Topology Optimization:
Background and User Manual

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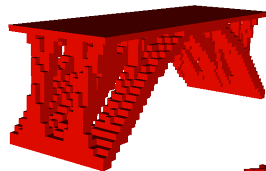

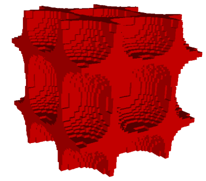

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Introduction to Structural Optimization

Structural optimization is a systematic approach based on mathematics for identifying the *best* solution to a structural design problem.

In mathematics, '*best'* is defined as a minimum (or maximum) of a function $f(x)$ occurring where the derivative is zero ($df/dx = 0$)

But what makes a structural design the '*best*'?

Quantitative: the lightest, the safest, and/or the most economical? Others?

Qualitative: the most visually striking, greatest emotional appeal, and/or best flow of space and light? Others?

It is impossible to rigorously define the '*best'* structural design by a mathematical function $f(\mathbf{x})$.

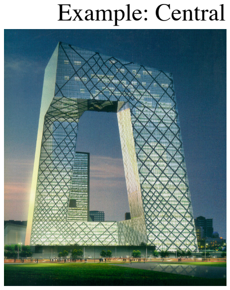
Structural optimization can thus be used as a design tool in the following manners…

A. Sizing Optimization

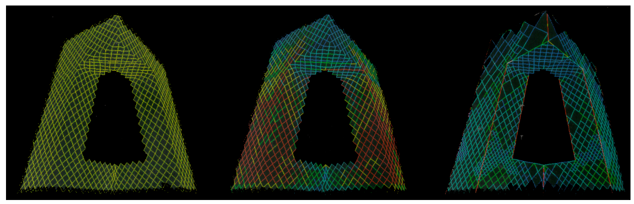
Given a conceptual design and framing plan, identify the optimal sizes of components in that framing plan. For example, minimize weight of a structure while satisfying stress and displacement constraints.

Optimized designs here are ‘optimal’ for the given conceptual design, which has presumably considered the qualitative objectives. However, the true optimality is limited by the appropriateness of the conceptual design.

Example: Central Chinese Television Tower



Conceptual Design



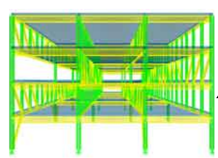
Uniform diagrid   *Stress distribution*   *Optimized diagrid to yield uniform stress distribution*

(images from Riley & Nordenson, 2004)

The diagrid pattern is optimized but is the cantilever form structurally optimal?

Sizing optimization is the most common form of structural optimization

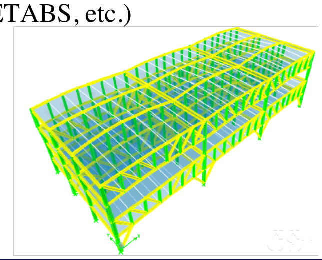
Determine optimal member sizes for a given connectivity



⋮
W18x130?
W18x119?
W18x106?
⋮

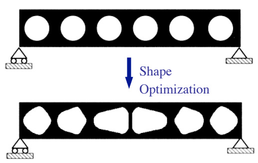
Benefits: Easiest form of structural optimization
Good and practical for optimizing a given design
Many commercial programs do this (ETABS, etc.)

Drawbacks: Requires and is **limited** to given design (connectivity)

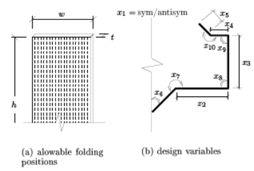


B. Shape Optimization

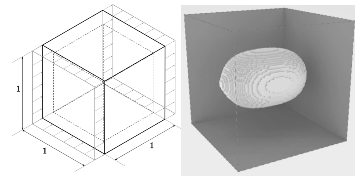
Given a body of fixed volume, identify the optimal shape of the boundary of that body - design the ‘skin’. For example, design the wing of an airplane.



(from Bendsøe & Sigmund 2003)



Optimizing folds in cold-formed steel cross-section (from Liu, Igusa, & Schafer 2004)

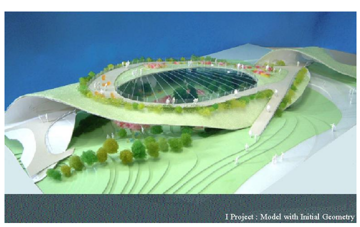


Minimizing drag on a body in a flow field (from Challis & Guest 2010)

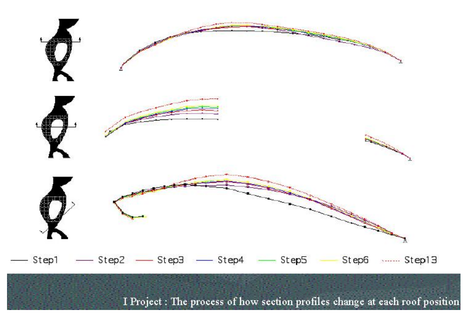
Benefits: Good for materials that are easily folded
Good for shaping the “skin” of structures
Some commercial programs do this (Abaqus, etc.)

Drawbacks: Restricted to the given body
Can’t introduce structural features (holes, bracing) into domain

Shape Optimization: applied to concrete shells



Given shell with designed opening,
optimize shell height to minimize strain energy.
Design iterations:



Courtesy of
Prof., Dr. Eng., Mutsuro Sasaki
Hosei University, Japan

C. Topology Optimization

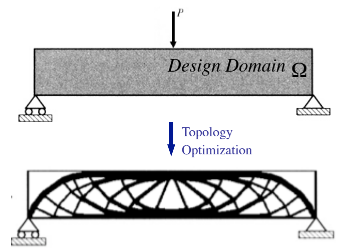
Given a design space, identify the optimal distribution of material within that space. In other words, find the optimal connectivity, size, & shape of features.

The algorithm is thus given a *blank canvas* and a material 'dropper' to *place material at any point in space* within that canvas.

Topology optimization is closest to 'conceptual design' and can be used to generate ideas for structural form. Qualitative objectives can be incorporated through designer interaction and tweaking optimized solutions.

Benefits: Most general approach
Good for solution of new problems
Good for finding the true optimal solution (not limited to given connectivity, etc.)

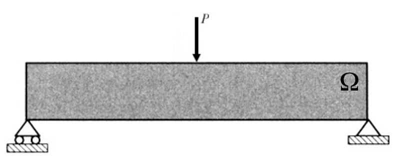
Drawbacks: Most difficult to solve
Very few programs available



Introduction to Topology Optimization

The topology optimization design problem

Given the design domain Ω, loads, and boundary conditions, find the optimal layout of material subject to design constraints (*e.g.*, limited volume *V*, allowable deflection, *etc.*)



Design problem: maximize stiffness of a reduced weight beam (50% void ratio)

Conventional low-weight design
(*from Bendsøe and Sigmund 2003*)

Topology optimized design is 40% stiffer for same weight (*from Guest et al. 2004*)

Introduction to Topology Optimization

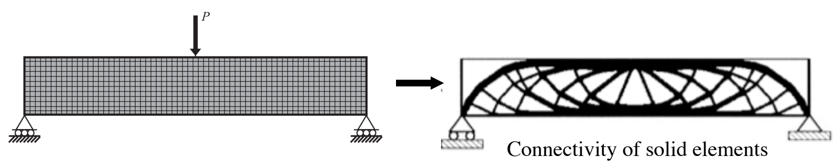
Origins of topology optimization

The Limits of Economy of Material in Frame-structures (Michell 1904)

Material distribution method (Bendsøe & Kikuchi 1988)

Discretize the continuum design domain and determine whether each element contains material (a solid) or does not contain material (a void). Define a element volume fraction ρ^e such that:

$$\rho^e(\mathbf{x}) = \begin{Bmatrix} 1 & if \;\; \mathbf{x} \in solid\ element \\ 0 & if \;\; \mathbf{x} \in void\ element \end{Bmatrix}$$



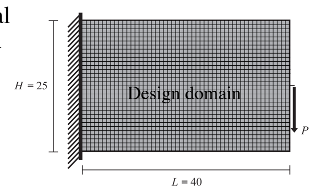
Discretize the design domain Ω

Connectivity of solid elements defines the structural geometry

Example: Maximum Stiffness Design

We can maximize stiffness for given volume of material *V* by minimizing the external work done by the applied loads (*force*displacement*). Or equivalently minimizing the strain energy in the structure.

This is called the minimum compliance problem.



cantilever beam problem

Optimization formulation in finite element notation:

(compliance)

structural stiffness matrix

displacements

applied loads

$$\min_{\boldsymbol{\rho}^e, \mathbf{d}} \quad \mathbf{f}^T \mathbf{d}$$

subject to $\mathbf{K}(\boldsymbol{\rho}^e)\, \mathbf{d} = \mathbf{f}$ (equilibrium condition)

$$\sum_{e \in \Omega} \rho^e \, v^e \leq V \quad \text{(volume constraint)}$$

$$\rho^e(\mathbf{x}) = \begin{Bmatrix} 1 & if \;\; \mathbf{x} \in solid\ element \\ \rho^e_{min} & if \;\; \mathbf{x} \in void\ element \end{Bmatrix}$$

(design variable constraints)

where

$\mathbf{f}$ = *applied nodal forces*

$\mathbf{d}$ = *nodal displacements*

$\mathbf{K}(\boldsymbol{\rho}^e) = \underset{e}{\mathrm{A}}\ \mathbf{k}^e(\rho^e)$

$\mathbf{k}^e = \rho^e\, \mathbf{k}^e_0$

v^e = *volume of element e*

V = *volume of material available*

ρ^e_{min} = *small positive number*

Challenges in Topology Optimization

We solve this problem using optimization algorithms. However, these problems are **extremely difficult** to solve!

1. Large design space
   - Proof-of-concept: ~ 10,000-65,000 design variables
   - Real design: > 100,000 design variables

3. Discrete (makes derivatives hard to come by)
   - Design variables are binary (or integer)
   - Constraints may be discrete

4. Nonlinear - simplest are bilinear in design and state variables

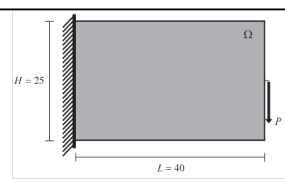
5. Continuous relaxed versions are nonconvex, meaning lots of peaks and valleys, making it impossible to find the absolute best solution.

6. Structural stiffness problems are ill-posed, leading to numerical instabilities

Details will not be presented but be aware that we can sometimes get stuck!

(see Bendsøe and Sigmund 2003 for a review of these issues)

Design Progression



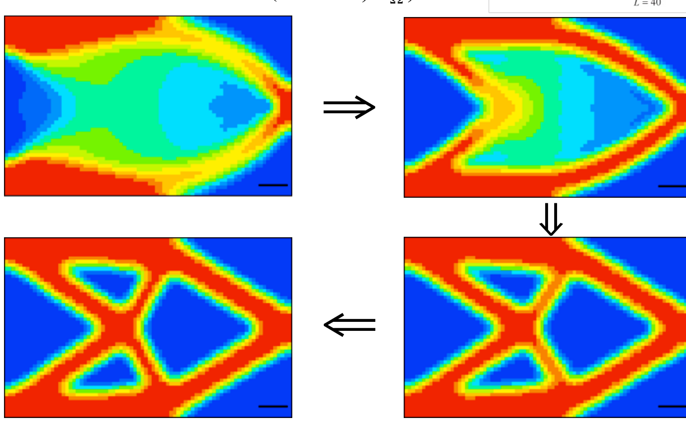
Cantilever Beam Example

Desired volume fraction = 50% $(V = 0.5\,)V_{\Omega}\,)$

Stage I ⇒ *Stage II*

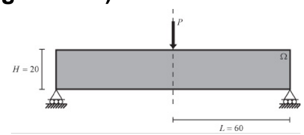
⇓

Stage V ⇐ *Stage III*



We can also control the minimum feature size (length scale)

There is an option in the program to input the minimum length scale (diameter d_{min}) of structural members. The *smaller the length scale the more intricate and stiffer the design*. However, smaller length scales require finer meshes, increasing computational run time.



$d_{min} = 4.0, C = 1.80\alpha$

$d_{min} = 2.0, C = 1.76\alpha$

$d_{min} = 1.5, C = 1.73\alpha$

$d_{min} = 1.0, C = 1.70\alpha$

$\alpha = P^2/E$

The HPM Program for Topology Optimization

We have provided a simple topology optimization demonstration program for minimum compliance (maximum stiffness) design.

Note: to make this simple to use on a Windows PC, we have had to sacrifice significant speed… so run times will be longer than usual.

The program is based on the Heaviside Projection Method (HPM) for topology optimization (Guest et al. 2004) and uses SIMP (Bendsøe 1989) as the material interpolation model and the Method of Moving Asymptotes (MMA) as the optimization solver (Svanberg 1987).

Note: This software is meant for educational purposes only. Users are asked to reference the following work if they publish results using this software:

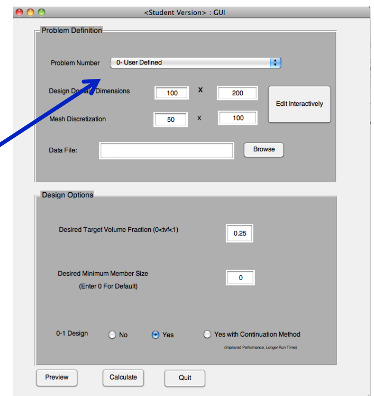
Guest JK, Prévost JH, Belytschko T. (2004). Achieving minimum length scale in topology optimization using nodal design variables and projection functions. *International Journal for Numerical Methods in Engineering* **61**(2): 238 - 254.

Running the executable…

Note: If you have not previously installed Mathwork's MCRInstaller you will need to do this before running the executable. Open the *MCRInstaller.exe* file and follow the on-screen instructions – this is a one-time process that installs MATLAB compiled libraries on your computer.

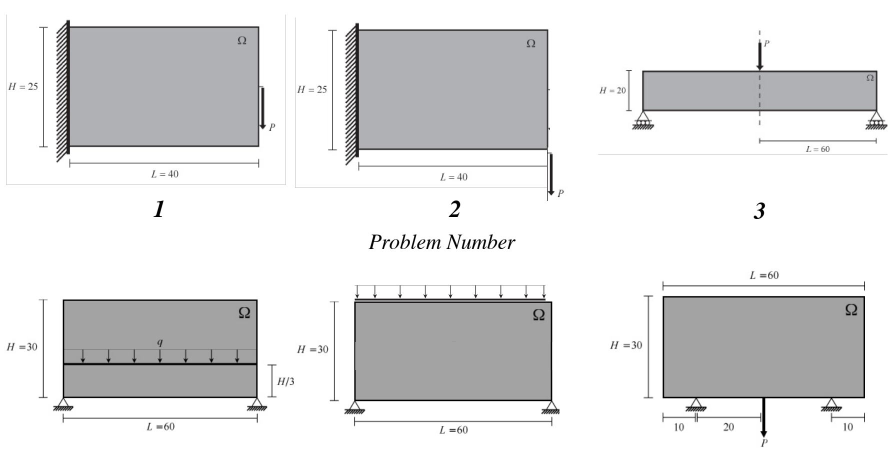
1. Open the executable HPMtopopt.exe
   This should open the window at right.

2. The first step is to choose the problem you would like to solve from the drop-down menu:
   - Select 0 to enter a new problem
   - Select 1-6 to solve a pre-programmed problem (shown on next page)



Running the executable…

Pre-programmed design domains 1-6:



1 *2* *3*

Problem Number

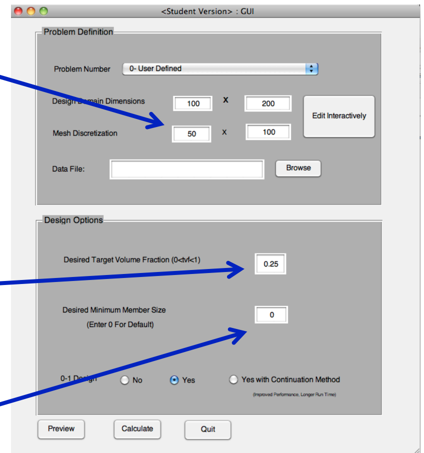
4 *5* *6*

Running the executable…

3. Enter the desired finite element mesh discretization (number of finite elements to be used in the x- and y- directions). The program uses 4-node quadrilateral elements. More elements means smoother boundaries but increased computational expense!

4. Enter the desired allowable volume fraction of material (e.g., 0.30 = 30% of domain may contain material). This will be a design constraint.

5. Enter the desired minimum allowable structural member size (length scale). Enter '0' to use the smallest possible for the given mesh discretization. (see slide 13, Guest et al. 2004 for examples)



Running the executable…

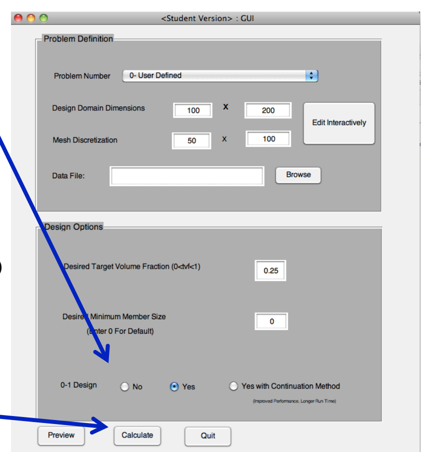
6. Enter whether a discrete solution is desired.

   No = functionally graded structures are desired (e.g., stage I of slide 12).

   Yes = 0-1 solution is desired (e.g., slide 13).

   Yes with Continuation Method = 0-1 solution desired and structural performance is preferred over computational expense. The exponent will be slowly increased (as in slide 12) to avoid local minima.

7. If solving a pre-programmed problem, select 'Calculate' to run the program. Else, continue…



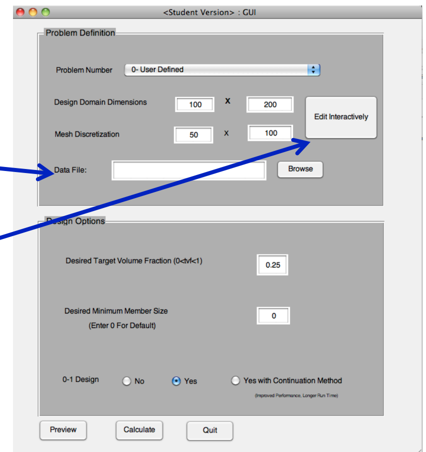
Running the executable…

8. Entering a new design problem requires specifying the applied loads and supports boundary conditions. This can be done using either:
   - A datafile (see enclosed example). Enter name of datafile here.

     The datafile must be in the same directory as the executable.

   OR
   - Interactive Menus.

     Choosing this option will open a new window where loads and support locations may be entered.

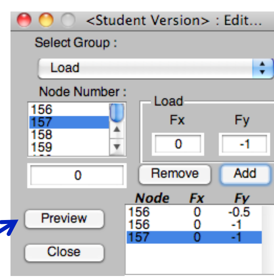


Running the executable…

Interactive Option

Add a load

Enter/Select the node number and the load magnitude in the x- and y-directions



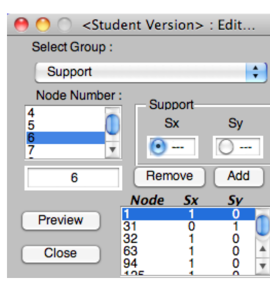
*Nodes are numbered from left to right, bottom to top. So the bottom left node is number 1 and the top right node is (nex+1) * (ney+1).*

Preview the design problem

Current loads and supports

Add a support

Enter/Select the node number and the degrees of freedom that are restrained (x- and/or y-displacements).



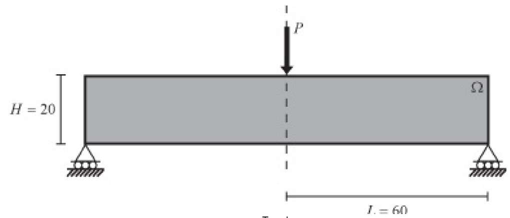
Once completed, choose 'Close' and then 'Calculate'

Running the executable…

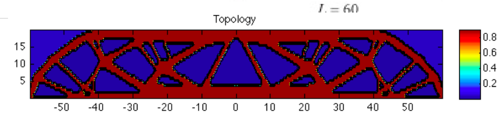
Note: **You must enter support boundary conditions that will yield a kinematically stable structure. If your support boundary conditions are not appropriate, you may see the following error:**

Warning: Divide by zero.

The optimization iteration history and design progression will be displayed.



A final solution for problem 3 is shown here:



Optimized topology - color bar shows coloring for magnitude of element volume fraction (0=void, 1=solid, in between is a mixture of the two)

Please send comments, questions and bugs to: topopt@jhu.edu

Acknowledgements

- Reza Lotfi, JHU Graduate Student, for his assistance in developing the MATLAB GUI.
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IABSE Foundation

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