
3	θ_3, $\Delta\theta$	M_2	0	$\Delta\theta$
4	θ_3, $\Delta\theta$	M_3, ΔM	∞	$\Delta\theta$, ΔM
5	θ_4	M_3, ΔM	0	θ_4, ΔM
6	θ_4	0	∞	θ_4

Model 2: post-peak plateau and stiffness loss

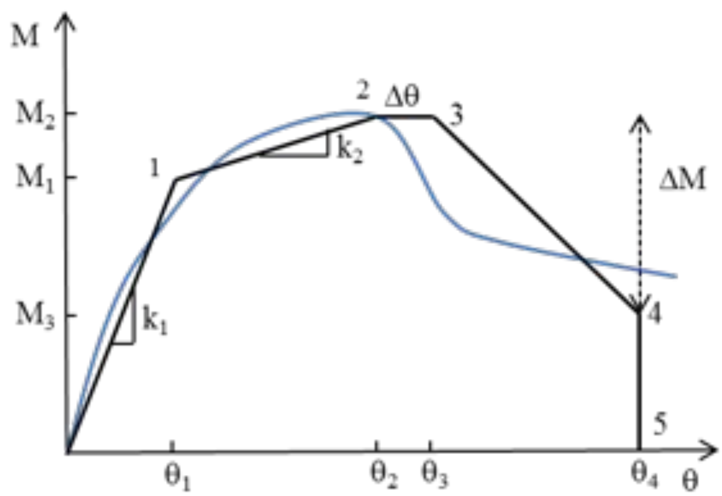


Figure 10: Model 2 backbone curve

Table 7: Variables defining M-θ curve of Model 2

point no	rotation	moment	stiffness	parameters selected
1	θ_1	M_1	k_1	M_1, k_1
2	θ_2	M_2	k_2	M_2, k_2
3	θ_3, $\Delta\theta$	M_2	0	$\Delta\theta$
4	θ_4	M_3, ΔM	k_3	θ_4, ΔM
5	θ_4	0	∞	θ_4

The shape of Model 2 is differentiated from Model 1 by the post-peak moment degradation. The post-peak region employs a post-peak plateau and stiffness loss (Fig. 10). The aim is to reflect the real behavior of CFS beams. This curve is composed of five critical points, which are defined in Table 7.

Model 1a: post-peak bilinear stiffness loss

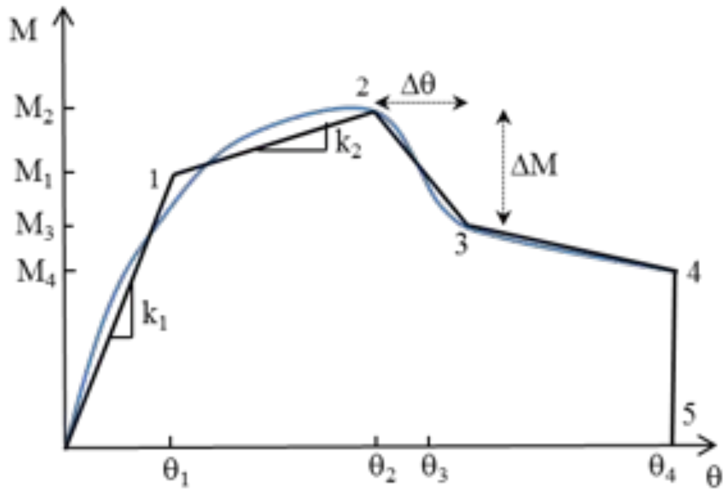


Figure 11: Model 1a backbone curve

The post-peak strength loss is composed of a bilinear stiffness loss curve in Model 1a (Fig. 11). The critical points to define this shape are given Table 8. An additional parameter (M_4) is needed to characterize Model 1a. The vector of controlling parameters is revised as following:

$$p = [M_1\ k_1\ M_2\ k_2\ \Delta\theta\ \Delta M\ \theta_4\ M_4]$$

Accordingly, constraints and initial conditions are added to as following:

$$M_3 > M_4 > 0$$
$$\Delta\theta_i = (\theta_{4i} - \theta_{2i})/2$$
$$\Delta M_i = (M_t(\text{at } \theta_{4i}) - \max(M_t))/2$$

note, $M_{4i} = M_t(\text{at } \theta_{4i})$

Table 8: Variables defining M-θ curve of Model 1a

point no	rotation	moment	stiffness	parameters selected
1	θ_1	M_1	k_1	M_1, k_1
2	θ_2	M_2	k_2	M_2, k_2
3	θ_3, $\Delta\theta$	M_3, ΔM	k_3	$\Delta\theta$, ΔM
4	θ_4	M_4	k_4	θ_4, M_4
5	θ_4	0	∞	θ_4

Characterization Results

The multi-linear ASCE 41-like models (Model 1, Model2, Model 1a) were fit separately to the down-sampled data generated from the tests of Yu and Schafer (2003 and 2006) and the FE models of Shifferaw and Schafer (2010). Several "fits" were pursued, four are detailed here. Two of the "fits" use all available data and the others limit the data to only $M_{postpeak}$ >50%$M_{t\text{-}postpeak}$. For both, "fits" are realized by either minimizing sum squared error on all 7 model parameters termed the "full fit", or by fitting only k_2, $\Delta\theta$, and ΔM termed the "const. fit". The constrained fit (abbreviated "const. fit") constrains the initial stiffness (k_1) and the peak (θ_2, M_2) as well as the final moment (M_4) to be the same as the test, also only in Model 1a final rotation (θ_4) is also fixed to be the same as the test in the "const. fit". In summary, the four examined "fits" are:

Using all available data

1. fit all 7 model parameters "full fit"
2. fit only k_2 and $\Delta\theta$ and ΔM (others fixed) "const. fit"

Given the arbitrary nature of the maximum θ available two more "fits" are also explored.

Using data $M_{postpeak}$ >50%$M_{t\text{-}postpeak}$

3. fit all 7 model parameters "full fit"
4. fit only k_2 and $\Delta\theta$ and ΔM (others fixed) "const. fit"

Typical M-θ fits for Model 1, Model 2 and Model 1a for the local buckling test 8C068-4E5W (Table 1), termed L11 here, is provided in the plots of Fig. 12. M-θ fits to all the local and distortional buckling test data of Yu and Schafer (2003, 2006) are provided in Appendices 3-5.

Although all models equate pre- and post- peak energy, Model 1a and Model 2 do not fit the shape of the observed post-peak M-θ response for either the local nor distortional buckling test data of Yu and Schafer. Model 1 (see Fig. 12, Appendix 5) provides the best efficiency for equating both the M-θ shape and the energy dissipated to produce general design expressions.

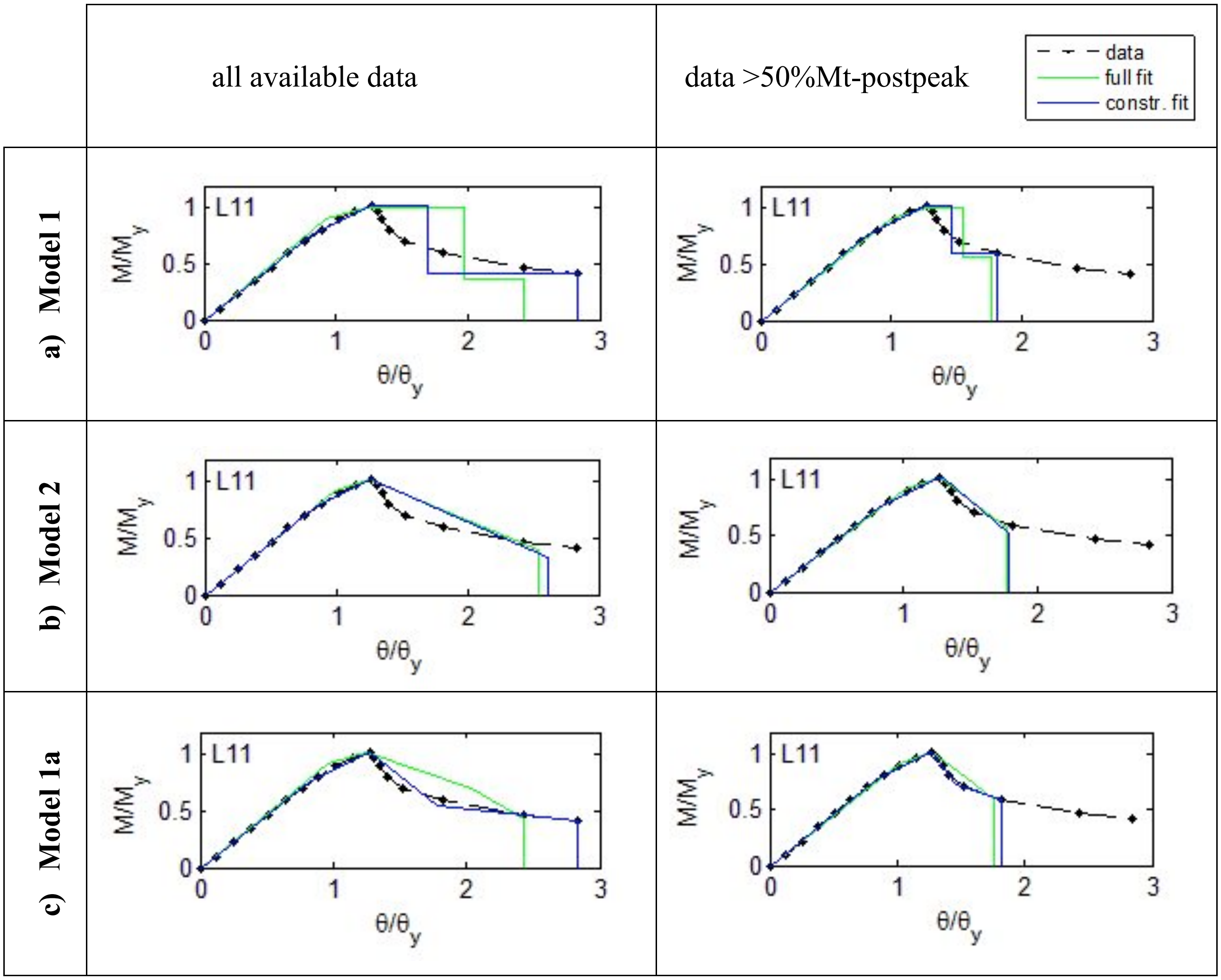


Figure 12: Typical fits for local buckling test result of 8C068-4E5W

Recommendation: Model 1

Even if Model 1a seems to provide more reliable characterization of M-θ behavior for the four point bending tests and simulations, there is no suitable way to predict M_4, the post-peak moment capacity of Model 1a. Model 1 gives more applicable results as error residuals are reasonable (generally less than 1×10^{-10}) and M-θ backbone follows a similar path to the available data. Therefore, adaptation of Model 1 is recommended.

5 Design Parameterization and Prediction for CFS-NEES Model 1

The goal of this Chapter is to develop a systematic design method for predicting the parameters of the Model 1 M-θ backbone curve (Fig. 13), applicable to all CFS beams failing in either local or distortional buckling. The Model 1 parameters are "fit" to available test data as described in Chapter 4. 'Fit 4' is employed for the parameterization conducted in this Chapter. 'Fit 4' uses only the post-peak data up to 50% of the tested post-peak moment and leaves only k_2, Δθ and ΔM as free parameters (necessary for matching the energy in the test vs. the model), all other parameters are set to exactly match the observed result in the test, see Chapter 4 and Appendix 2 for further discussion.

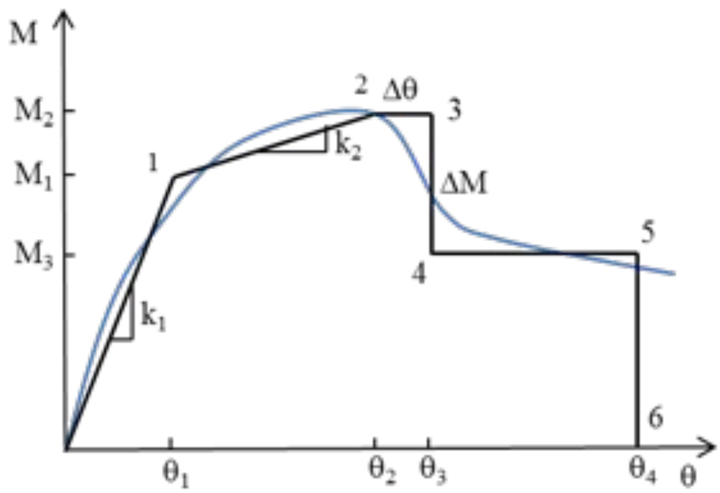


Figure 13: CFSNEES Model 1 – recommened backbone curve

Local Buckling

Due to the large range of observed M-θ behavior it is not possible to provide fixed values for the Model 1 parameters (as is typical in ASCE 41). However, existing design does provide insights on how to predict many of the Model 1 parameters. For example, the peak moment capacity (M_2) is known to be well predicted by the Direct Strength Method (DSM) of AISI-S100. DSM uses local cross-section slenderness (λ_ℓ) as the key variable for predicting strength, where:

$$\lambda_\ell = \sqrt{\frac{M_y}{M_{cr\ell}}}$$

and M_y is the elastic yield moment and $M_{cr\ell}$ is the elastic critical local buckling moment. Specifically, if the peak moment M_2 is set to $M_{n\ell}$ in the existing DSM provisions, then

$$\frac{M_2}{M_y} = \begin{cases} 1+\left(1-\frac{1}{C_{y\ell}^2}\right)\frac{\left(M_p - M_y\right)}{M_y} \text{ and } C_{y\ell} = \sqrt{\frac{0.776}{\lambda_\ell}} \le 3 & \text{if } \lambda_\ell < 0.776 \\ \left(1-0.15\left(\frac{1}{\lambda_\ell^2}\right)^{0.4}\right)\left(\frac{1}{\lambda_\ell^2}\right)^{0.4} & \text{if } \lambda_\ell \ge 0.776 \end{cases}$$

Note, the provisions for $\lambda_\ell < 0.776$ were adopted in AISI-S100 in February 2011 based on the work of Shifferaw and Schafer (2011). Performance of these expressions against the available data is provided in Fig. 14. Schafer 2008 provides additional discussion and validation of the DSM approach.

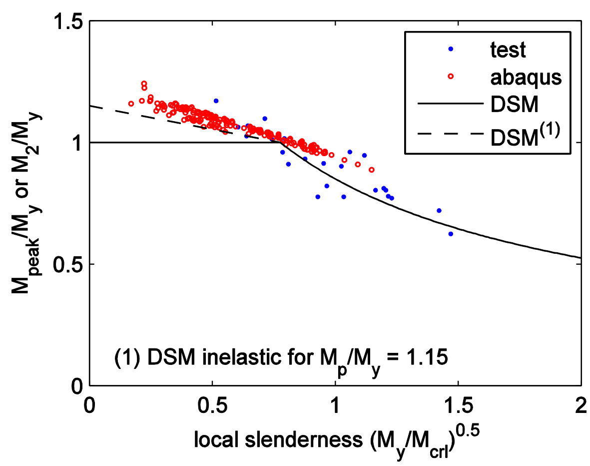


Figure 14: Peak moment strength as a function of local slenderness

A key parameter for CFS beams in Model 1 is the rotation at the peak moment (M_2). It is known that locally slender cross-sections have a reduced stiffness (see Section 3 for example) so the rotation at peak (θ_2) can be significantly larger than the elastic rotation (i.e. M_2/k_1 where k_1 is the initial elastic stiffness, also known as k_e). Fig. 15 provides θ_2 normalized by the yield rotation θ_y (θ_y=M_y/k_1 or M_y/k_e) as a function of local slenderness. Somewhat remarkably, the available data exhibits a clear trend with local slenderness and a simple expression is proposed as shown in the Figure:

$$\frac{\theta_2}{\theta_y} = \frac{1}{\lambda_\ell}$$

This simple expression provides a means to determine the reduced stiffness that occurs due to local buckling. Unlike existing stiffness predictions (Section 3) this stiffness method is decoupled from the strength prediction.

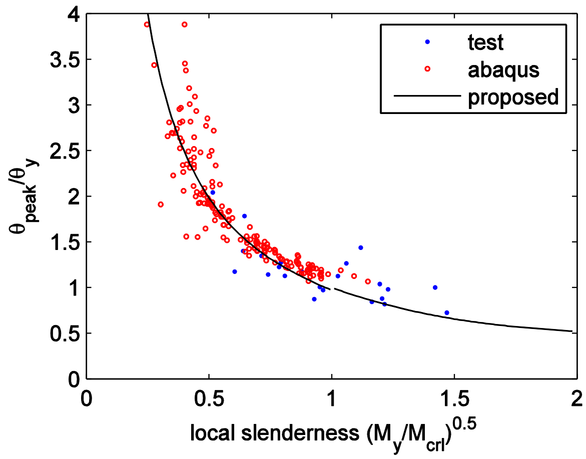


Figure 15: Peak rotation ($\theta_{peak}=\theta_2$) as a function of local slenderness

With the peak point anchored (i.e, θ_2 and M_2 known) the development of the design method may now turn to other Model 1 parameters. Specifically, the pre-peak behavior must be completed, by determining either M_1 or θ_1 – it is assumed k_1 (the elastic stiffness) is known. It is typical in current CFS beam design to determine the moment at which a section becomes "partially effective", for Model 1, this moment is M_1. Therefore, M_1 is explored directly here, as shown in Fig. 16.

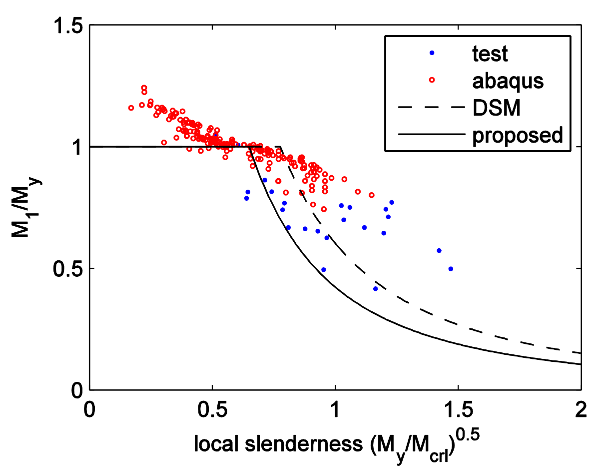


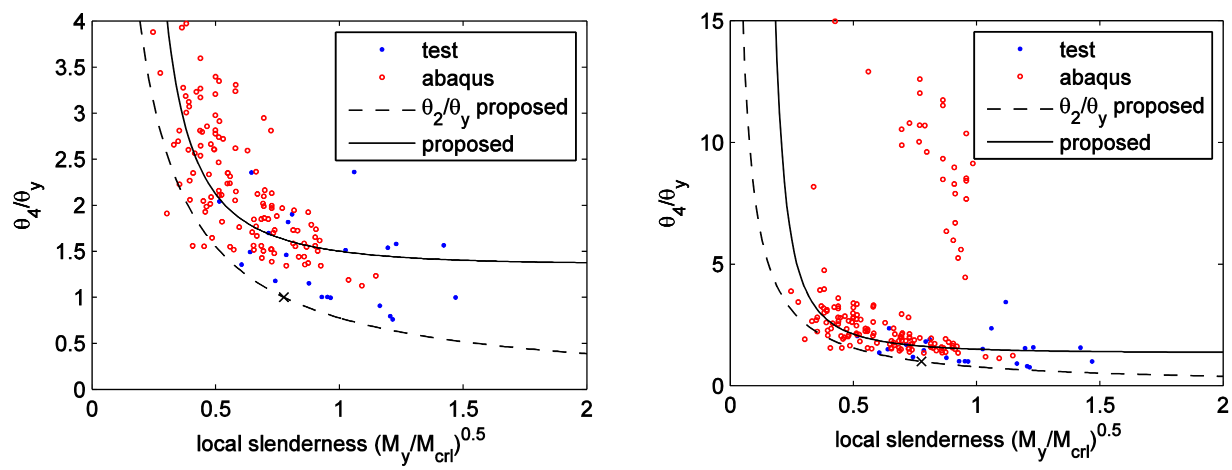
Figure 16: "Fully effective" moment (M_1) as a function of local slenderness

The scatter in prediction of M_1 (Fig. 16) is greater than for M_2 (Fig. 15). Nonetheless, the trend with respect to local slenderness remains. A simple expression is fit to the data:

$$\frac{M_1}{M_y} = \left\{ \begin{array}{cc} 1 & \text{if } \lambda_\ell < 0.650 \\ \left(\dfrac{0.650}{\lambda_\ell}\right)^2 & \text{if } \lambda_\ell \geq 0.650 \end{array} \right\} \leq \frac{M_2}{M_y}$$

The proposed relation between M_1 and local slenderness is a departure from current practice (Section 3) because (a) it disconnects the stiffness prediction from the strength prediction, and (b) it implies that the local slenderness (λ_ℓ) must be as small as 0.650 for the section to be fully effective. Current design assumes that when the strength reaches M_y (i.e., $\lambda_\ell = 0.776$) the section is fully effective. In the proposed expressions a CFS beam must exhibit moderate inelastic reserve capacity if it is to be fully effective (elastic) up to its peak moment.

The post-peak performance has greater scatter in the observed data than the peak and pre-peak behavior. Figure 17 provides θ_4 for the available data versus local slenderness.



a) for limeted number b) for all data

Figure 17: Maximum rotation as a function of local slenderness

Based on Fig. 17 it is proposed that θ_4 is

$$\frac{\theta_4}{\theta_y} = \left\{ \begin{array}{cc} 1.5\dfrac{1}{\lambda_\ell} & \text{if } \lambda_\ell > 1 \\ 1.5\left(\dfrac{1}{\lambda_\ell}\right)^{1/4\lambda_l} & \text{if } \lambda_\ell \leq 1 \end{array} \right\}$$

Finally, this leaves the post-peak parameters Δθ and ΔM in need of prediction expressions. In general Δθ is intended to capture post-peak yielding, theoretically this is only significant for sections with inelastic reserve. Fig. 18 provides the post-peak yielding Δθ as a function of local slenderness for the available data.

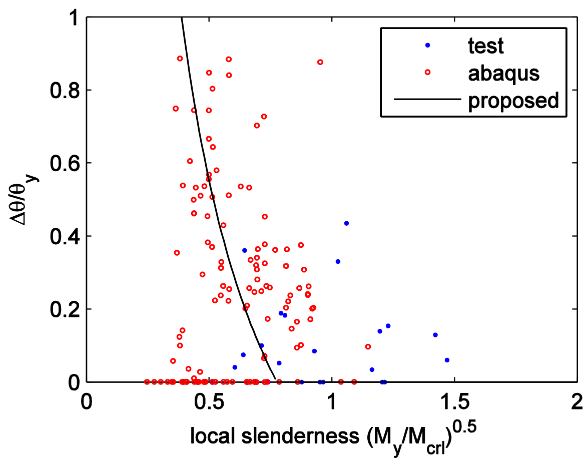


Figure 18: Post-peak yielding (Δθ) as a function of local slenderness

The scatter is large in Fig 18 and many sections that have strength below M_y exhibit some post-peak yielding. However, for simplicity it is proposed that only sections with strength greater than M_y be predicted to have nonzero Δθ. The following expressions are proposed for use and shown in Fig. 18:

$$\frac{\Delta\theta}{\theta_y} = \begin{cases} \left(\dfrac{0.776}{\lambda_\ell}\right) - 1 & \text{if } \lambda_\ell < 0.776 \\ 0 & \text{if } \lambda_\ell \geq 0.776 \end{cases}$$

Finally, the post-peak moment drop (ΔM) is explored. Note (Δθ,ΔM) + (θ_2,M_2) = (θ_3,M_3), so determination of ΔM is the final necessary parameter for Model 1. The post-peak moment drop is provided as a function of local slenderness for the available data in Fig. 19. For some of the data little or no moment drop is observed, this occurs in models where sufficient post-peak rotation was not explored (either the test or the FE model was stopped before reaching high post-peak rotations). Thus, the data with post-peak moment drop is the most important. In the absence of a definitive theory it is presumed that a 50% moment drop exists for all sections with some local buckling strength reduction (λ_ℓ>0.776) otherwise the moment drop increases from zero as the local slenderness increases via:

$$\frac{\Delta M}{M_2} = 1 - 1/\left(\frac{\lambda_\ell}{0.776} + 1\right)^{1.1} \leq 0.5$$

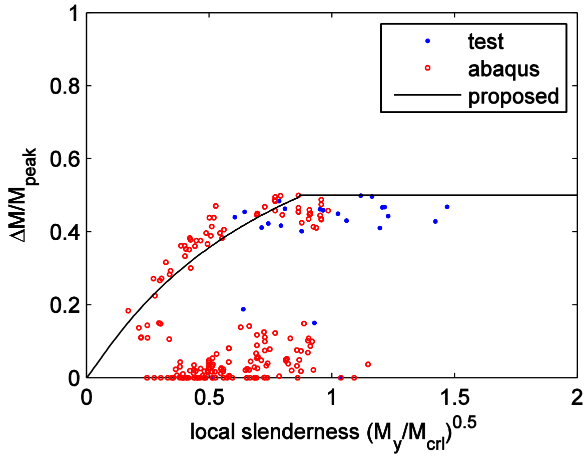


Figure 19: Post-peak moment drop (ΔM) as a function of local slenderness (note M_{peak}=M_2)

Taken together the prediction method for developing the CFS-NEES Model 1 backbone curve in local buckling is provided in Table 9.

Table 9: Design expressions for local buckling

	Local
	$\lambda_\ell = \sqrt{\frac{M_y}{M_{cr\ell}}}$
rotations	$\frac{\theta_1}{\theta_y} = \frac{M_1}{k_1\theta_y} = \frac{M_1}{k_e\theta_y} = \frac{M_1}{M_y}$ (note, M_1 given below)
	$\frac{\theta_2}{\theta_y} = \frac{1}{\lambda_\ell} \le \frac{M_2}{k_e}$
	$\theta_3 = \theta_2 + \Delta\theta$, where $\Delta\theta$ is: $\frac{\Delta\theta}{\theta_y} = \begin{cases} \left(\frac{0.776}{\lambda_\ell}\right) - 1 & \text{if } \lambda_\ell < 0.776 \\ 0 & \text{if } \lambda_\ell \ge 0.776 \end{cases}$
	$\frac{\theta_4}{\theta_y} = \begin{cases} 1.5\frac{1}{\lambda_\ell} & \text{if } \lambda_\ell > 1 \\ 1.5\left(\frac{1}{\lambda_\ell}\right)^{1/4\lambda_\ell} & \text{if } \lambda_\ell \le 1 \end{cases}$
moments	$\frac{M_1}{M_y} = \left\{ \begin{matrix} 1 & \text{if } \lambda_\ell < 0.650 \\ \left(\frac{0.650}{\lambda_\ell}\right)^2 & \text{if } \lambda_\ell \ge 0.650 \end{matrix} \right\} \le \frac{M_2}{M_y}$
	$\frac{M_2}{M_y} = \frac{M_{n\ell}}{M_y}$ where $M_{n\ell}$ is per AISI-S100, i.e.: $\frac{M_2}{M_y} = \begin{cases} 1 + \left(1 - \frac{1}{C_{y\ell}^2}\right)\frac{(M_p - M_y)}{M_y} \text{ and } C_{y\ell} = \sqrt{\frac{0.776}{\lambda_\ell}} \le 3 & \text{if } \lambda_\ell < 0.776 \\ \left(1 - 0.15\left(\frac{1}{\lambda_\ell^2}\right)^{0.4}\right)\left(\frac{1}{\lambda_\ell^2}\right)^{0.4} & \text{if } \lambda_\ell \ge 0.776 \end{cases}$
	$M_3 = M_2 - \Delta M$, where ΔM is: $\frac{\Delta M}{M_2} = 1 - 1/\left(\frac{\lambda_\ell}{0.776} + 1\right)^{1.1} \le 0.5$

Ancillary expressions useful for defining the complete curve include

$$k_2 = \frac{M_2 - M_1}{\theta_2 - \theta_1}$$

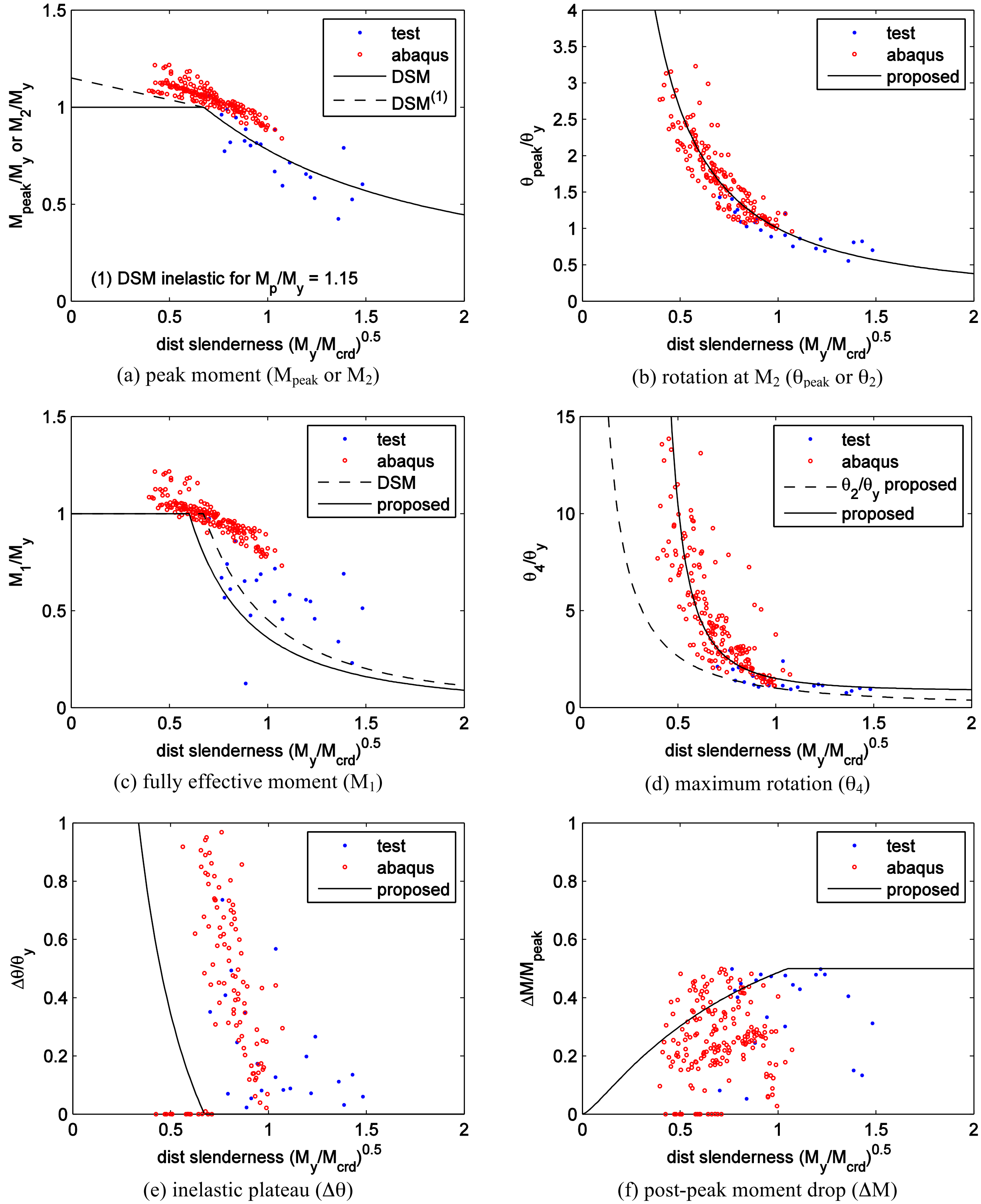
Distortional Buckling

Distortional buckling is evaluated in the same manner as local buckling and similar design expressions are developed. Fig. 20 provides the same information as Figures 14-19 for local buckling. Table 10 provides a summary of the proposed design expressions and Table 11 summarizes the quantitative performance of the method.

Figure 20a indicates that DSM may be employed to predict the peak strength. Figure 20b shows again that the rotation at the peak moment may be readily predicted as a function of cross-section (distortional in this case) slenderness. The rotation at peak moment (θ_2) in the distortional buckling data (Fig. 20b) is slightly greater than the local buckling data (Fig. 15), so the proposed expression (see Table 10) reflects this. The notion that distortional buckling modes experience greater stiffness reductions than local buckling failures is not commonly recognized in the literature. The fully effective moment, M_1, Figure 20c, exhibits significant scatter and similar to the local buckling case (Figure 16) a convenient expression that generally provides an M_1 slightly below M_2 is selected as shown in Figure 20c and reported in Table 10.

The post-peak Model 1 parameters are captured in Figure 20d-f and are arrived at in a similar fashion to the local buckling results. The maximum rotation (θ_4, Figure 20d) is set equal to 1.5 times the rotation at peak moment (θ_2) when λ_d>1, exactly the same as in the local buckling case. The inelastic plateau ($\Delta\theta$, Figure 20e) is only allowed for members predicted to have strength greater than M_y, and otherwise follows available data as closely as possible. The moment drop expression (ΔM, Fig. 20f) follows the same basic expression as local buckling and assumes a 50% drop in moment for sections which experience any reduction in strength due to distortional buckling (i.e., $M_{nd}<M_y$, $\lambda_d>0.673$).

Overall the quantitative performance of the method is summarized in Table 10. In general the approach is a more conservative predictor than for local buckling, but provides an appropriate method for design.



(a) peak moment (M_{peak} or M_2)

(b) rotation at M_2 (θ_{peak} or θ_2)

(c) fully effective moment (M_1)

(d) maximum rotation (θ_4)

(e) inelastic plateau ($\Delta\theta$)

(f) post-peak moment drop (ΔM)

Figure 20: CFS-NEES Model 1a parmaters for available data as a function of distortional slenderness, proposed design expressions indicated by solid lines

Table 10: Design expressions for distortional buckling

	Distortional
	$\lambda_d = \sqrt{\frac{M_y}{M_{crd}}}$
rotations	$\frac{\theta_1}{\theta_y} = \frac{M_1}{k_1\theta_y} = \frac{M_1}{k_e\theta_y} = \frac{M_1}{M_y}$
	$\frac{\theta_2}{\theta_y} = \left(\frac{1}{\lambda_d}\right)^{1.4}$
	$\theta_3 = \theta_2 + \Delta\theta$, where $\Delta\theta$ is: $\frac{\Delta\theta}{\theta_y} = \begin{cases} \left(\frac{0.673}{\lambda_d}\right) - 1 & \text{if } \lambda_d < 0.673 \\ 0 & \text{if } \lambda_d \geq 0.673 \end{cases}$
	$\frac{\theta_4}{\theta_y} = \begin{cases} 1.5\left(\frac{1}{\lambda_d}\right)^{1.4} & \text{if } \lambda_d > 1 \\ 1.5\left(\frac{1}{\lambda_d}\right)^{1.4/\lambda_d} & \text{if } \lambda_d \leq 1 \end{cases}$
moments	$\frac{M_1}{M_y} = \left\{ \begin{matrix} 1 & \text{if } \lambda_\ell < 0.600 \\ \left(\frac{0.600}{\lambda_d}\right)^2 & \text{if } \lambda_\ell \geq 0.600 \end{matrix} \right\} \leq \frac{M_2}{M_y}$
	$\frac{M_2}{M_y} = \frac{M_{nd}}{M_y}$ where M_{nd} is per AISI-S100, i.e.:
	$\frac{M_2}{M_y} = \begin{cases} 1 + \left(1 - \frac{1}{C_{yd}^2}\right)\frac{(M_p - M_y)}{M_y} \text{ and } C_{yd} = \sqrt{\frac{0.673}{\lambda_d}} \leq 3 & \text{if } \lambda_d < 0.673 \\ \left(1 - 0.22\left(\frac{1}{\lambda_\ell^2}\right)^{0.5}\right)\left(\frac{1}{\lambda_\ell^2}\right)^{0.5} & \text{if } \lambda_d \geq 0.673 \end{cases}$
	$M_3 = M_2 - \Delta M$, where ΔM is: $\frac{\Delta M}{M_2} = 1 - 1/\left(\frac{\lambda_d}{0.673} + 1\right)^{1.4} \leq 0.5$

Ancillary expressions useful for defining the complete curve include

$$k_2 = \frac{M_2 - M_1}{\theta_2 - \theta_1}$$

Accuracy of design expressions

The accuracy of the prediction method for M-θ is qualitatively provided in Figures 14-19 for local buckling and in Fig. 20 for distortional buckling, a quantitative assessment of the accuracy of the prediction method is provided in Table 11 (see Appendix 6 for comparison of all available data). Consistent with the figures, variation (standard deviation) can sometimes be significant; however, taken in total the method performs surprisingly well. Exploration of Figures 19 and 20f show that statistics for moment drop which are greater than 20% of M_2 produce better results as shown in the last column of Table 11.

Table 11: Test-to-predicted statistics for proposed design method for generating CFS-NEES Model 1 backbone curves

			ratio of test (or FE) - to - predicted for							
			Energy		fully eff. limit	eff. k	peak		drop	
			Pre-peak	Post-peak	M_1	k_{sec}	θ_2	M_2	ΔM	for ΔM>0.20M2
local	tests	mean	1.00	1.03	1.36	1.00	1.06	1.03	0.84	0.97
		st. dev.	0.32	0.61	0.13	0.15	0.20	0.08	0.32	0.09
	FE models	mean	1.18	1.09	1.21	1.01	1.06	1.046	0.400	1.07
		st. dev.	0.71	1.06	0.14	0.15	0.20	0.024	0.467	0.10
	all data	mean	1.16	1.08	1.23	1.01	1.06	1.04	0.45	1.06
		st. dev.	0.66	1.01	0.14	0.15	0.20	0.03	0.45	0.10
distortional	tests	mean	0.89	0.84	1.26	1.01	0.98	0.98	0.75	0.86
		st. dev.	0.26	0.41	0.00	0.16	0.16	0.13	0.31	0.21
	FE models	mean	1.10	1.56	1.21	1.08	1.07	1.10	0.73	0.91
		st. dev.	0.55	0.81	0.07	0.19	0.40	0.04	0.37	0.27
	all data	mean	1.08	1.48	1.21	1.08	1.06	1.08	0.73	0.90
		st. dev.	0.52	0.77	0.07	0.19	0.39	0.06	0.37	0.26

A statistical summary comparing EWM and DSM I_{eff} to the measured data is provided in Table 5. The test-to-predicted ratio for the pre-peak secant stiffness (I_{eff}) is compared at the ten load levels explored in the local and distortional buckling tests. It was shown that neither method provides highly accurate stiffness predictions.

The effectiveness of the new design expressions for pre-peak secant stiffness is compared with DSM and EWM I_{eff} in Table 13. Mean values of test-to-predicted stiffness ratio are provided for all data obtained from both the experiments and the FE models. EWM and DSM provide reasonable stiffness predictions for lower load levels; however, Table 13 shows that both methods are lacking particularly when compared to the FE models. The mean test-to-predicted ratio becomes as small as 0.60 for these two methods (when focusing on predicting the secant stiffness at peak strength of FE models). The new design expressions denoted "D.Exp." in the table are a significant improvement and provide reliable predictions across all load levels, though are generally more conservative at low load levels. The new expressions are simple in form and provide much improved accuracy over the available approaches. These new expressions are recommended for design.

Table 12: Comparison of design expressions results with EWM and DSM for pre-peak stiffness

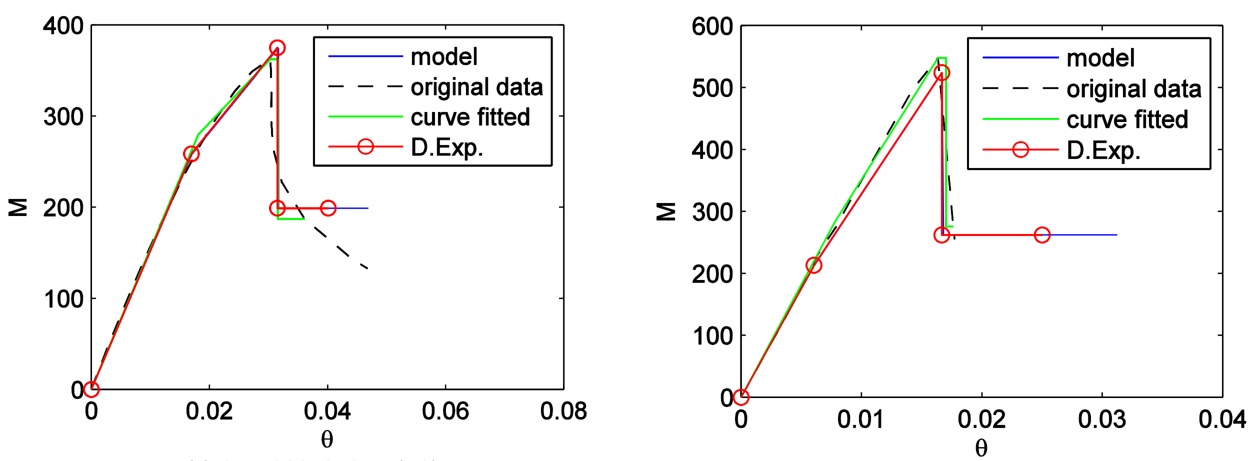
		$k_{secant\text{-}measured}/k_{secant\text{-}predicted}$ at									
		δ_{peak}	$0.9\delta_{peak}$	$0.8\delta_{peak}$	$0.7\delta_{peak}$	$0.6\delta_{peak}$	$0.5\delta_{peak}$	$0.4\delta_{peak}$	$0.3\delta_{peak}$	$0.2\delta_{peak}$	$0.1\delta_{peak}$
mean	***LOCAL BUCKLING FE models***										
	DSM	0.62	0.68	0.74	0.79	0.84	0.88	0.92	0.95	0.97	1.00
	EWM	0.61	0.67	0.73	0.79	0.84	0.89	0.92	0.95	0.97	1.00
	D.Exp.	0.98	1.03	1.06	1.06	1.03	1.01	1.00	1.01	1.02	1.02
	LOCAL BUCKLING tests										
	DSM	0.97	1.01	1.02	1.02	1.01	0.99	0.98	0.98	0.99	1.00
	EWM	1.13	1.17	1.16	1.15	1.11	1.07	1.03	1.00	0.99	1.00
	D.Exp.	1.00	1.03	1.04	1.02	1.00	0.98	0.98	0.96	0.96	1.00
	DIST BUCKLING FE models										
	DSM	0.71	0.77	0.83	0.88	0.91	0.94	0.96	0.98	0.99	1.00
	EWM	0.65	0.71	0.77	0.83	0.89	0.93	0.96	0.97	0.99	1.00
	D.Exp.	1.07	1.12	1.15	1.15	1.12	1.06	1.01	1.00	1.00	1.00
	DIST BUCKLING tests										
	DSM	0.97	1.00	1.01	1.01	0.99	0.98	0.97	0.96	0.97	1.00
	EWM	1.03	1.06	1.07	1.07	1.04	1.02	0.99	0.99	0.98	1.00
	D.Exp.	1.02	1.05	1.06	1.06	1.03	1.00	0.97	0.97	0.97	1.00

6 Design Example: Development of M-θ for CFS Section

In this Chapter an idealization of the beam behavior of Fig. 5a is realized with the nonlinear spring model of Fig. 5b. The nonlinearity of the beam is confined to the springs. Therefore, spring characteristics are defined according to the predicted moment-rotation curve for CFS beams developed herein. The moment and rotation values defining critical points of the predicted curve are calculated with design expressions for each section.

In the example provided here ABAQUS has been adopted as the computational tool, but the model is a simple lumped parameter nonlinear spring model and could be completed in a variety of software. The rigid bars are modeled as having their actual cross-section, but with the elastic modulus defined as 1000 times greater than actual to provide the desired rigid bar behavior (all flexibility is lumped into the equivalent springs). The nonlinear spring is defined by giving pairs of moment and rotation values. Hence, the spring stiffness simulates the real beam behavior. The load is applied as a displacement. An example input file is provided in Appendix 7.

Two local buckling tests (see Table 1) and two distortional buckling tests (see Table 2) are chosen to demonstrate that the modeling exercise is possible and agrees with the expected response.

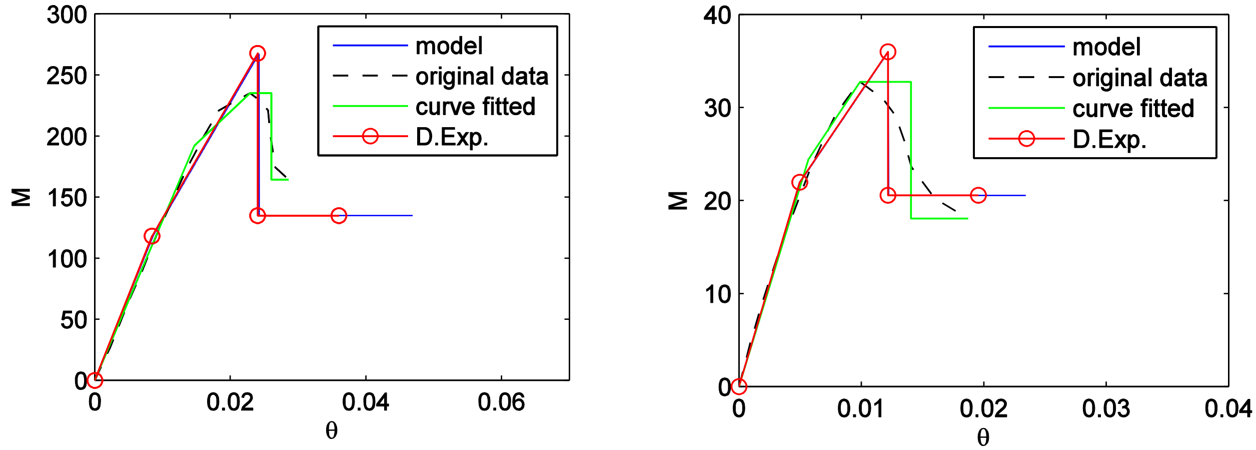


(a) 8.5Z092-4E2W (L3)

(b) 11.5Z082-2E1W (L23)

Figure 21: Verification of M-θ curve for local buckling test specimens

The results obtained from ABAQUS are compared in Figures 23 and 24. The moment-rotation curve of the ABAQUS bar-spring model perfectly matches the curve assigned to it from the design expressions. These figures also demonstrate how the design expressions compare to the actual and Model 1 fitted data.



(a) D8.5Z082-4E3W (D4)

(b) D8Z033-1E2W (D16)

Figure 22: Verification of M-θ curve for distortional buckling test specimens

7 Future Research

Significant future work remains, most notably

a) performing additional cyclic testing to verify and expand the proposed design method based on monotonic testing,

b) further implementing the proposed expressions in an analysis framework such that ASCE 41 style pushover analysis can be explored in real structures, and

c) developing companion expressions that address moment-curvature instead of moment-rotation to provide a more fundamental set of expressions for implementation in analysis.

For (a) research is planned at Virginia Tech by Professors Moen and Eatherton as an AISI sponsored companion to the CFS-NEES project to develop this data. For (b) the senior author is collaborating in the ongoing development of ASCE 41 and continues to actively seek the best software platform for the implementation.

8 Conclusions

Knowledge of the moment-rotation (M-θ) response of cold-formed steel beams is fundamental to the success of cold-formed steel structures. Existing monotonic test and finite element data provide a characterization of the backbone M-θ response of cold-formed steel beams failing in local and distortional buckling limit states. Simplified multi-linear models in the spirit of ASCE 41 formulations are fit to existing data by insuring pre-peak and post-peak energy balance is maintained between the model and the original data. The derived model parameters, e.g. the moment at which pre-peak nonlinear stiffness engages (M_1) or the available rotation at a pos-peak moment level 50% of the peak value (θ_4) are then examined to determine if a simple method may be used in their prediction. It is found that local and distortional cross-sectional slenderness are adequate explanatory variables for parameterizing the simplified M-θ model parameters – and simple design expressions are developed for predicting unique M-θ curves for all cold-formed steel cross-sections in local or distortional buckling. The developed expressions are shown to adequately predict the available data and provide an improvement for pre-peak stiffness prediction when compared to existing methods. In addition, for the first time, post-peak predictions of ductility are available for cold-formed steel beams. Much work remains, but the research demonstrates the viability of a significant expansion of the Direct Strength Method philosophies to the prediction of post-peak member behavior and provides a tool for further exploring the nonlinear response of cold-formed steel systems.

9 Acknowledgments

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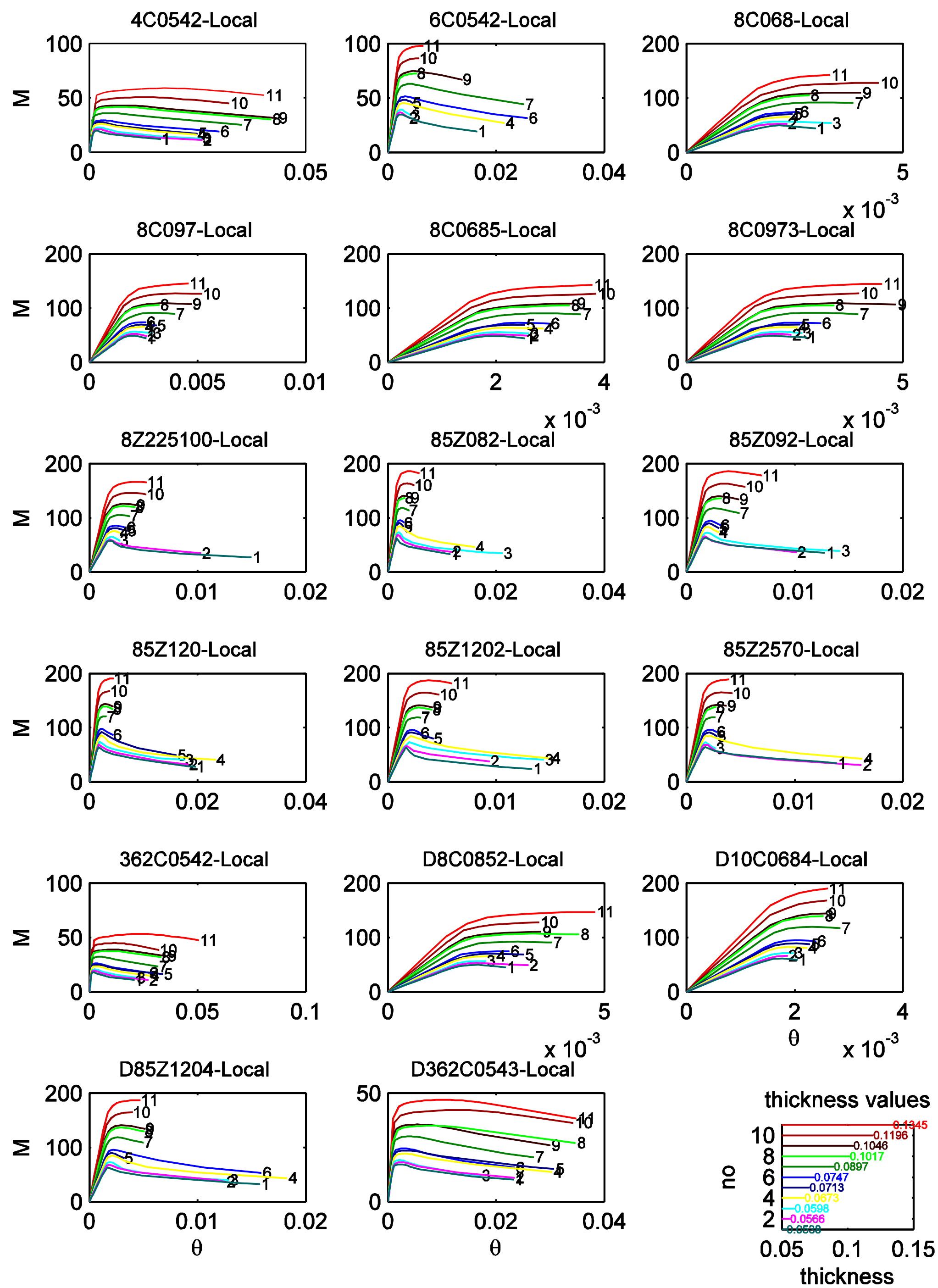
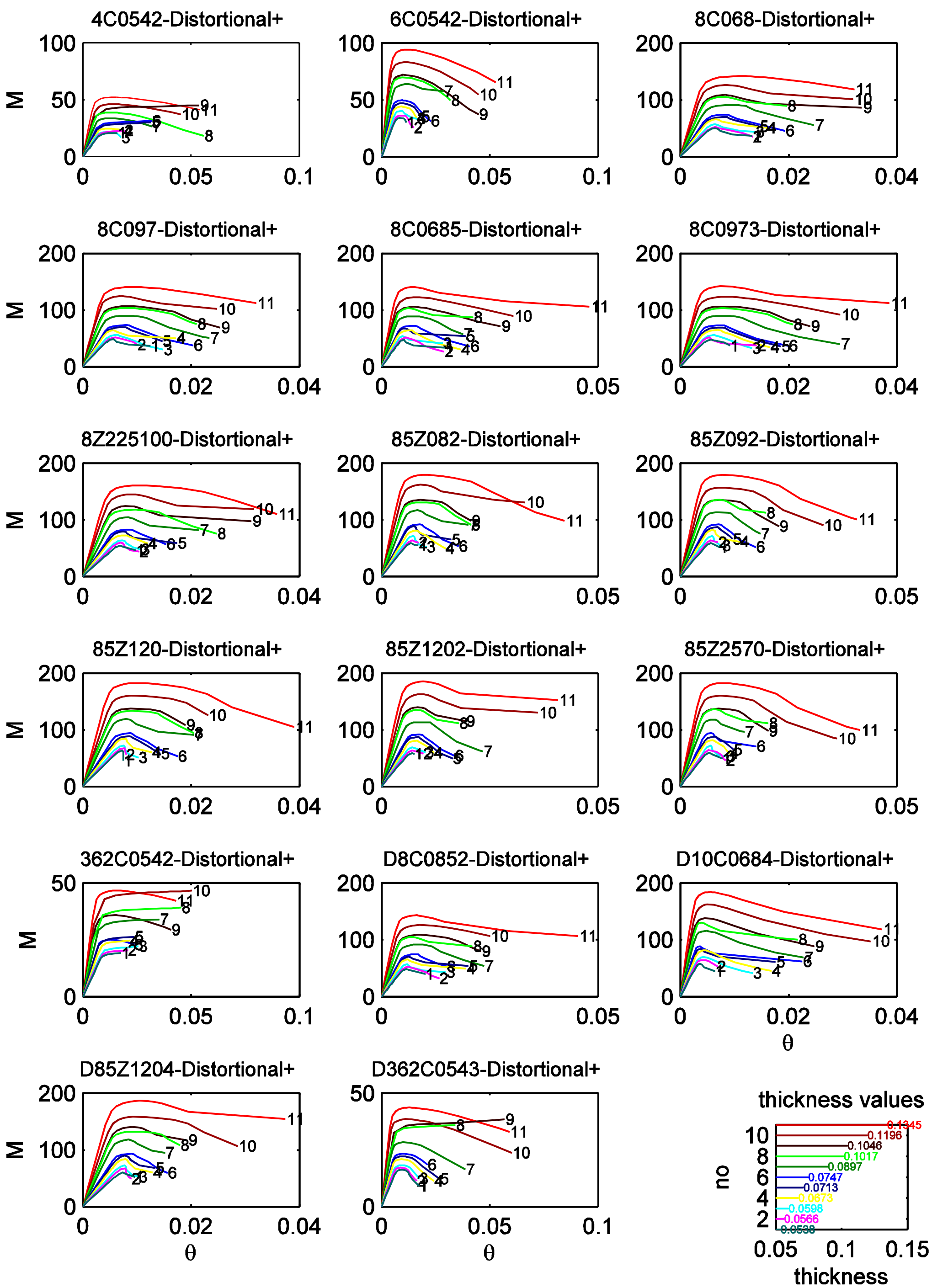
Appendices

Appendix 1: M-θ curves for FE models




4C0542-Distortional+
6C0542-Distortional+
8C068-Distortional+
8C097-Distortional+
8C0685-Distortional+
8C0973-Distortional+
8Z225100-Distortional+
85Z082-Distortional+
85Z092-Distortional+
85Z120-Distortional+
85Z1202-Distortional+
85Z2570-Distortional+
362C0542-Distortional+
D8C0852-Distortional+
D10C0684-Distortional+
D85Z1204-Distortional+
D362C0543-Distortional+
M
θ
thickness values
no
thickness

Appendix 2: MATLAB routines to solve optimization problem

This optimization problem is solved with MATLAB routines which include following:

ASCE41model.m	gives θ and M based on model parameters
ASCE41model_fitter.m	handles fitting to model includes selected err. residual
ASCE41model_driver.m	program that runs fitter and saves the results
ASCE41model_viewer.m	plotting of all M-θ and other plots (manual switching in here)
ASCE41model_qpts.m	supplementary function that based on parameters gives θ_1-θ_4

ASCE41model.m

```
function [Mfit,Error] = ASCE41model1(p,q,Mt)
%ASCE41model Moment output
%   p is the parameters defining the curve
%   p = [M1 k1 M2 k2 deltaq deltaM q4]
%   q is the rotation at which the model output is generated
%   Mt is the moment values at q that the fit is compared against
%Parameters
%-1-%
M1=p(1);
k1=p(2);
q1=M1/k1;
%-2-%
M2=p(3);
k2=p(4);
q2=q1+(M2-M1)/k2;
%-3-%
deltaq=p(5);
q3=q2+deltaq;
%-4-%
deltaM=p(6);
M3=M2-deltaM;
%-5/6-%
q4=p(7);

%
%Function evaluations
for i=1:length(q)
    %if q ins not monotonic enforce it to be so
    if i==1
        qi=q(i);
    elseif q(i)<q(i-1)
        qi=qi; %stays the same, don't let it decrease
    else
        qi=q(i);
    end
    %curve
    if qi<=q1
        Mi=k1*qi;
    elseif qi<q2
        Mi=M1+k2*(qi-q1);
    elseif qi<q3
        Mi=M2;
```

```
    elseif qi<=q4
        Mi=M3;
    else
        Mi=0;
    end
    Mfit(i,1)=Mi;
end
%
%Error Measures
if Mt==0 | Mt==NaN
    Error=zeros(1,6);
else
    %E1: Numerical energy (area) error measure
    A=sum(diff(q).*diff(Mt)/2 + diff(q).*Mt(1:length(Mt)-1));
    Afit=sum(diff(q).*diff(Mfit)/2 + diff(q).*Mfit(1:length(Mfit)-1));
    E1=(A-Afit)^2;
    %E2 SSE (sum squared error) error measure
    SSE=sum(Mfit-Mt).^2;
    E2=SSE;
    %E3 Combined E1+E2 error measure (unit dependent, a little odd)
    E3=E1+E2;
    %E4: True energy (area) error measure
    Afittrue=1/2*M1*q1 + (q2-q1)*(M2+M1)/2 + M2*(q3-q2) + M3*(q4-q3);
    E4=(A-Afittrue)^2;
    %E5: Pre and post separated area measure
    [Mmax,iMmax]=max(Mt);
    [qmax,iqmax]=max(q);
    Apre =sum([diff(q(1:iMmax)).*diff(Mt(1:iMmax))/2 ;
diff(q(1:iMmax)).*Mt(1:iMmax-1)]);
    if iqmax>iMmax %some post region exists
        Apost=sum([diff(q(iMmax+1:iqmax)).*diff(Mt(iMmax+1:iqmax))/2 ;
diff(q(iMmax:iqmax)).*Mt(iMmax:iqmax-1)]);
    else %no post region
        Apost=0;
    end
    Afitpre=1/2*M1*q1 + (q2-q1)*(M2+M1)/2;
    Afitpost=M2*(q3-q2) + M3*(q4-q3);
    E5=(Apre-Afitpre)^2+(Apost-Afitpost)^2;
    %E6: Energy pre, Complementary Energy post
    if iqmax>iMmax
        ACpost=sum([diff(q(iMmax+1:iqmax)).*diff(Mt(iMmax+1:iqmax))/2 ;
diff(Mt(iMmax:iqmax)).*q(iMmax:iqmax-1)]);
    else
        ACpost=0;
    end
    ACfitpost=M2*(q3) + M3*(q4-q3);
    E6=(Apre-Afitpre)^2+(ACpost-ACfitpost)^2;
    %E7: Energy pre SSE, on q post
    qfit=q;
    for i=iMmax:length(Mt)
        if Mt(i)>=M2
            qfit(i)=q(i);
        elseif Mt(i)<M2 & Mt(i)>=M3
            qfit(i)=q3;
        elseif Mt(i)<M3;
            qfit(i)=q(i);
        end
```

```
    end
    E7=E5;%(Apre-Afitpre)^2+sum((qfit-q).^2);
    %All error measures
    Error=[E1 E2 E3 E4 E5 E6 E7];
    %Penalties for constrained optimization
    r=0;
    if q1<0
        r=r+abs(q1)*100;
    elseif q2<q1
        r=r+abs(q2-q1)*100;
    elseif q3<q2
        r=r+abs(q3-q2)*100;
    elseif q4<q3
        r=r+abs(q4-q3)*100;
    end
    if M1<0
        r=r+abs(M1)*10;
    elseif M2<M1
        r=r+abs(M2-M1)*10;
    elseif M3>M2
        r=r+abs(M3-M2)*10;
    elseif M3<0;
        r=r+abs(M3)*10;
    end
    Error=Error+r;
end
```

ASCE41model_fitter.m

```
function [p]=ASCE41model1_fitter(q,M,pi)
%This function performs the curve fitting to ASCE41model1
% with defined M4-q4 likely to Model 1a
%inputs are
%   q rotation
%   M moment
%   pi initial model parameters for ASCE41model1
%Parameters
%-1-
M1i=pi(1);
k1i=pi(2);
q1i=M1i/k1i;
%-2-%
M2i=pi(3);
k2i=pi(4);
q2i=q1i+(M2i-M1i)/k2i;
%-3-%
deltaqi=pi(5);
q3i=q2i+deltaqi;
%-4-%
deltaMi=pi(6);
M3i=M2i-deltaMi;
%-5/6-%
q4i=pi(7);
```

```
%
%Perform fit across all model parameters
pf=fminsearch(@modelfit1,pi);
    function [E]=modelfit1(p)
        [Mfit,Error]=ASCE41model1(p,q,M);
        E=Error(5);
        watchiterations=0;
        if watchiterations
            %plot
            figure(1)
            h1=plot(q,M,'b.-');,hold on
            qpts1=qpoints(p);
            [Mfit1,temp]=ASCE41model1(p,qpts1,0);
            h3=plot(qpts1,Mfit1,'g.--'); hold off
            title(['fitting across all model parameters, Error=',num2str(E)])
        end
    end
    if watchiterations
        pause
    end
%
%
%Perform fit across subset of parameters
%Set M2 k1 q4 as fixed values then optimize
p2i=[M1i k2i deltaqi deltaMi];
p2f=fminsearch(@modelfit2,p2i);
    function [E]=modelfit2(p2)
        p=[p2(1) k1i M2i p2(2) p2(3) p2(4) q4i];
        [Mfit,Error]=ASCE41model1(p,q,M);
        E=Error(5);
        watchiterations=0;
        if watchiterations
            %plot
            figure(1)
            h1=plot(q,M,'b.-');,hold on
            qpts1=qpoints(p);
            [Mfit1,temp]=ASCE41model1(p,qpts1,0);
            h3=plot(qpts1,Mfit1,'g.--'); hold off
            title(['fitting with M2 k1 q4 fixed Error=',num2str(E)])
        end
   end
    if watchiterations
        pause
    end
pf2=[p2f(1) k1i M2i p2f(2) p2f(3) p2f(4) q4i];
%
%
%Perform annother fit across subset of parameters
%Set k1 q2 M2 q4 and deltaM as fixed values then optimize
[Mmax,indexMmax]=max(M);
q2i=q(indexMmax);
p3i=[M1i deltaqi];
p3f=fminsearch(@modelfit3,p3i);
    function [E]=modelfit3(p3)
        k2i=(M2i-p3(1))/(q2i-p3(1)/k1i);
        p=[p3(1) k1i M2i k2i p3(2) deltaMi q4i];
        [Mfit,Error]=ASCE41model1(p,q,M);
```

```
        E=Error(5);
        watchiterations=0;
        if watchiterations
            %plot
            figure(1)
            h1=plot(q,M,'b.-');,hold on
            qpts1=qpoints(p);
            [Mfit1,temp]=ASCE41model1(p,qpts1,0);
            h3=plot(qpts1,Mfit1,'g.--'); hold off
            title(['fitting with k1 M2 (q2) k2 q4 and deltaM fixed,
Error=',num2str(E)])
        end
    end
    if watchiterations
        pause
    end
k2f=(M2i-p3f(1))/(q2i-p3f(1)/k1i);
pf3=[p3f(1) k1i M2i k2f p3f(2) deltaMi q4i];
%
%
p=[ pf
    pf2
    pf3];
%
%
%
%Function to help plot ASCE41model could be broken out into another m file
    function [qpts]=qpoints(p)
        M1=p(1);
        k1=p(2);
        M2=p(3);
        k2=p(4);
        deltaq=p(5);
        deltaM=p(6);
        q1=M1/k1;
        q2=q1+(M2-M1)/k2;
        q3=q2+deltaq;
        q4=p(7);
        qpts=[0 q1 q2 q3-10*eps q3 q4 q4+10*eps];
    end

end %function
```

ASCE41model_driver.m

```
%This is the main script file for generating the
%cruve fit to the available data against ASCE41model1
clear all
close all
%
%
%Load the data to be fit
%NOTE this is highly processed data already reduced
```

```
%to 10 pre-peak and ~10 post-peak points by Ayhan programs
%
load test_local
% load test_dist%
% load abaqus_local
% load abaqus_dist+

%**don't forget to change the save file below**
%**also need to change some of the plotting if that is being used**
% Your variables (after loading) are:
% Ix            Md
% Ix_eff        My
% Ixe           k_elastic_q
% M             k_secant_q
% M_05          q
% M_08          q_05
% M_el          q_08
% M_postMd      q_postMd
% M_preMd       qd
% Mcrd
% Mcrl
%
for i=1:length(q) %q is a structure thus this loop is over all specimens
    ['fitting ',int2str(i)]
    %-------------------------------------------
    %Curve fit to all available data
    qt=q{i}';
    Mt=M{i}';
    %   so we won't truncate q and M here at all.
    %Establish the initial guesses for ASCE41model1
    [Mtmax,iMtmax]=max(Mt);
    qtMtmax=qt(iMtmax);
    %guesses
    M1i=0.9*Mtmax;
    k1i=Mt(5)/qt(5); %could polyfit, implies data is structured
    M2i=max(Mt);
        q2i=qtMtmax;
    k2i=(M2i-M1i)/(q2i-M1i/k1i);
    q4i=qt(length(qt));
    deltaqi=(q4i-q2i)/2;
    deltaMi=Mtmax-Mt(length(Mt));

    pi=[M1i k1i M2i k2i deltaqi deltaMi q4i];
    %Call the ASCE41 fitter
    %Three curves will actually be fit
    %the output are the ASCE41model1 parameters
    %p(:,1) is a full fit with no restrictions
    %P(:,2) sets M2 k1 q4 as fixed values then optimize
    %p(:,3) sets k1 q2 M2 q4 as fixed values then optimize
    [p{i}]=ASCE41model1_fitter(qt,Mt,pi);
    %
    %-------------------------------------------
    %Curve fit to data up to 50% M drop
    qt=q{i}';
    Mt=M{i}';
    endindex=max(find(Mt>0.50*max(Mt)));
    qt=qt(1:endindex);
```

```
    Mt=Mt(1:endindex);
    %Establish the initial guesses for ASCE41model1
    [Mtmax,iMtmax]=max(Mt);
    qtMtmax=qt(iMtmax);
    %guesses
    M1i=0.9*Mtmax;
    k1i=Mt(5)/qt(5); %could polyfit, implies data is structured
    M2i=max(Mt);
        q2i=qtMtmax;
    k2i=(M2i-M1i)/(q2i-M1i/k1i);
    q4i=qt(length(qt));
    deltaqi=(q4i-q2i)/2;
    deltaMi=Mtmax-Mt(length(Mt));

    pi=[M1i k1i M2i k2i deltaqi deltaMi q4i];
    %Call the ASCE41 fitter
    %Three curves will actually be fit
    %the output are the ASCE41model1 parameters
    %p(:,1) is a full fit with no restrictions
    %P(:,2) sets M2 k1 q4 as fixed values then optimize
    %p(:,3) sets k1 q2 M2 q4 as fixed values then optimize
    [p50{i}]=ASCE41model1_fitter(qt,Mt,pi);
    %----------------
end
%
% save test_local_ASCE41model1fit
% save test_dist_ASCE41model1fit
% save abaqus_local_ASCE41model1fit
% save abaqus_dist+_ASCE41model1fit
%
%
%
%
%The visualization routine below was moved to ASCE41model1_viewer...
if 1
    %Visualization of the curve fits
    for j=1:2 %loop over the two types of fites
        if j==1
            %titlestring=['ASCE41model1, local tests, all data'];
            titlestring=['ASCE41model1, dist. tests, all data'];
            pplot=p;
        elseif j==2
            %titlestring=['ASCE41model1, local tests, all data,
M_{postpeak}>50%M_{peak}'];
            titlestring=['ASCE41model1, dist. tests, all data,
M_{postpeak}>50%M_{peak}'];
            pplot=p50;
        end
        for i=1:length(q)
            Myield=My(i);
            qyield=Myield/k_elastic_q(i);
            %let's try 12 per plot to start
            figi=ceil(i/12);
            f=figure((j-1)*100+figi);
            ploti=i-(figi-1)*12; %suplotindex
            %
            if ploti==1
```

```
                width=6.5; %inches
                height=9; %inches
                left=1; %inch from the left edge of the screen
                bottom=1; %inch from the bottom of the screen
                set(f,'Units','Inches','Position',[left bottom width height])
                set(f,'PaperSize',[width height])
           %NOTE! the preceding is not controllling the plot size the way
           %I want when I use the print command to create the eps files I
                %will need to look into this further!
            end
            subplot(6,2,ploti)
            %
            h1=plot(q{i}'/qyield,M{i}'/Myield,'k.--');,hold on
            %fit1
            qpts1=ASCE41model1_qpts(pplot{i}(1,:));
            Mfit1=ASCE41model1(pplot{i}(1,:),qpts1,0);
            h3=plot(qpts1/qyield,Mfit1/Myield,'g-');
            %fit3
            qpts3=ASCE41model1_qpts(pplot{i}(3,:));
            Mfit3=ASCE41model1(pplot{i}(3,:),qpts3,0);
            h5=plot(qpts3/qyield,Mfit3/Myield,'b-');
            hold off
            %plot details
            axis([0 25 0 1.2])
            %axis([0 4 0 2])
            %text(0.1,1.0,['L',int2str(i)])
            text(0.1,1.0,['D',int2str(i)])
            %text(0.1,1.0,['L_a',int2str(i)])
            %text(0.1,1.0,['D_{a+}',int2str(i)])
            %text(0.1,1.0,['D_{a-}',int2str(i)])
            if ploti==2
    legend([h1 h3 h5],'data','full fit','constr.fit','Location','Northeast')
            end
            if max(ploti==[1 3 5 7 9 11])
                ylabel('M/M_y')
            end
            if max(ploti==[11 12]) | i>=length(q)-1
                xlabel('\theta/\theta_y')
            end
            if ploti==1&i==1
                title(titlestring)
            elseif ploti==1
                title([titlestring,' (cont.)'])
            end
        end
    end
end
```

ASCE41model_viewer.m

```
%Allows for viewing of the ASCE41model1 fits
clear all
close all
%
```

```
%test_local
load test_local_ASCE41model1fit
titlestring=['local tests'];
filestring=['test_local_ASCE41modl1_mq_'];
plotlabel=['L'];

%test_dist
%load test_dist_ASCE41model1fit
%titlestring=['dist. tests, all data'];
%filestring=['test_dist_ASCE41modl1_mq_'];
%plotlabel=['D'];

%abaqus_local
% load abaqus_local_ASCE41model1fit
% titlestring=['abaqus local'];
% filestring=['abaqus_local_ASCE41modl1_mq_'];
% plotlabel=['AL'];

%abaqus_distortional+
%load abaqus_dist+_ASCE41model1fit
%titlestring=['abaqus dist.+'];
%filestring=['abaqus_dist+_ASCE41modl1_mq_'];
%plotlabel=['AD+'];

%
%-------------------------------------------------
%
%Visualizaton of each individual ASCE41model1 fit
%
%-------------------------------------------------
if 1 %this is just a simple manual switch as to whether or not we want this
plot
    for j=1:2 %loop over the two types of fites
        if j==1
            pplot=p;
        elseif j==2
            titlestring=[titlestring,' M_{postpeak}>50%M_{peak}'];
            pplot=p50;
        end
        for i=1:length(q)
            Myield=My(i);
            qyield=Myield/k_elastic_q(i);
            %let's try 12 subplots (11 plots) per figure to start
            figi=ceil(i/11);
            f=figure((j-1)*100+figi);
            if rem(i-1,11)==0 %restart the suplotindex
                ploti=1;
            end
            %
            if ploti==1
                width=6.5; %inches
                height=9; %inches
                left=1; %inch from the left edge of the screen
                bottom=1; %inch from the bottom of the screen
                set(f,'Units','Inches','Position',[left bottom width height])
                set(f,'PaperPosition',[0 0 width height])
```

```
            %NOTE! the preceding is not controllling the plot size the way
            %I want when I use the print command to create the eps files I
                %will need to look into this further!
            end
            subplot(6,2,ploti)
            if ploti==2 %this will be the legend and title only
                h1=plot(0,0,'k.--');,hold on
                h3=plot(0,0,'g-');
                h5=plot(0,0,'b-');
                axis off
                legend([h1 h3 h5],'data','full fit','constr.
fit','Location','NorthEast')
                if i==2
                    title(titlestring)
                else
                    title([titlestring,' (cont.)'])
                end
                ploti=ploti+1;
                subplot(6,2,ploti)
            end
            %
            h1=plot(q{i}'/qyield,M{i}'/Myield,'k.--');,hold on
            %fit1
            qpts1=ASCE41model1_qpts(pplot{i}(1,:));
            Mfit1=ASCE41model1(pplot{i}(1,:),qpts1,0);
            h3=plot(qpts1/qyield,Mfit1/Myield,'g-');
            %fit3
            qpts3=ASCE41model1_qpts(pplot{i}(3,:));
            Mfit3=ASCE41model1(pplot{i}(3,:),qpts3,0);
            h5=plot(qpts3/qyield,Mfit3/Myield,'b-');
            hold off
            %plot details
            axis([0 3 0 1.2])
            text(0.1,1.0,[plotlabel,int2str(i)])
            if max(ploti==[1 3 5 7 9 11])
                ylabel('M/M_y')
            end
            if max(ploti==[11 12]) | i>=length(q)-1
                xlabel('\theta/\theta_y')
            end
            %print out the plots
            %print this figure to eps with tiff preview
            if j==1
   print('-depsc','-loose','-tiff','-r600',[filestring,'all_',int2str(figi)])
            else
   print('-depsc','-loose','-tiff','-r600',[filestring,'50p_',int2str(figi)])
            end
            %increment to the next plot
            ploti=ploti+1;
        end
    end
end


%Exploration of test_local
if 0
```

```
load test_local_ASCE41model1fit
    %------------------------------------------------------
    %
    %Simple DSM strength plot to verify data looks fine
    %
    %------------------------------------------------------
    for i=1:length(q)
        Mpeak(i)=max(M{i})
    end
    f=figure(1)
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
    bottom=1; %inch from the bottom of the screen
    set(f,'Units','Inches','Position',[left bottom width height])
    set(f,'PaperPosition',[0 0 width height])
    h1=plot((My./Mcrl).^0.5,Mpeak./My,'.')
    axis([0 2 0 1.5])
    xlabel('local slenderness (M_y/M_{crl})^{0.5}')
    ylabel('M_{peak}/M_y or M_2/M_y')
    %add abaqus data
    load abaqus_local_ASCE41model1fit
    for i=1:length(q)
        Mpeak(i)=max(M{i})
    end
    hold on
    h2=plot((My./Mcrl).^0.5,Mpeak./My,'ro','MarkerSize',2)
    %DSM fit
    x=[0 0.776 0.78:0.01:2];
    y=(1-0.15*(1./x).^2.^0.4).*(1./x).^2.^0.4;
    y(1:2)=[1 1];
    hold on
    h3=plot(x,y,'k-');
    xin=[0 0.776];
    yin=[1.15 1.0];
    h4=plot(xin,yin,'k--');
    %legend, hold off print and reload for next plot
    legend([h1 h2 h3 h4],'test','abaqus','DSM','DSM^{(1)}')
    text(0.1,0.1,'(1) DSM inelastic for M_p/M_y = 1.15')
    hold off
    load test_local_ASCE41model1fit
    print('-depsc','-loose','-tiff','-r600','local01_DSM')
    %------------------------------------------------------
    %
    %Rotation at peak strength
    %
    %------------------------------------------------------
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
    end
    f=figure(2)
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
```

```
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,qpeak./qy,'.')
axis([0 2 0 10])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('\theta_{peak}/\theta_y')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
end
hold on
h2=plot((My./Mcrl).^0.5,qpeak./qy,'ro','MarkerSize',2)
%***New curve fit***
x=(0:0.01:2);
y=1./(x./1.0);
h3=plot(x,y,'k-')
%h4=plot(0.776,1,'kx') %implied anchor point baed on DSM itself
%legend and clean up for next plot...
legend([h1 h2 h3],'test','abaqus','proposed')
hold off
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local02_peakrotation')
%--------------------------------------------------
%
%Ratio of secant stiffness to elatic stiffness
%
%--------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
end
f=figure(3)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,ksecant./k_elastic_q,'.')
axis([0 2 0 1])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('k_{secant}/k_{elastic}')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
```

```
    ksecant(i)=Mpeak(i)/qpeak(i);
end
hold on
h2=plot((My./Mcrl).^0.5,ksecant./k_elastic_q,'ro','MarkerSize',2)
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak qpeak qy ksecant
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local03_secanttoelastic')
%-----------------------------------------------------------
%
%Post peak moment drop when using up the 50%M post-peak data
%
%-----------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaM(i)=p50{i}(3,6)
end
f=figure(4)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,deltaM./Mpeak,'.')
axis([0 2 0 1])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('\DeltaM/M_{peak}')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaM(i)=p50{i}(3,6)
end
hold on
h2=plot((My./Mcrl).^0.5,deltaM./Mpeak,'ro','MarkerSize',2)
%***New curve fit***
x=0:0.01:2;
y=(1-1./(x/0.776+1)).^1.1;
y(88:201)=0.5;
h3=plot(x,y,'k-')
%legend and clean up for the next plot
legend([h1 h2 h3],'test','abaqus','proposed')
hold off
clear Mpeak qpeak qy ksecant deltaM x y x1 y1
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local04_deltaMdrop')
```

```
%------------------------------------------------------------
%
%Post peak delta q when using up the 50%M post-peak data
%
%------------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
f=figure(5)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,deltaq./qy,'.')
axis([0 2 0 1])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('\Delta\theta/\theta_y')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
hold on
h2=plot((My./Mcrl).^0.5,deltaq./qy,'ro','MarkerSize',2)
%**simplified curve fit
x1=(0:0.01:0.776);
y1=1./(x1/0.776)-1;
x2=[0.776 2];
y2=[0 0];
h3=plot(x1,y1,'k-');
h4=plot(x2,y2,'k-');
%legend and cleanup
legend([h1 h2 h3],'test','abaqus','proposed')
hold off
clear Mpeak qpeak qy ksecant deltaq
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local05_deltaq')
%------------------------------------------------------------
%
%Model41 stiffness loss pre-peak k2/k1
%
%------------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
```

```
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5);
    k1(i)=p50{i}(3,2);
    k2(i)=p50{i}(3,4);
end
f=figure(6)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,k2./k1,'.')
axis([0 2 0 1])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('k_2/k_1')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5);
    k1(i)=p50{i}(3,2);
    k2(i)=p50{i}(3,4);
end
hold on
h2=plot((My./Mcrl).^0.5,k2./k1,'ro','MarkerSize',2)
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak qpeak qy ksecant deltaq k1 k2
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local06_k2onk1')
%-----------------------------------------------------------
%
%Theta 4 -max rot - when using up the 50%M post-peak data
%
%-----------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    q4(i)=p50{i}(3,7)
end
f=figure(7)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,q4./qpeak,'.')
```

```
    axis([0 2 0 4])
    xlabel('local slenderness (M_y/M_{crl})^{0.5}')
    ylabel('\theta_4/\theta_{peak} = \theta_4/\theta_2')
    %add abaqus data
    load abaqus_local_ASCE41model1fit
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        q4(i)=p50{i}(3,7)
    end
    hold on
    h2=plot((My./Mcrl).^0.5,q4./qpeak,'ro','MarkerSize',2)
    legend([h1 h2],'test','abaqus')
    hold off
    clear Mpeak qpeak qy q4
    load test_local_ASCE41model1fit
    print('-depsc','-loose','-tiff','-
r600','local07_theta4maxrot_thetapeaknorm')
    %-----------------------------------------------------------
    %
    %Theta 4 -max rot- different norm - when using up the 50%M post-peak data
    %
    %-----------------------------------------------------------
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        q4(i)=p50{i}(3,7)
    end
    f=figure(8)
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
    bottom=1; %inch from the bottom of the screen
    set(f,'Units','Inches','Position',[left bottom width height])
    set(f,'PaperPosition',[0 0 width height])
    h1=plot((My./Mcrl).^0.5,q4./qy,'.')
    axis([0 2 0 4])
    xlabel('local slenderness (M_y/M_{crl})^{0.5}')
    ylabel('\theta_4/\theta_y')
    %add abaqus data
    load abaqus_local_ASCE41model1fit
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        q4(i)=p50{i}(3,7)
    end
    hold on
    h2=plot((My./Mcrl).^0.5,q4./qy,'ro','MarkerSize',2)
    %new fit
    %q2fit
    x=(0:0.01:2);
```

```
    yq2=1./(x./0.776);
    h3=plot(x,yq2,'k--')
    h4=plot(0.776,1,'kx')
%     yq4=1.5.*yq2;
    yq4=1.5*(1./(x./1.0)).^((1.0./(4.*x)));
    if x>1
        yq4=1.5*(1./(x./1.0)).^(1.0);
    end
    h5=plot(x,yq4,'k-')
    %legend and cleanup for next plot
legend([h1 h2 h3 h5],'test','abaqus','\theta_2/\theta_y proposed','proposed')
    hold off
    clear Mpeak qpeak qy q4
    load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local08_theta4maxrot_thetaynorm')


    %-----------------------------------------------------------
    %
    %Post peak moment drop when using all data
    %
    %-----------------------------------------------------------
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        ksecant(i)=Mpeak(i)/qpeak(i);
        deltaM(i)=p{i}(3,6)
    end
    f=figure(9)
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
    bottom=1; %inch from the bottom of the screen
    set(f,'Units','Inches','Position',[left bottom width height])
    set(f,'PaperPosition',[0 0 width height])
    h1=plot((My./Mcrl).^0.5,deltaM./Mpeak,'.')
    axis([0 2 0 1])
    xlabel('local slenderness (M_y/M_{crl})^{0.5}')
    ylabel('\DeltaM/M_{peak} (all data)')
    %add abaqus data
    load abaqus_local_ASCE41model1fit
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        ksecant(i)=Mpeak(i)/qpeak(i);
        deltaM(i)=p{i}(3,6)
    end
    hold on
    h2=plot((My./Mcrl).^0.5,deltaM./Mpeak,'ro','MarkerSize',2)
    %***New curve fit***
    x=0:0.01:2;
    y=(1-(1./(x/0.776+1)).^1);
```

```
h3=plot(x,y,'k-')
%legend and clean up for the next plot
legend([h1 h2 h3],'test','abaqus','proposed','Location','Northwest')
hold off
clear Mpeak qpeak qy ksecant deltaM
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local09_deltaMdrop_alldata')
%------------------------------------------------------------
%
%Post peak delta q when using all post-peak data
%
%------------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p{i}(3,5)
end
f=figure(10)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,deltaq./qy,'.')
axis([0 2 0 1])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('\Delta\theta/\theta_y (all data)')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p{i}(3,5)
end
hold on
h2=plot((My./Mcrl).^0.5,deltaq./qy,'ro','MarkerSize',2)
%**proposed fit**
%legend and cleanup
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak qpeak qy ksecant deltaq x y x2 y2
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local10_deltaq_alldata')
%---------------------------------------------------
%
%M1 model fit (end of elatic regime)..
%
%---------------------------------------------------
for i=1:length(q)
    Mpeak(i)=max(M{i})
```

```
    M1(i)=p50{i}(3,1)
end
f=figure(11)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,M1./My,'.')
axis([0 2 0 1.5])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('M_1/M_y')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    Mpeak(i)=max(M{i})
    M1(i)=p50{i}(3,1)
end
hold on
h2=plot((My./Mcrl).^0.5,M1./My,'ro','MarkerSize',2)
%**new curve fit
x=[0 0.776 0.777:0.01:2];
y=(0.776./x).^2;
y(1:2)=[1 1];
h3=plot(x,y,'k--');
x2=[0 0.65 0.65:0.01:2];
y2=(0.65./x2).^2;
y2(1:2)=[1 1];
h4=plot(x2,y2,'k-')
%legend and cleanup
legend([h1 h2 h3 h4],'test','abaqus','DSM','proposed')
hold off
clear Mpeak M1
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local11_M1')
%--------------------------------------------------
%
%M1 model fit (end of elatic regime).. norm to M2
%
%--------------------------------------------------
for i=1:length(q)
    Mpeak(i)=max(M{i});
    M1(i)=p50{i}(3,1);
    M2(i)=p50{i}(3,3);
end
f=figure(12)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,M1./M2,'.')
axis([0 2 0 1.5])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('M_1/M_2')
```

```
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    Mpeak(i)=max(M{i});
    M1(i)=p50{i}(3,1);
    M2(i)=p50{i}(3,3);
end
hold on
h2=plot((My./Mcrl).^0.5,M1./M2,'ro','MarkerSize',2)
%**new curve fit
%legend and cleanup
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak M1 M2
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local12_M1normtoM2')
%------------------------------------------------------------
%
%Post peak delta q when using up the 50%M post-peak data - no norm!
%
%------------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
f=figure(13)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrl).^0.5,deltaq,'.')
axis([0 2 0 .02])
xlabel('local slenderness (M_y/M_{crl})^{0.5}')
ylabel('\Delta\theta')
%add abaqus data
load abaqus_local_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
hold on
h2=plot((My./Mcrl).^0.5,deltaq,'ro','MarkerSize',2)
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak qpeak qy ksecant deltaq
load test_local_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','local13_deltaq_nonorm')
```

```
end

%---------------------------------------------------------------------------
%---------------------------------------------------------------------------
%Exploration of test_distortional
%---------------------------------------------------------------------------
%---------------------------------------------------------------------------
if 0
load test_dist_ASCE41model1fit
    %---------------------------------------------------
    %
    %Simple DSM strength plot to verify data looks fine
    %
    %---------------------------------------------------
    for i=1:length(q)
        Mpeak(i)=max(M{i})
    end
    f=figure(1)
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
    bottom=1; %inch from the bottom of the screen
    set(f,'Units','Inches','Position',[left bottom width height])
    set(f,'PaperPosition',[0 0 width height])
    h1=plot((My./Mcrd).^0.5,Mpeak./My,'.')
    axis([0 2 0 1.5])
    xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
    ylabel('M_{peak}/M_y or M_2/M_y')
    %add abaqus data
    load abaqus_dist+_ASCE41model1fit
    for i=1:length(q)
        Mpeak(i)=max(M{i})
    end
    hold on
    h2=plot((My./Mcrd).^0.5,Mpeak./My,'ro','MarkerSize',2)
    %DSM fit
    x=[0 0.673 0.68:0.01:2];
    y=(1-0.22*(1./x).^2.^0.5).*(1./x).^2.^0.5;
    y(1:2)=[1 1];
    hold on
    h3=plot(x,y,'k-');
    xin=[0 0.673];
    yin=[1.15 1.0];
    h4=plot(xin,yin,'k--');
    %legend, hold off print and reload for next plot
    legend([h1 h2 h3 h4],'test','abaqus','DSM','DSM^{(1)}')
    text(0.1,0.1,'(1) DSM inelastic for M_p/M_y = 1.15')
    hold off
    load test_dist_ASCE41model1fit
    print('-depsc','-loose','-tiff','-r600','dist01_DSM')
    %---------------------------------------------------
    %
    %Rotation at peak strength
    %
    %---------------------------------------------------
    for i=1:length(q)
        [m,mi]=max(M{i});
```

```
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
end
f=figure(2)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,qpeak./qy,'.')
axis([0 2 0 4])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('\theta_{peak}/\theta_y')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
end
hold on
h2=plot((My./Mcrd).^0.5,qpeak./qy,'ro','MarkerSize',2)
%***New curve fit***
x=(0:0.01:2);
y=(1./(x./1.0)).^1.4;
h3=plot(x,y,'k-')
%h4=plot(0.776,1,'kx') %implied anchor point baed on DSM itself
%legend and clean up for next plot...
legend([h1 h2 h3],'test','abaqus','proposed')
hold off
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist02_peakrotation')
%--------------------------------------------------
%
%Ratio of secant stiffness to elatic stiffness
%
%--------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
end
f=figure(3)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,ksecant./k_elastic_q,'.')
axis([0 2 0 1])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
```

```
ylabel('k_{secant}/k_{elastic}')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
end
hold on
h2=plot((My./Mcrd).^0.5,ksecant./k_elastic_q,'ro','MarkerSize',2)
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak qpeak qy ksecant
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist03_secanttoelastic')
%-----------------------------------------------------------
%
%Post peak moment drop when using up the 50%M post-peak data
%
%-----------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaM(i)=p50{i}(3,6)
end
f=figure(4)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,deltaM./Mpeak,'.')
axis([0 2 0 1])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('\DeltaM/M_{peak}')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaM(i)=p50{i}(3,6)
end
hold on
h2=plot((My./Mcrd).^0.5,deltaM./Mpeak,'ro','MarkerSize',2)
%***New curve fit***
x=0:0.01:2;
y=(1-1./(x/0.673+1)).^1.4;
y(107:201)=0.5;
```

```
h3=plot(x,y,'k-')
%legend and clean up for the next plot
legend([h1 h2 h3],'test','abaqus','proposed')
hold off
clear Mpeak qpeak qy ksecant deltaM x y x1 y1
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist04_deltaMdrop')
%------------------------------------------------------------
%
%Post peak delta q when using up the 50%M post-peak data
%
%------------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
f=figure(5)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,deltaq./qy,'.')
axis([0 2 0 1])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('\Delta\theta/\theta_y')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
hold on
h2=plot((My./Mcrd).^0.5,deltaq./qy,'ro','MarkerSize',2)
%**simplified curve fit
x1=(0:0.01:0.673);
y1=1./(x1/0.673)-1;
x2=[0.674 2];
y2=[0 0];
h3=plot(x1,y1,'k-');
h4=plot(x2,y2,'k-');
%legend and cleanup
legend([h1 h2 h3],'test','abaqus','proposed')
hold off
clear Mpeak qpeak qy ksecant deltaq
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist05_deltaq')
%------------------------------------------------------------
```

```
%
%Model41 stiffness loss pre-peak k2/k1
%
%-----------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5);
    k1(i)=p50{i}(3,2);
    k2(i)=p50{i}(3,4);
end
f=figure(6)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,k2./k1,'.')
axis([0 2 0 1])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('k_2/k_1')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5);
    k1(i)=p50{i}(3,2);
    k2(i)=p50{i}(3,4);
end
hold on
h2=plot((My./Mcrd).^0.5,k2./k1,'ro','MarkerSize',2)
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak qpeak qy ksecant deltaq k1 k2
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist06_k2onk1')
%-----------------------------------------------------------
%
%Theta 4 -max rot - when using up the 50%M post-peak data
%
%-----------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    q4(i)=p50{i}(3,7)
end
f=figure(7)
```

```
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
    bottom=1; %inch from the bottom of the screen
    set(f,'Units','Inches','Position',[left bottom width height])
    set(f,'PaperPosition',[0 0 width height])
    h1=plot((My./Mcrd).^0.5,q4./qpeak,'.')
    axis([0 2 0 4])
    xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
    ylabel('\theta_4/\theta_{peak} = \theta_4/\theta_2')
    %add abaqus data
    load abaqus_dist+_ASCE41model1fit
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        q4(i)=p50{i}(3,7)
    end
    hold on
    h2=plot((My./Mcrd).^0.5,q4./qpeak,'ro','MarkerSize',2)
    legend([h1 h2],'test','abaqus')
    hold off
    clear Mpeak qpeak qy q4
    load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist07_theta4maxrot_thetapeaknorm')
    %------------------------------------------------------------
    %
    %Theta 4 -max rot- different norm - when using up the 50%M post-peak data
    %
    %------------------------------------------------------------
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        q4(i)=p50{i}(3,7)
    end
    f=figure(8)
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
    bottom=1; %inch from the bottom of the screen
    set(f,'Units','Inches','Position',[left bottom width height])
    set(f,'PaperPosition',[0 0 width height])
    h1=plot((My./Mcrd).^0.5,q4./qy,'.')
    axis([0 2 0 15])
    xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
    ylabel('\theta_4/\theta_y')
    %add abaqus data
    load abaqus_dist+_ASCE41model1fit
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        q4(i)=p50{i}(3,7)
```

```
    end
    hold on
    h2=plot((My./Mcrd).^0.5,q4./qy,'ro','MarkerSize',2)
    %new fit
    %q2fit
    x=(0:0.01:2);
    yq2=(1./(x./1.0)).^1.4;
    h3=plot(x,yq2,'k--')
    yq4=1.5*(1./(x./1.0)).^(1.4./x);
    if x>1
        yq4=1.5*(1./(x./1.0)).^(1.4);
    end
    h5=plot(x,yq4,'k-')
    %legend and cleanup for next plot
legend([h1 h2 h3 h5],'test','abaqus','\theta_2/\theta_y proposed','proposed')
    hold off
    clear Mpeak qpeak qy q4
    load test_dist_ASCE41model1fit
    print('-depsc','-loose','-tiff','-r600','dist08_theta4maxrot_thetaynorm')

    %------------------------------------------------------------
    %
    %Post peak moment drop when using all data
    %
    %------------------------------------------------------------
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        ksecant(i)=Mpeak(i)/qpeak(i);
        deltaM(i)=p{i}(3,6)
    end
    f=figure(9)
    width=4; %inches
    height=3; %inches
    left=1; %inch from the left edge of the screen
    bottom=1; %inch from the bottom of the screen
    set(f,'Units','Inches','Position',[left bottom width height])
    set(f,'PaperPosition',[0 0 width height])
    h1=plot((My./Mcrd).^0.5,deltaM./Mpeak,'.')
    axis([0 2 0 1])
    xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
    ylabel('\DeltaM/M_{peak} (all data)')
    %add abaqus data
    load abaqus_dist+_ASCE41model1fit
    for i=1:length(q)
        [m,mi]=max(M{i});
        Mpeak(i)=M{i}(mi);
        qpeak(i)=q{i}(mi);
        qy(i)=My(i)/k_elastic_q(i);
        ksecant(i)=Mpeak(i)/qpeak(i);
        deltaM(i)=p{i}(3,6)
    end
    hold on
    h2=plot((My./Mcrd).^0.5,deltaM./Mpeak,'ro','MarkerSize',2)
    %***New curve fit***
```

```
x=0:0.01:2;
y=(1-1./(x/0.673+1)).^1.4;
y(107:201)=0.5;
h3=plot(x,y,'k-')
%legend and clean up for the next plot
legend([h1 h2 h3],'test','abaqus','proposed','Location','Northwest')
hold off
clear Mpeak qpeak qy ksecant deltaM
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist09_deltaMdrop_alldata')
%------------------------------------------------------------
%
%Post peak delta q when using all post-peak data
%
%------------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p{i}(3,5)
end
f=figure(10)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,deltaq./qy,'.')
axis([0 2 0 1])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('\Delta\theta/\theta_y (all data)')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p{i}(3,5)
end
hold on
h2=plot((My./Mcrd).^0.5,deltaq./qy,'ro','MarkerSize',2)
%**proposed fit**
%legend and cleanup
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak qpeak qy ksecant deltaq x y x2 y2
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist10_deltaq_alldata')
%---------------------------------------------------
%
%M1 model fit (end of elatic regime)..
%
```

```
%-------------------------------------------------
for i=1:length(q)
    Mpeak(i)=max(M{i})
    M1(i)=p50{i}(3,1)
end
f=figure(11)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,M1./My,'.')
axis([0 2 0 1.5])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('M_1/M_y')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    Mpeak(i)=max(M{i})
    M1(i)=p50{i}(3,1)
end
hold on
h2=plot((My./Mcrd).^0.5,M1./My,'ro','MarkerSize',2)
%**new curve fit
x=[0 0.673 0.673:0.01:2];
y=(0.673./x).^2;
y(1:2)=[1 1];
h3=plot(x,y,'k--');
x2=[0 0.6 0.6:0.01:2];
y2=(0.6./x2).^2;
y2(1:2)=[1 1];
h4=plot(x2,y2,'k-')
%legend and cleanup
legend([h1 h2 h3 h4],'test','abaqus','DSM','proposed')
hold off
clear Mpeak M1
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist11_M1')
%-------------------------------------------------
%
%M1 model fit (end of elatic regime).. norm to M2
%
%-------------------------------------------------
for i=1:length(q)
    Mpeak(i)=max(M{i});
    M1(i)=p50{i}(3,1);
    M2(i)=p50{i}(3,3);
end
f=figure(12)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,M1./M2,'.')
```

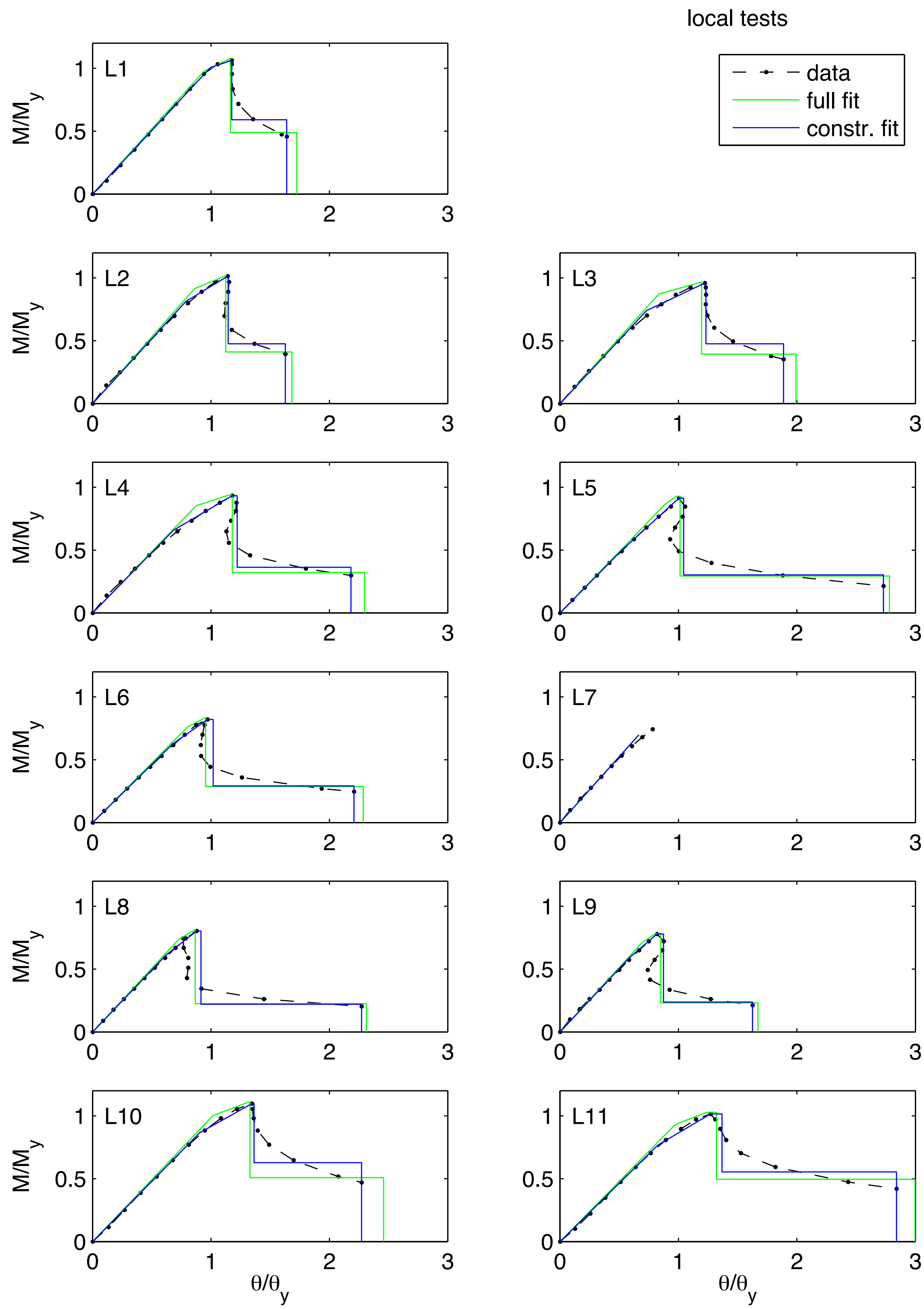
```
axis([0 2 0 1.5])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('M_1/M_2')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    Mpeak(i)=max(M{i});
    M1(i)=p50{i}(3,1);
    M2(i)=p50{i}(3,3);
end
hold on
h2=plot((My./Mcrd).^0.5,M1./M2,'ro','MarkerSize',2)
%**new curve fit
%legend and cleanup
legend([h1 h2],'test','abaqus')
hold off
clear Mpeak M1 M2
load test_dist_ASCE41model1fit
print('-depsc','-loose','-tiff','-r600','dist12_M1normtoM2')
%------------------------------------------------------------
%
%Post peak delta q when using up the 50%M post-peak data - no norm!
%
%------------------------------------------------------------
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
f=figure(13)
width=4; %inches
height=3; %inches
left=1; %inch from the left edge of the screen
bottom=1; %inch from the bottom of the screen
set(f,'Units','Inches','Position',[left bottom width height])
set(f,'PaperPosition',[0 0 width height])
h1=plot((My./Mcrd).^0.5,deltaq,'.')
axis([0 2 0 .02])
xlabel('dist slenderness (M_y/M_{crd})^{0.5}')
ylabel('\Delta\theta')
%add abaqus data
load abaqus_dist+_ASCE41model1fit
for i=1:length(q)
    [m,mi]=max(M{i});
    Mpeak(i)=M{i}(mi);
    qpeak(i)=q{i}(mi);
    qy(i)=My(i)/k_elastic_q(i);
    ksecant(i)=Mpeak(i)/qpeak(i);
    deltaq(i)=p50{i}(3,5)
end
hold on
h2=plot((My./Mcrd).^0.5,deltaq,'ro','MarkerSize',2)
legend([h1 h2],'test','abaqus')
hold off
```

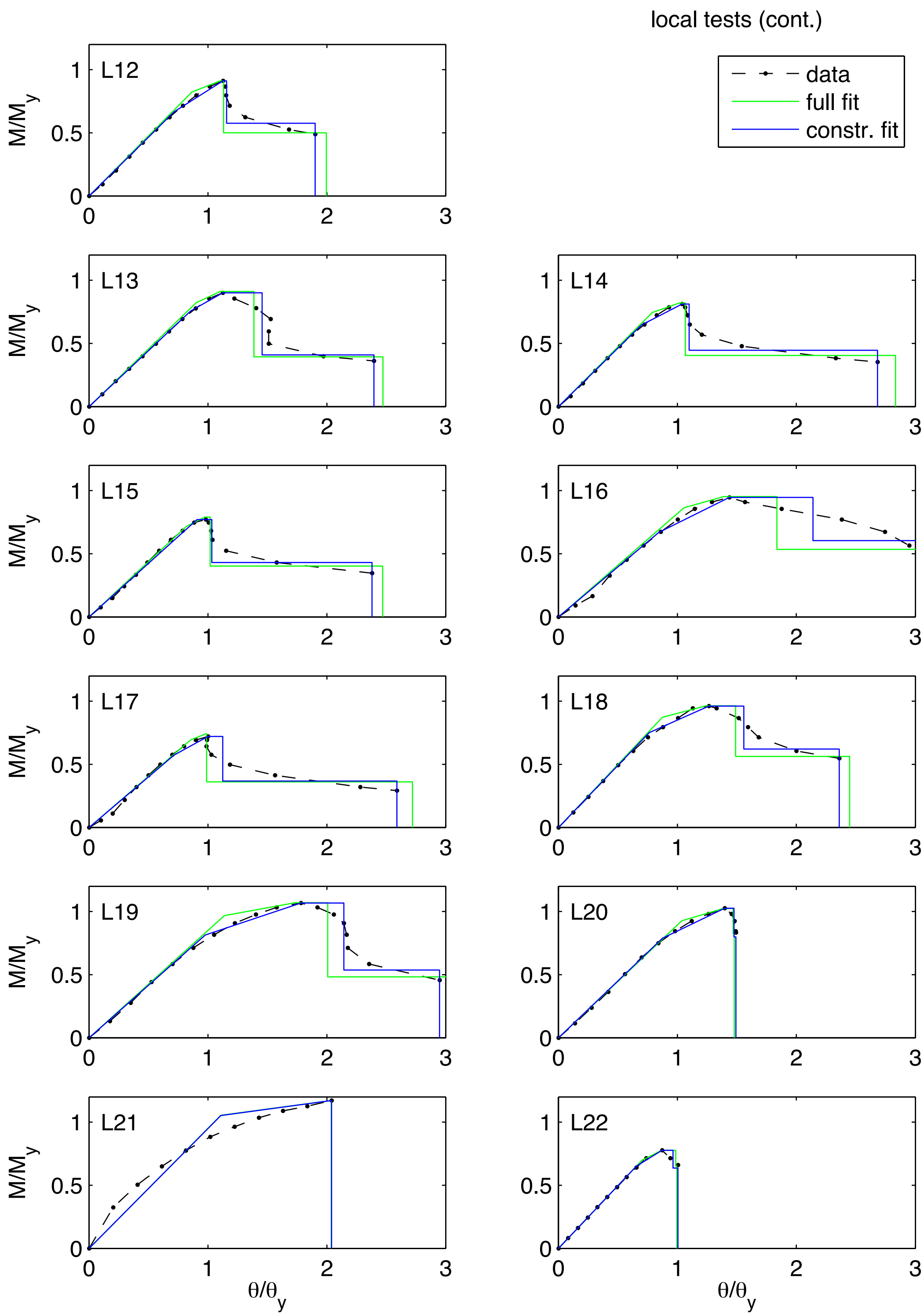
```
    clear Mpeak qpeak qy ksecant deltaq
    load test_dist_ASCE41model1fit
    print('-depsc','-loose','-tiff','-r600','dist13_deltaq_nonorm')
end
```

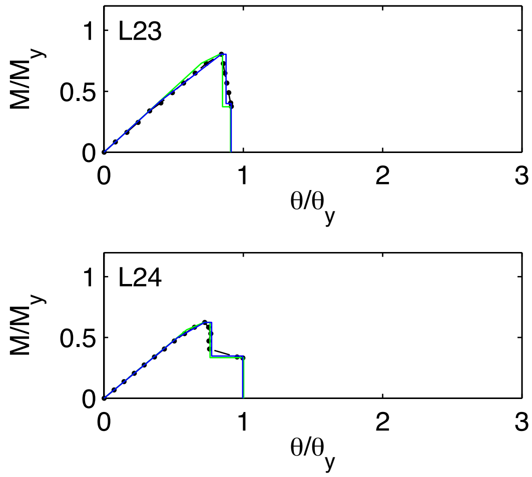
ASCE41model_qpts.m

```
function [qpts] = ASCE41model1_qpts(p)
%Given model parameter vector p, this provides
%the key rotations q for ASCE41model1
%   p is the parameters defining the curve
%   p = [M1 k1 M2 k2 deltaq deltaM q4]
%
M1=p(1);
k1=p(2);
M2=p(3);
k2=p(4);
deltaq=p(5);
deltaM=p(6);
q1=M1/k1;
q2=q1+(M2-M1)/k2;
q3=q2+deltaq;
q4=p(7);
qpts=[0 q1 q2 q3-10*eps q3 q4 q4+10*eps];
```

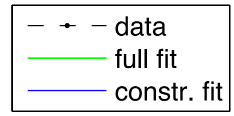
Appendix 3: Model 1 Fit with Yu and Schafer Experiments

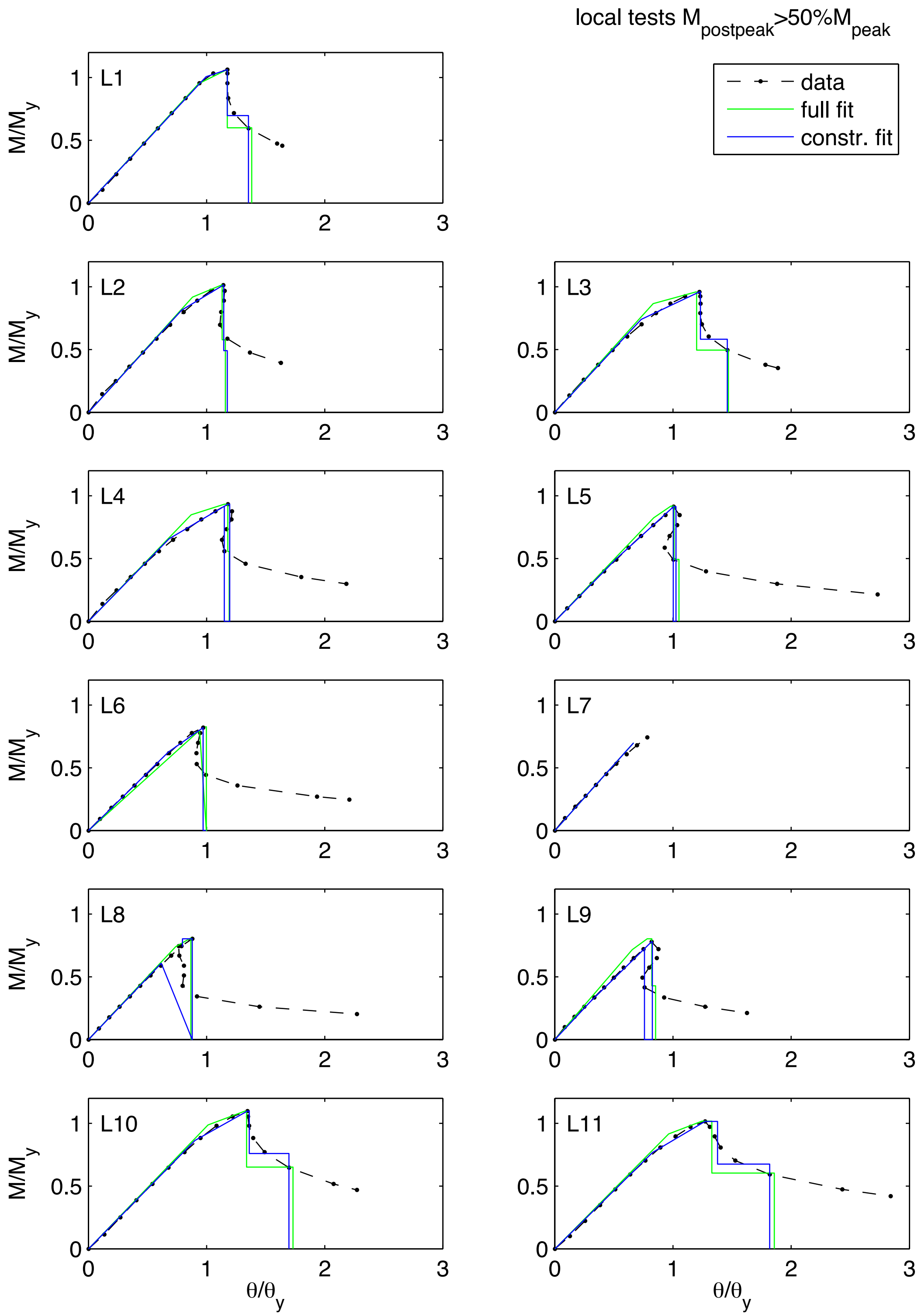


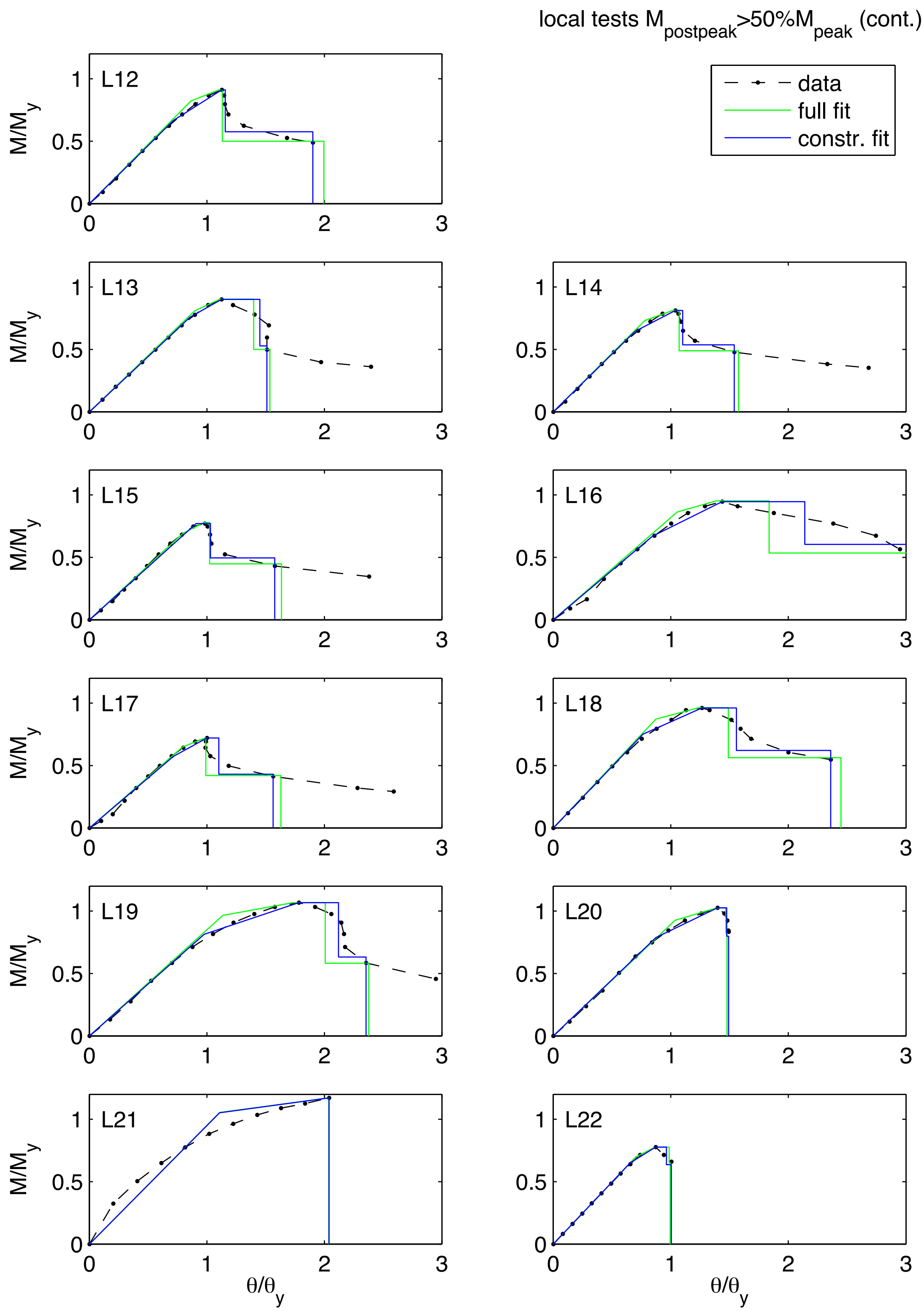

local tests (cont.)
data
full fit
constr. fit
L12
L13
L14
L15
L16
L17
L18
L19
L20
L21
L22
M/M_y
θ/θ_y

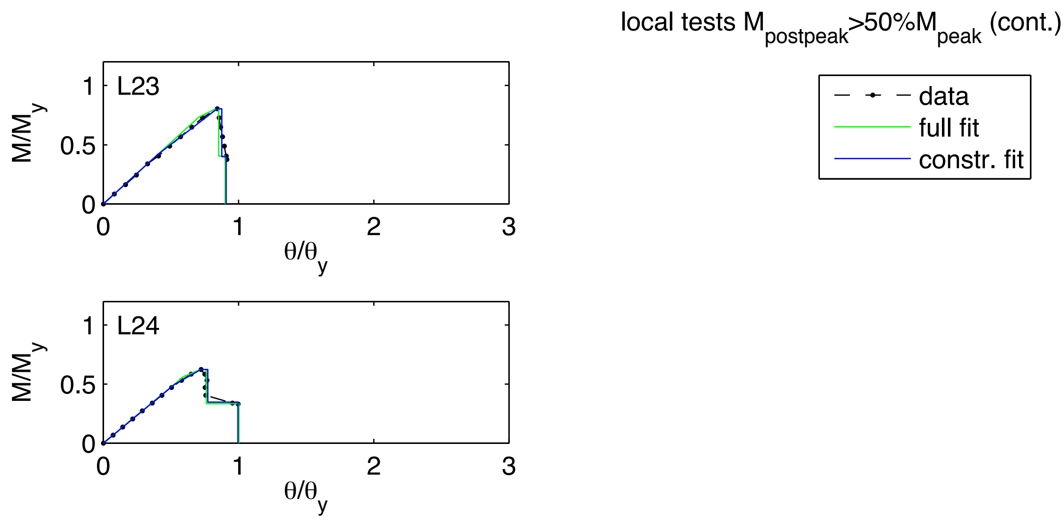


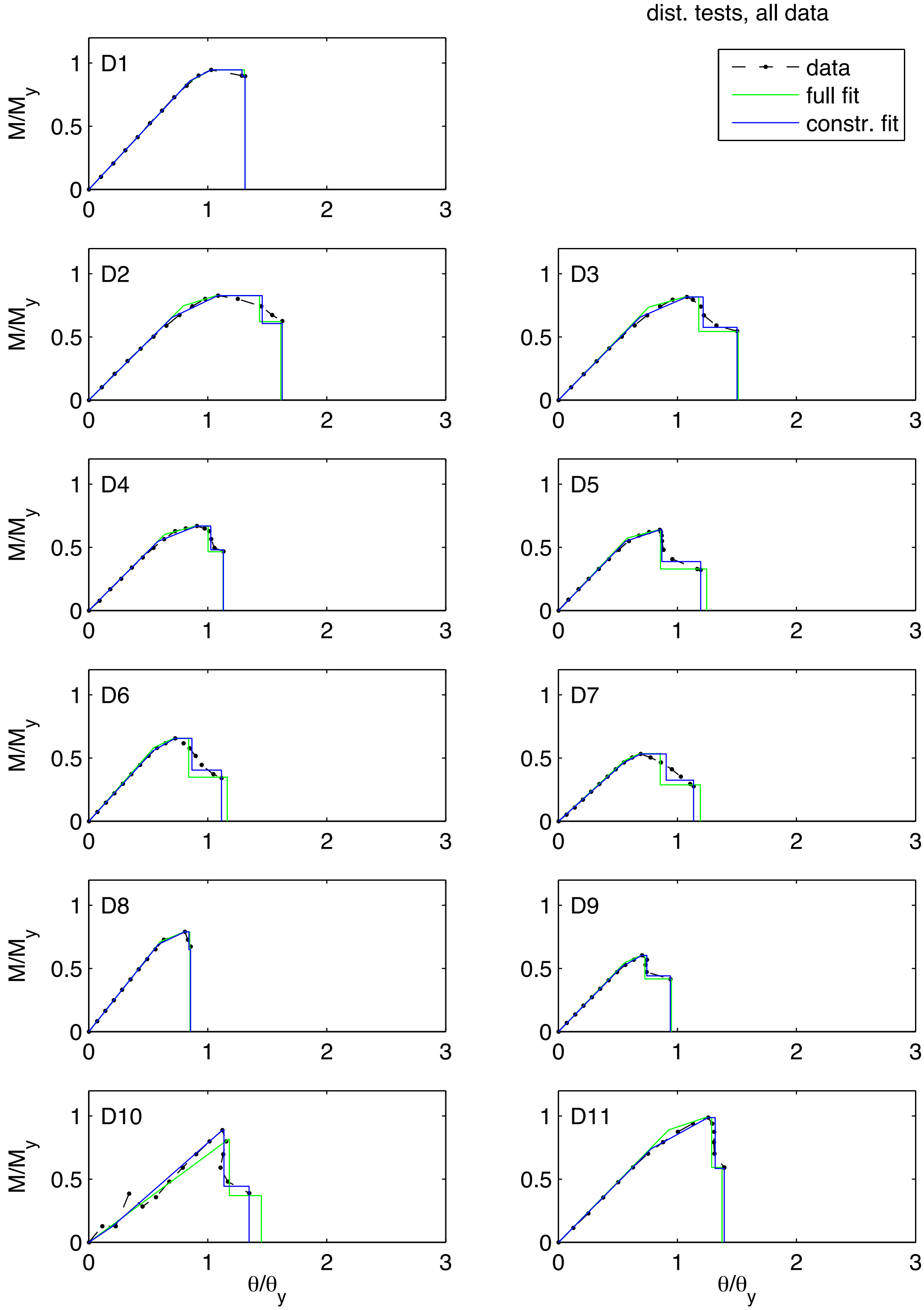
local tests (cont.)



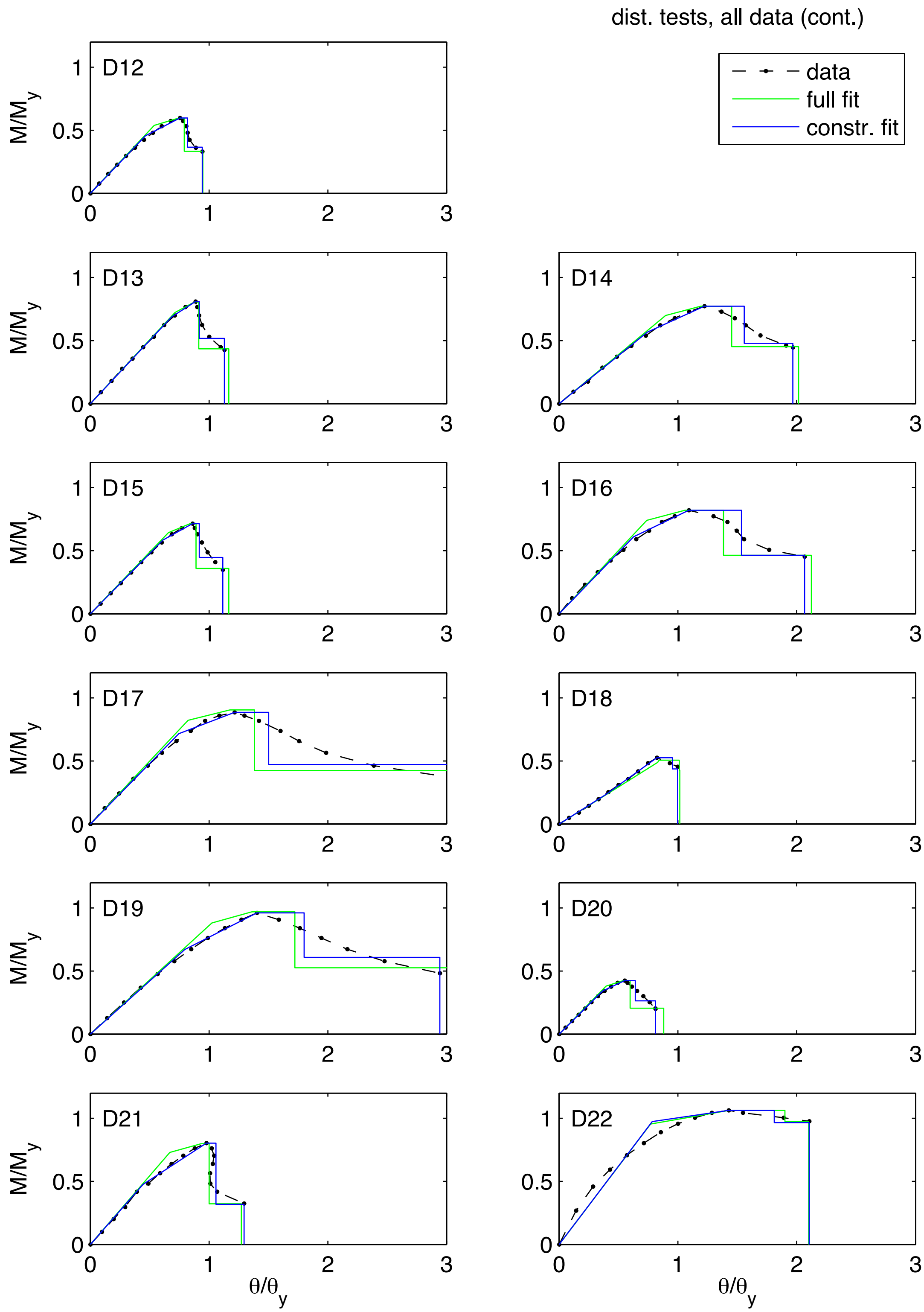

local tests M_postpeak>50%M_peak
data
full fit
constr. fit
L1
L2
L3
L4
L5
L6
L7
L8
L9
L10
L11
M/M_y
θ/θ_y

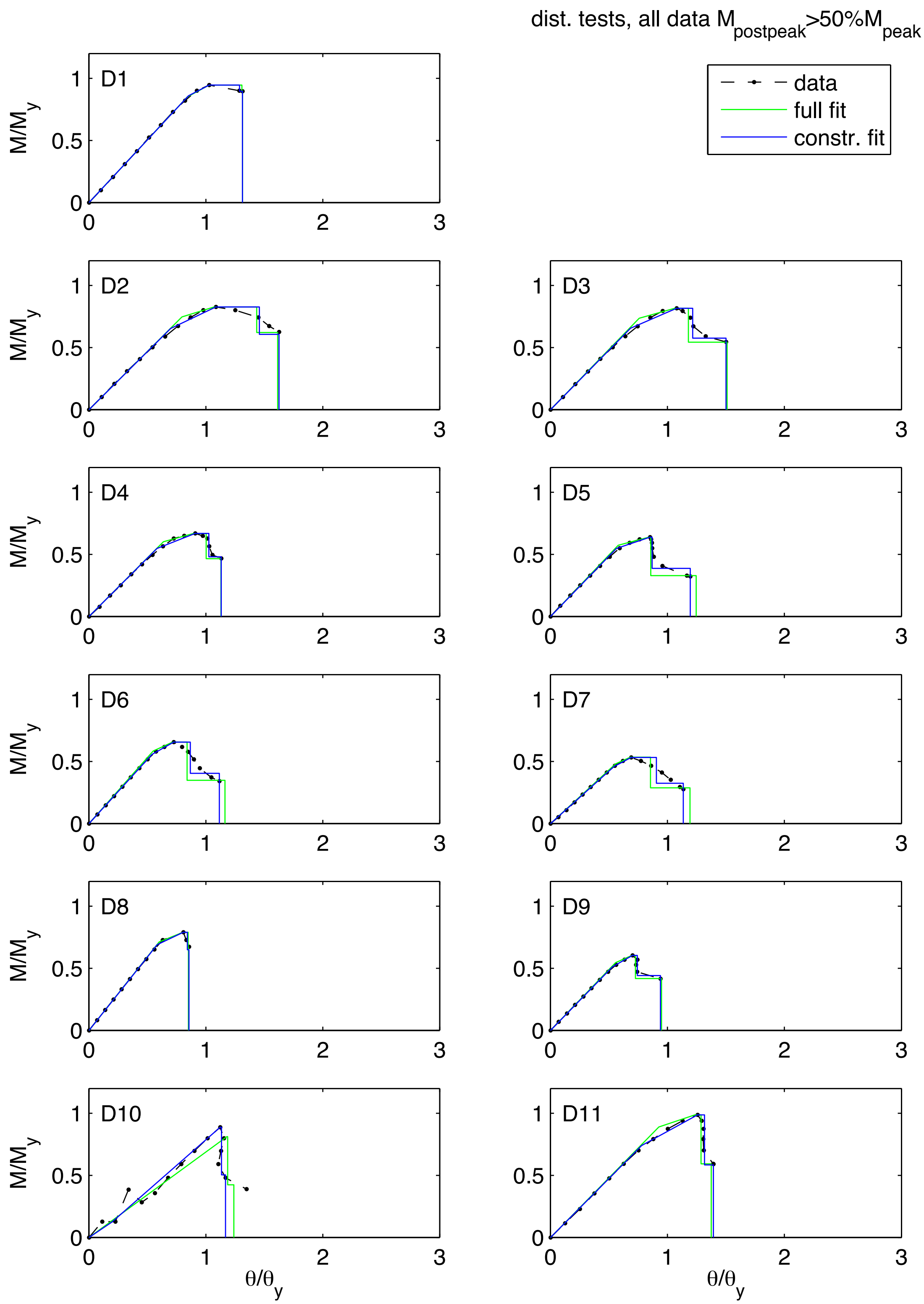

local tests $M_{postpeak}$>50%M_{peak} (cont.)
data
full fit
constr. fit
L12
L13
L14
L15
L16
L17
L18
L19
L20
L21
L22
M/M_y
θ/$θ_y$

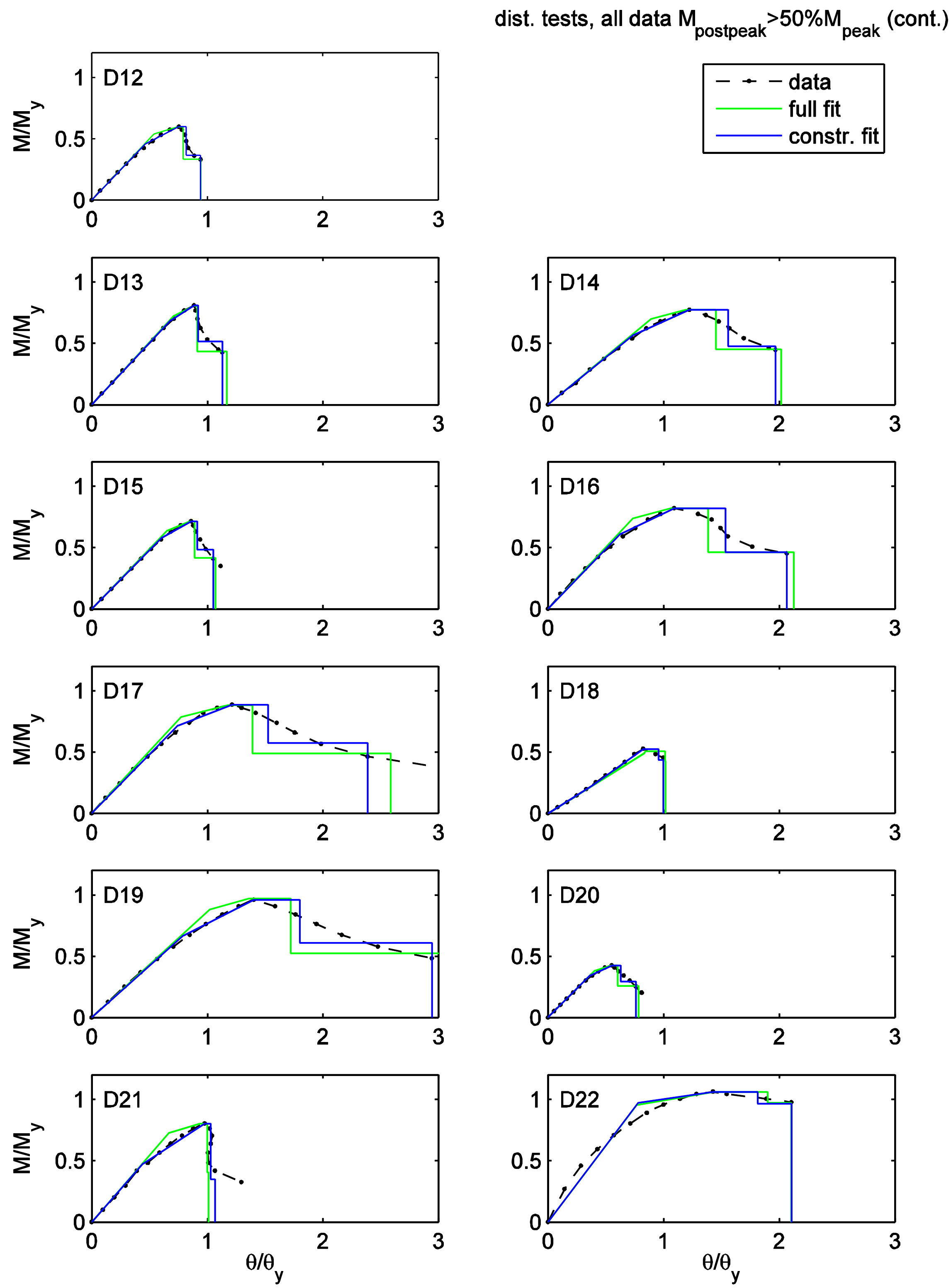

local tests M_postpeak>50%M_peak (cont.)
L23
L24
M/M_y
θ/θ_y
0
0.5
1
0
1
2
3
data
full fit
constr. fit



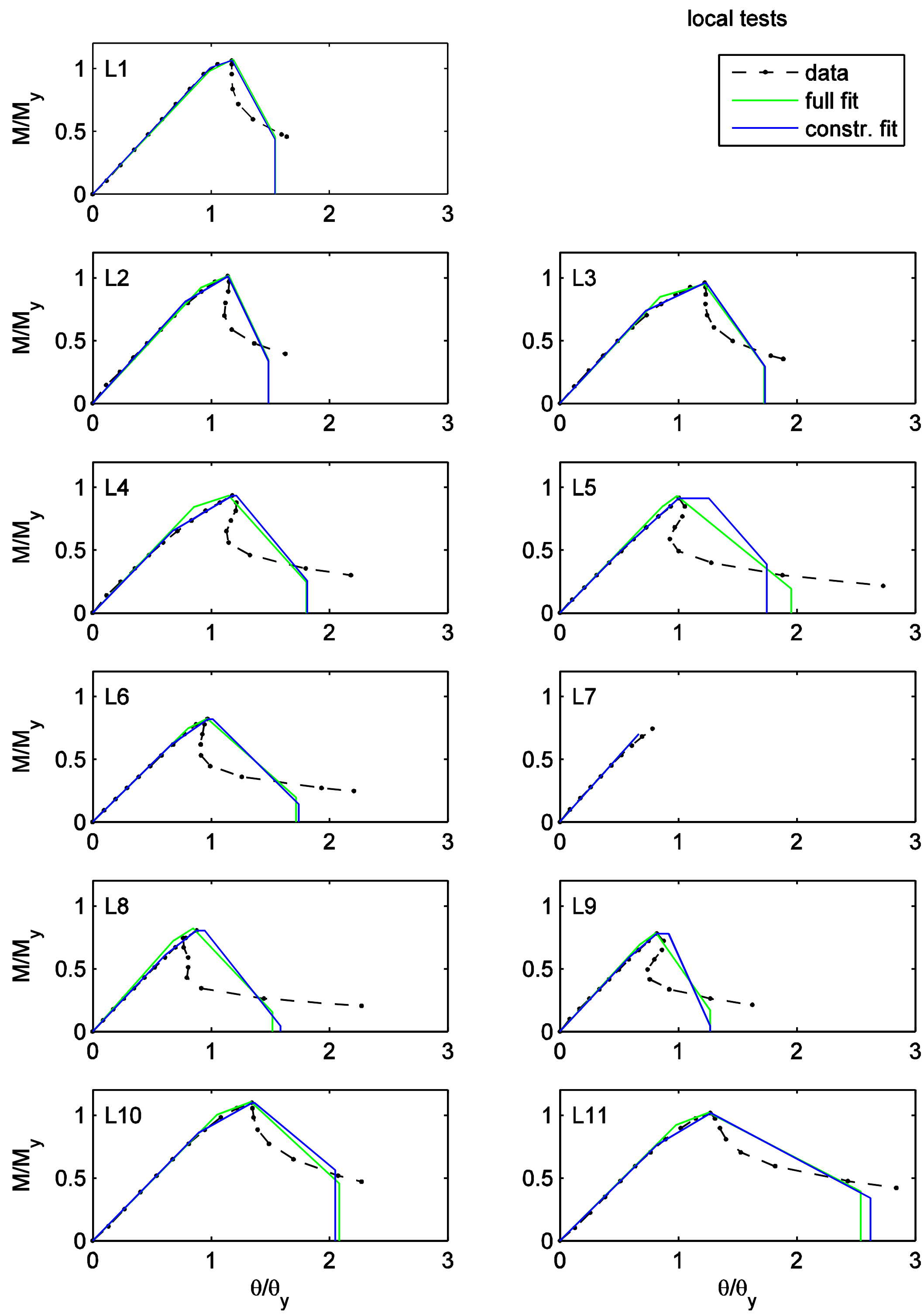
dist. tests, all data
data
full fit
constr. fit
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
M/M_y
θ/θ_y
0
0.5
1
2
3

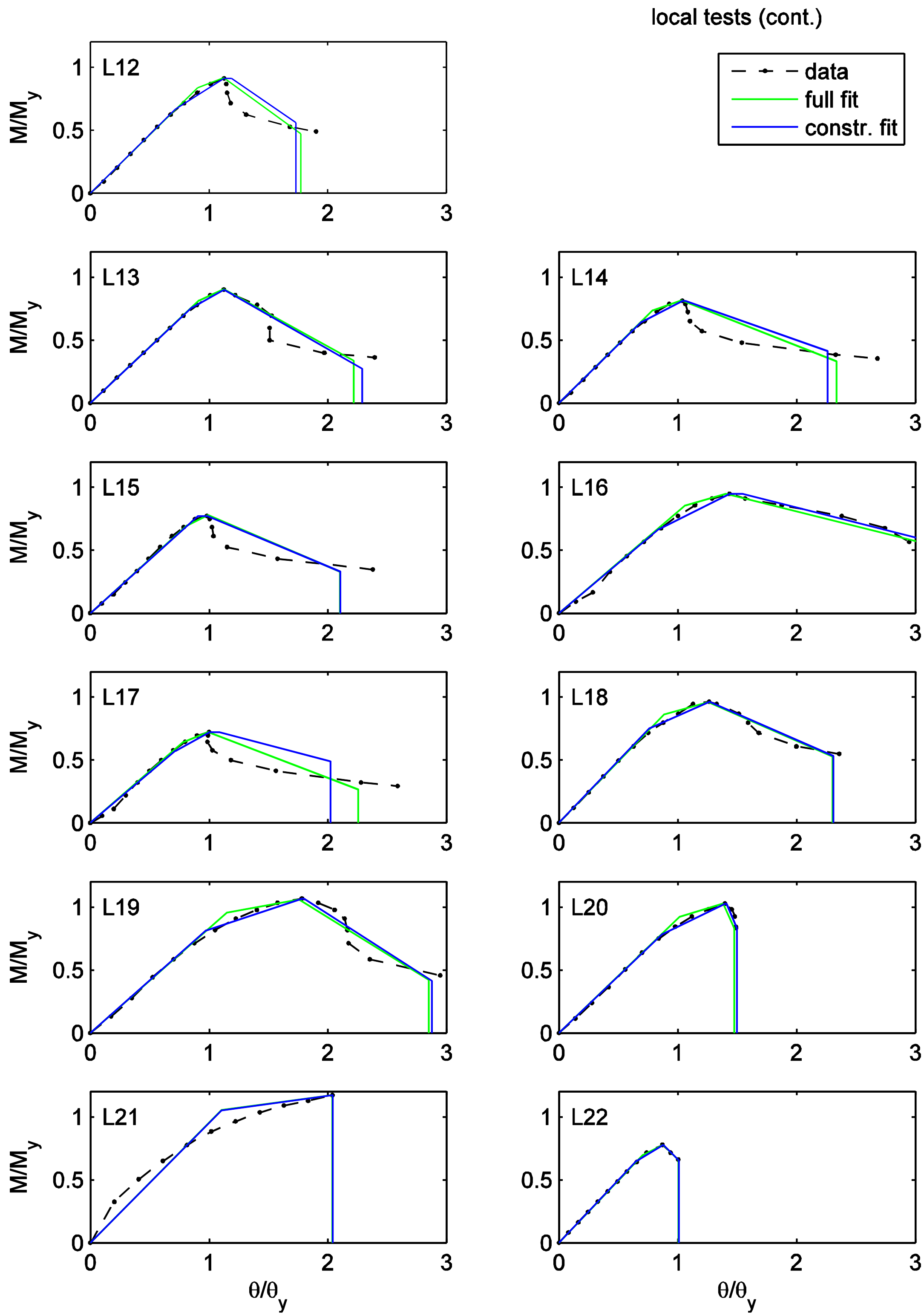

dist. tests, all data (cont.)
data
full fit
constr. fit
D12
D13
D14
D15
D16
D17
D18
D19
D20
D21
D22
M/M_y
θ/θ_y
0
0.5
1
2
3


dist. tests, all data $M_{postpeak}$>50%M_{peak}
data
full fit
constr. fit
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
M/M_y
θ/θ_y

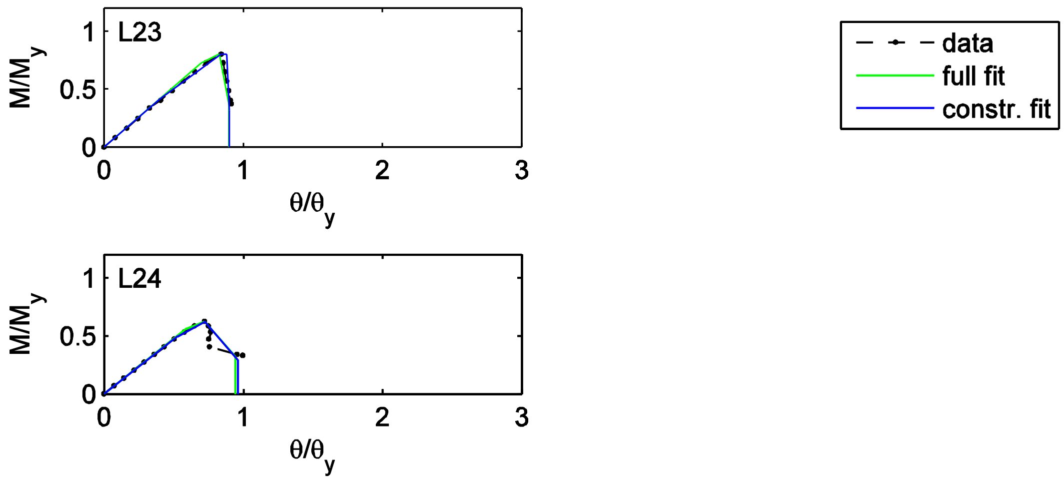

dist. tests, all data M_postpeak>50%M_peak (cont.)
data
full fit
constr. fit
D12
D13
D14
D15
D16
D17
D18
D19
D20
D21
D22
M/M_y
θ/θ_y

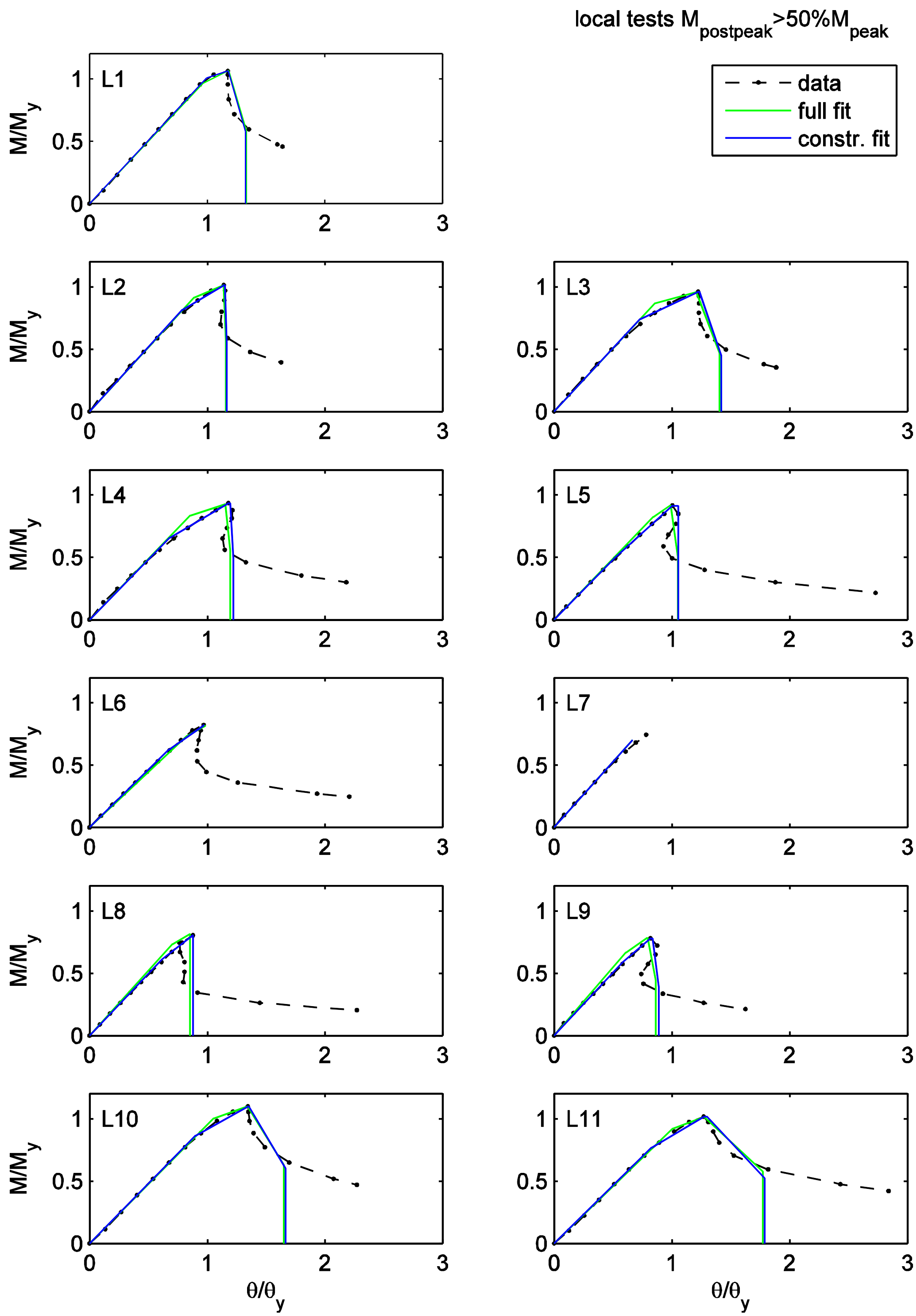
Appendix 4: Model 2 Fit with Yu and Schafer Experiments

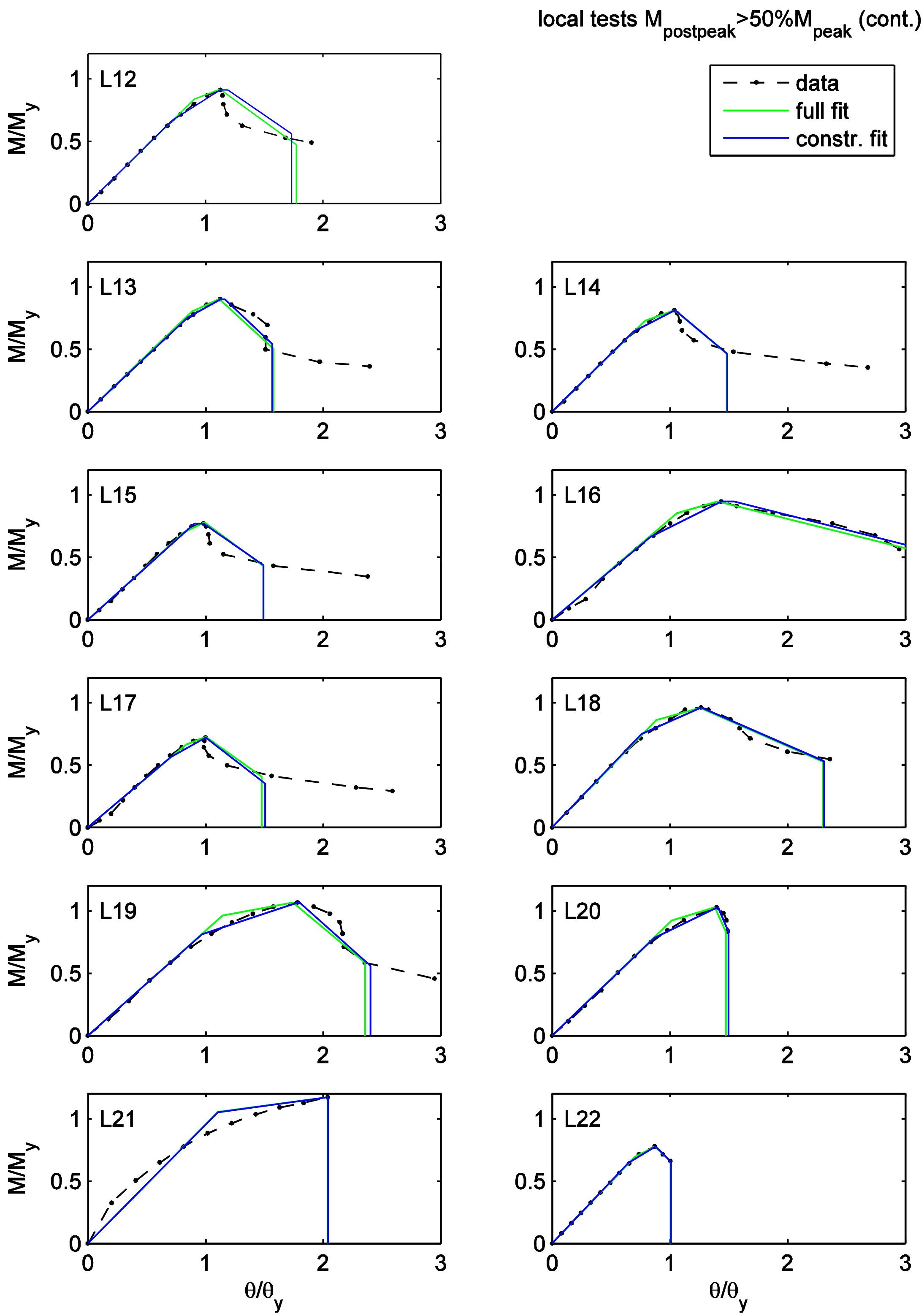



local tests (cont.)
data
full fit
constr. fit
L12
L13
L14
L15
L16
L17
L18
L19
L20
L21
L22
M/My
θ/θy

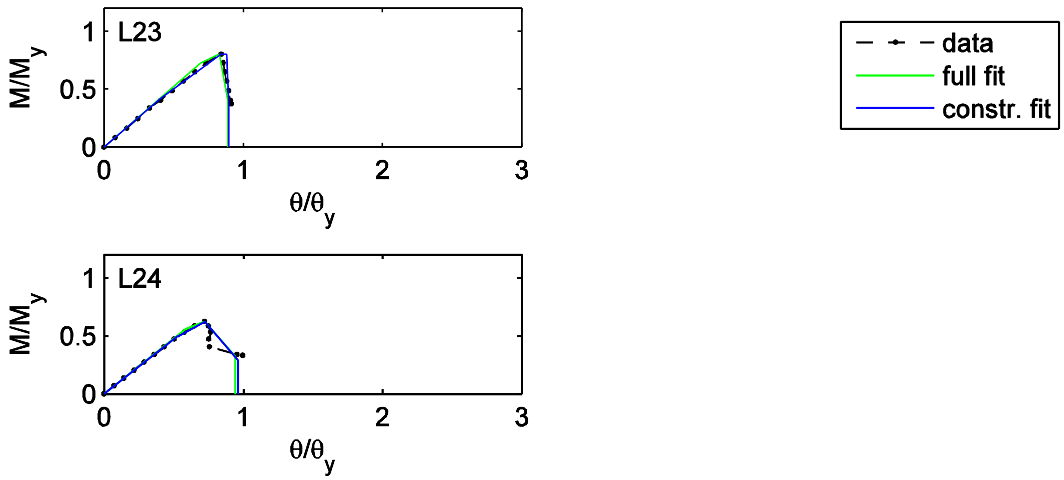
local tests (cont.)

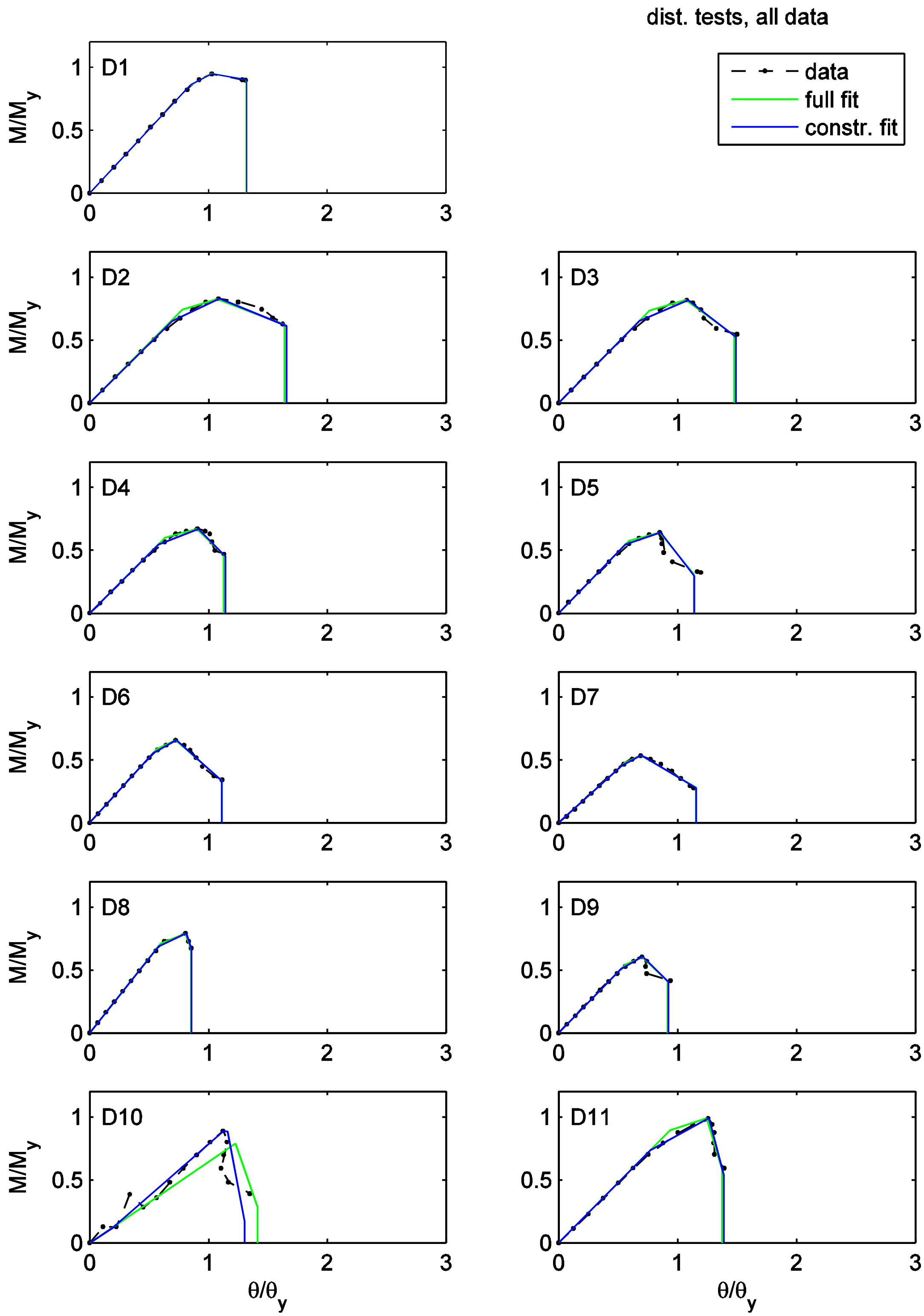


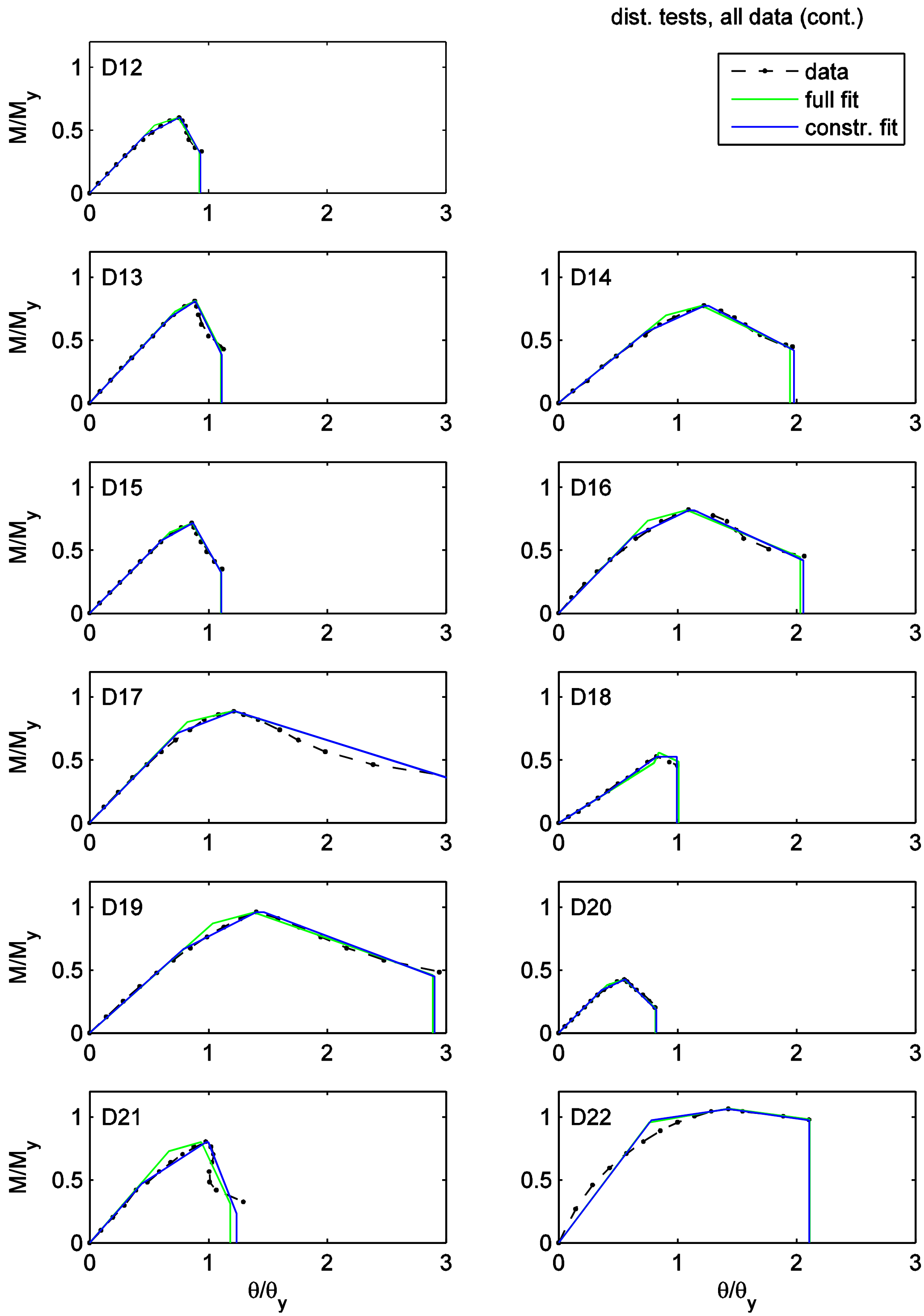

local tests $M_{postpeak}$>50%M_{peak}
data
full fit
constr. fit
L1
L2
L3
L4
L5
L6
L7
L8
L9
L10
L11
M/M_y
θ/θ_y

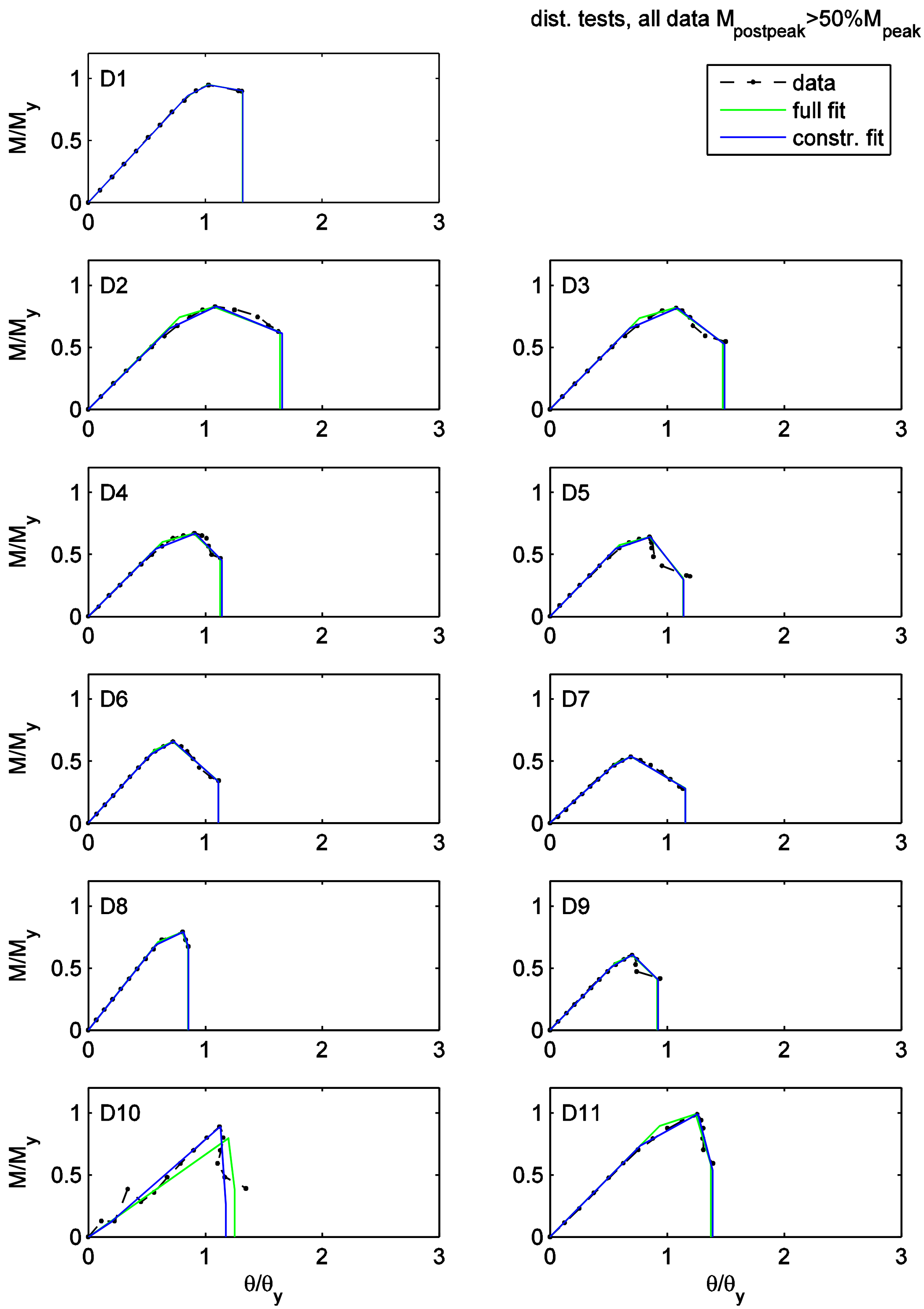

local tests $M_{postpeak}$>50%M_{peak} (cont.)
data
full fit
constr. fit
L12
L13
L14
L15
L16
L17
L18
L19
L20
L21
L22
M/M_y
θ/$θ_y$
0
0.5
1
0
1
2
3

local tests $M_{postpeak}$>50%M_{peak} (cont.)

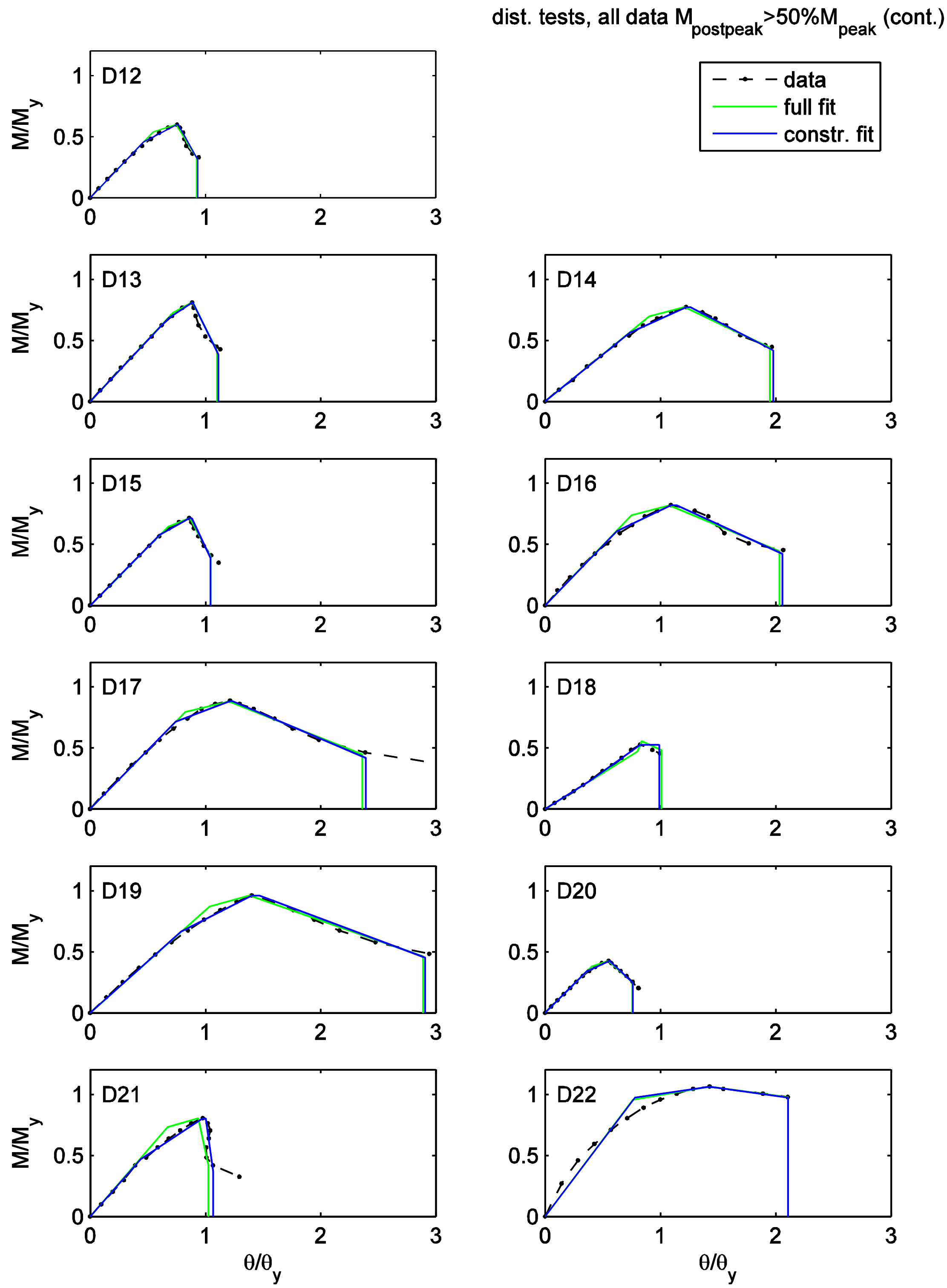



dist. tests, all data
data
full fit
constr. fit
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
M/M$_y$
θ/θ$_y$

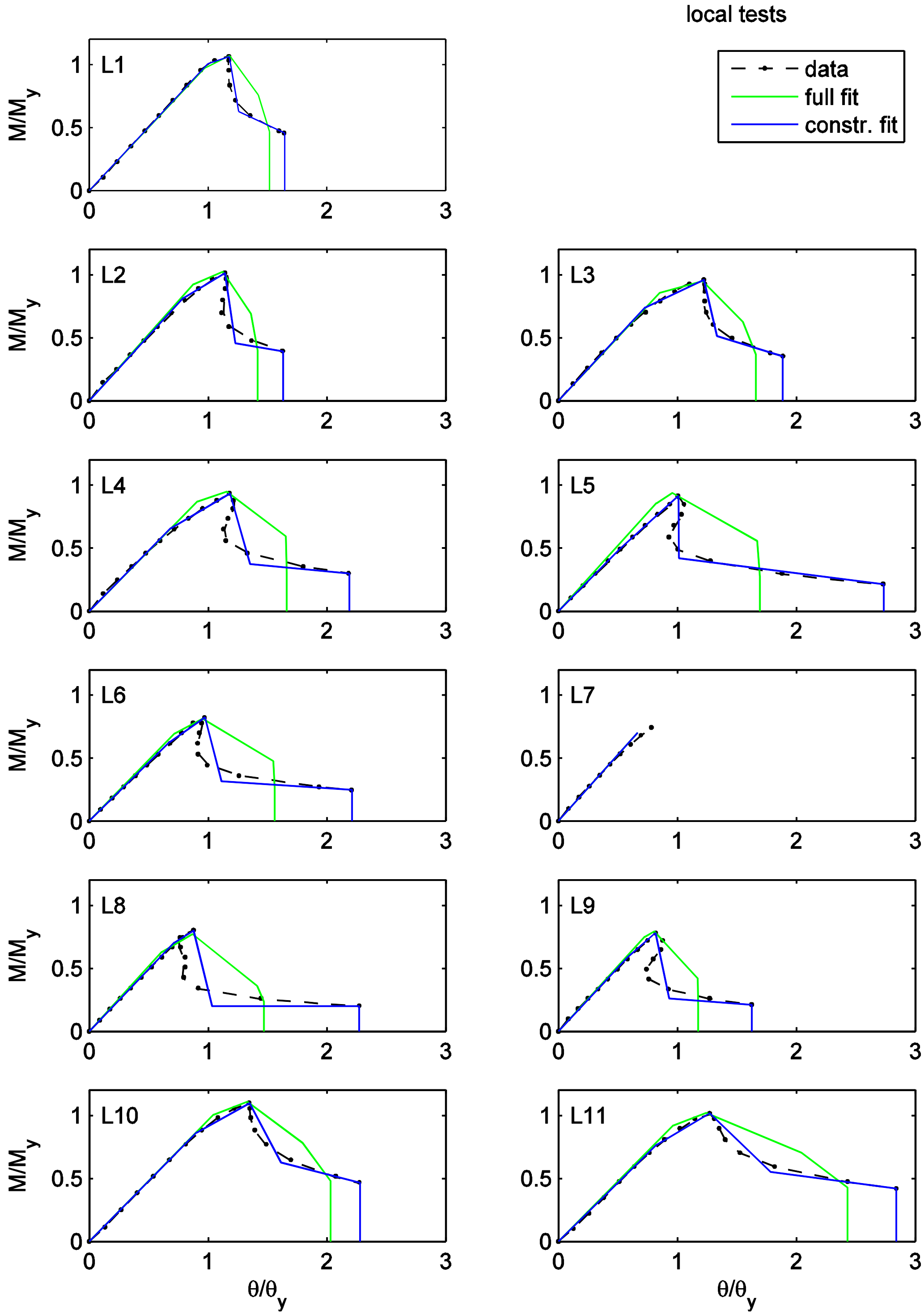

dist. tests, all data (cont.)
data
full fit
constr. fit
D12
D13
D14
D15
D16
D17
D18
D19
D20
D21
D22
M/M_y
θ/θ_y

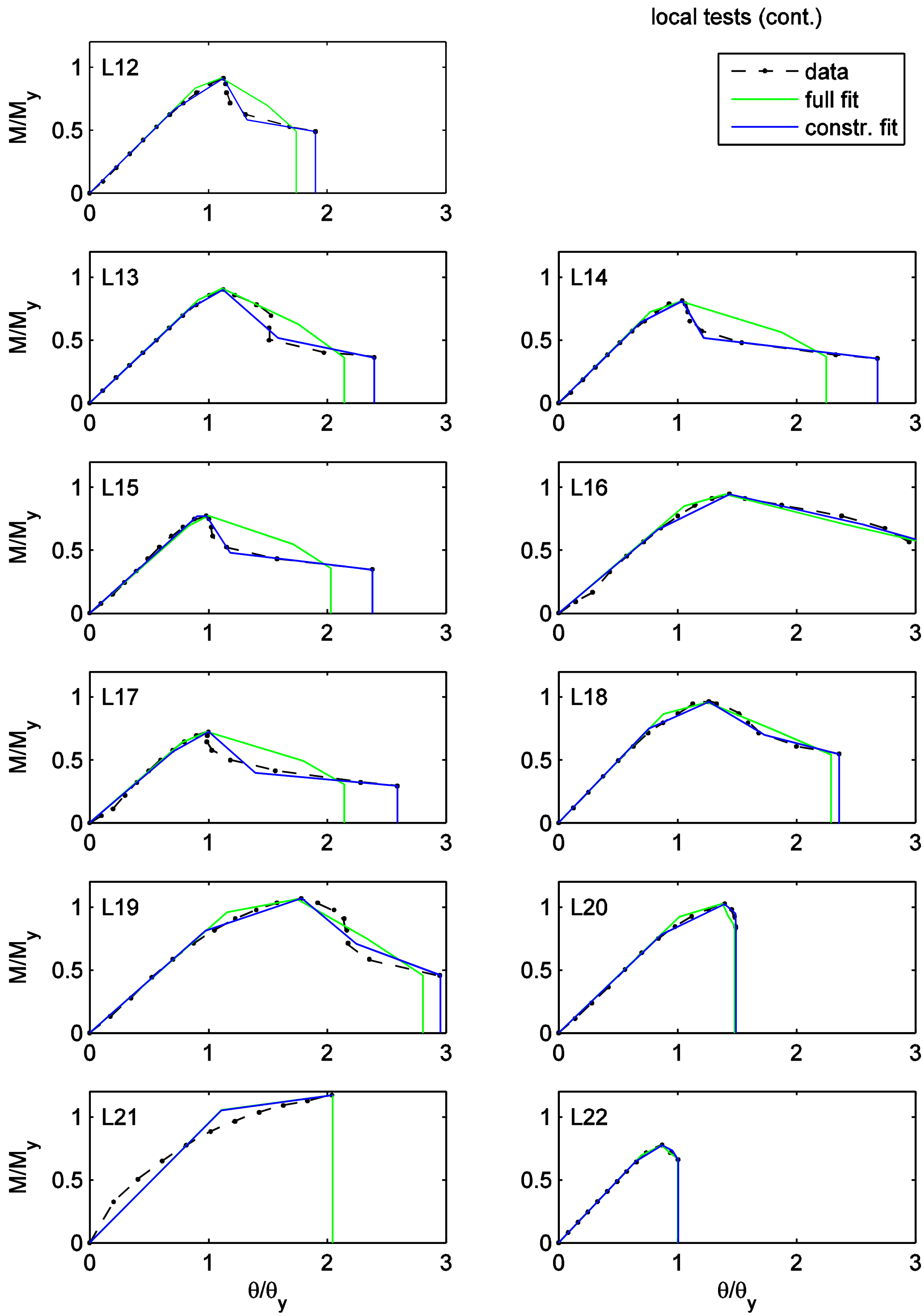


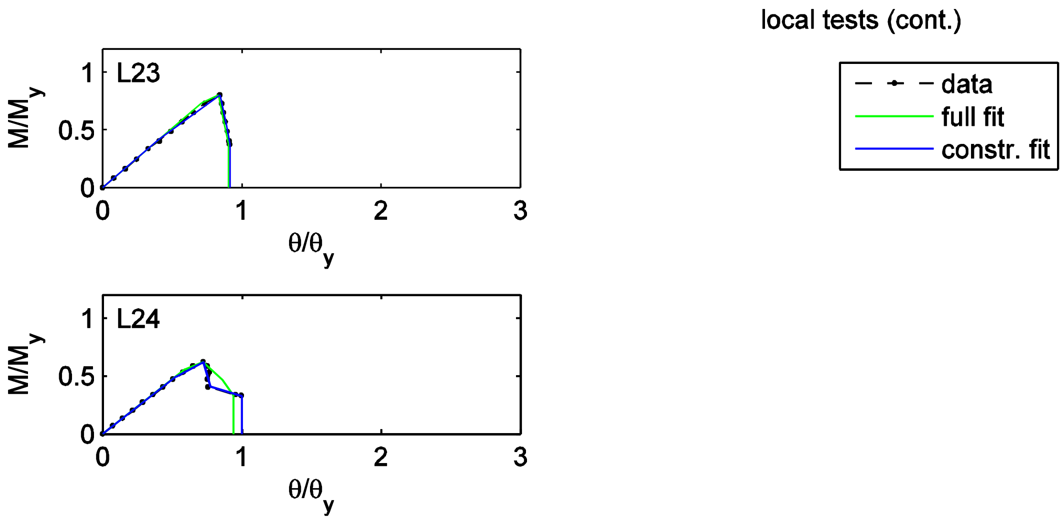
dist. tests, all data M_postpeak>50%M_peak
data
full fit
constr. fit
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
M/M_y
θ/θ_y
0
0.5
1
2
3

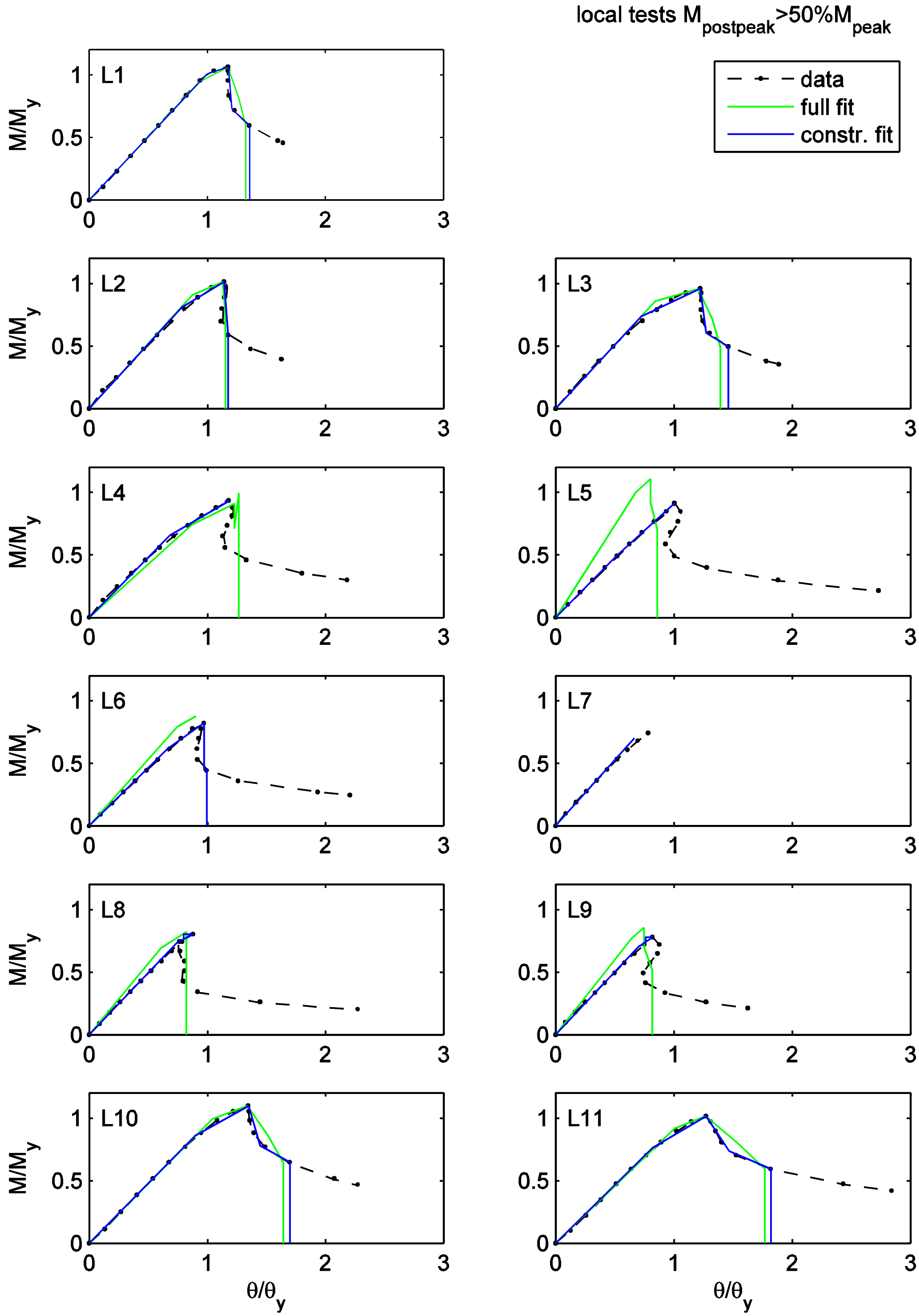

dist. tests, all data $M_{postpeak}$>50%M_{peak} (cont.)
data
full fit
constr. fit
D12
D13
D14
D15
D16
D17
D18
D19
D20
D21
D22
M/M_y
θ/$θ_y$

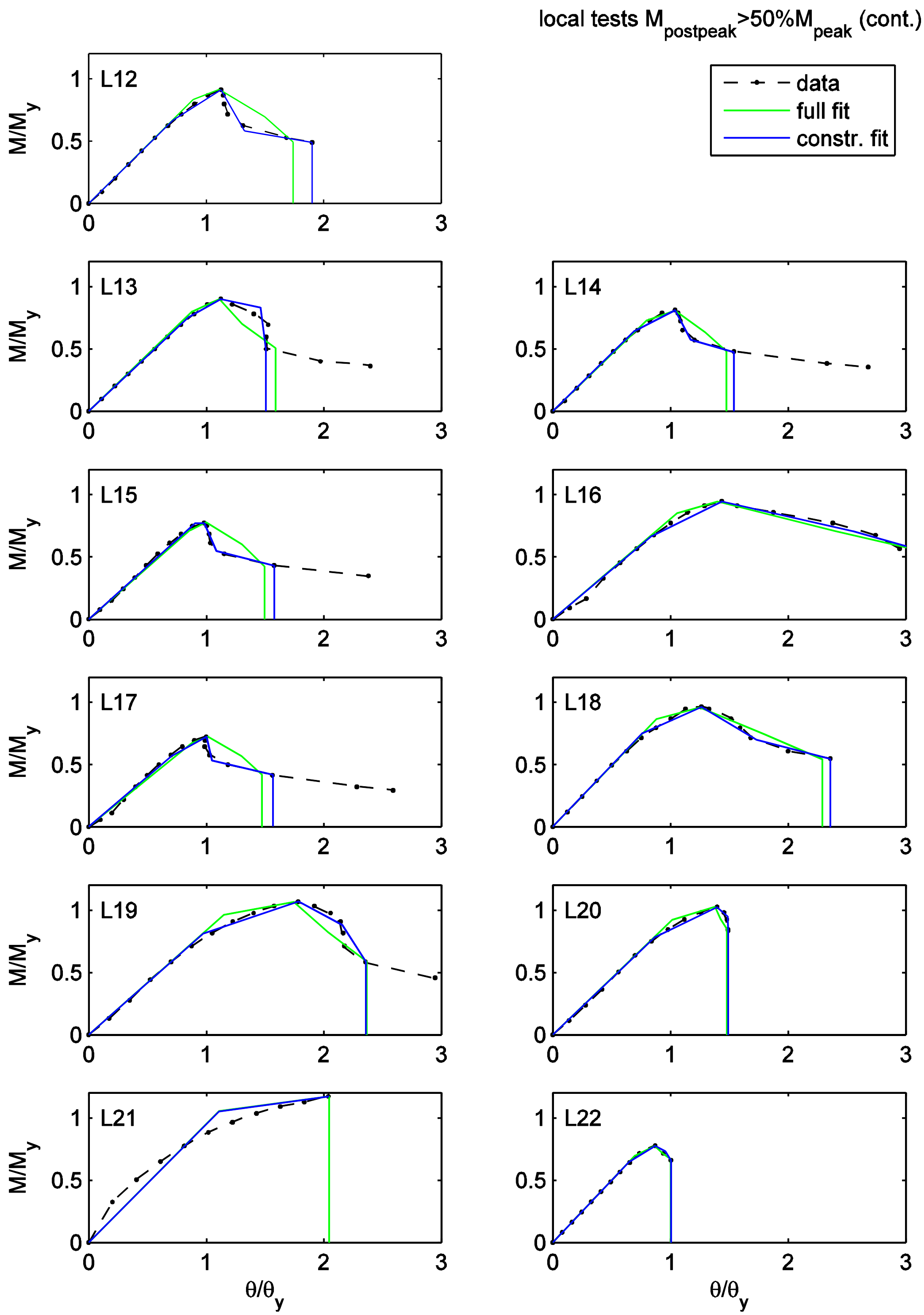
Appendix 5: Model 1a Fit with Yu and Schafer Experiments



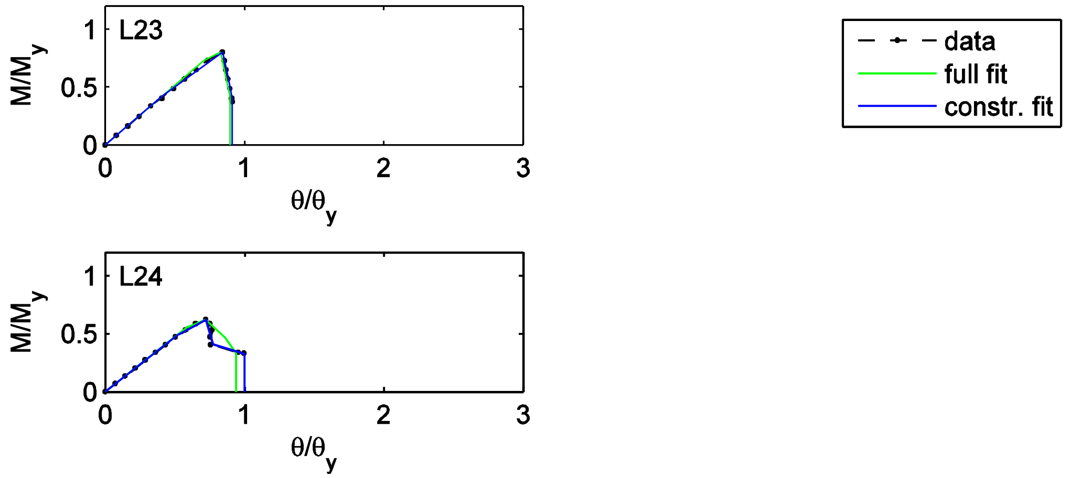

local tests (cont.)
data
full fit
constr. fit
L12
L13
L14
L15
L16
L17
L18
L19
L20
L21
L22
M/M_y
θ/θ_y

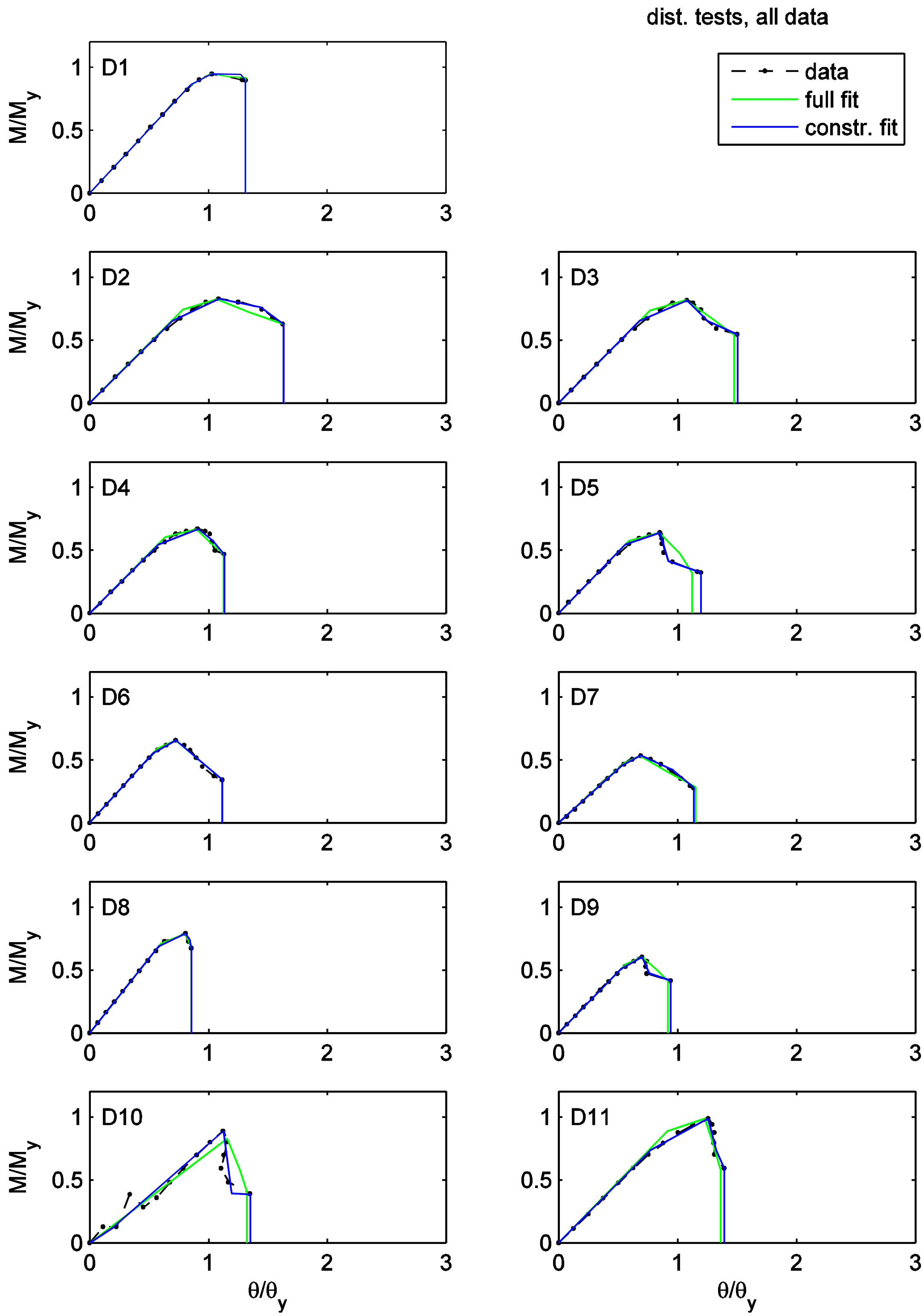

local tests (cont.)
data
full fit
constr. fit
L23
L24
M/My
θ/θy
0
0.5
1
0
1
2
3

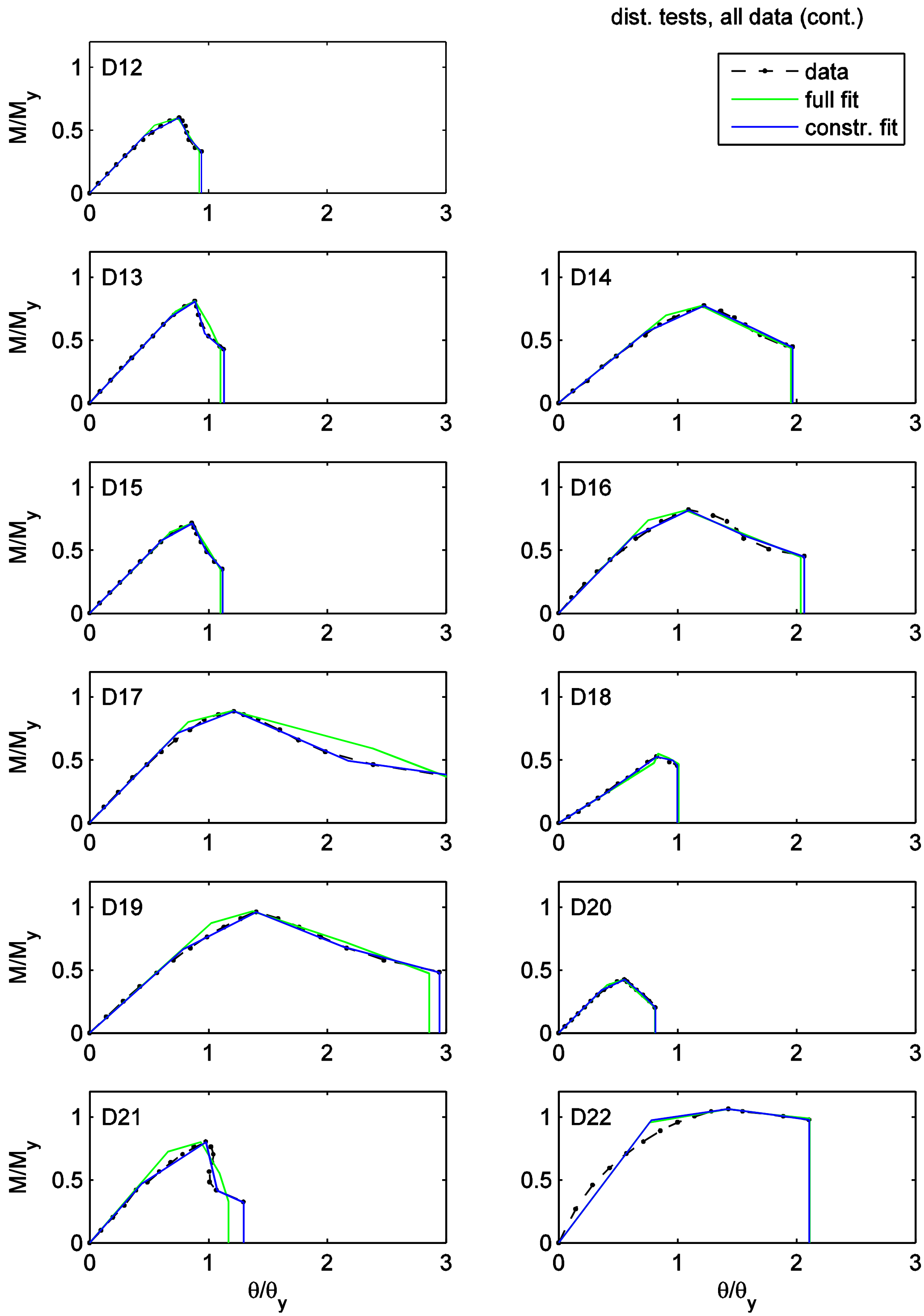

local tests $M_{postpeak}$>50%M_{peak}
data
full fit
constr. fit
L1
L2
L3
L4
L5
L6
L7
L8
L9
L10
L11
M/M_y
θ/θ_y

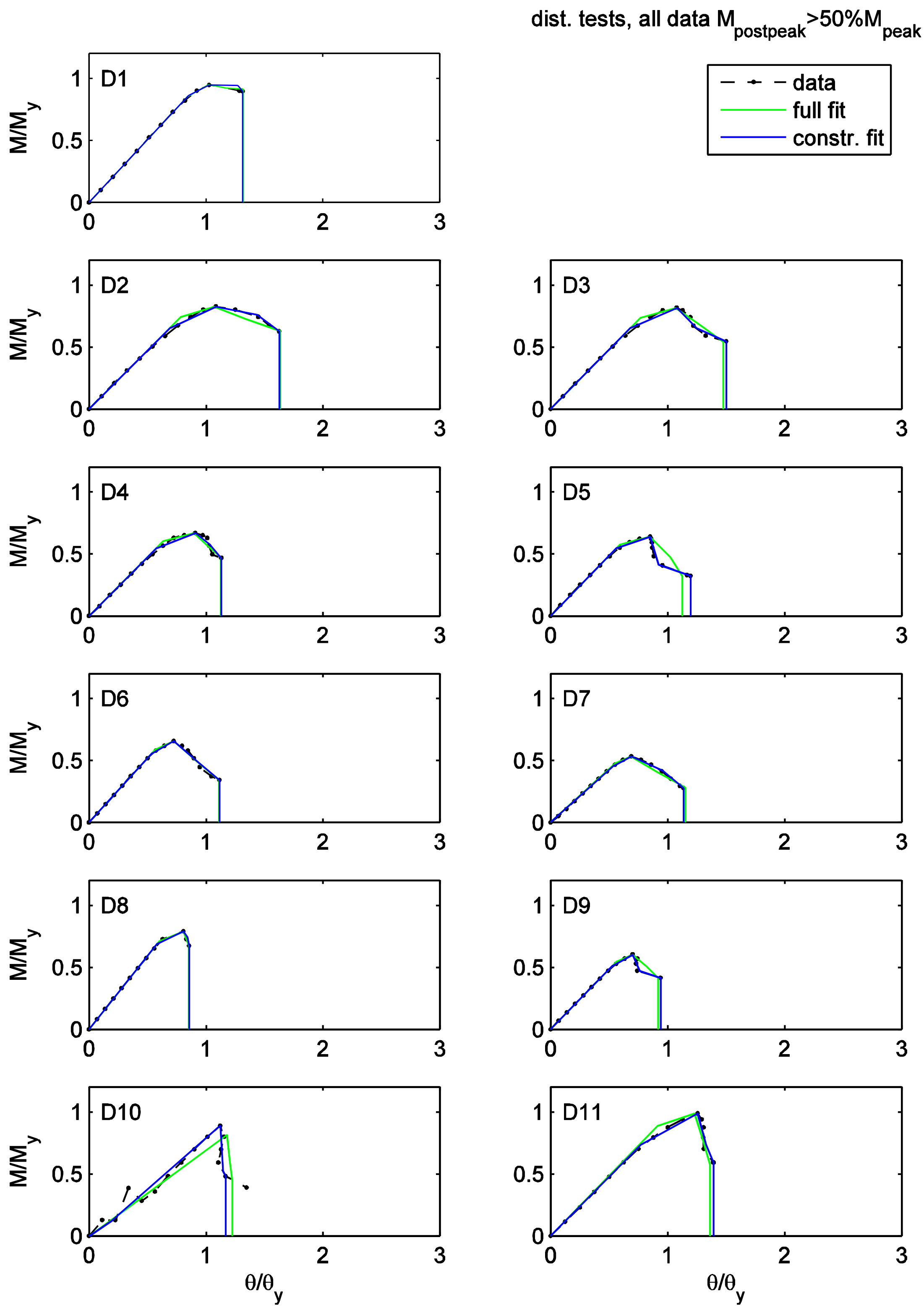

local tests $M_{postpeak}$>50%M_{peak} (cont.)
data
full fit
constr. fit
L12
L13
L14
L15
L16
L17
L18
L19
L20
L21
L22
M/M_y
θ/$θ_y$

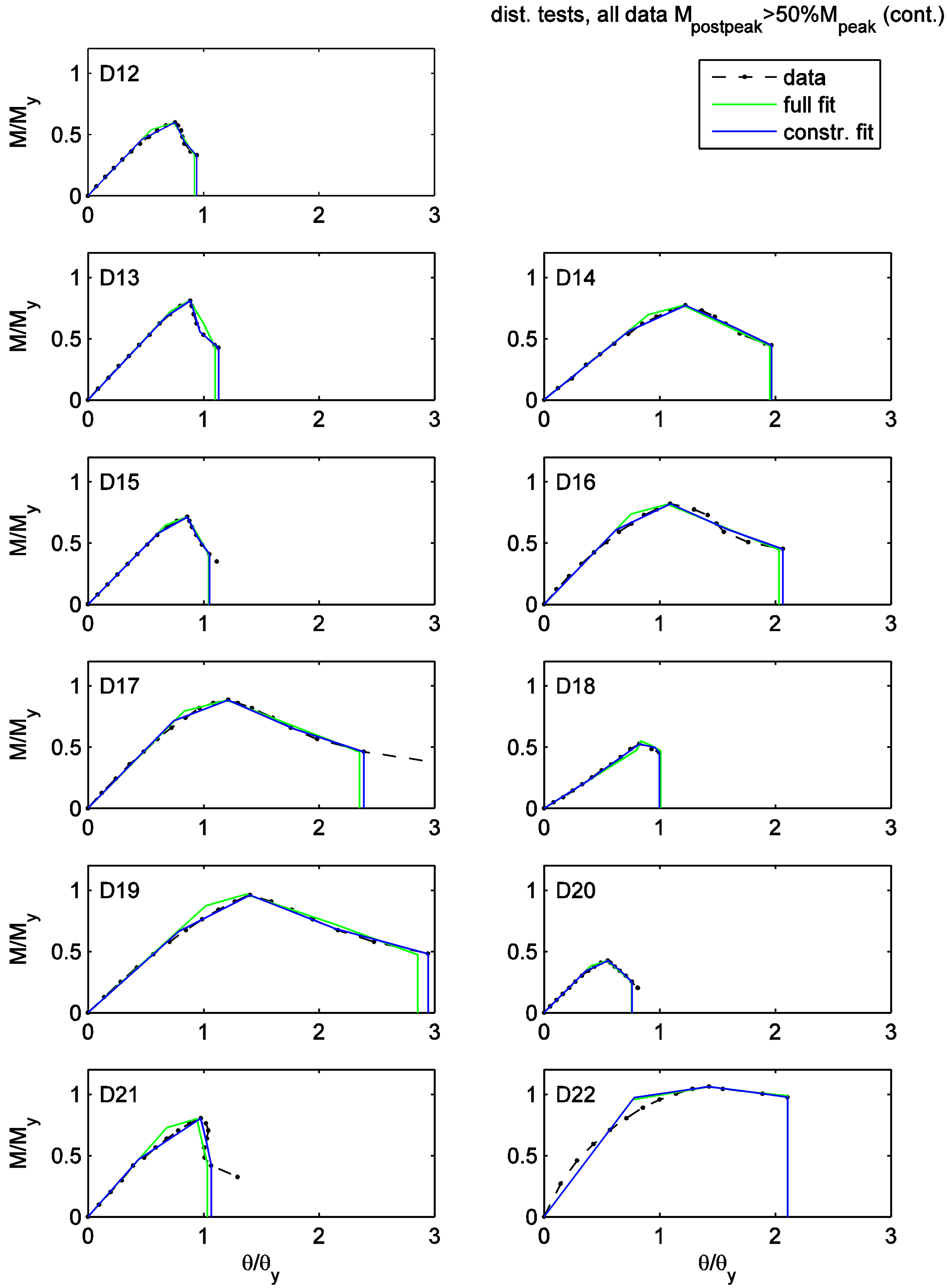
local tests $M_{postpeak}$>50%M_{peak} (cont.)



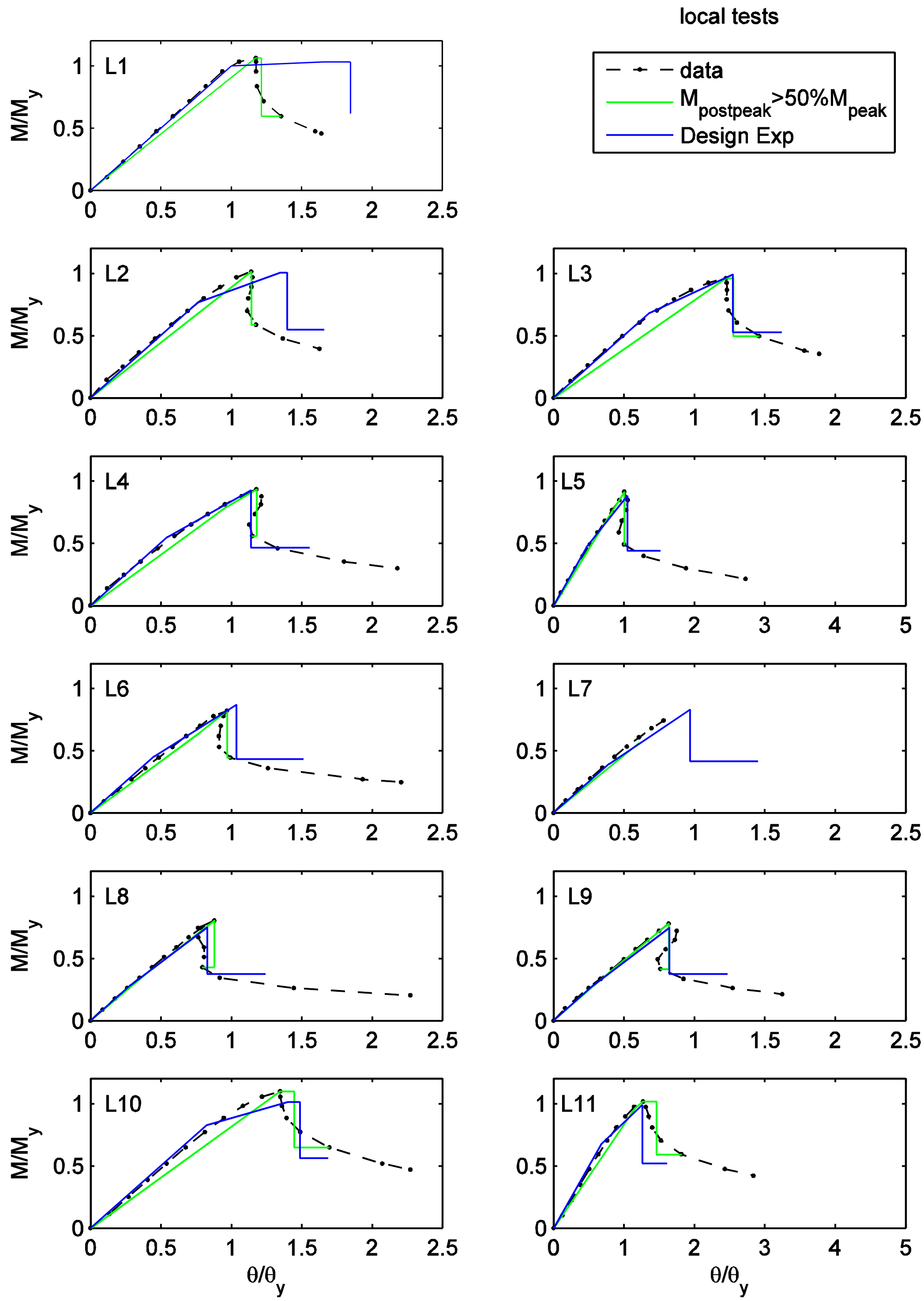

dist. tests, all data
data
full fit
constr. fit
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
M/M_y
θ/θ_y

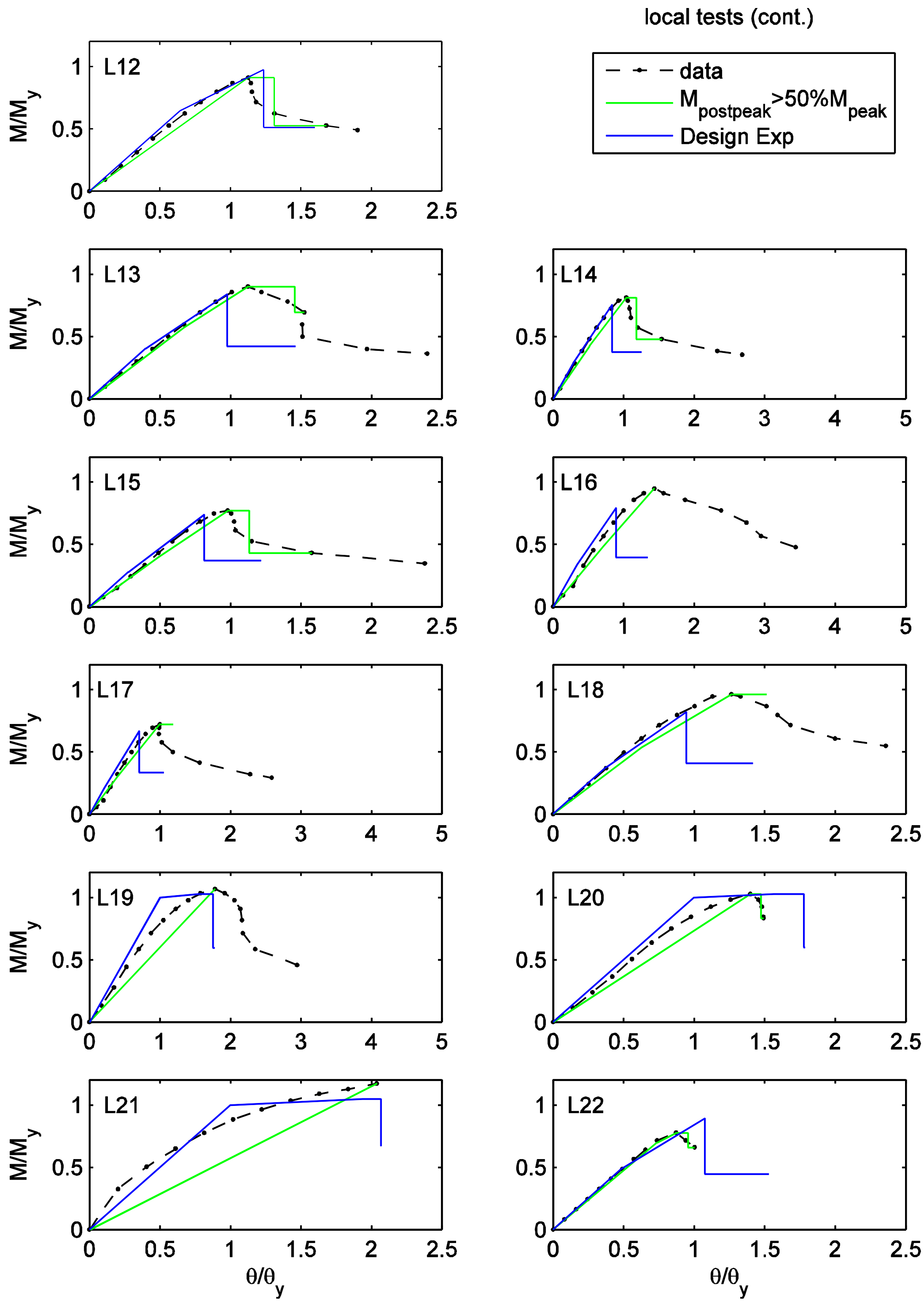

dist. tests, all data (cont.)
data
full fit
constr. fit
D12
D13
D14
D15
D16
D17
D18
D19
D20
D21
D22
M/My
θ/θy

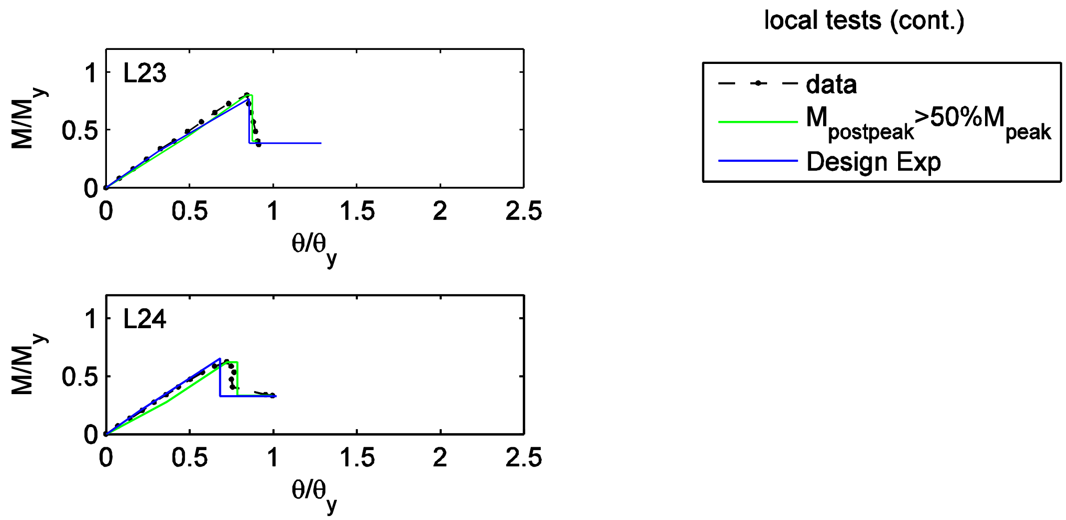

dist. tests, all data $M_{postpeak}$>50%M_{peak}
data
full fit
constr. fit
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
M/M_y
θ/$θ_y$
0
0.5
1
2
3

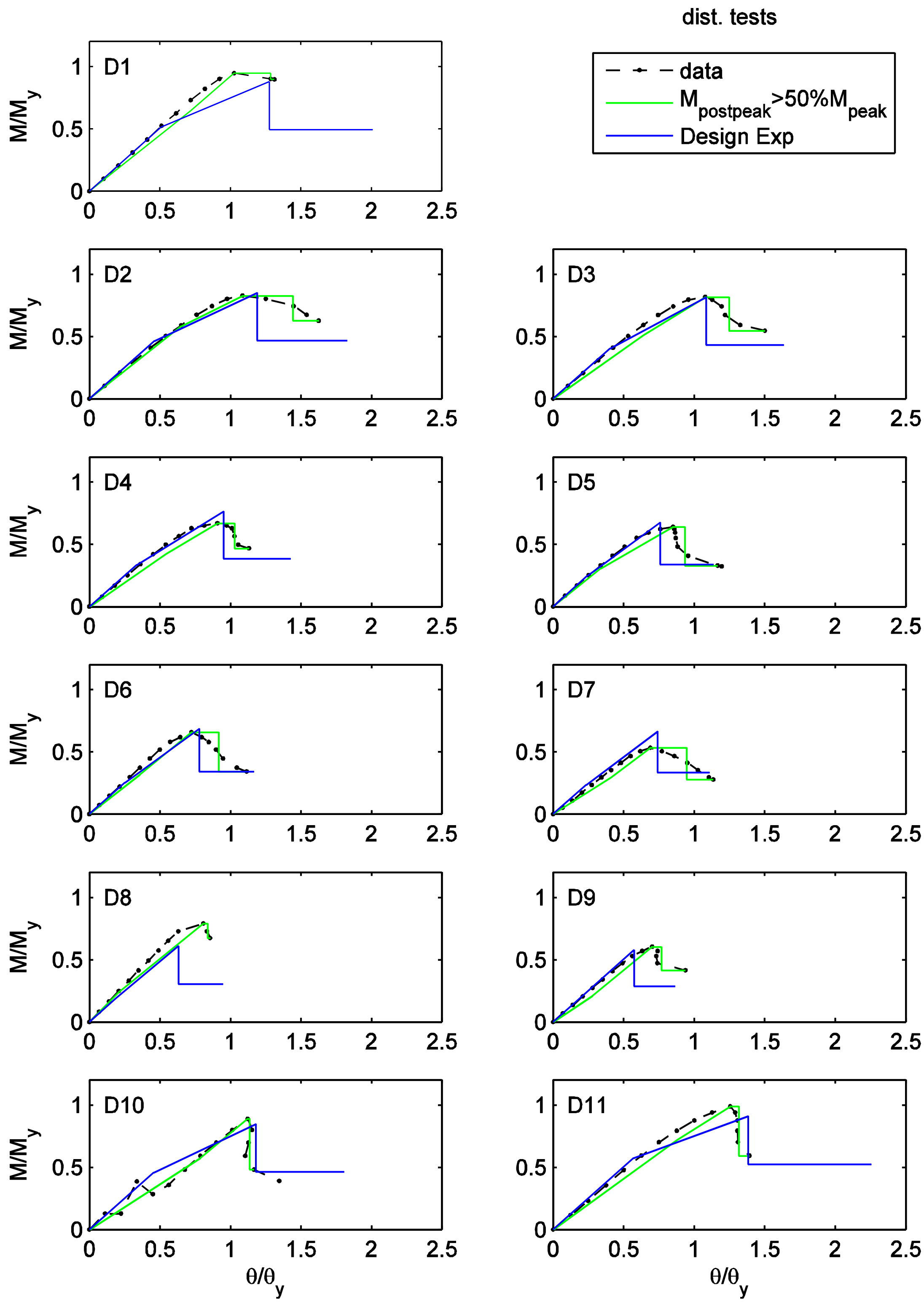

dist. tests, all data M_postpeak>50%M_peak (cont.)
data
full fit
constr. fit
D12
D13
D14
D15
D16
D17
D18
D19
D20
D21
D22
M/M_y
θ/θ_y

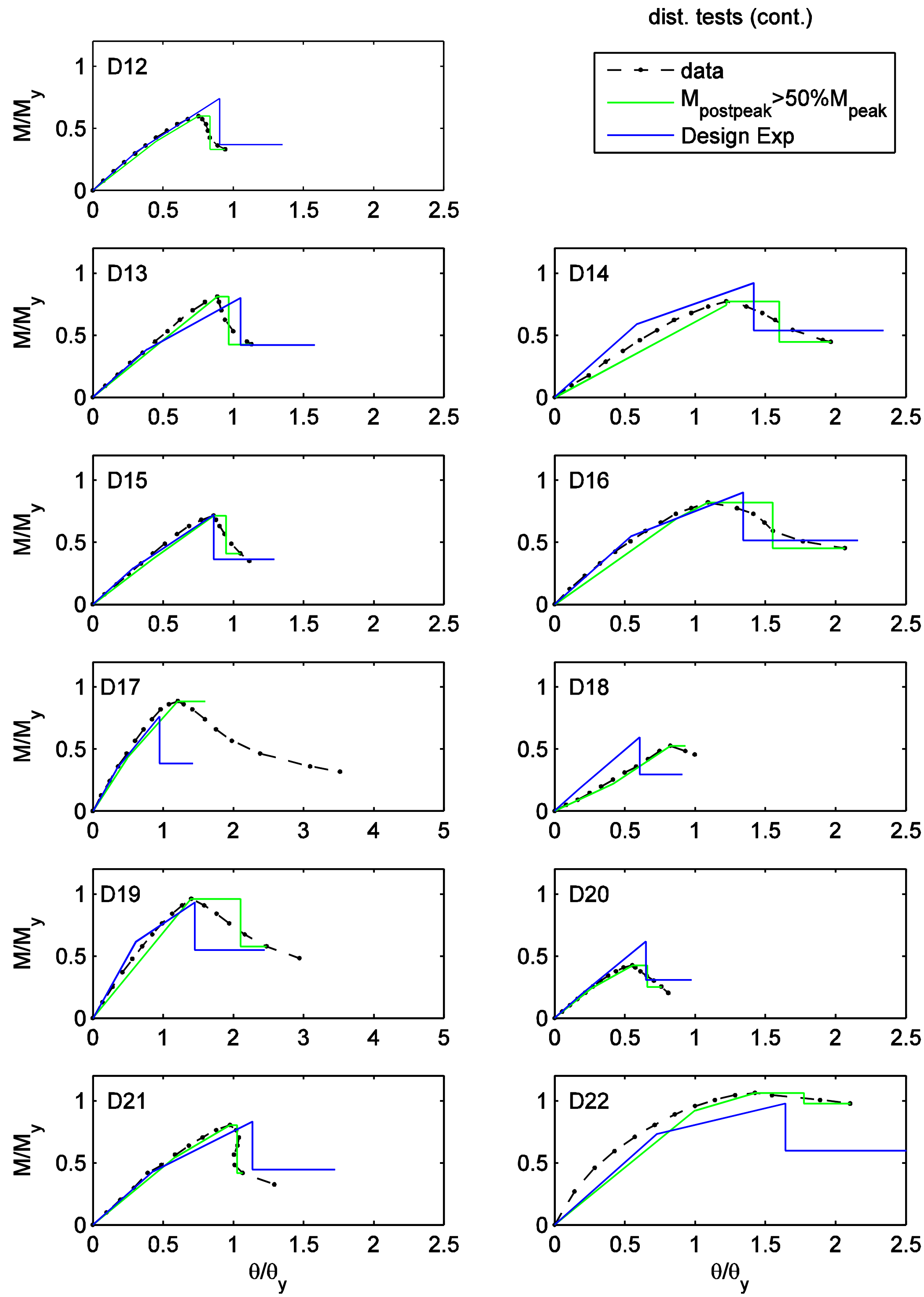
Appendix 6: Design Expressions curves for all available data (tests and FE models)



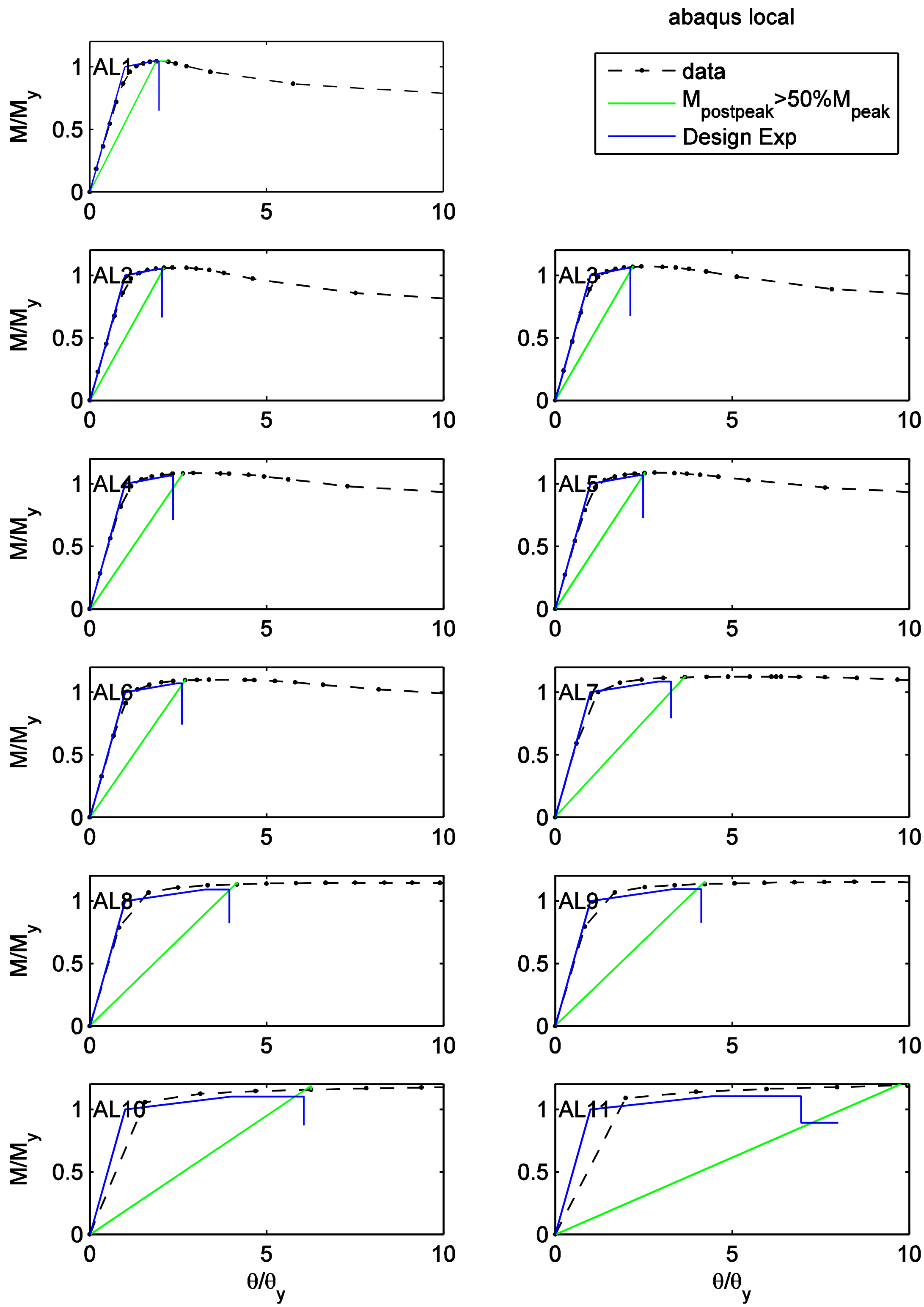

local tests (cont.)
data
M_postpeak>50%M_peak
Design Exp
L12
L13
L14
L15
L16
L17
L18
L19
L20
L21
L22
M/M_y
θ/θ_y

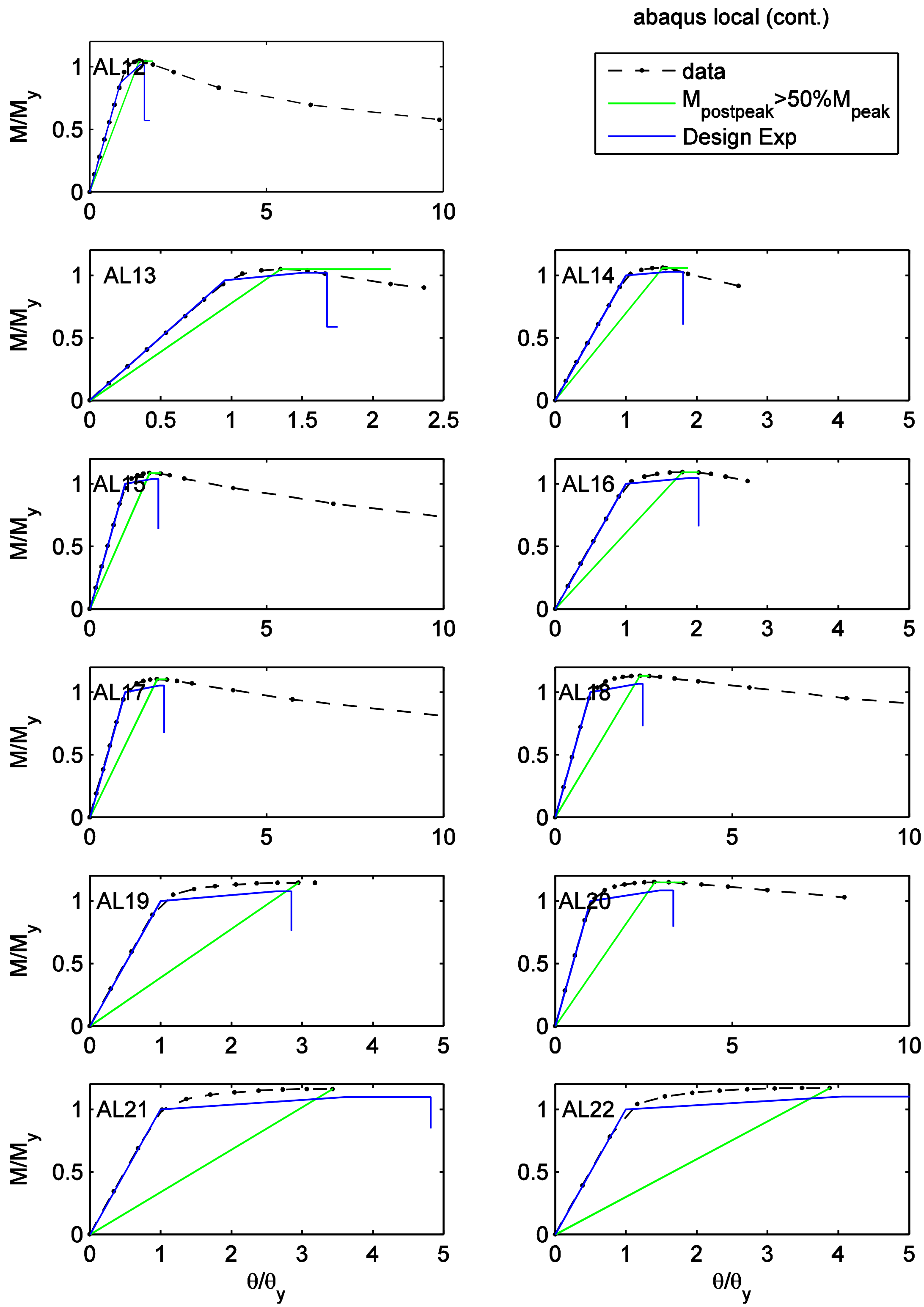

local tests (cont.)
L23
L24
M/M_y
θ/θ_y
0
0.5
1
1.5
2
2.5
data
M_postpeak>50%M_peak
Design Exp


dist. tests
data
$M_{postpeak}$>50%M_{peak}
Design Exp
D1
D2
D3
D4
D5
D6
D7
D8
D9
D10
D11
M/M_y
θ/$θ_y$

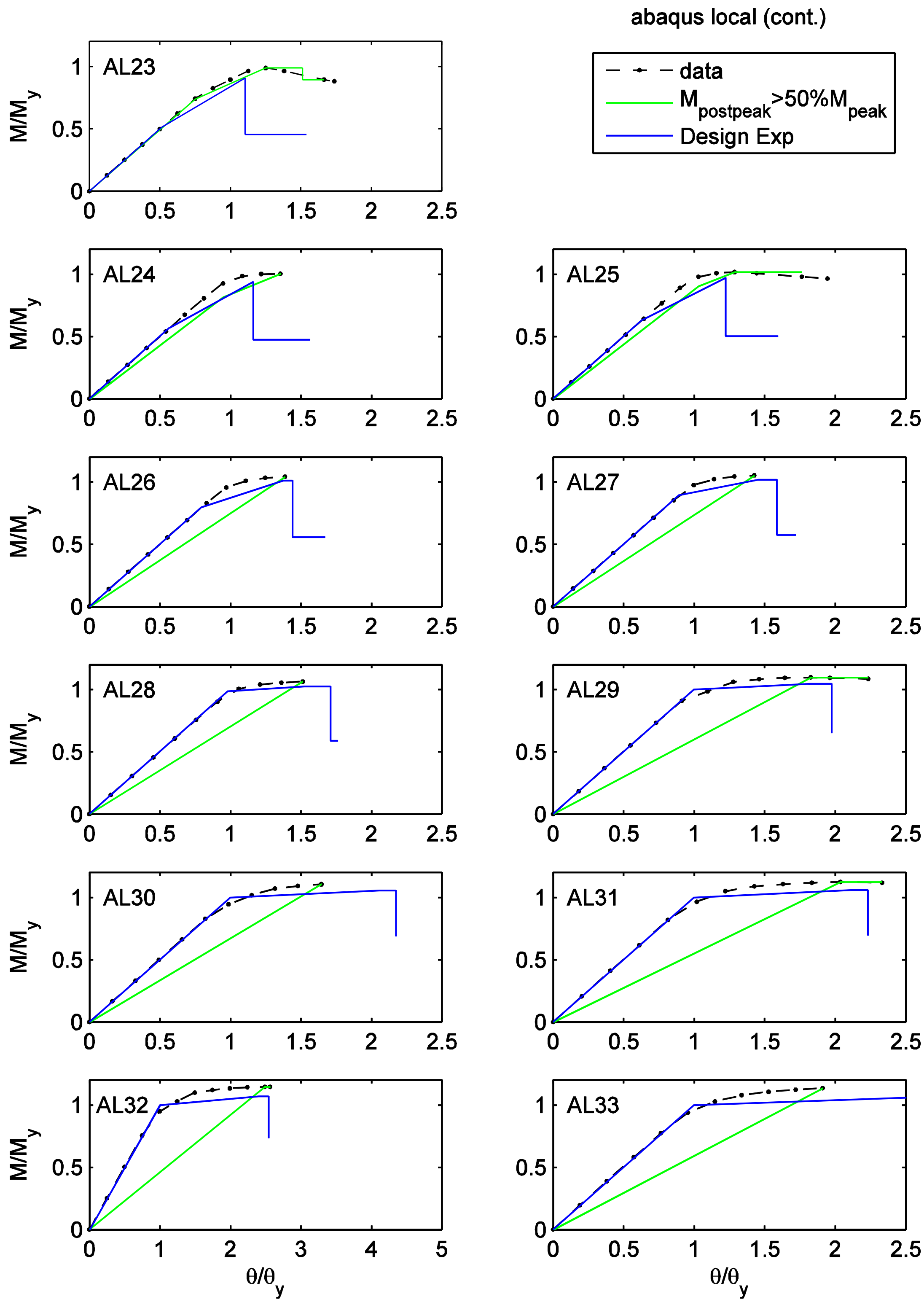


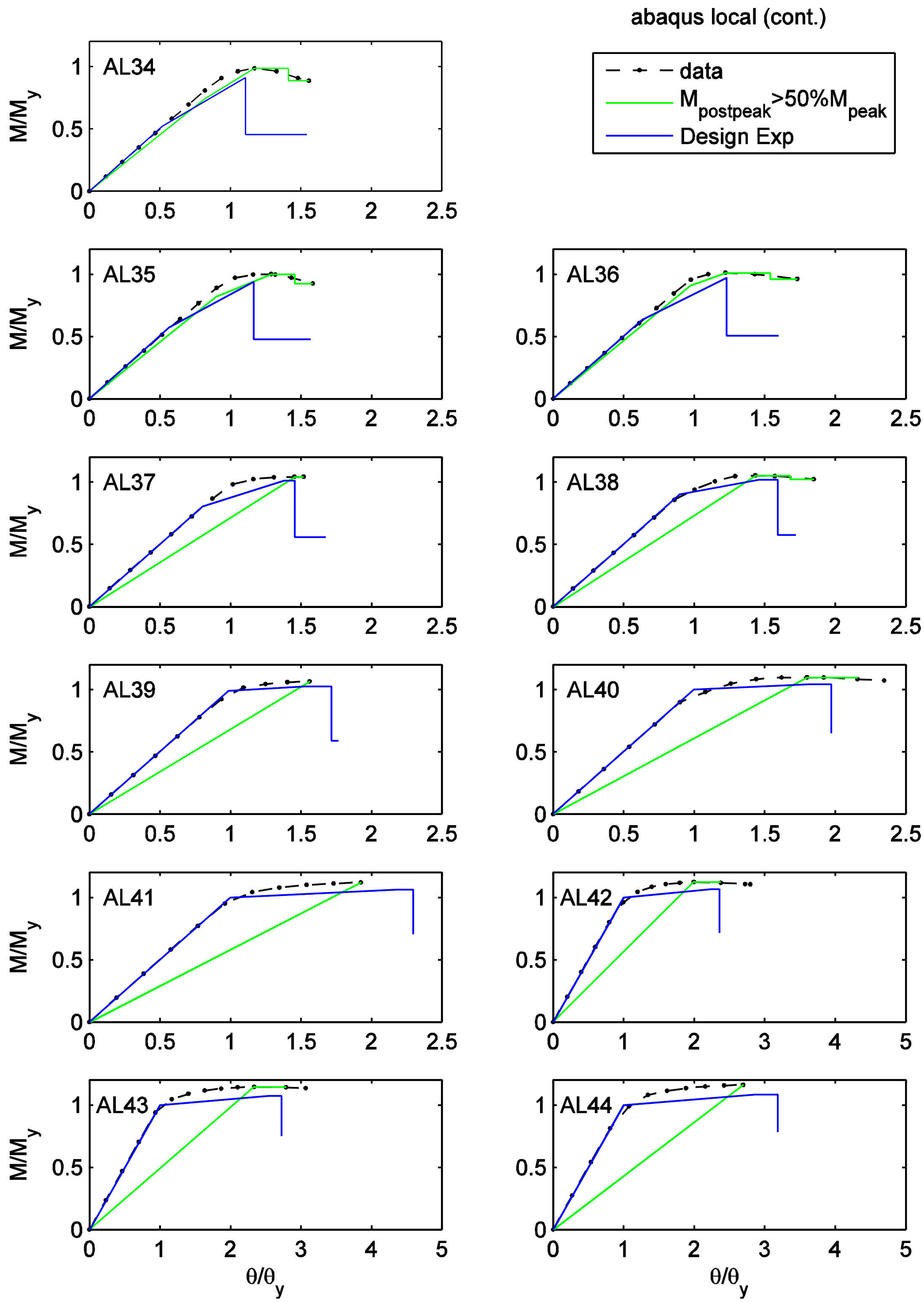
dist. tests (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
D12
D13
D14
D15
D16
D17
D18
D19
D20
D21
D22
M/My
θ/θy


abaqus local
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AL1
AL2
AL3
AL4
AL5
AL6
AL7
AL8
AL9
AL10
AL11
M/M_y
θ/$θ_y$
0
0.5
1
5
10

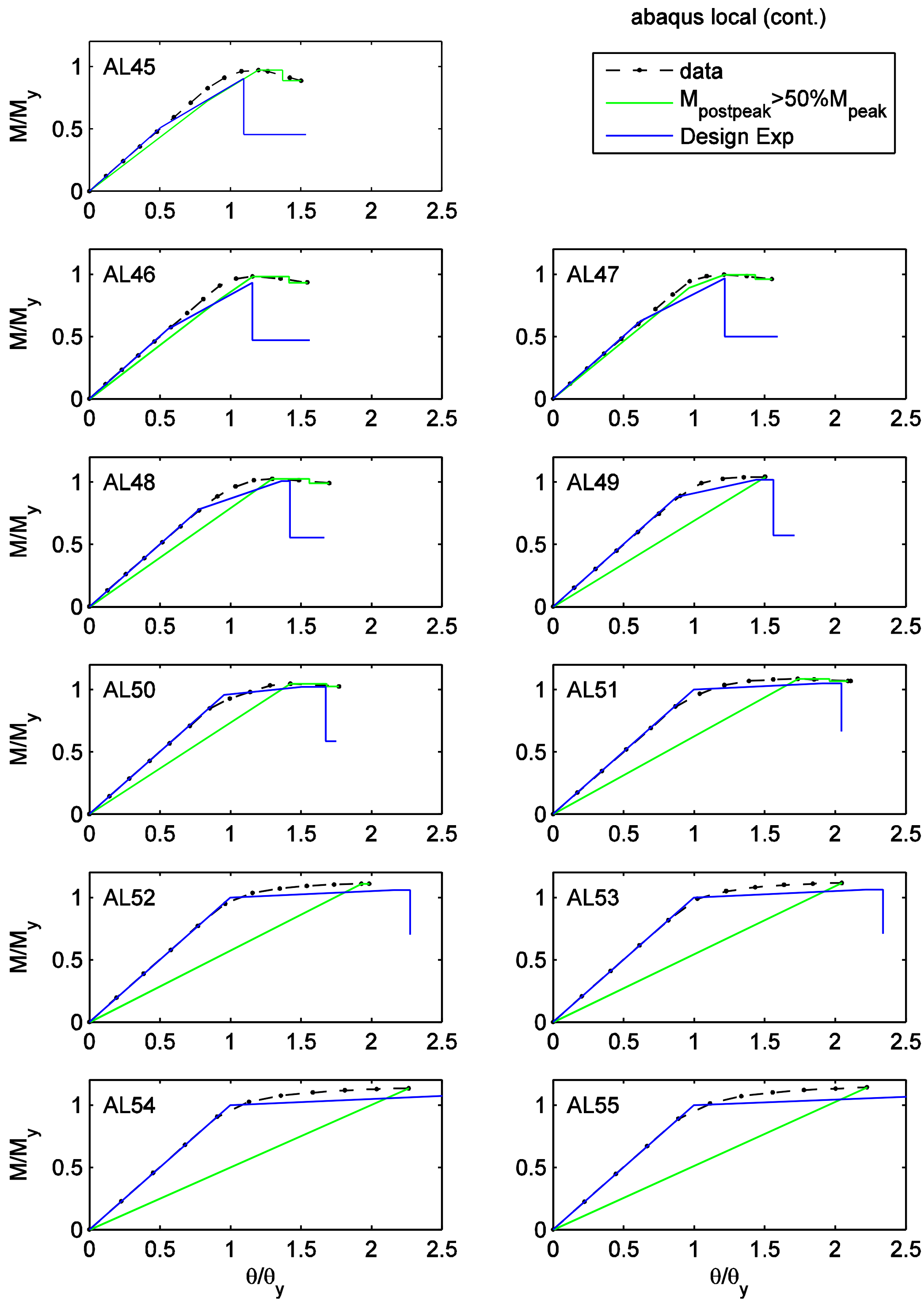


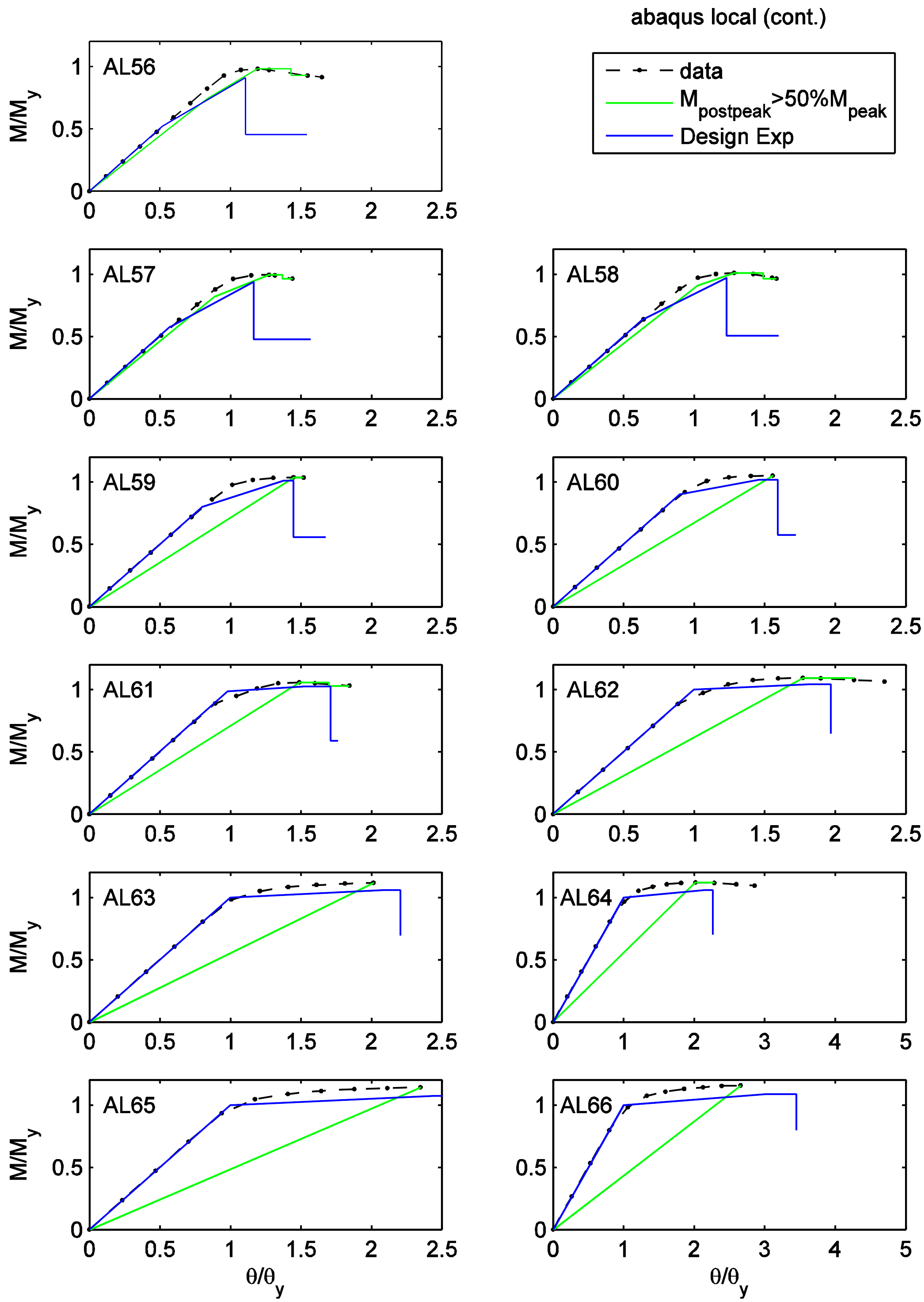
abaqus local (cont.)
data
M_postpeak>50%M_peak
Design Exp
AL12
AL13
AL14
AL15
AL16
AL17
AL18
AL19
AL20
AL21
AL22
M/M_y
θ/θ_y

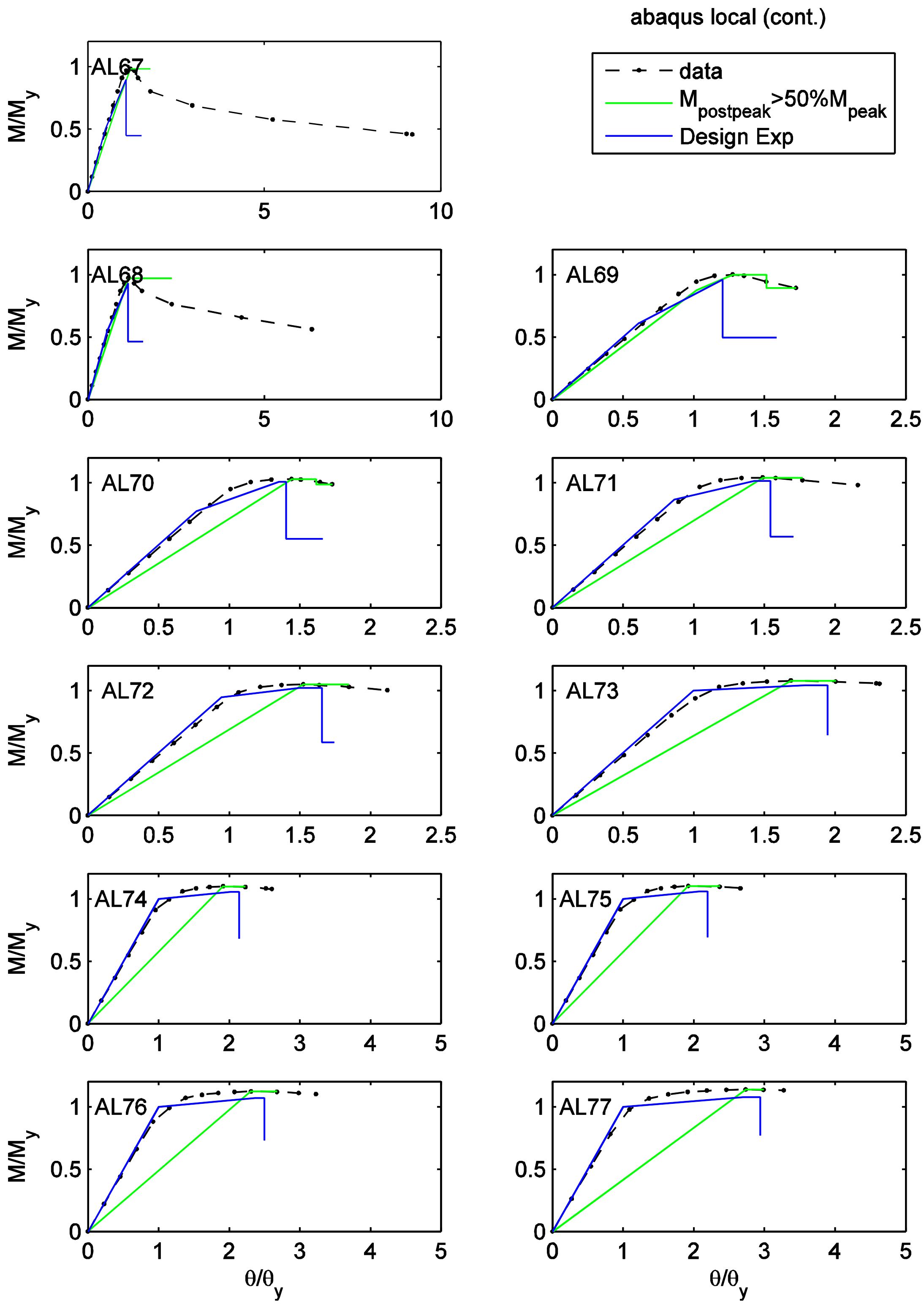

abaqus local (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AL23
AL24
AL25
AL26
AL27
AL28
AL29
AL30
AL31
AL32
AL33
M/M_y
θ/$θ_y$

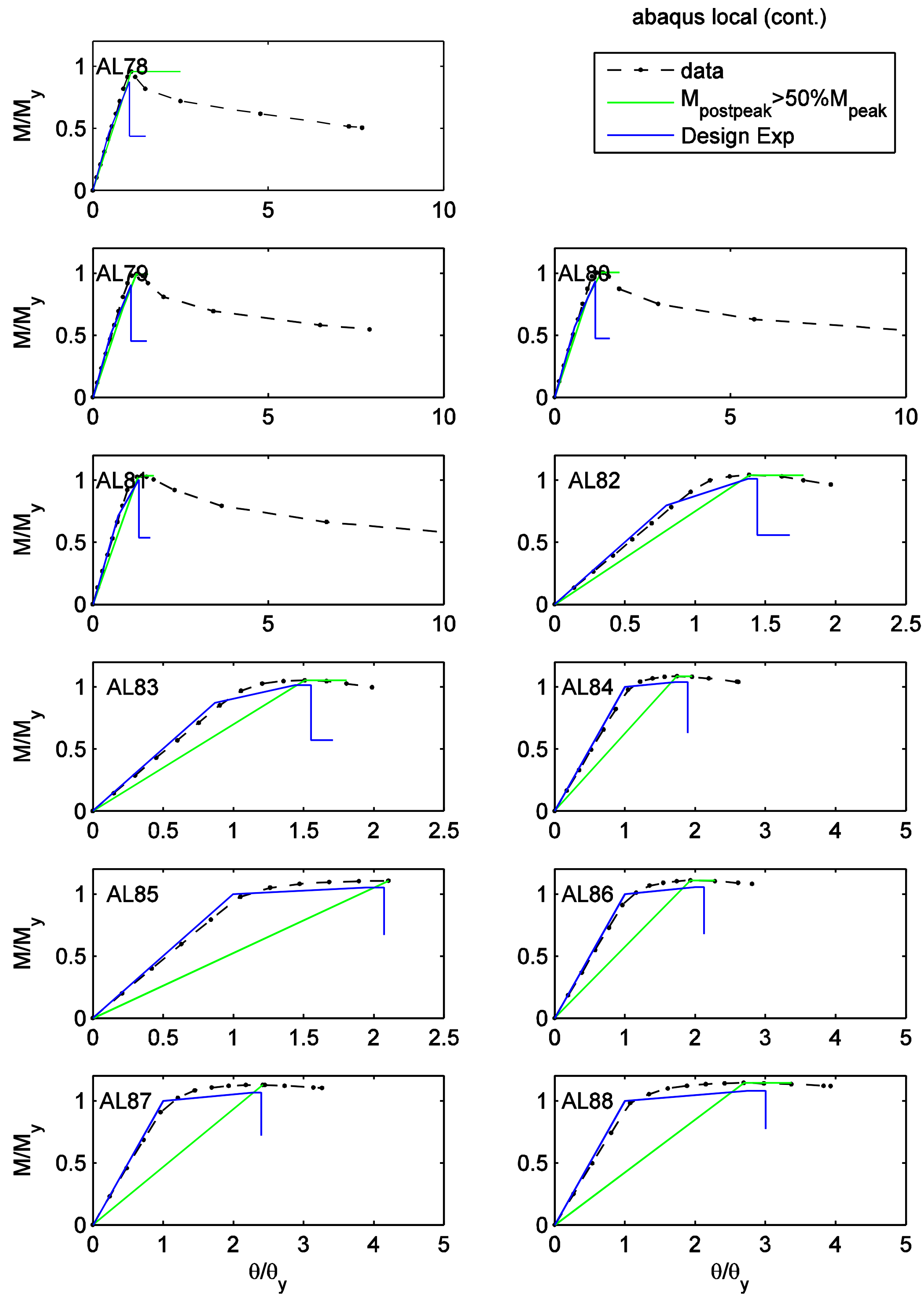


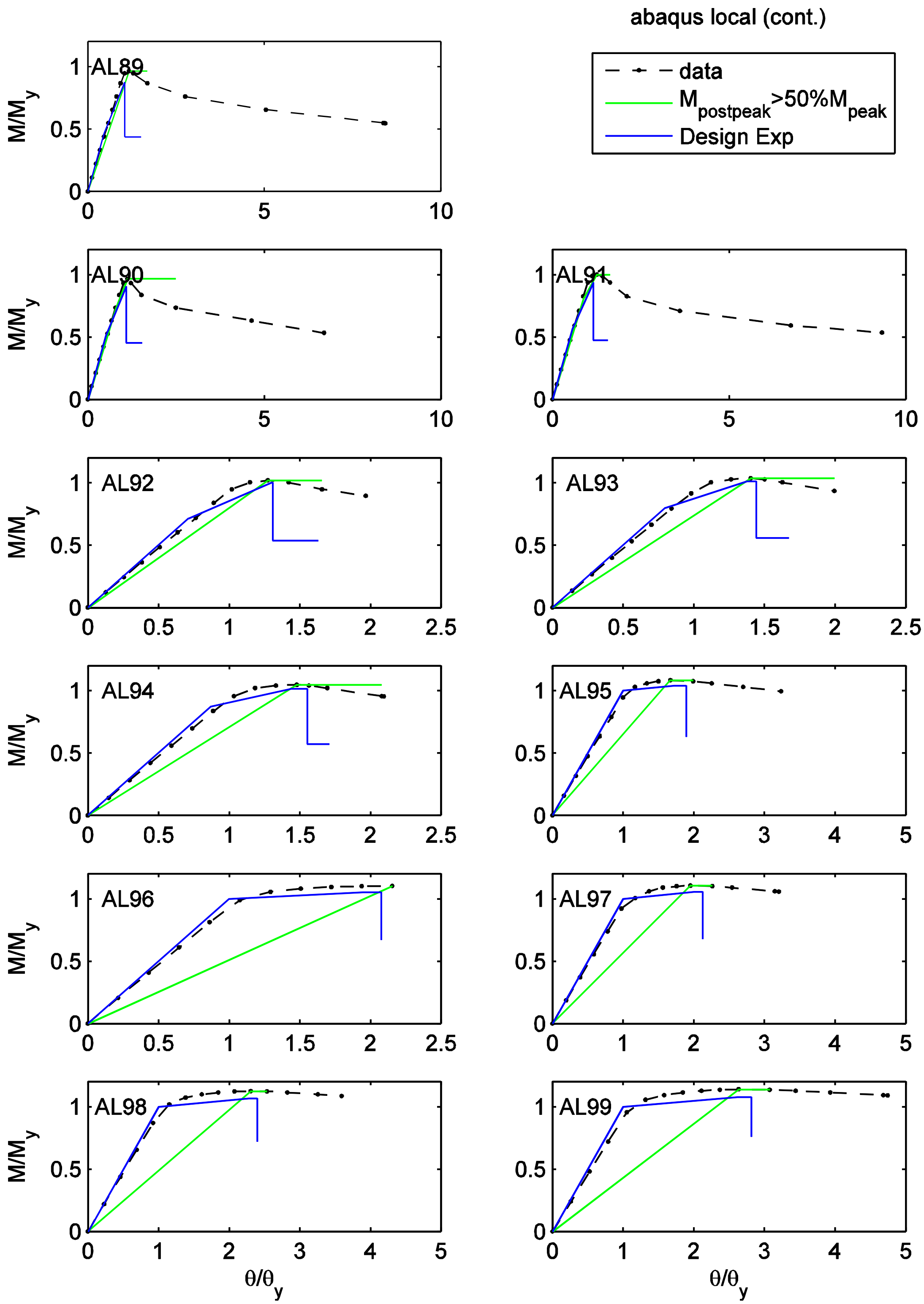
abaqus local (cont.)
data
M postpeak>50%M peak
Design Exp
AL34
AL35
AL36
AL37
AL38
AL39
AL40
AL41
AL42
AL43
AL44
M/M y
θ/θ y

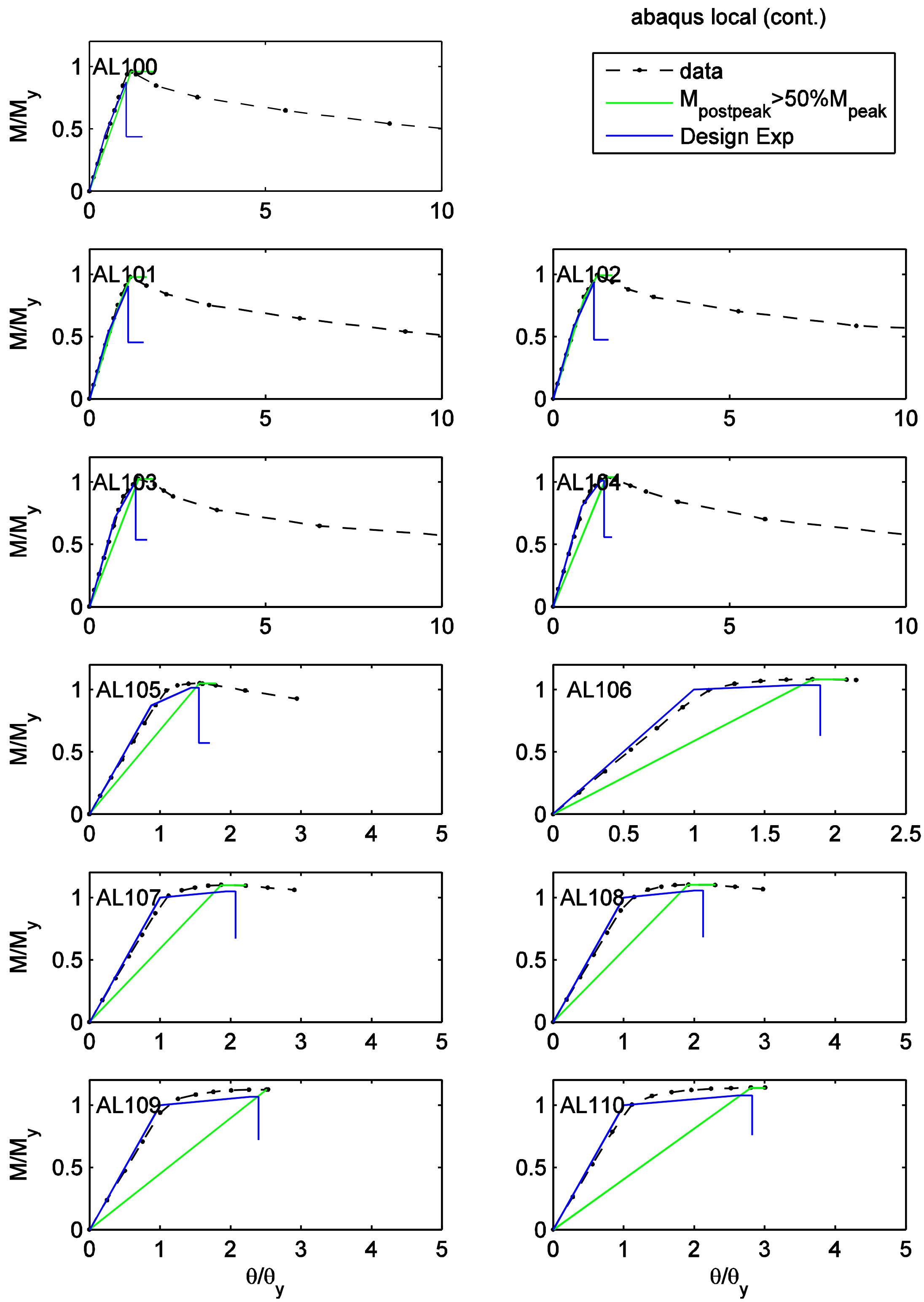

abaqus local (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AL45
AL46
AL47
AL48
AL49
AL50
AL51
AL52
AL53
AL54
AL55
M/M_y
θ/θ_y

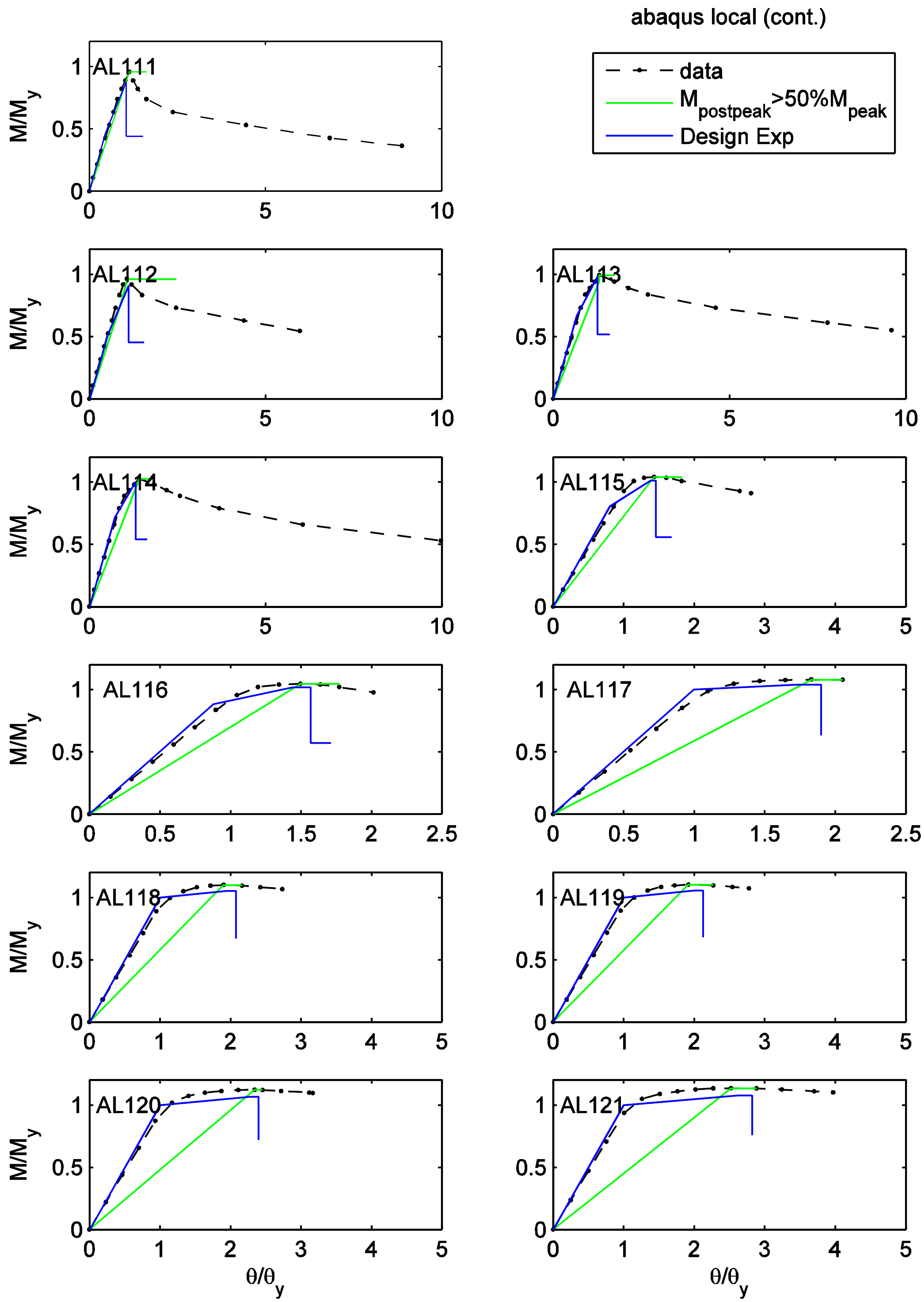

abaqus local (cont.)
data
M_postpeak>50%M_peak
Design Exp
AL56
AL57
AL58
AL59
AL60
AL61
AL62
AL63
AL64
AL65
AL66
M/My
θ/θy


abaqus local (cont.)
data
M_postpeak>50%M_peak
Design Exp
AL67
AL68
AL69
AL70
AL71
AL72
AL73
AL74
AL75
AL76
AL77
M/M_y
θ/θ_y

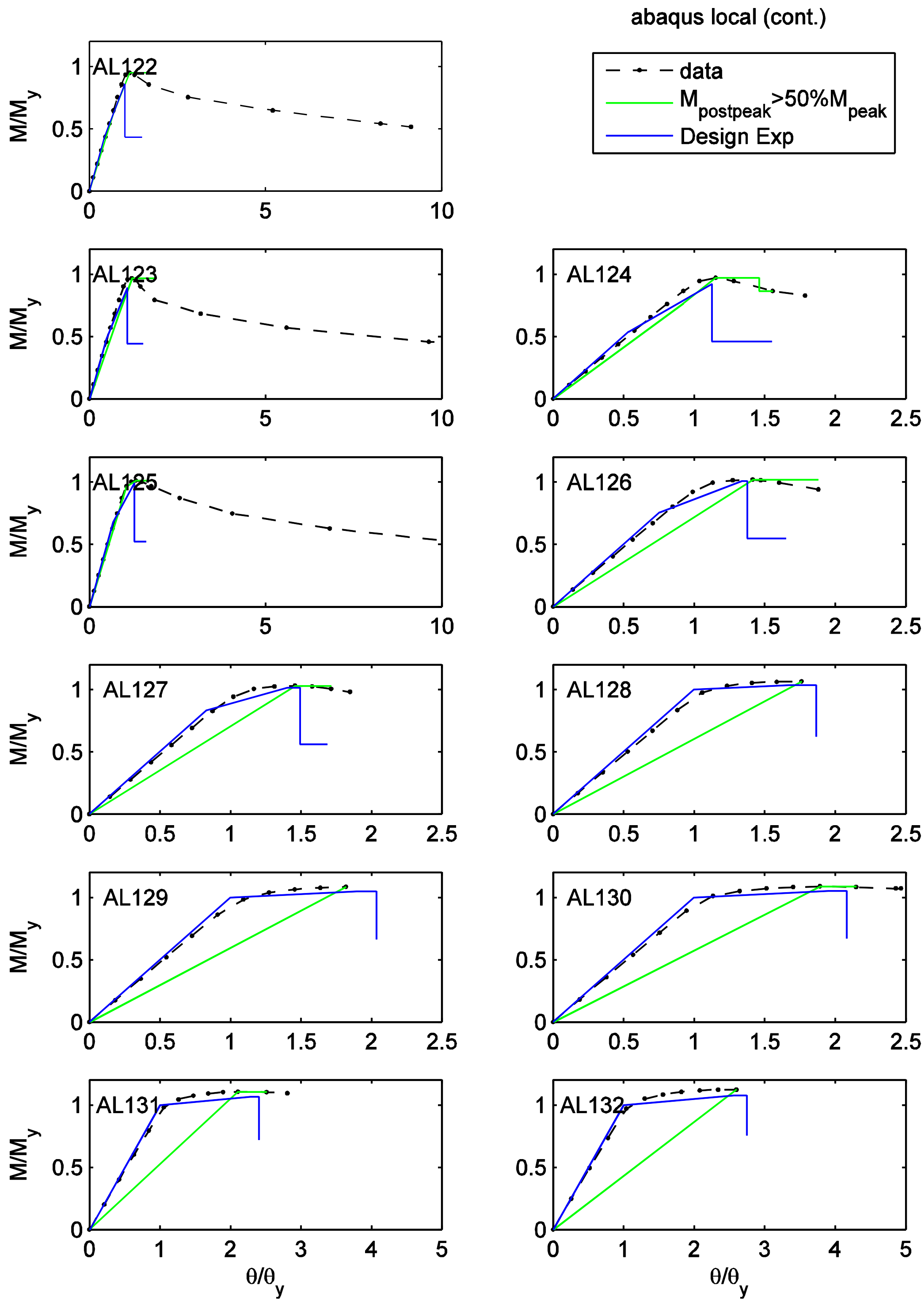

abaqus local (cont.)
data
M_postpeak>50%M_peak
Design Exp
AL78
AL79
AL80
AL81
AL82
AL83
AL84
AL85
AL86
AL87
AL88
M/M_y
θ/θ_y

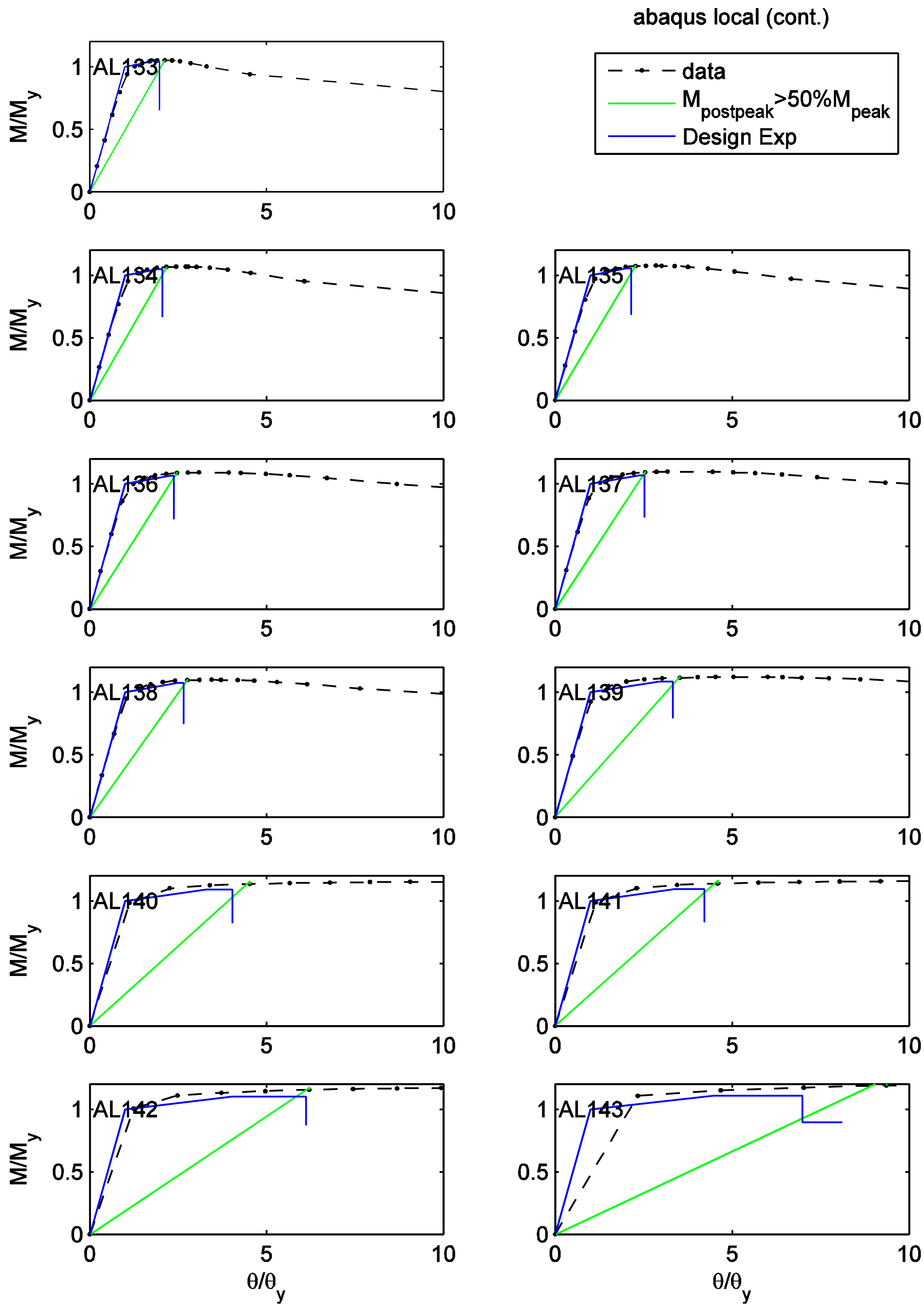

abaqus local (cont.)
data
M postpeak >50%M peak
Design Exp
AL89
AL90
AL91
AL92
AL93
AL94
AL95
AL96
AL97
AL98
AL99
M/M y
θ/θ y

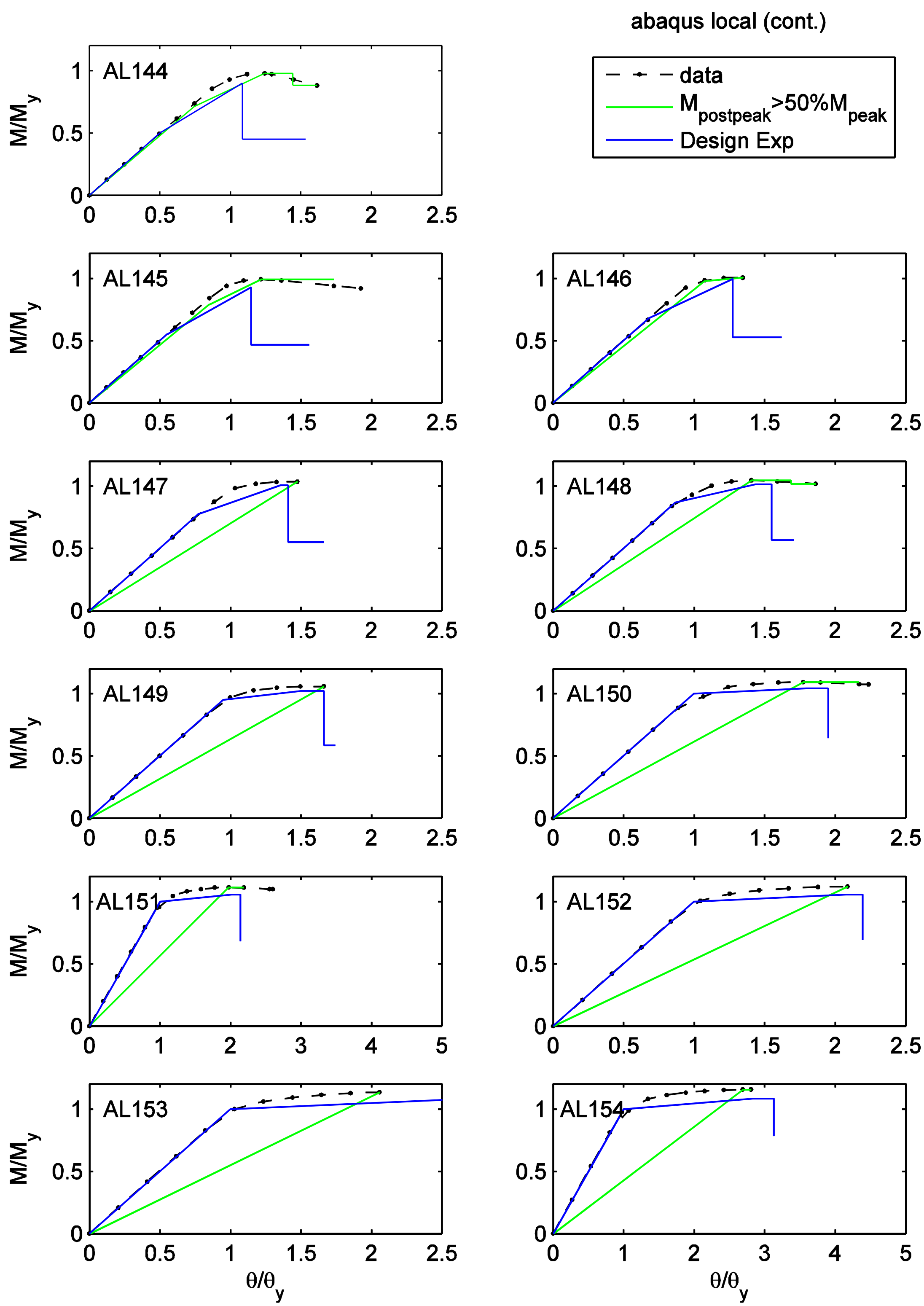

abaqus local (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AL100
AL101
AL102
AL103
AL104
AL105
AL106
AL107
AL108
AL109
AL110
M/M_y
θ/$θ_y$

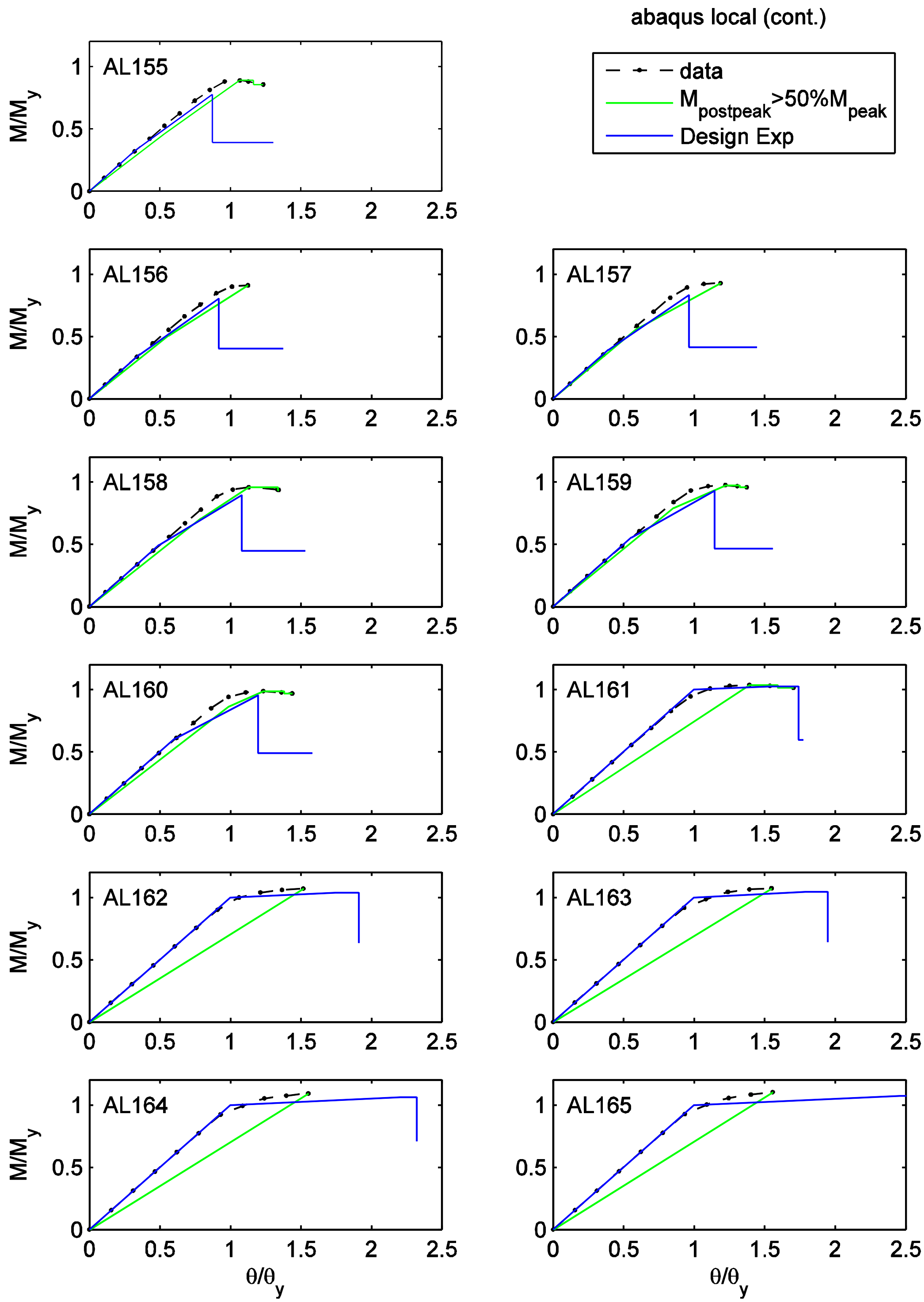


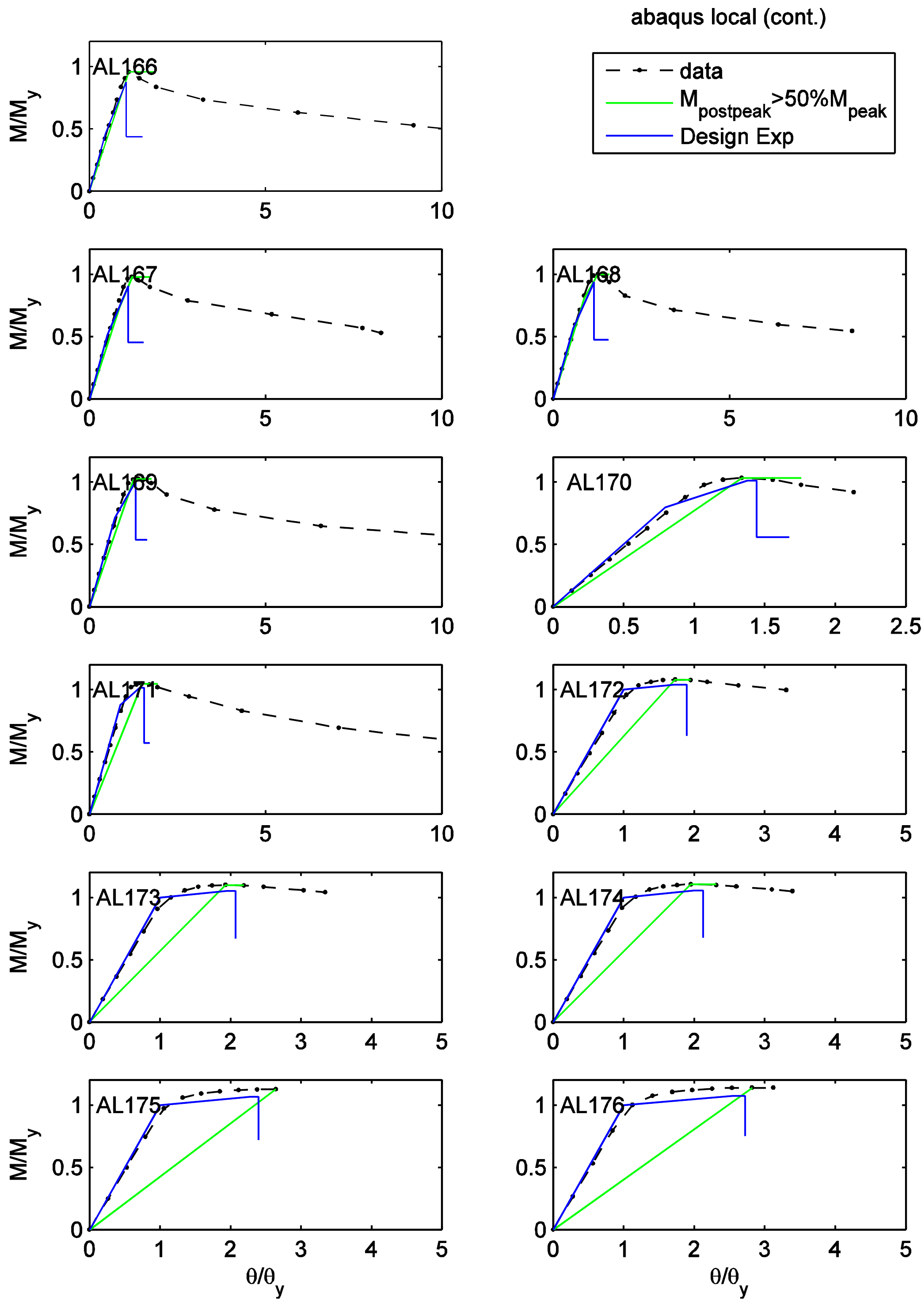
abaqus local (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AL111
AL112
AL113
AL114
AL115
AL116
AL117
AL118
AL119
AL120
AL121
M/M_y
θ/θ_y

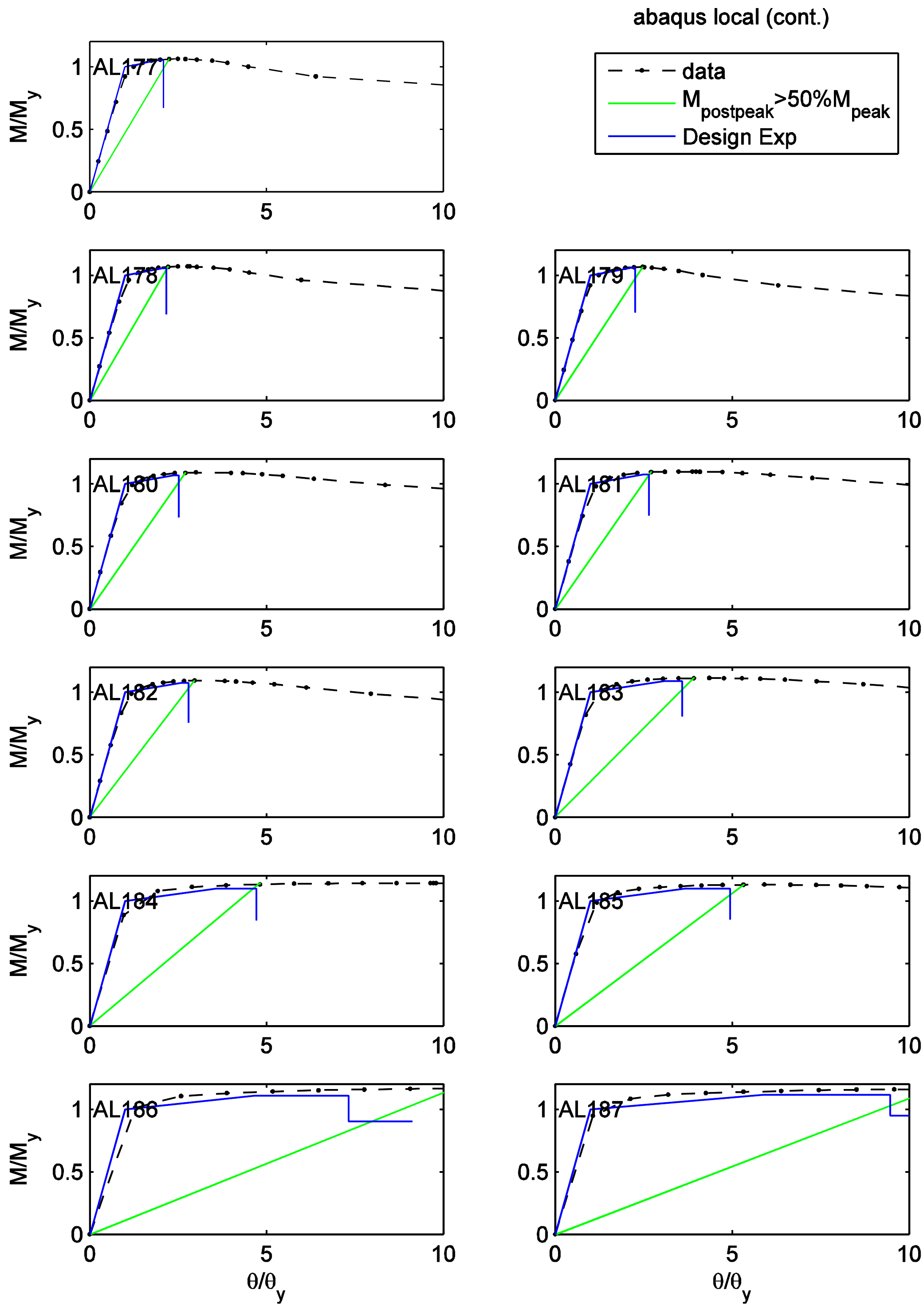

abaqus local (cont.)
data
M_postpeak>50%M_peak
Design Exp
AL122
AL123
AL124
AL125
AL126
AL127
AL128
AL129
AL130
AL131
AL132
M/M_y
θ/θ_y

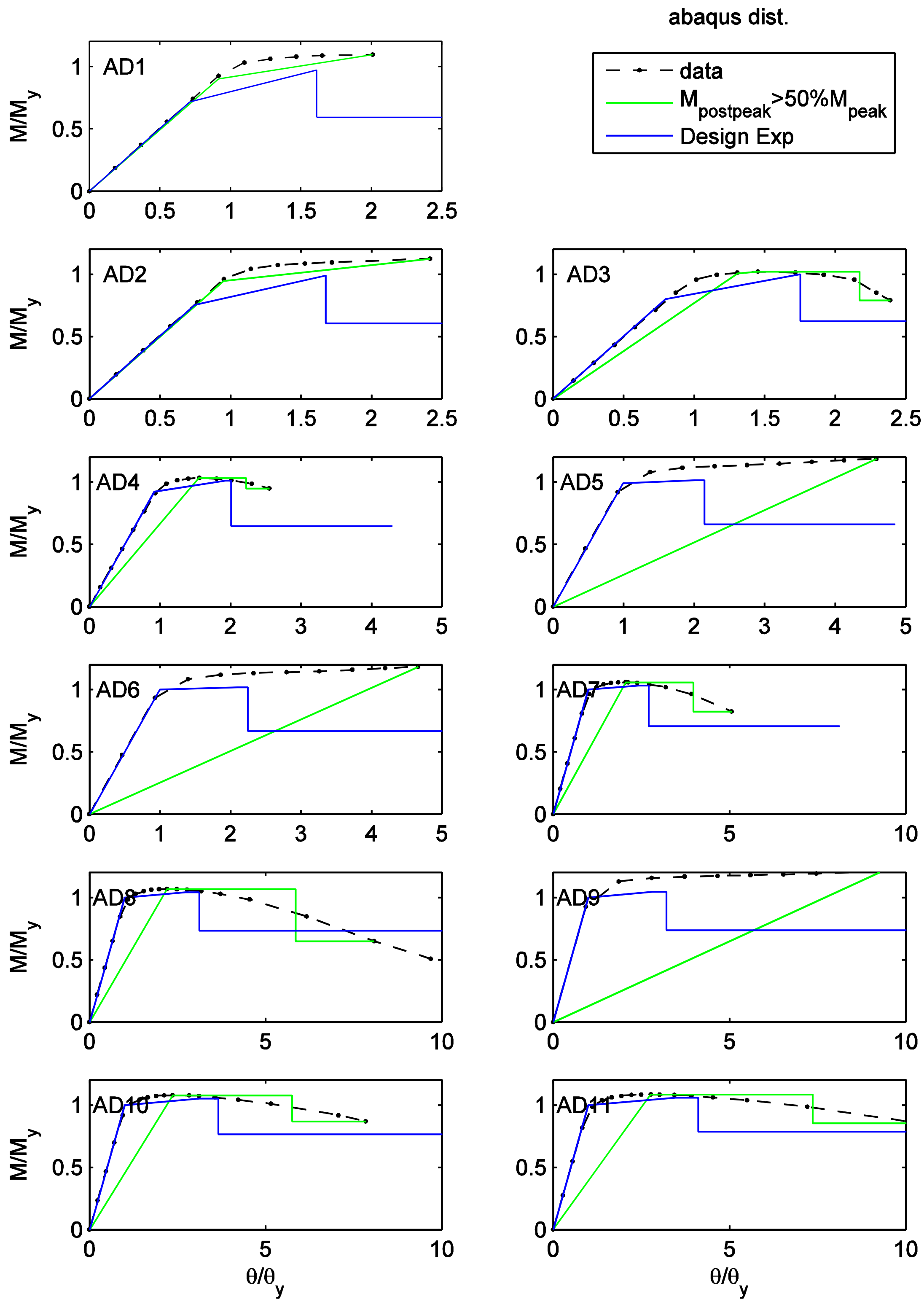

abaqus local (cont.)
data
M_postpeak>50%M_peak
Design Exp
AL133
AL134
AL135
AL136
AL137
AL138
AL139
AL140
AL141
AL142
AL143
M/M_y
θ/θ_y
0
0.5
1
5
10

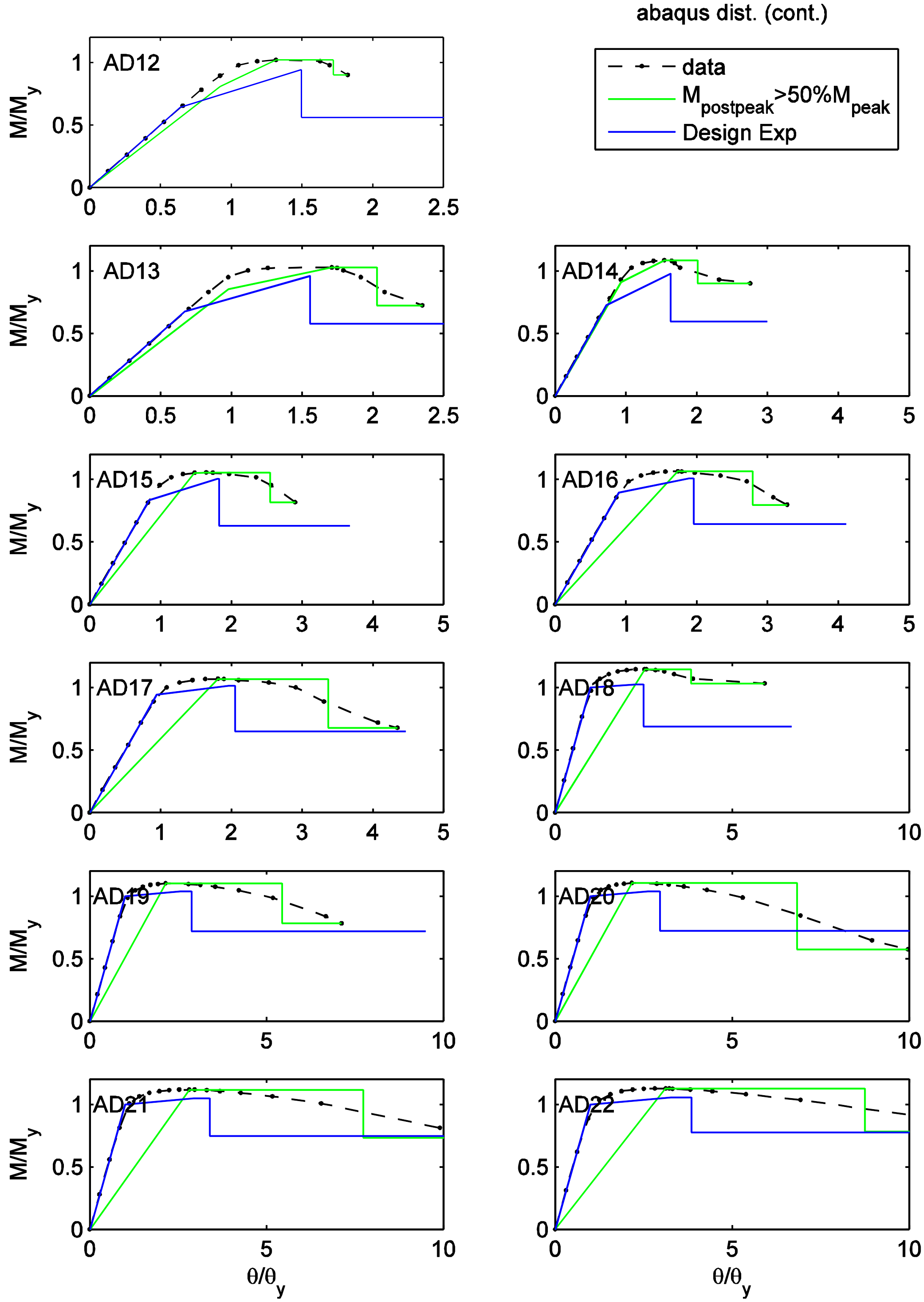

abaqus local (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AL144
AL145
AL146
AL147
AL148
AL149
AL150
AL151
AL152
AL153
AL154
M/M_y
θ/θ_y


abaqus local (cont.)
data
M postpeak>50%M peak
Design Exp
AL155
AL156
AL157
AL158
AL159
AL160
AL161
AL162
AL163
AL164
AL165
M/M y
θ/θ y

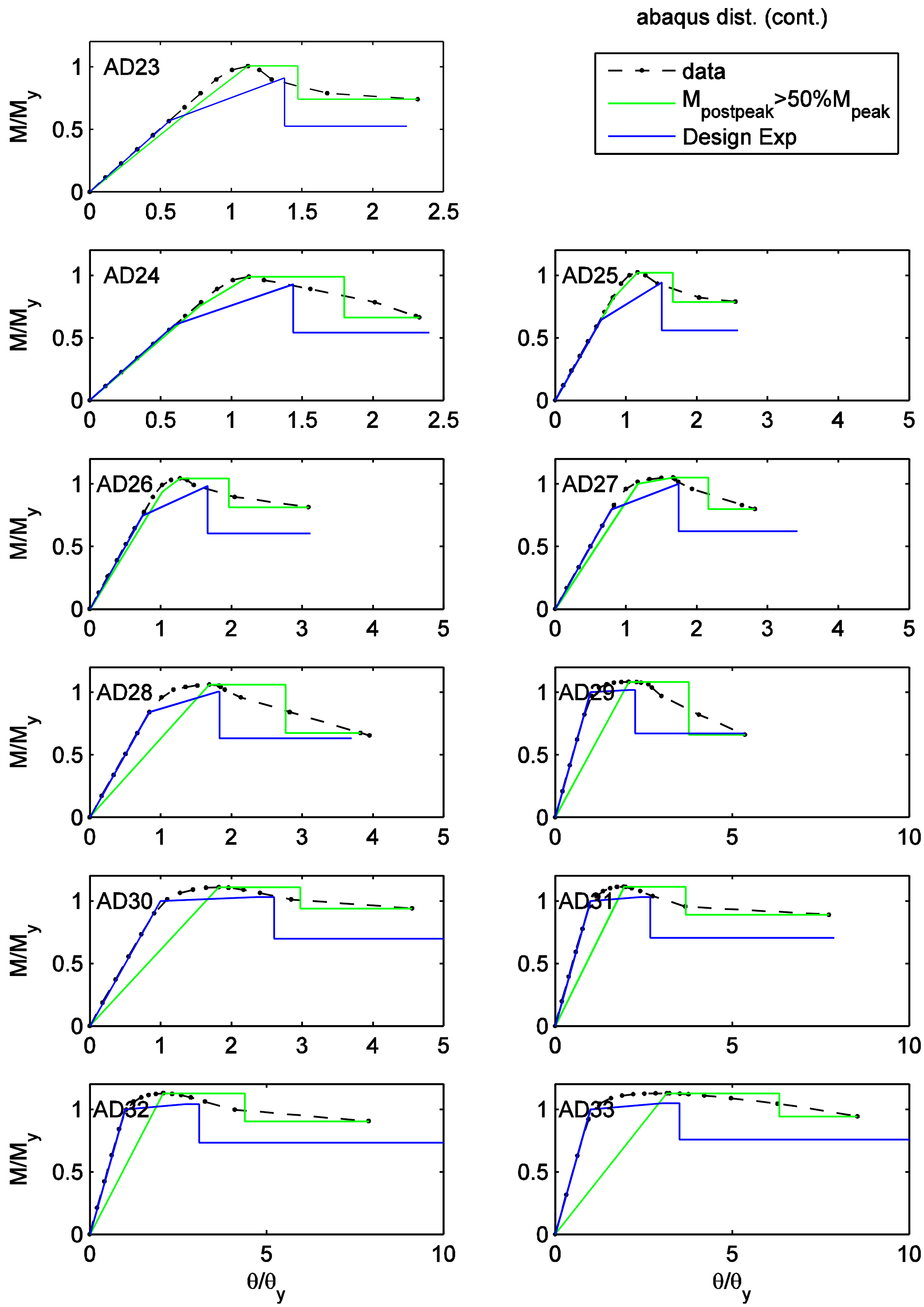

abaqus local (cont.)
data
M_postpeak>50%M_peak
Design Exp
AL166
AL167
AL168
AL169
AL170
AL171
AL172
AL173
AL174
AL175
AL176
M/M_y
θ/θ_y

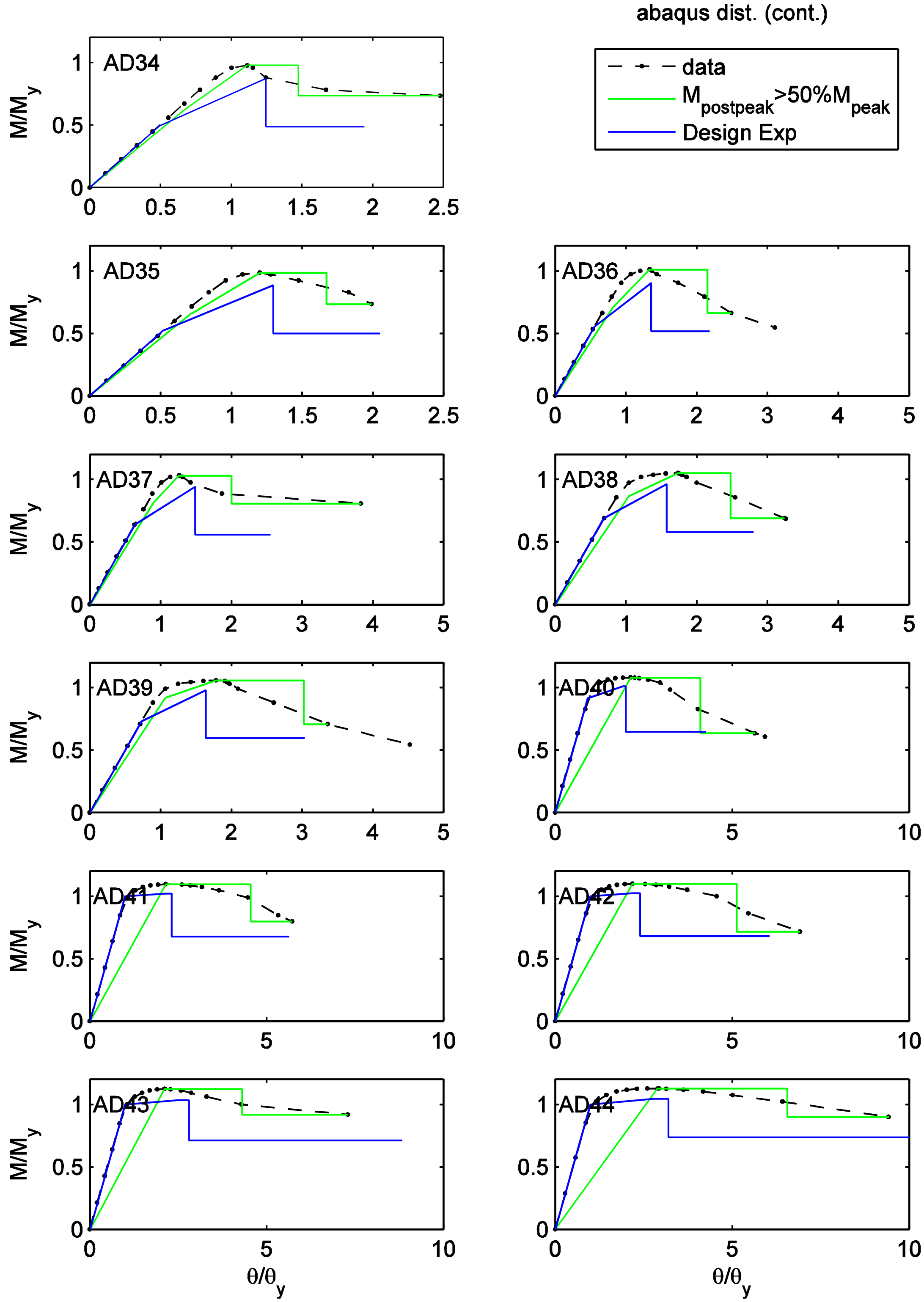

abaqus local (cont.)
data
M postpeak >50%M peak
Design Exp
AL177
AL178
AL179
AL180
AL181
AL182
AL183
AL184
AL185
AL186
AL187
M/M y
θ/θ y

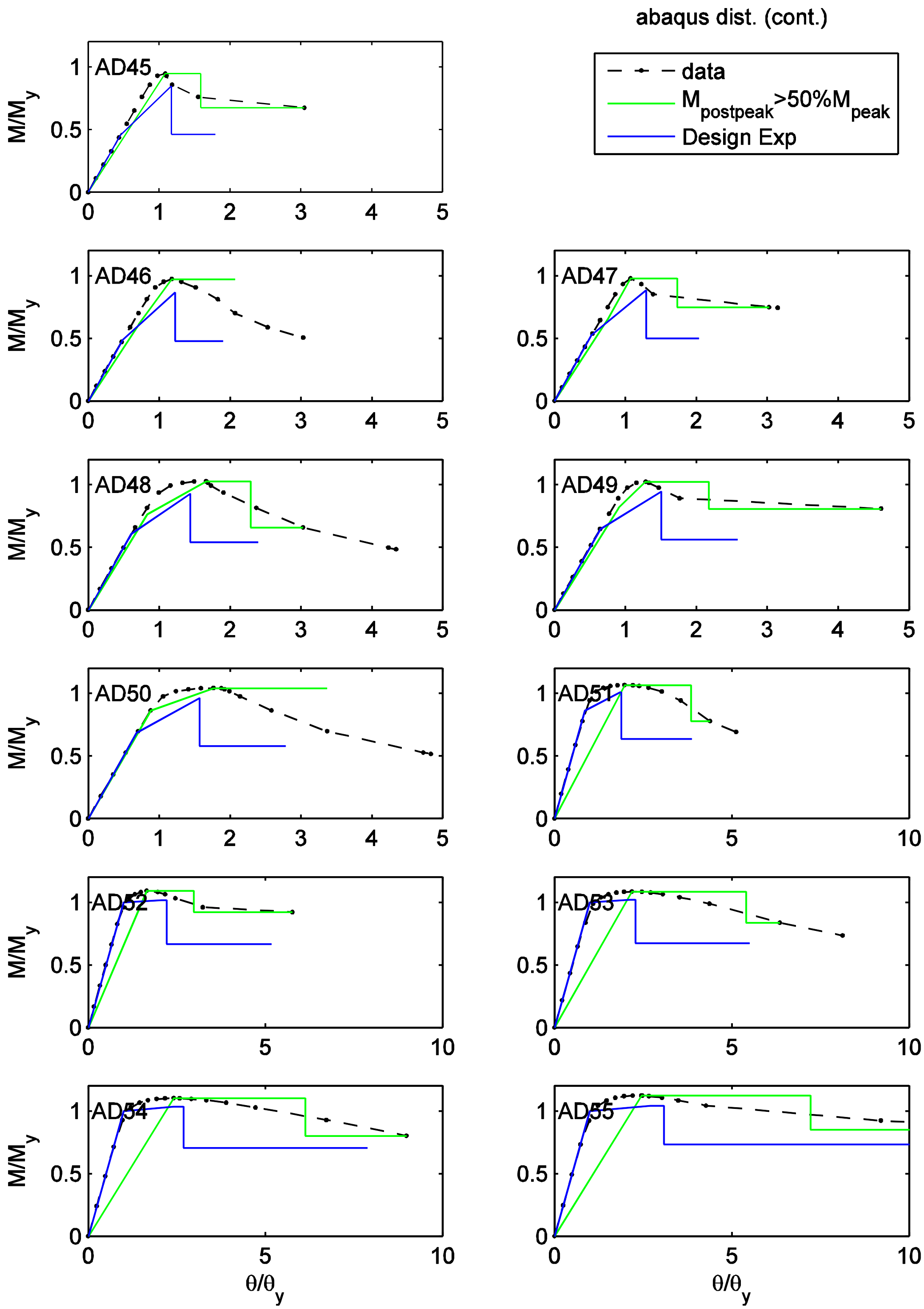

abaqus dist.
data
M_postpeak>50%M_peak
Design Exp
AD1
AD2
AD3
AD4
AD5
AD6
AD7
AD8
AD9
AD10
AD11
M/M_y
θ/θ_y

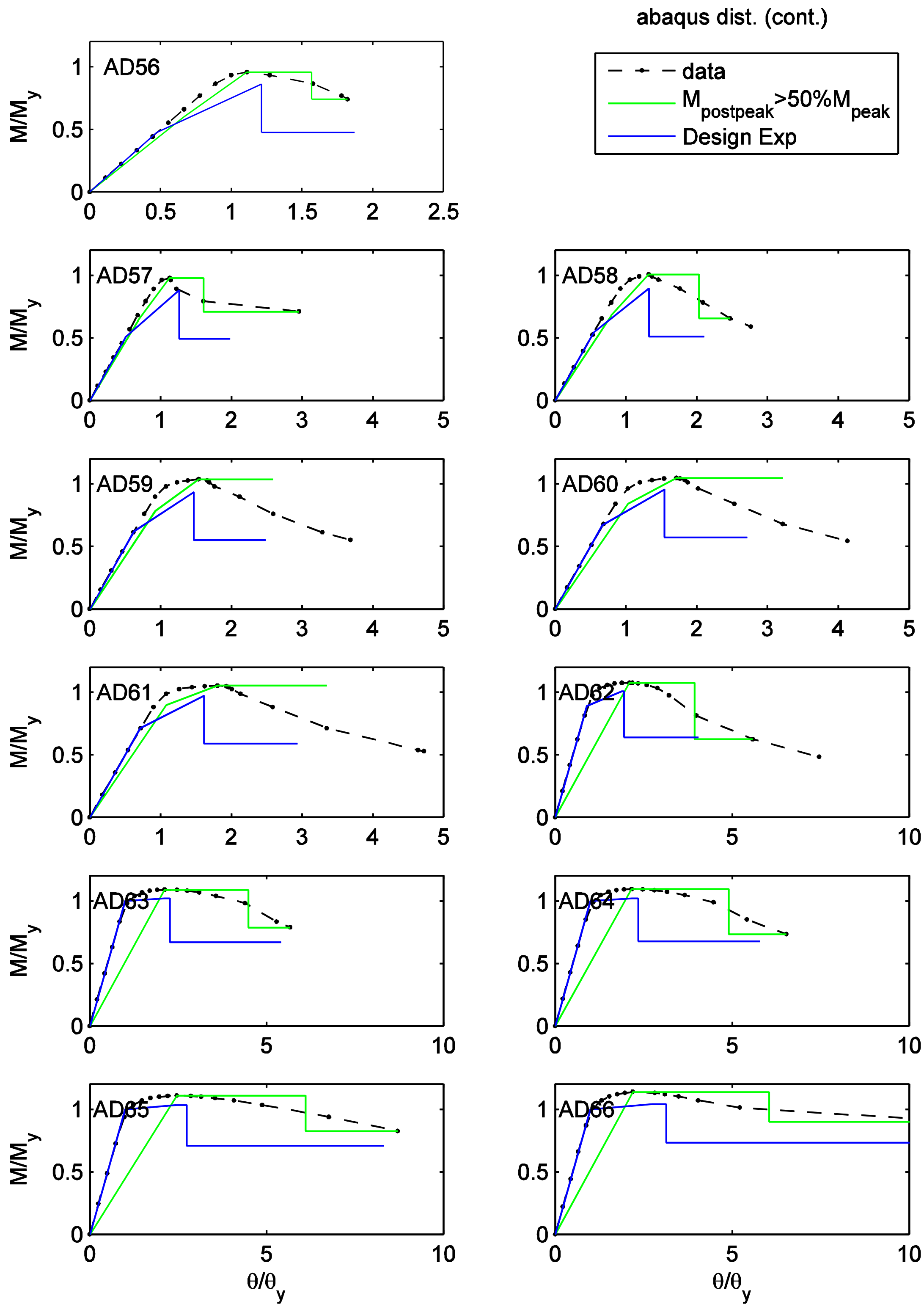


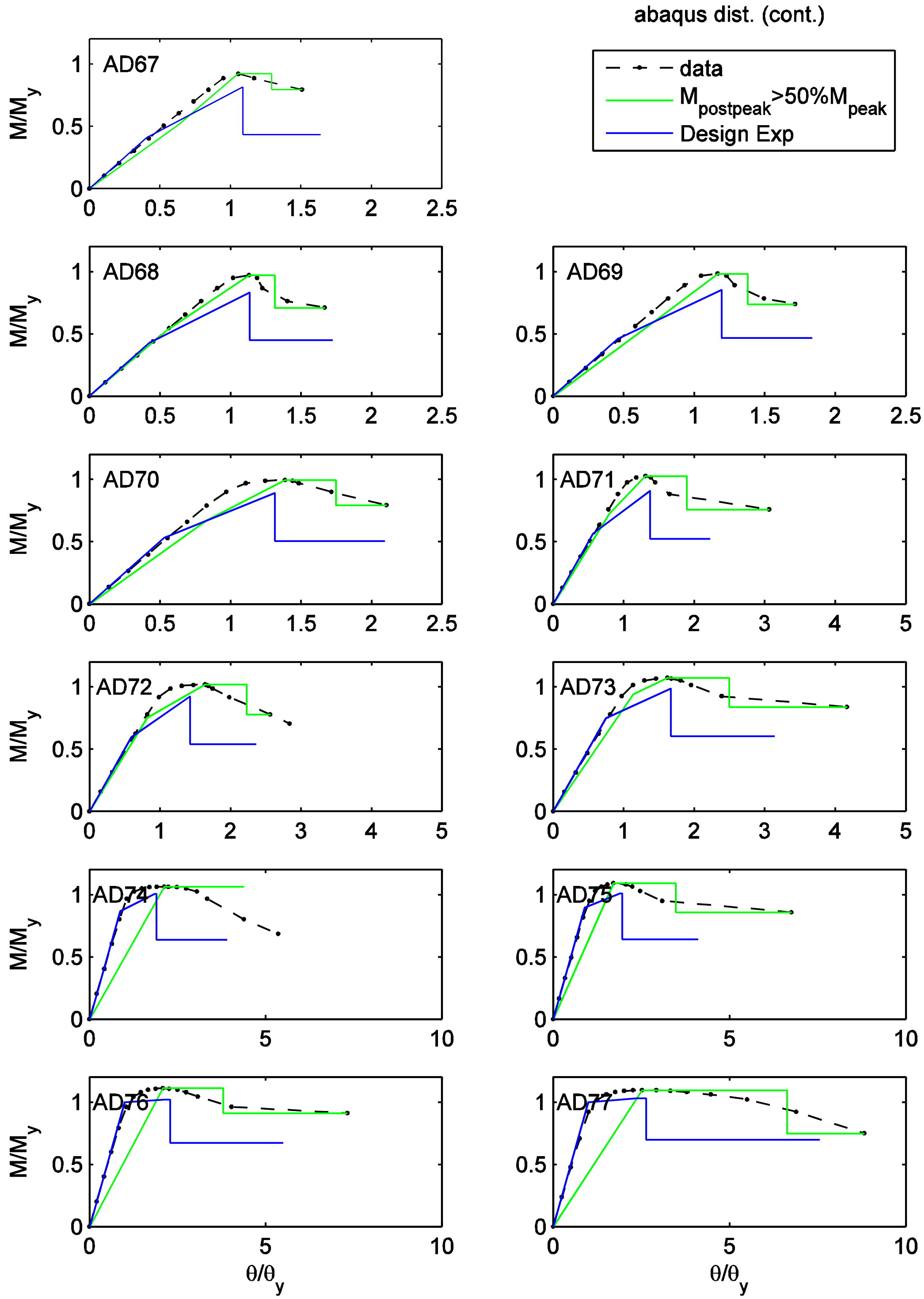
abaqus dist. (cont.)
data
M postpeak>50%M peak
Design Exp
AD12
AD13
AD14
AD15
AD16
AD17
AD18
AD19
AD20
AD21
AD22
M/My
θ/θy

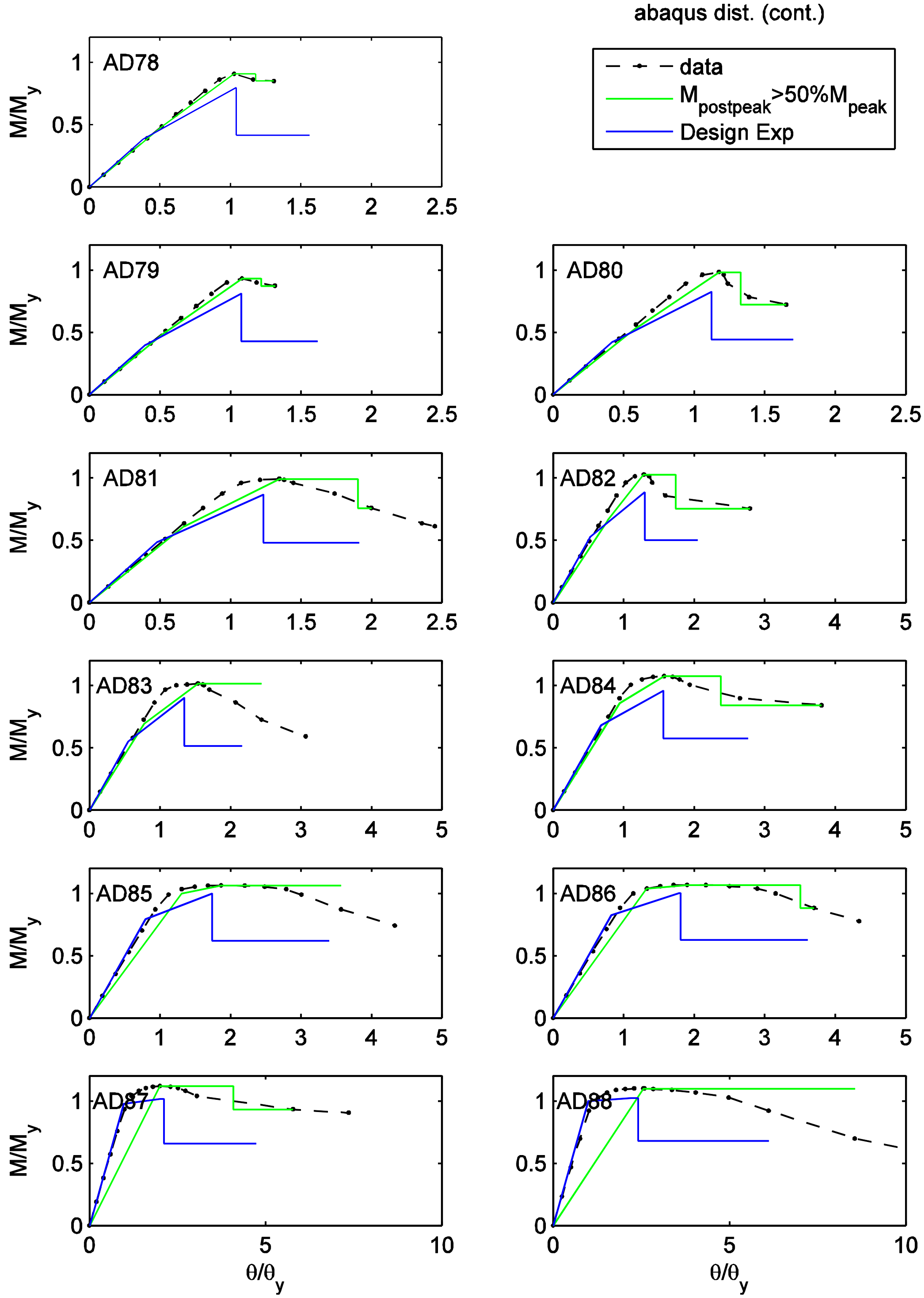

abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD23
AD24
AD25
AD26
AD27
AD28
AD29
AD30
AD31
AD32
AD33
M/M y
θ/θ y
0
0.5
1
1.5
2
2.5
3
4
5
10


abaqus dist. (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AD34
AD35
AD36
AD37
AD38
AD39
AD40
AD41
AD42
AD43
AD44
M/M_y
θ/$θ_y$

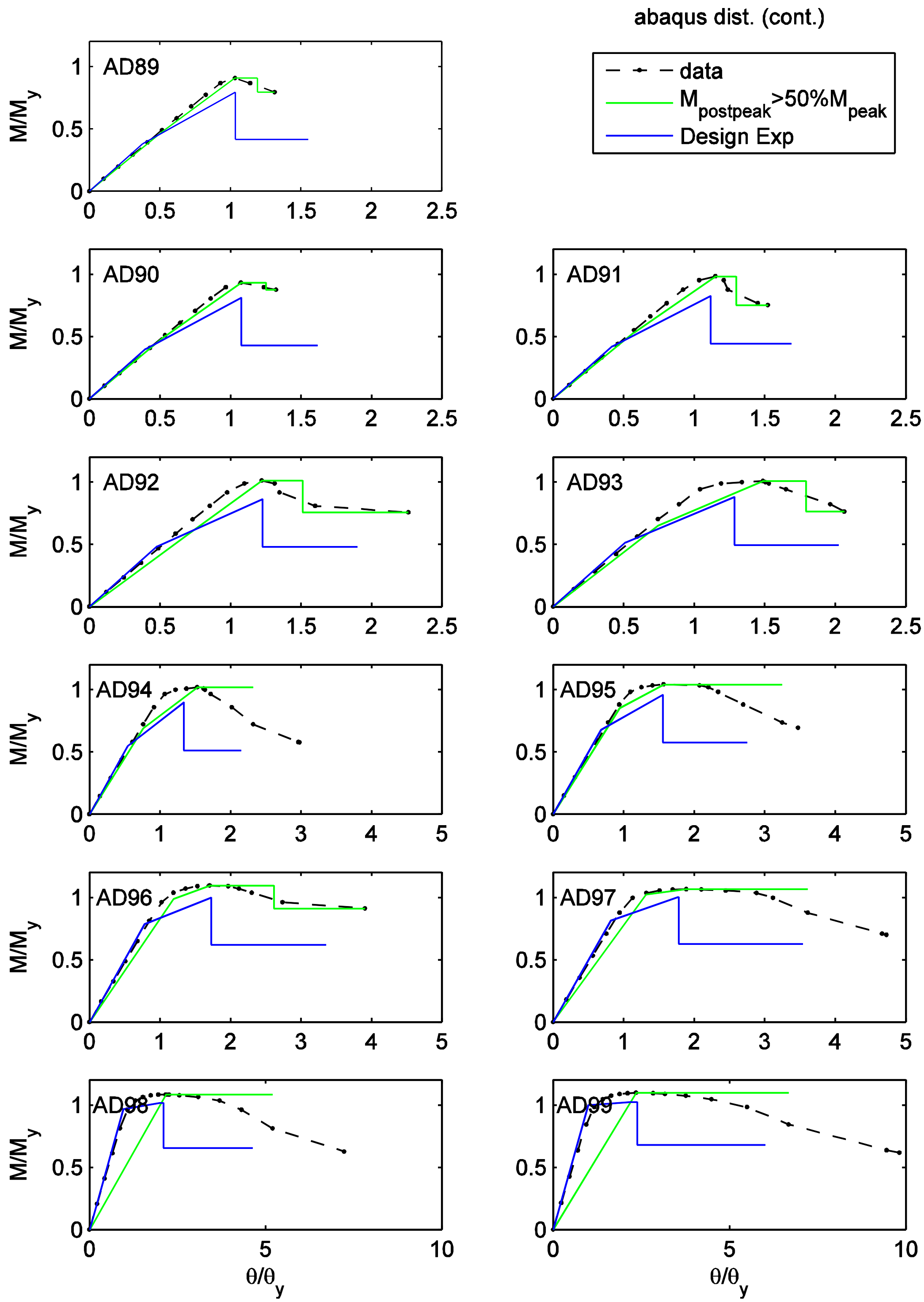

abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD45
AD46
AD47
AD48
AD49
AD50
AD51
AD52
AD53
AD54
AD55
M/M y
θ/θ y

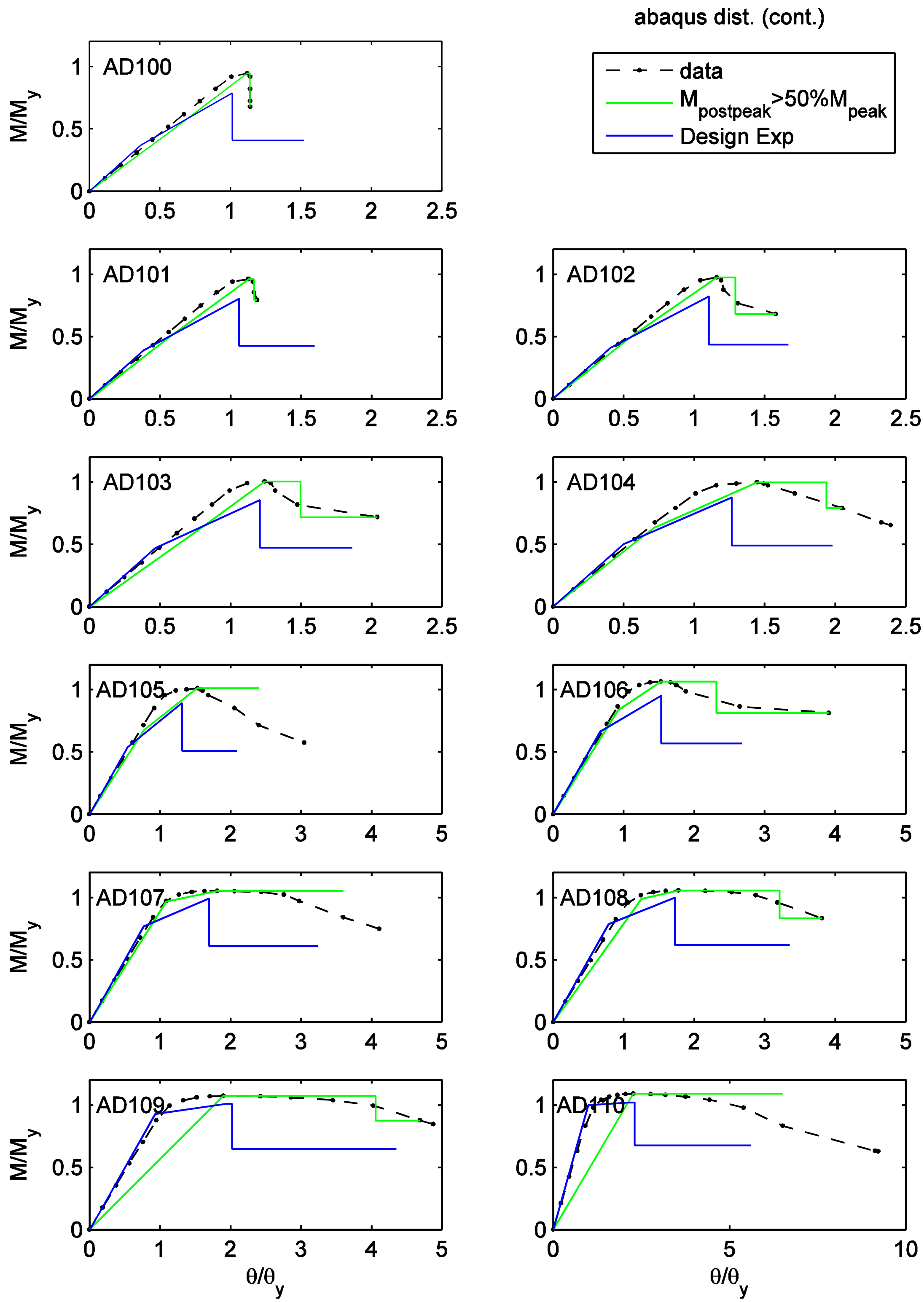

abaqus dist. (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AD56
AD57
AD58
AD59
AD60
AD61
AD62
AD63
AD64
AD65
AD66
M/M_y
θ/θ_y

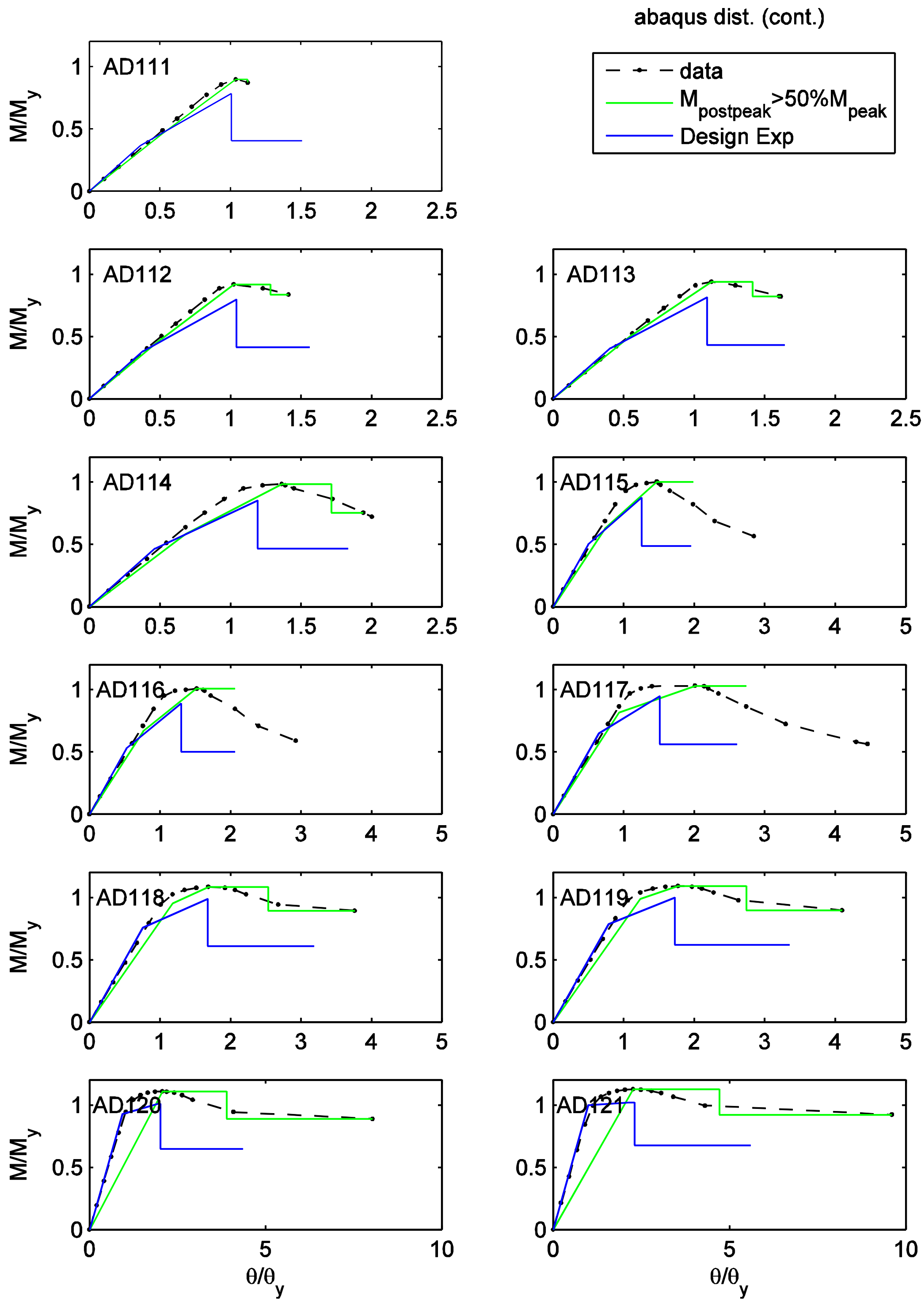

abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD67
AD68
AD69
AD70
AD71
AD72
AD73
AD74
AD75
AD76
AD77
M/M y
θ/θ y

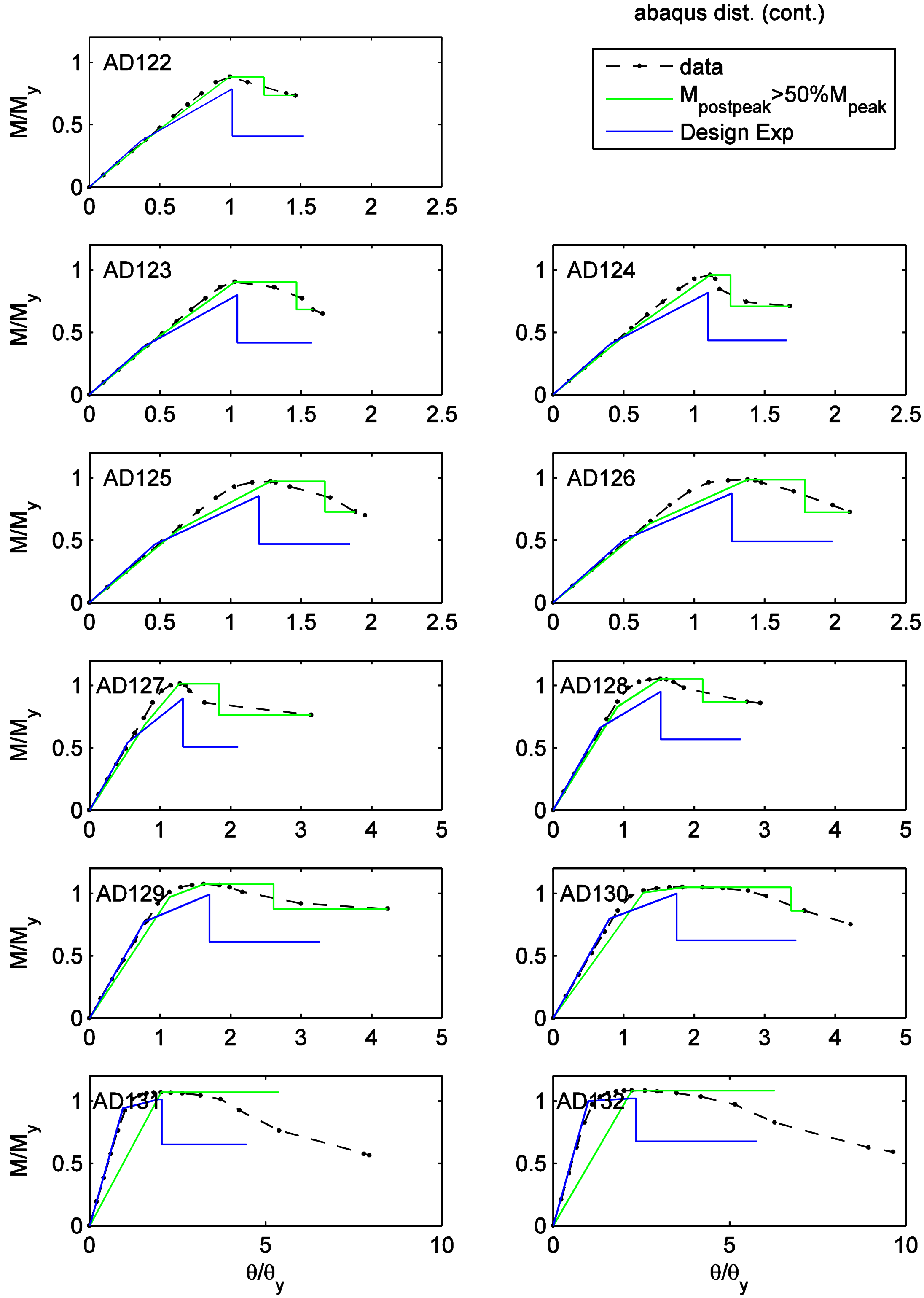


abaqus dist. (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AD78
AD79
AD80
AD81
AD82
AD83
AD84
AD85
AD86
AD87
AD88
M/M_y
θ/θ_y

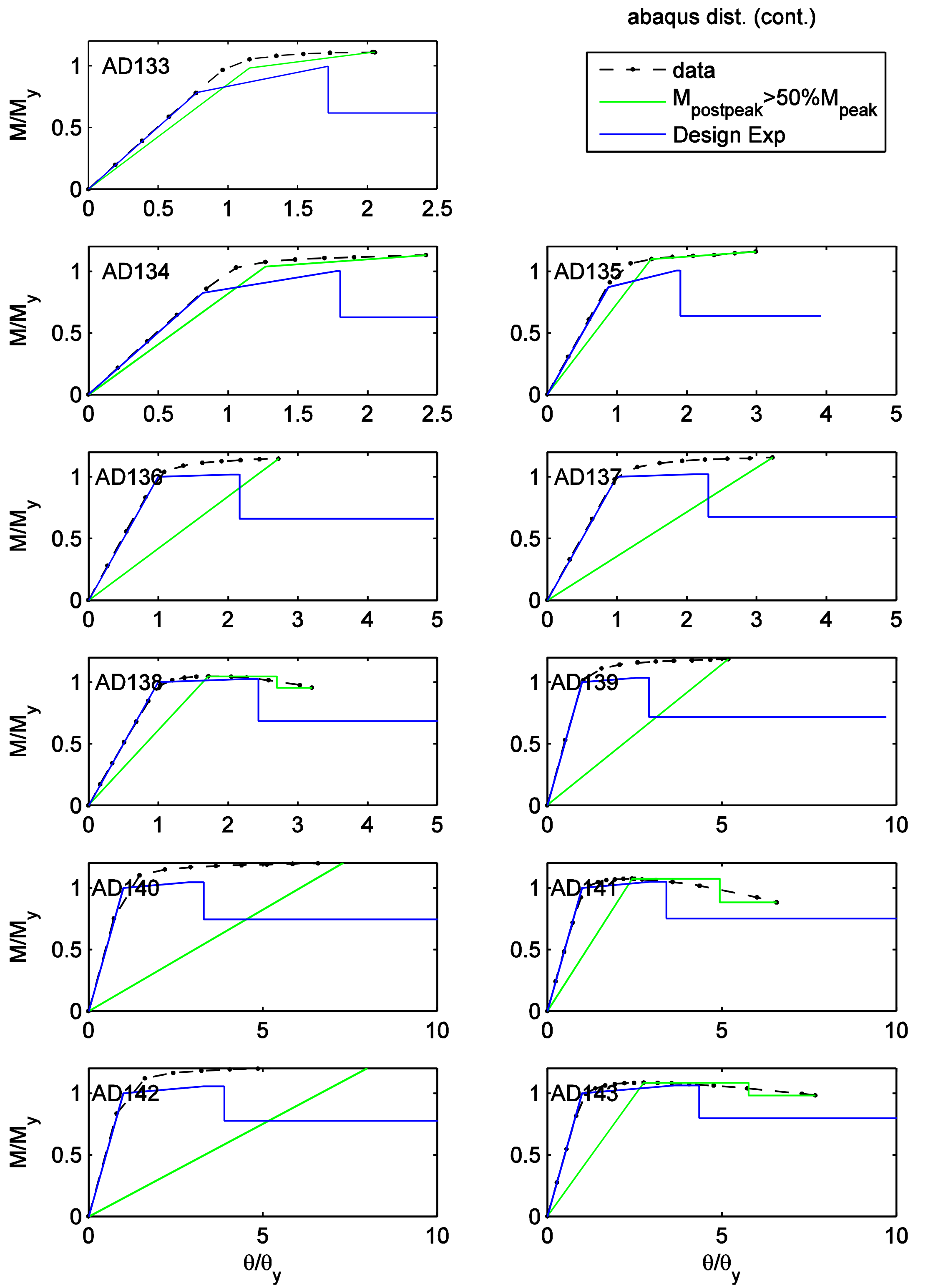

abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD89
AD90
AD91
AD92
AD93
AD94
AD95
AD96
AD97
AD98
AD99
M/M y
θ/θ y

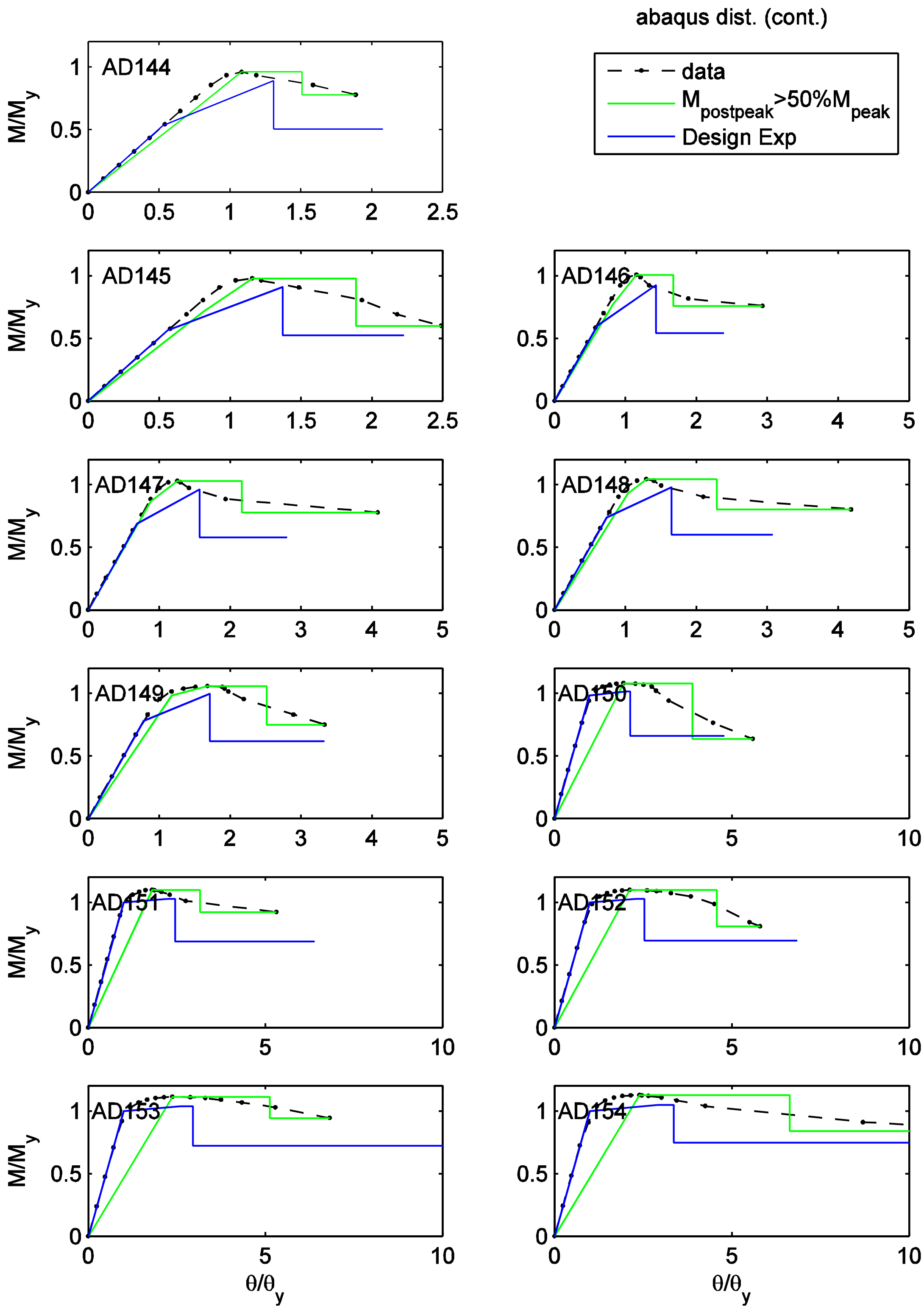

abaqus dist. (cont.)
data
$M_{postpeak}$>50%M_{peak}
Design Exp
AD100
AD101
AD102
AD103
AD104
AD105
AD106
AD107
AD108
AD109
AD110
M/M_y
θ/θ_y


abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD111
AD112
AD113
AD114
AD115
AD116
AD117
AD118
AD119
AD120
AD121
M/M y
θ/θ y

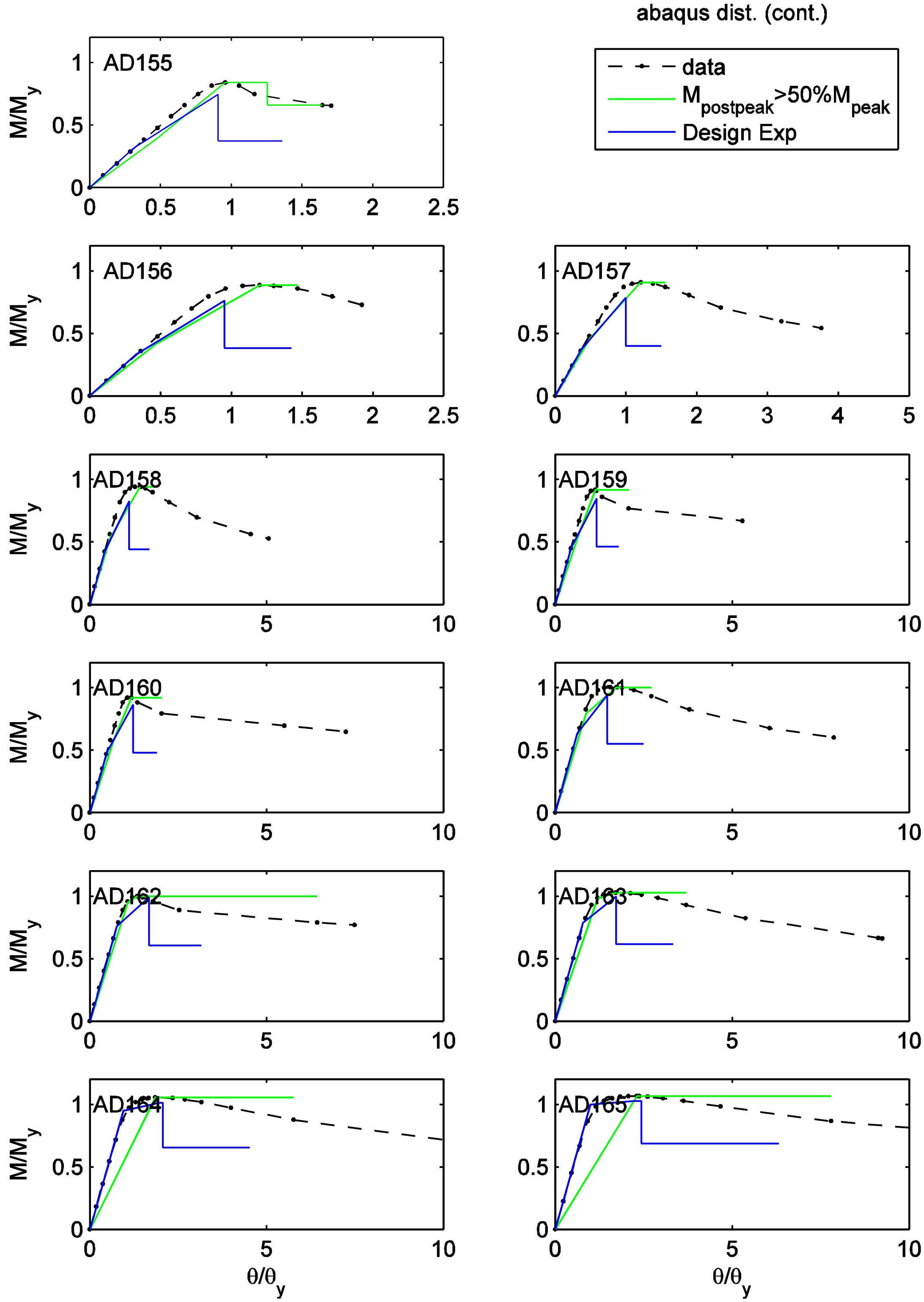


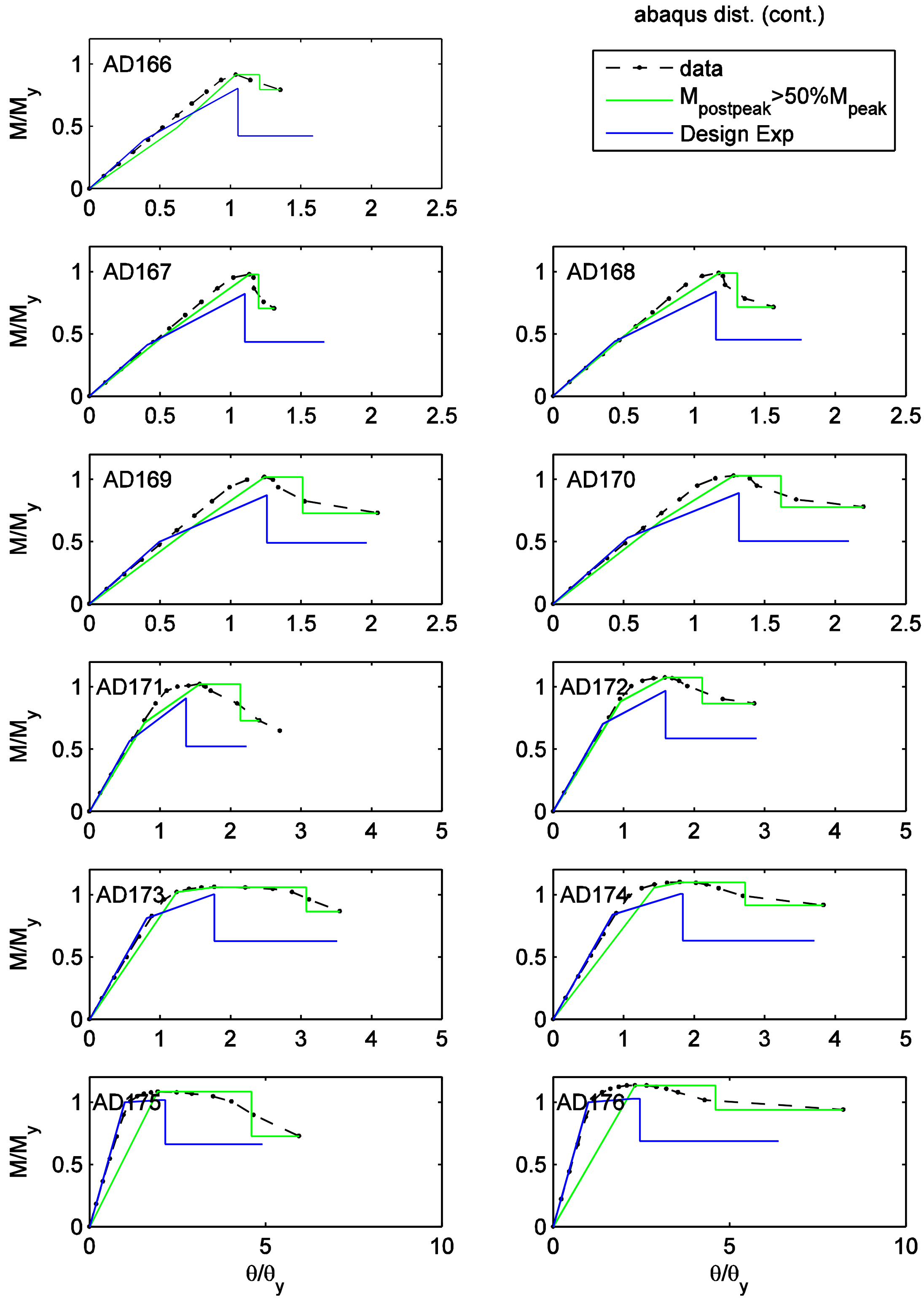
abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD122
AD123
AD124
AD125
AD126
AD127
AD128
AD129
AD130
AD131
AD132
M/My
θ/θy

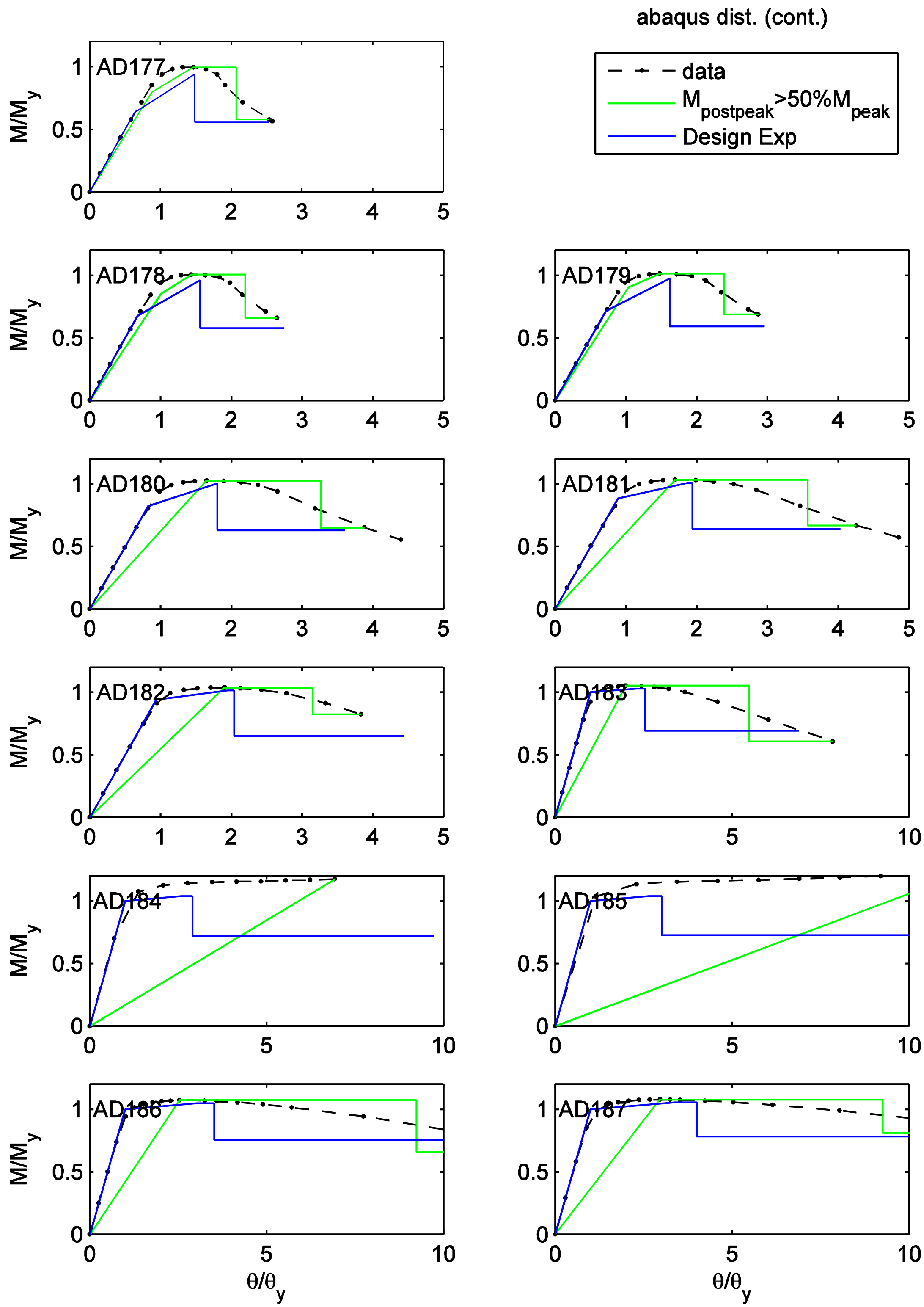

abaqus dist. (cont.)
data
M_postpeak>50%M_peak
Design Exp
AD133
AD134
AD135
AD136
AD137
AD138
AD139
AD140
AD141
AD142
AD143
M/M_y
θ/θ_y



abaqus dist. (cont.)
data
M_postpeak>50%M_peak
Design Exp
AD144
AD145
AD146
AD147
AD148
AD149
AD150
AD151
AD152
AD153
AD154
M/M_y
θ/θ_y


abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD155
AD156
AD157
AD158
AD159
AD160
AD161
AD162
AD163
AD164
AD165
M/M y
θ/θ y


abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD166
AD167
AD168
AD169
AD170
AD171
AD172
AD173
AD174
AD175
AD176
M/M y
θ/θ y


abaqus dist. (cont.)
data
M postpeak >50%M peak
Design Exp
AD177
AD178
AD179
AD180
AD181
AD182
AD183
AD184
AD185
AD186
AD187
M/My
θ/θy

Appendix 7: ABAQUS input routine for design example

```
*Heading
** Job name: BarSpring Model name: BarSpring
*Preprint, echo=NO, model=NO, history=NO, contact=NO
*Node,NSET=ALL
     1,        0,        0
     2,       64,        0
     3,       64,        0
     4,      128,        0
     5,      128,        0
     6,      192,        0
** Element: beam
*Element, type=B23,ELSET=ALL
 1,  1,  2
 2,  3,  4
 3,  5,  6
** Section: GENERAL BEAM SECTION
*BEAM GENERAL SECTION, SECTION=GENERAL, POISSON=0.3, ELSET=ALL
3.4767, 36.278, 10.45, 5.64, 0.016, 0., 35
0.,0.,-1
2950000.,11000.
*EQUATION
2
2,1,1,3,1,-1
2
2,2,1,3,2,-1
2
4,1,1,5,1,-1
2
4,2,1,5,2,-1
** define the nonlinear spring
*Element, type=spring2,elset=nspring
11, 2, 3
12, 4, 5
*Elset, elset=nspring, generate
11,12,1
*Spring,nonlinear, elset=nspring
6, 6
-198.82, -0.03150000000001
-374.98, -0.0315
-258, -0.0169
0,0
258, 0.0169
374.98, 0.0315
198.82, 0.03150000000001
```

```
*NSET, NSET=mergednodes
3
5
** BOUNDARY CONDITIONS
** Name: Displacement/Rotation
*Boundary
1, 1, 2
6, 2, 2
** STEP
*Step, nlgeom, INC= 1000
*Static,direct
0.001, , ,
**
** LOADS
*Boundary
3,2,,-3
5,2,,-3
**
** OUTPUT REQUESTS
*OUTPUT, FIELD, variable=PRESELECT
*NODE OUTPUT
U
*EL Print, elset=nspring
S11,E11
*Node Print, SUMMARY=NO, NSET=ALL
U
*END STEP
```