

## Constrained Rectilinear Motion

Dynamics Laboratory, Spring 2003

Pulley systems are commonly used (in cranes, for example) to gain a mechanical advantage, allowing heavy loads to be lifted by smaller forces moving over longer distances. For the three systems considered in this laboratory exercise, we are interested in relating the motions of two points in the system that are interconnected by an inextensible cord through a system of pulleys. Because the cord is inextensible, its length is constant, and this allows us to derive an equation of constraint relating the motions of the two points, as discussed in section 2/9 of the textbook.

### EXPERIMENT

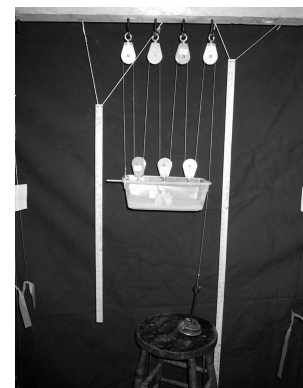
For each system, begin by making a schematic drawing of the system, which will be important in deriving the equation of constraint. In cases where two pulleys rotate about a common axis, the two pulleys should be drawn with different diameters so that the path of the cord is clearly shown. Free-hand drawings are fine during the laboratory session, but are not acceptable for submission with your laboratory report. Guidelines for the figures to be presented in the report are given with the Laboratory Report Requirements below.

**System 1:** We are interested in relating the motion of the cart on the inclined track to the motion of the vertically suspended mass. To determine the relationship between the displacements of these points, move one of them a prescribed distance (measured on the yardstick), and measure the distance traveled by the other. To obtain more reliable results, you should take at least three sets of measurements, with different values of prescribed displacement. Also, measure the angle of inclination of the track. *Note that the displacement on the inclined track is measured in metric units, while the vertical displacement is measured in traditional units, so don't forget to convert the measurements appropriately.*

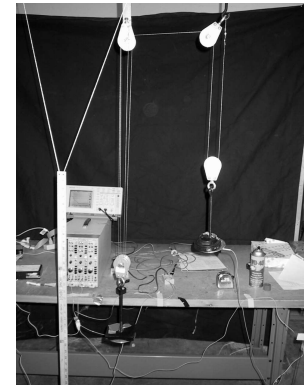


*Note that the displacement on the inclined track is measured in metric units, while the vertical displacement is measured in traditional units, so don't forget to convert the measurements appropriately.*

**System 2:** We are interested in relating the motion of the vertically suspended mass to the motion of the threaded rod at the opposite end of the suspended bucket. To determine the relationship between the displacements of these points, move one of them a prescribed distance (measured on the yardstick), and measure the distance traveled by the other. As before, you should take at least three sets of measurements with different values of prescribed displacement



**System 3:** We are interested in relating the motions of the two vertically suspended masses. To determine the relationship between the displacements of these points, move one of them a prescribed distance (measured on the yardstick) and measure the distance traveled by the other. As with the other systems, you should take at least three sets of measurements with different values of prescribed displacement. For this system, the masses are also instrumented with accelerometers, which allow us to measure the relationship between their accelerations.



#### Accelerometer Calibration:

Accelerometers give a voltage output that is proportional to acceleration, and to scale the voltage to units of acceleration, it is necessary to determine the constant of proportionality, a procedure referred to as calibration. The inner workings of an accelerometer can be understood as a spring-mounted mass moving along a fixed axis (this type of oscillator will be studied near the end of the semester), and the voltage output of the instrument is proportional to the displacement of this internal mass relative to the casing of the instrument. The internal spring-mass system has a high value of viscous damping and is tuned so that its relative displacement (and consequently the voltage output) is proportional to the acceleration of the instrument over a wide range of frequencies.

This internal spring-mounted mass deflects under the action of gravity, giving a constant nonzero voltage output even when the instrument is not accelerating. If the accelerometer is aligned with its positive axis pointed vertically upwards, the voltage output will be the same as if the instrument were accelerating at  $1g$  ( $9.81 \text{ m/s}^2$ ) along its axis in the positive direction, without the action of gravity. Similarly, if the accelerometer is aligned with its positive axis pointed vertically downwards, the voltage output will be the same as if the instrument were accelerating at  $1g$  along its axis in the negative direction. If the accelerometer is aligned with its positive axis oriented horizontally, there will be no static deflection of the internal spring-mounted mass, and zero voltage output. These observations can be used to calibrate the accelerometers, since gravitational acceleration is accurately known.

The voltage output of the accelerometers is recorded on a two-channel oscilloscope (record the manufacturer and model number to present in your report). To calibrate the accelerometers, set the sensitivity of each channel to  $500 \text{ mV/division}$  by adjusting the knob labeled “VOLTS/DIV” for each channel. The current value of sensitivity for each channel is displayed at the bottom of the screen (e.g., CH1 500mV CH2 500mV), and this value gives the scale on the vertical axis (the voltage difference between two horizontal gridlines). The vertical position of each trace can be adjusted using the knob labeled “POSITION” for each channel. Before taking measurements, it is convenient to adjust each trace to coincide with one of the horizontal gridlines, so that changes in voltage can be more easily computed. To calibrate the accelerometers, record the voltage when the accelerometer is hanging in its initial vertical position (this measurement can be set to zero and used as a reference for the other measurements), when it is oriented horizontally, and when it is oriented vertically upside-down. Record these three voltages for both accelerometers; these values will be used for determining the calibration factors.

Also record the manufacturer and model number of the accelerometers to present in your report.

#### Acceleration Measurement

To determine the relationship between the accelerations of the two masses, you will lift the heavier mass until its pulley is about 3 inches from the upper pulley, and then release both masses, allowing them to fall freely until they hit the foam cushions. For these measurements, increase the sensitivity of each channel to 100 mV/division by adjusting the knob labeled "VOLTS/DIV" for each channel (Be sure to account for this change in sensitivity in your calculations). In order to achieve a relatively constant acceleration during free-fall, it is important to minimize the influence of friction by making sure that the cord is properly seated in the pulleys and that the cord is not twisted, and by lubricating the pulleys (using WD40) before running the test. After the masses have hit the cushions, you can press the "RUN/STOP" button to freeze the traces on the oscilloscope, and you can print the display by pressing "HARDCOPY". You will need the printout to determine the ratio between the accelerations of the two masses. Be sure to press "RUN/STOP" again to resume plotting by the oscilloscope before running another test. To ensure that the accelerations during free-fall are captured well, it is good to wait until the oscilloscope trace is near the left-hand edge of the screen, and to press "RUN/STOP" quickly after impact so that the trace is not overwritten. Perform the experiment three times with three different values of mass attached to the heavier side; in each case, measure the mass on each side of the pulley system using the electronic scale.

### LABORATORY REPORT REQUIREMENTS

1. The laboratory report should have a cover page listing the names of each group member, along with the signature of each member. Each group member must contribute to the preparation of the report, so you should decide how to divide the work or agree on a time to meet together to work on the report.
2. Make a schematic drawing of each system showing the coordinates used to represent the displacement of the points of interest. In cases where two pulleys rotate about a common axis, the two pulleys should be drawn with different diameters so that the path of the cord is clearly shown. Prepare your drawing neatly using either computer drawing tools or using a straight edge and circle templates.
3. Describe the experimental procedures used (including the displacement measurement, accelerometer calibration, and acceleration measurement) and the equipment used, giving manufacturers and model numbers for the oscilloscope and accelerometers
4. Derive the constraint equation for each system that relates the displacements of the points of interest. Constants representing physical dimensions of the system (e.g., the radius of a pulley) can be grouped together as in Sample Problem 2/14 in the text, and numerical values for these constants do not need to be reported. For system 1, discuss the influence of the inclination angle on the ratio between the displacements of the points of interest.
5. For system 3, use the constraint equation for displacements to derive an equation relating the accelerations of the two points.

6. For each system, present all displacement measurements in tabular format. Compute the ratio between the displacements of the two points for each of the three (or more) sets of measurements, and compute an average of these values of the displacement ratio. Compare this value with the ratio predicted by the theoretically derived constraint equation. Compute the largest deviation of the three (or more) displacement ratios from the average value, and report this deviation as a percentage of the average value, as a measure of the uncertainty in your measurements. Discuss the sources of experimental error.
7. Use your measurements from tilting the accelerometers to determine the calibration factor for each instrument (in  $g/mV$ ). Present your measurements in tabular format and discuss the method used to determine the calibration factor.
8. For system three, determine the average acceleration of each mass during free-fall by fitting the best flat line through the accelerations measured while the masses were falling, as recorded on the print-out, and scale these voltages to accelerations using the calibration factors determined previously. Present the measured accelerations in tabular format, along with the measured values of mass on each side of the pulley system. Compute the ratio between the accelerations of the two points for each of the three sets of measurements, and compute an average of these values of the acceleration ratio. Compare this value with the ratio predicted by the theoretically derived constraint equation. Compute the largest deviation of the three acceleration ratios from the average value, and report this deviation as a percentage of the average value, as a measure of the uncertainty in your measurements. Discuss the sources of experimental error.