From: Experiments in Gothic Structure by Robert Mark

parametric term applies to both the full-scale prototype (represented by the subscript p) and the scaled model (represented by subscript m):

$$f_p \cdot L_p/F_p = f_m \cdot L_m/F_m \qquad (2)$$

and this identity yields:

$$f_p = f_m \cdot (F_m/F_p)^{-1} \cdot (L_m/L_p) \qquad (3)$$

where

(F_m/F_p) = loading scale factor (model to prototype)
(L_m/L_p) = dimensional scale factor (model to prototype)

Note that equation (3), relating the prototype to behavior observed in the model, applies to any ratio of model-to-prototype loading. The model's loading is merely limited to a range that will preserve it from appreciable distortion. Hence the magnitude of the loads may be selected to yield convenient experimental data. An example of the application of the scaling law to analyze a structural element of Amiens Cathedral is given at the end of the following section.

PHOTOELASTIC MODELING

The development of reliable, accurate experimental stress analysis techniques was given impetus during the Second World War by the demand for increasingly sophisticated aircraft components and in the decades after the war by the requirements of the nuclear power industry. Probably the most important single advance came with the introduction of electric strain gauges, which can very accurately measure response in both a full-scale structure and a model. Yet because gauges provide information only at the points where they are applied, they are not as advantageous for structural modeling as the somewhat less precise but far more informative full-field optical methods. These techniques give a clear view of *overall* structural behavior and facilitate recognition of previously unsuspected critical regions within the structure. Moreover they permit the investigation of highly localized stress concentrations even with small models. These advantages have led to the refinement and simplification of the most venerable, but probably the most powerful, of the optical modeling techniques, photoelasticity, based on observation in polarized light of transparent models.[18]

By the mid-1960s the advantages and the general reliability of photoelastic modeling for the analysis of mechanical components was well understood and accepted. The 1964 edition of the *Boiler and Pressure Vessel Code*, published by the American Society of Mechanical Engineers, specified that the technique was to be used for the structural analysis of nuclear reactor containment vessels.[19] Yet photoelasticity had rarely been used in this country to study building structures until we began, as part of our research using small-scale models, to apply it to determine the structural behavior of thin-shell concrete roofs,[20] and following the success of this experience, to apply the technique in the analysis of a series of Gothic buildings.

Photoelastic modeling is applicable for planar (two-dimensional) or three-dimensional structural studies. The model loading method described is that generally used for three-dimensional models; however its advantages led also to its use for the two-dimensional models discussed in this text.

All the models are fabricated from stress-free epoxy plastic. Cast sheets are used for the two-dimensional models, which are then formed using a contour router.[21] More complex three-dimensional elements, such as the vault models formed from cylindrical castings as described in chapter 8, are shaped with conventional, metal-working machine tools. The overall size of the planar model sections is about 25 cm (10 in) square for the smaller buildings and about 40 cm (16 in) square for the larger ones. The size of the base enclosed by the three-dimensional model used for the vault study is 28 cm (11 in) square.

Following their fabrication, the models are loaded with weights scaled to represent the loadings acting on the full-scale building. Figure 15 illustrates the technique for dead-weight loading of a two-dimensional model. In the figure, the model of Beauvais Cathedral is partially loaded to represent the effect of vault, buttress, and upper pier loadings (the effect of the weight of the lower portions of the piers was accounted for analytically). To simplify the experiment, the total load applied to the Beauvais model, 4.40 kg (9.70 lbs), was inverted, that is, the model was placed in tension rather than in the compression to which the actual structure is subjected. The *distribution* of forces within the model is exactly the same whether it is in tension or compression, and the additional bracing needed to prevent possible out-of-plane buckling of the thin model under compression can be dispensed with. The analyst need only bear in mind that, with the inverted loading, compression in the model corresponds to tension in the actual building.

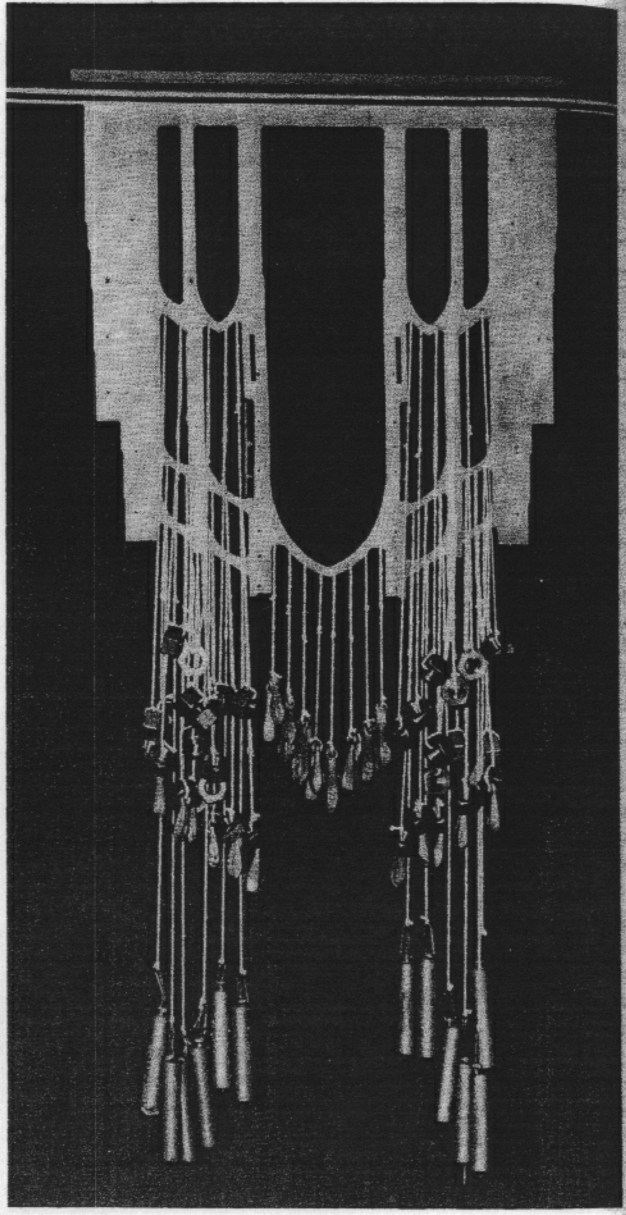
The actual, distributed deadweight and wind loadings are considered to act at discrete points. Figure 14 illustrates how the wind loading acting on a bay of Amiens Cathedral is reduced in scale and simulated by an array of discrete loadings—with the total loading chosen to produce good optical activity without unduly distorting the model. The model loads shown in this figure, representing 1:135,000 of the full-scale wind loading, are simulated on the model in figure 16.

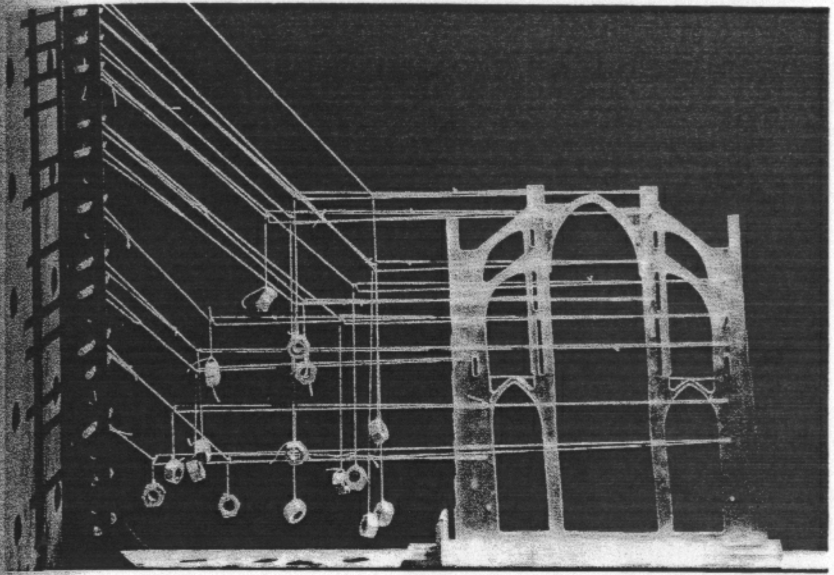
The loaded models are placed in an oven as the first step in a process known as "stress-freezing." They are gradually heated to about 150° Celsius. At this temperature, the epoxy changes from its room temperature "glassy" condition to a "rubbery" state and the loadings cause it to deform. These deformations are locked in when the model is slowly cooled, and the epoxy returns to its glassy state. When the model is cool, the loadings are removed and the effect of the locked-in strains can be detected as patterns of light and dark (or of color when a white-light source is used) with the use of a polarizing-light instrument called a polariscope. These patterns can be read as contour maps of strain that, since strain is directly related to force intensity, can be interpreted as showing the force distributions within the model. The maps, or interference patterns, can then be photographed to provide a comprehensive record of the model's behavior.[22] Once examined and photographed, the models can be used again with different loadings because any prior pattern is erased when they are reheated with a new loading array. Three-dimensional models cannot be readily reused, however, because, in order to examine patterns internal to the structure, slices must be cut from the model for observation in the polariscope.

The principles of the polariscope used to observe the interference patterns in the model are illustrated in figure 17 and a polariscope, which I designed with a dual light source (for white or monochromatic green light), is illustrated in figure 18.[23] Under white light illumination each area in the interference pattern is characterized by a distinctive color indicating a specific order of interference (see, for example, plate 2) and representing a different magnitude of force intensity: the color of the dark background field is order zero, blue is order one, magenta is two, and so on. In monochromatic light, the interference pattern appears as successive dark and light lines (see figure 31).

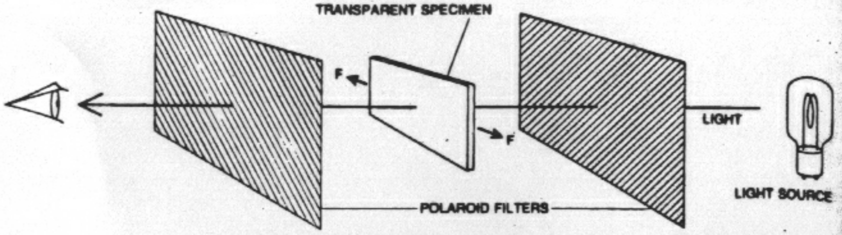
Both the monochromatic and color photoelastic interference patterns may be interpreted both qualitatively and quantitatively. Examples of qualitative determination are: generally regions of highest local stress are indicated by close spacing of the lines (for example, at the base of the Amiens piers where $N = 6.0$ in figure 31); bending patterns are typically characterized by almost parallel lines along the axis of the members[24] (for example, in the piers and flying buttresses of figure 31); and, conversely, freedom from bending

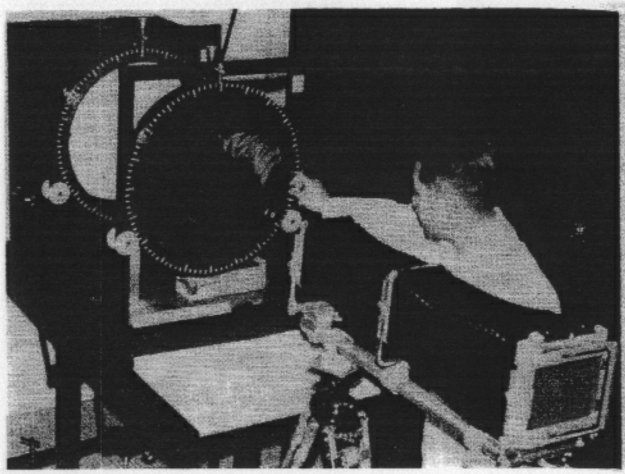
15 Beauvais Cathedral model under simulated partial dead-weight loading. Photo by K. Bakhtar.





16 Amiens Cathedral model under simulated wind loading. Vertically hung weights attached to 45-degree spring lines produce the horizontal model loadings. Photo by M. Goro, *Life*.





17 Principle of photo-elastic observation. The stretched, transparent specimen (acted upon by forces, F), placed between crossed polarizing filters, displays to the viewer an interference pattern related to stress distribution.

18 A 45-cm-diameter field, dual light-source polariscope designed by the author. Photo courtesy of the Naval Civil Engineering Laboratory.

is indicated by the absence of lines (for example, almost pure axial loading (compression) is indicated in the piers of Palma Cathedral in plate 10).

For quantitative analysis, photographs of the interference patterns permit the analyst to identify and give fractional numerical values to points that lie between the interference contours (this is best accomplished with the monochromatic patterns), thereby refining the model data. The intensity of force at any point on the model (f) is then derived by multiplying the order of interference (N) by a calibration factor (K) obtained from a specimen of the model material subjected to a known force distribution under the same test conditions as the model.[25]

An illustration of the application of these formulations is given by the analysis for stress in the Amiens pier buttress under wind loading (figure 31). Since the force along the edge of the model (f_m) is given by multiplying the interference order (N) by the calibration factor (K), equation (3) may be rewritten as

$$f_p = N \cdot K \cdot (F_m/F_p)^{-1} \cdot (L_m/L_p) \qquad (4)$$

The stress at a point in the buttress is then derived from dividing the force per unit length (f_p) by the width of the buttress at that point. At the point in the pier where the corresponding model interference order (N) is 2.0—and from calibration it is known that the model material has a sensitivity (K) of 0.360 kg/order-cm (2.01 lbs/order-in)—the model loading scale factor is 1:135,000, and the dimensional scale factor is 1:136, equation (4) yields f_p = 715 kg/cm (3990 lbs/in). The stress in the pier buttress, which is 180 cm (70 in) wide at this point, is therefore 715 kg/cm divided by 180 cm, yielding 4.0 kg/cm² (57 psi). Figure 32, which shows the forces along the entire edge of the pier buttress of Amiens Cathedral, was derived in this manner.

The techniques described are those that would be employed in the engineering analysis of complex modern structures. Such analyses are based on detailed knowledge of the building's configuration, the behavior of its construction materials, and the environmental conditions to which it is subject. Although quantitative methods of this type are not usually applied to the study of historic architecture, the accounts that follow demonstrate the power of the methodology particularly for buildings having the structural complexity of a High Gothic cathedral.

NOTES

An introductory explanation of the basic behavior of structures and their analysis is given in the glossary. Recommended simple texts on this subject, in order of increasing analytical complexity, are Mario Salvadori and Robert Heller, *Structure in Architecture* (Englewood Cliffs, N.J.: Prentice-Hall, 1963); William Morgan, *The Elements of Structure* (London: Pitman, 1964); and Daniel L. Schodek, *Structures* (Englewood Cliffs, N.J.: Prentice-Hall, 1980).

For further information on photoelastic modeling and other optical stress analysis techniques, see James W. Dally and William F. Riley, *Experimental Stress Analysis*, 2nd ed. (New York: McGraw-Hill, 1978); on dimensional analysis, see P. W. Bridgeman, *Dimensional Analysis (New Haven: Yale University Press*, 1931); and on wind loading, see Emil Semiu and Robert H. Scanlan, *Wind Effects on Structures* (New York: Wiley-Interscience, 1978).

1 Studies of the cathedrals of Chartres and Metz based on the general approach developed in this chapter but using finite-element computer modeling, as described in chapter 8, were carried out by Lutz Kübler ("Computeranalyse der Statik zweier gotischen Kathedralen," *Architectura* 4.2 [1974]:97–111), who reported good agreement with our earlier photoelastic model study of Chartres.

2 See Ronald C. Smith, *Materials of Construction* (New York: McGraw-Hill, 1973), p. 165. Henri Masson ("Le Rationalisme dans l'Architecture du Moyen Age," *Bulletin Monumental* LXXVI [1912]), reports tests of medieval mortar giving tensile strengths of 2 kg/cm² (30 psi).

3 John Fitchen ("Appendix G: The Slow Setting Time of Medieval Mortars and its Consequences," *The Construction of Gothic Cathedrals* [Oxford: Clarendon Press, 1961], pp. 262–265) cites the lowest figure, "a minimum of six months," from H. and E. Ranquet, "Origine française du berceau roman," *Bulletin Monumental* XC (1931):45. The highest is contained in the claim that "in walls of great thickness, centuries can pass before the final set is acquired" (Pol Abraham, "Les données plastiques et fonctionnelles du problème de l'ogive," *Recherche No. 1: Le Problème de l'ogive* [Paris, 1939]:36.

A mortar has hydraulic properties if it will set under water. Roman pozzolana cements and modern Portland cement are hydraulic. Medieval mortars that were made by burning limestone containing clay, blue lias limestone, chalk marls, or grey chalk could have had hydraulic or semihydraulic properties (see Ministry of Public Building and Works, *Lime for Building*, 3rd ed. [London: HMSO, 1970]). The likely medieval building practice of keeping hydrated lime in tubs with a water layer at the top during a period of construction, however, guaranteed that the mortar would lose even semihydraulic properties during storage.

4 Freshly quarried stone is "green" with ground water—"quarry sap," which evaporates during its exposure to the atmosphere. See Robert J. Schaffer, *The Weathering of Natural Building Stones* (Watford: Garston, 1972), p. 15. Rates of absorption for bricks and building stones vary according to their natural properties and moisture content; but all absorb considerable amounts of water and set a mortar under normal circumstances. See Sven Sahlin, *Structural Masonry* (Englewood Cliffs, N.J.: Prentice-Hall, 1971), pp. 12ff.

5 Sahlin, *Structural Masonry*, p. 19.

6 For a detailed discussion of the chemistry, see Frederick M. Lea, *The Chemistry of Cement and Concrete*, 3rd ed. (London: E. Arnold & Company, 1970), pp. 29ff.

7 Sahlin, *Structural Masonry*, pp. 52–56.

8 Robert Mark, "Photomechanical Model Analysis of Concrete Structures," in *Models for Concrete Structures* (Detroit: American Concrete Institute, 1970), pp. 187–214.

9 Solid dressed stone was used for the construction of many critical structural elements such as the flying buttresses and the piers of High Gothic churches. Some structural elements, however, were formed of a rubble and mortar core encased in dressed stone. For purposes of modeling, this distinction has not been taken into account as doing so would not greatly affect the resulting force distributions.

10 The true action of the ribbed vault is partially accounted for by deepening the modeled rib in order to have the same scaled stiffness as the total of all the transverse and diagonal ribs meeting the pier extension, each rib being attached to a strip of web four times its width.

11 This phenomenon also underlies the use of models made of certain viscoelastic plastics in order to predict elastic response. See Raymond D. Midlin, "A Mathematical Theory of Photoviscoelasticity," *Journal of Applied Physics* 20 (1949):206–216.

12 In this country, velocity data are now usually given by the quantity "fastest mile," the maximum velocity of a mile-long column of air passing a reference point. This is done to represent better the mean wind conditions and to make this information easier to combine with the gust factor in equation (1).

13 Alan G. Davenport, "The Relationship of Wind Structure to Wind Loading," in *Wind Effects on Buildings and Structures* (London: HMSO, 1963), pp. 53–103.

14 N. Chien, Y. Feng, H. H. Want, and T. T. Siao, "Wind Tunnel Studies of Pressure Distribution on Elementary Building Forms," Institute of Hydraulics Research, University of Iowa, Ames, Iowa, 1951.

15 Alan G. Davenport, "The Treatment of Wind Loads on Tall Buildings," *Tall Buildings*, edited by A. Coull and B. Stafford Smith (New York: Pergamon Press, 1967).

16 As with finding force distributions (rather than local stress distributions) from plastic models of reinforced concrete structures, these criteria may be interpreted as *gross* behavior. The best check on model-prototype compatibility is derived from comparing model predictions with measured prototype performance. This has been done with reinforced concrete structures (see note 8). No similar measurements are available for medieval masonry construction, but subsequent confirmations by on-site observation of model predictions of local tension provides a measure of confidence.

17 The force intensity (f), called a "stress resultant" in texts on shell theory, is defined as the integral of the stress over the thickness. Since the stress in a planar model is constant through the model thickness, it is most convenient when using optical models to measure this quantity directly and then to determine the model stress, if needed, merely by dividing f by the model thickness.

18 The photoelastic effect was first reported in the early nineteenth century by Sir David Brewster in *Philosophical Transactions* (1815), p. 60. James Clerk Maxwell read a paper in 1850 before the Royal Society of Edinburgh in which he described a photoelastic experiment using "jelly of isinglass" as a model material. Photoelasticity, however, did not become a practical engineering tool until the 1930s with the development of plastics that were optically sensitive and easily formed into models.

19 American Society of Mechanical Engineers, *ASME Boiler and Pressure Vessel Code*, Section III, "Nuclear Vessels" (New York: 1965), p. 107.

20 David P. Billington and Robert Mark, "Small Scale Model Analysis of Thin Shells," *Journal of the American Concrete Institute* 62 (June 1965):673–688.

21 In this process, an aluminum template of about 0.3 cm (one-eighth in) thickness is first fabricated. The template is then attached to the roughly-cut-to-form plastic sheet with double-sided contact tape in preparation for routing.

22 Monochromatic interference patterns were photographed using 102 × 127 mm (4 × 5) back camera with a long-focal-length process lens and an interference filter. Typical exposure time is 40 seconds at f/32 with Polaroid 3000 ASA film. For color photography, typical data are one-second exposure at f/8 with 25 ASA color film using the unfiltered white light source and a 35 mm camera.

23 Further instrument details may be found in Robert Mark, "Dual Light Source for a Large Field Diffused Light Polariscope," *Review of the Scientific Instruments* 35 (April 1964): 521–522.

24 The identification of tension or compression resulting from bending can usually be detected simply from the curvature of the model's edge (convex signifies bending tension; concave indicates compression). A more general method for determining tension or compression is based on observing the shift of the model interference patterns while rotating one of the polarizing filters of the polariscope (August J. Durelli and William F. Riley, *Introduction to Photomechanics* (Englewood Cliffs, N.J.: Prentice-Hall, 1965), pp. 92–94). This determination must be made directly with the model in the polariscope; the interference pattern photographs themselves do not contain sufficient information to indicate if the model forces are compressive or tensile.

25 This relationship applies only along the boundary of a member. But since the boundary forces in structures are critical in practically all instances, this limitation need not concern us. The dimensional units of the boundary force (f) are force/unit-length. Hence the units of the calibration factor (K) are force/order-unit-length. Calibration techniques are described in Dally and Riley, *Experimental Stress Analysis*.

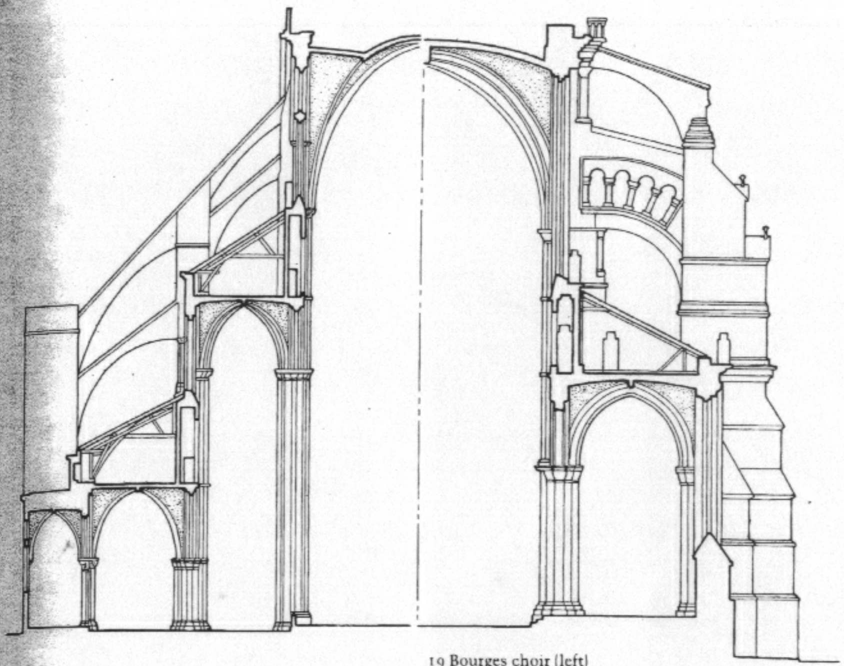
3

THE BEGINNING OF HIGH GOTHIC: THE CATHEDRALS OF CHARTRES AND BOURGES

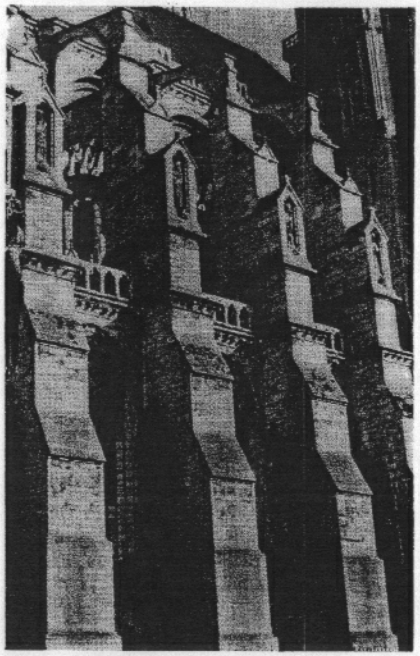
Construction of the cathedrals at Chartres and Bourges began almost simultaneously: 1194 for Chartres and 1195 for Bourges. At Bourges construction proceeded in the usual manner, from east to west. The choir was completed in 1214, although other construction was not completed for almost a century. The much more rapid pace of construction at Chartres brought work on the entire main vessel of the cathedral to a close in 1221.[1] Dimensions of the buildings are very similar; the height of the vaults of both is about 46 m (118 ft). But Chartres has three aisles and a transept between the nave and choir, whereas Bourges has five continuous aisles and no transept.

Chartres is a very impressive building, particularly in its details, and it has always been greatly admired. After some initial resistance, its form was accepted as the standard for Gothic church design, ending the period of wide experimentation with church building forms that characterized the twelfth century.[2] On the other hand, although Bourges has always been esteemed for its imposing size and beauty (plate 1), it has never attracted such an extensive following. The importance of Chartres is implicit in the emphasis placed on it in the literature on the Gothic cathedral. Bourges, often mentioned as an interesting footnote, has been the subject of only one complete modern study, by the architectural historian Robert Branner.[3] The main reason for the ascendancy of Chartres, according to Branner, was that it was imitable: its design could be replicated at any scale to suit almost any site; the Bourges spatial scheme could only be adopted whole and at a very large scale.

When Chartres and Bourges were in construction, exposed flying buttresses were a relatively new device. Although they were probably first employed before 1180 at Notre Dame in Paris, their full potential for allowing a great reduction of clerestory wall was only realized at Chartres and Bourges. Cross sections of the two buildings, however, reveal that their designers employed very different forms of buttressing (figure 19).[4] At Chartres



19 Bourges choir (left) and Chartres nave (right). Comparative cross sections.



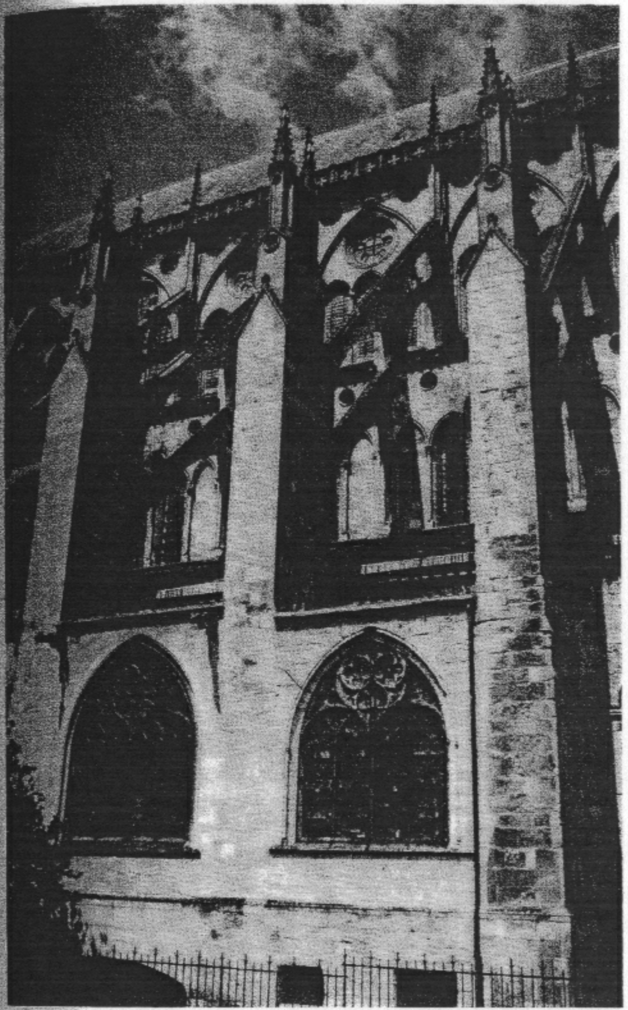
20 Chartres nave buttressing. Flying buttresses are almost hidden by the massive pier buttresses. Photo by J. Hart.

the entire system is massive except for the relatively light upper flyers (figure 20). Each of the tall pier buttresses, exclusive of its foundation, weighs about 1 million kg (2.2 million lbs). At Bourges, on the other hand, a series of fine, steeply sloped flyers is supported by low pier buttresses, each weighing but 400,000 kg (880,000 lbs) (figure 21).

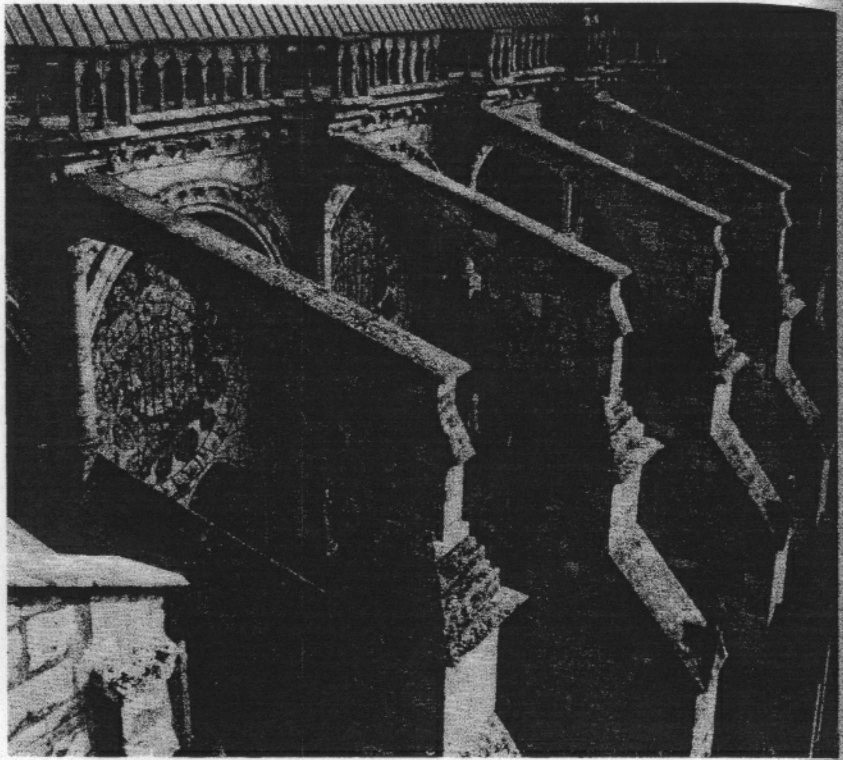
The present chapter is concerned with what lay behind these different approaches to structure in two giant cathedrals built in the same region and at the same time.

THE NAVE OF CHARTRES

As with every other feature of Chartres, much has been written of its structure, particularly its use of flying buttresses. According to Paul Frankl, "the master who rebuilt the cathedral at Chartres . . . was the first man to draw the logical consequences from the construction of flying buttresses."[5] In the same vein, George Henderson writes in a more recent text on Chartres: "The architect . . . recognized in the flying buttress a great new device whereby he might radically reorganize the whole appearance of a church interior, and at one stroke achieve simplicity, unity, coherence. From the moment when he took command of work at Chartres he had flying buttresses and their logical employment in mind."[6] The view of still another influential writer, Otto von Simson, is that "the flying buttresses of Chartres are the first to have been conceived, not only structurally but also aesthetically, as integral parts of the overall design."[7] Almost all the writers, however, question the role of the light upper flyers at roof level (figure 22). Some maintain that the upper flyers are purely decorative. Others believe that they function as structure, although there is disagreement about how they do so.



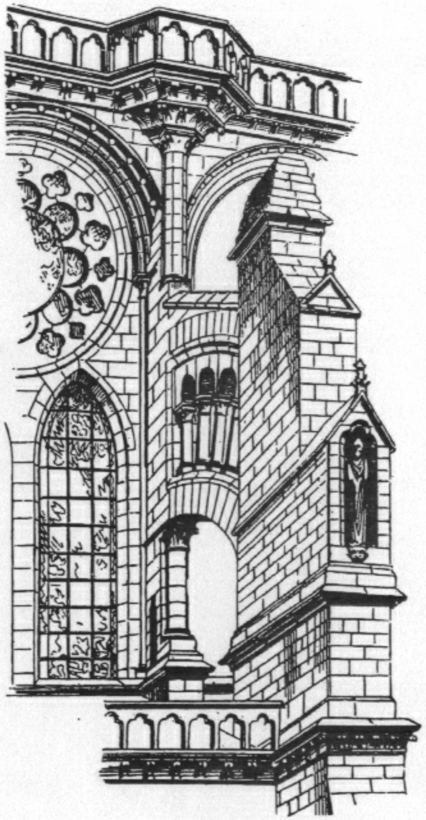
21 Bourges choir buttressing. The lightness of the buttressing system in a building of this size is unique.



22 Chartres nave. The upper flying buttresses were thought to have been added in the fourteenth century.

Viollet-le-Duc appears to have been the first to take the view that the upper flying buttresses at Chartres were not part of the original building and were therefore introduced to solve a problem in the structure that appeared later. In his article on the flying buttress, he discussed only the lower flyers and included a drawing that omitted the upper flyers altogether (figure 23).[8] His argument was based on a document written in 1316 by a group of experts who had gathered to examine the fabric of the century-old cathedral. Viollet-le-Duc and many after him have generally interpreted this document as a recommendation for the construction of additional flying buttresses.

Viollet-le-Duc's interpretation has, however, been challenged by others, including my co-worker, Alan Borg, who carefully reexamined the 1316 document and assembled archaeological evidence to indicate that the upper flyers at Chartres had been part of the original construction.[9] Even if this was the case, the functional reason for their existence still requires explanation. We know that they were not necessary to counteract the dead-load weight of the roof because medieval roof framing is tied between the parapets that support it by stout, pinned timbers. There is, therefore, no outward-acting horizontal thrust from the roof weight as there is from the vaulting. In the absence of this thrust, however, wind action will create lateral loads that are transmitted to the upper portion of the pier extensions. Thus Viollet-le-Duc's argument that the upper flyers were added after 1316 to correct a fault in the original configuration might be substantiated if it could be shown that they significantly reduced local tensile forces in the pier extension caused by wind loadings on the high roof. Tests of the model of the nave were therefore directed to an examination of the response of the pier extensions to wind action—with and without the upper flyers in place.



23 Chartres nave. Buttressing without the upper flyers (after Viollet-le-Duc).

To examine the pier extension response, one-hundred-year meteorological data for the Paris region were obtained (Chartres is 80 km from Paris). A scale model of a typical nave "frame" section of Chartres (as shown in figure 19) was fabricated without the upper flyers and tested under a loading that represented the actual distribution of wind pressure (and suction on the lee side).[10] The internal force distributions were determined from photoelastic observation as described in chapter 2. Following this first test, the model was annealed by heating to restore it to its undeformed condition and the scaled upper flyers were attached with high-temperature epoxy cement. A second test was then made with the same simulated wind loading to produce the interference pattern shown in plate 2.

In the final phase of the analysis, a calculation was made of the stress in the pier extensions caused by the dead-weight loading of the roof and its framing, the weight of that portion of the structure of the bay supported by the pier extensions above the critical sections, and the weight of the heavy longitudinal arches above the clerestory windows. Combining this dead-weight effect with the stresses arising from extreme wind forces gave the maximum stresses that could be anticipated in the structure.

All of the compression values in the buttress region were much lower than typical stresses at the pier bases. Hence they were not critical, and they required no further consideration. Low tensile stresses, however, were indicated at the windward edges of the windward pier extensions just above the middle tier of the flying buttresses at the base of the colonnettes that provide support to the upper flyers. The onset of this tension in the section without flying buttresses was found to correspond to gusts with a mean velocity at rooftop level of 70 km/hr (44 mph). When the upper wall of the clerestory was supported by the flying buttresses, however, no tension appeared until the wind reached 90 km/hr (56 mph). The buttresses were, therefore, effective in improving the resistance of the pier extensions to high winds.

But they were not totally effective. Under the most extreme gust conditions, with mean velocities of 105 km/hr (65 mph) near ground level and 135 km/hr (84 mph) at rooftop level, maximum tensile stress, which reached 4 kg/cm² (57 psi) in the pier extension of the section without the buttresses, still reached 2 kg/cm² (28 psi) in the buttressed section. Although these are relatively low magnitudes, cracking distress in mortar can occur with a tensile stress of only 2 kg/cm². One would expect, therefore, to find evidence of distress in the cathedral's fabric, even with the upper flying buttresses in place, although the relatively low values of the indicated stress and the nature of the bending stress distribution across the section of the pier extension would mean that the distress would most likely be confined to the outermost courses of stone.

This finding was borne out by observation of the cathedral itself. During a site visit in 1971, soon after the analysis was performed, I discovered that stone in the critical region at the base of the colonnettes had recently been replaced. The supervising architect for the cathedral at that time, M. Louis Esnault, confirmed that this was part of a systematic program of repair in that area of the fabric.[11]

It is possible to conclude, then, that in the absence of the upper flying buttresses, the pier extensions would have shown tension under the effect of relatively lower wind velocities and the probability of distress in the piers would certainly have been greater. But, as it has been necessary to replace the stones, the upper flyers do not entirely eliminate the problem of tension. Evidently they are too light a structure to have been a deliberate addition intended to rectify an obvious structural flaw.

Under these circumstances, it becomes difficult to believe that the experts of 1316 would have suggested the addition of this type of buttress. By that date, architects had considerable practical experience with buttressing and they would hardly have proposed the difficult and expensive addition of extra flyers unless they were convinced that they would be effective. It therefore seems more likely that Viollet-le-Duc was in error and that the upper flyers were, in fact, part of the original construction, a conclusion borne out by the archaeological evidence.

The significance of the analysis, however, goes beyond the apparently minor issue of the date and purpose of the upper flyers at Chartres. The findings lead to the conclusion that the architect of Chartres was uncertain about buttressing. This conclusion is corroborated by the entirely different picture that emerges when one examines the choir at Bourges, built at the same time as Chartres but with light, open buttresses that offer a strong contrast to the heavy, even ponderous, system at Chartres.

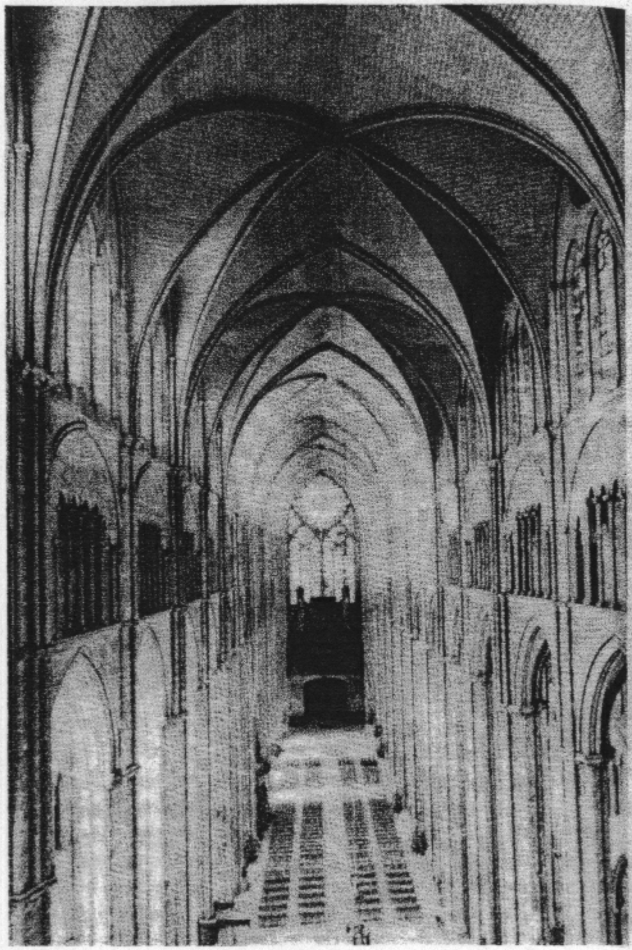
THE CHOIR OF BOURGES

In planning the model test to determine the function and effectiveness of the flying buttresses of Bourges, it was necessary to account for its sexpartite vaulting (figure 24). Unlike the quadripartite vaults of Chartres, which distribute equal loads to all the interior piers, sexpartite vaults transmit alternating high and low loadings to primary and secondary piers along the interior aisle.[12] At Bourges the secondary piers are somewhat lighter than the primary piers, but the remainder of the structural system is the same at both primary and secondary sections. Hence it can be assumed that the structure at a primary section is subjected to greater stress than that of a secondary section. For this reason, only the primary section was modeled. This model was tested under both dead-weight loading (plate 3) and simulated wind loads.[13]

Meteorological data similar to those used for the Chartres studies were not available for Bourges. The best data applicable to Bourges were for Chateauroux, some 50 km to the southwest, for a 10-year period. These data indicate that Bourges is in a more sheltered area than Chartres. The maximum mean wind velocity at the elevation of the cathedral roof was calculated as 105 km/hr (65 mph), compared to 135 km/hr (84 mph) for Chartres. Since wind forces on a building are produced as a function of the square of the wind velocity, the calculated maximum total force acting on each bay of the cathedral was only 55,000 kg (120,000 lbs) for Bourges, significantly less than the figure of 100,000 kg (220,000 lbs) at Chartres.

Under the action of combined dead-weight and wind loading, the stress levels throughout the section were found to be quite low. The highest compression stress, at the base of the main piers, was found to be 21 kg/cm² (300 psi), or about two-thirds of the maximum levels usually found in High Gothic buildings. Part of this reduction is attributable to the lower ambient wind speeds, but even more of it is due to the building's lightweight structure and the broader profile of its transverse section. Indeed the findings are mainly attributable to the boldness of the choir's designer.

Since the choir at Bourges has no upper flyer at roof level, the relatively slender, unsupported, main pier extension was scrutinized for possible signs of tensile cracking. It was seen that the thrust of the vault is entirely carried by the lower of the two tiers of flying buttresses that support the clerestory wall. Hence the higher tier of flyers must have been placed to provide support against roof and parapet wind loading. Why were they not brought up close to the roof as were the flyers at Chartres or, for that matter, the seven piers in the nave at Bourges that were constructed after 1232? The answer



24 Bourges Cathedral. Interior as seen from the nave.

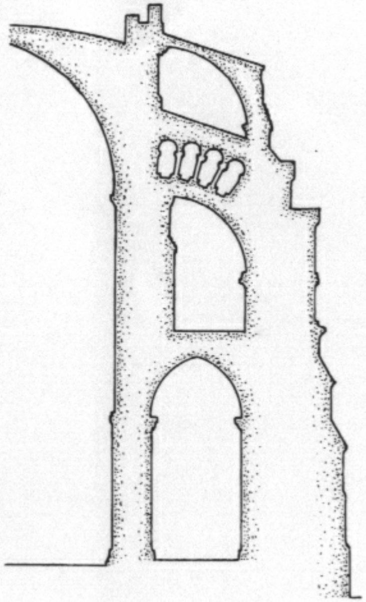
can be seen in the choir clerestory if one examines the intersection of the higher flying buttress with the pier extension (figure 21). At this point of greatest bending, the pier extension is reinforced by the lower part of the parapet to form a stout T-section. The effectiveness of this combination was demonstrated in the model tests, which revealed that the onset of tensile stress—which occurs in the windward upper pier extension just at the top of the upper flying buttress—does not take place until gusts reach a mean wind velocity at rooftop level of 92 km/hr (57 mph). Even under the most severe wind conditions, tension is less than 17 kg/cm^2 (10 psi), well below the tensile strength of the mortar.[14]

A quantitative comparison of the effectiveness of the buttressing systems of Chartres and Bourges can now be made. The onset of tensile stress in the nave of Chartres occurs with a rooftop wind velocity of 90 km/hr. The corresponding value for Bourges is 92 km/hr. It can be concluded that the light structure of the choir at Bourges provides a measure of safety that is comparable to that afforded by the much heavier configuration of the nave at Chartres.

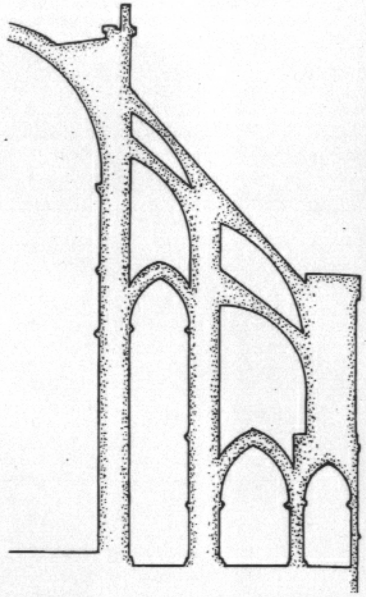
Additional light is shed on the achievement of the architect of Bourges by a structural critique of the Late Gothic church of St. Ouen in Rouen published in 1902 by the French architectural authority Julien Guadet (see chapter 7).[15] He proposed a hypothetical alternate design for St. Ouen in which the original interior configuration is unaltered but the buttressing system is considerably lightened by substituting steeply inclined parabolic arches for the Gothic flying buttresses. Besides requiring less material, the alternative design implies a simpler construction process. Guadet included a graphical force analysis, taking into account the effect of gravity loadings to substantiate his design. But that early method of analysis has certain limitations not shared by the photoelastic model method.[16] A model test of Guadet's configuration under gravity loading was performed, and although some further modifications might be necessary if wind forces were taken into account, the test showed his design to be reasonable.[17]

Guadet's alternate design for St. Ouen can be considered a theoretically more advanced Gothic structural form that represents how the form might have evolved if some techniques of scientific analysis had been available to the builders. Hence it is particularly interesting to juxtapose the sections of the three buildings, which are all of similar size, to reveal an obvious hierarchy (figure 25). The great reduction in the amount of materials used for both Bourges and the hypothetical St. Ouen is achieved by carrying the vault and roof forces more directly to the foundations by raising the angle of the flying buttresses and consequently lowering the height of the pier buttresses. With only primitive machinery available for the quarrying, transportation, shaping, and lifting of the huge stones into place, the 60-percent reduction in weight of the pier buttresses at Bourges must have represented a tremendous economy in construction.

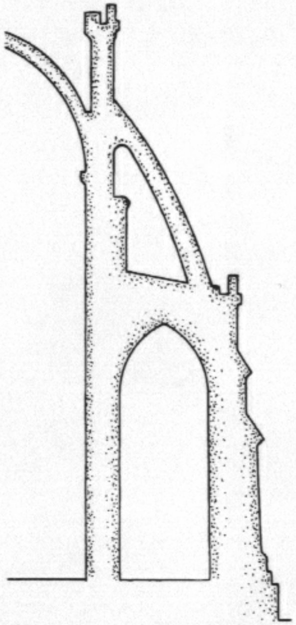
It could be argued that the more efficient buttressing of Bourges, with its highly sloped flyers and low pier buttresses, came about because of the necessity to span an additional aisle with the flying buttresses rather than because of the original master's superior understanding of structural design. But this argument is refuted by reviewing the five-aisled churches that were strongly influenced by Bourges—Branner's so-called "school of Bourges"—the abbey of St. Martin at Tours and the cathedrals of Burgos, Le Mans, Toledo, and Coutances, all designed in the second quarter of the thirteenth century. The extant buildings of this group—all except the abbey of St. Martin—show no appreciation of the sophistication of the structure of Bourges and demonstrate that the technical achievement of the original master of



Chartres



Bourges



St. Ouen (modified)

25 Comparative sections of Chartres, Bourges, and Guadet's St. Ouen indicate a design progression to lighter buttressing.

Bourges could not just have been the fortuitous result of the solution to a five-aisled building program.[18] For example, the bifurcated array of buttresses supporting the hemicycle (the rounded end portion of the choir) at Le Mans, one of the most visually striking structures of the era (figure 26), can hardly be justified on structural engineering grounds when it is compared with the relatively simple buttress system supporting the even higher hemicycle of Bourges.

The only legitimate structural heir to the Bourges choir is the nave of Bourges itself. Constructed between 1225 and 1255 by the successors of the original master, it deserves closer examination as a member of the "school of Bourges," since it is the only case in which the structural example of the choir of Bourges has been closely followed.

THE NAVE OF BOURGES

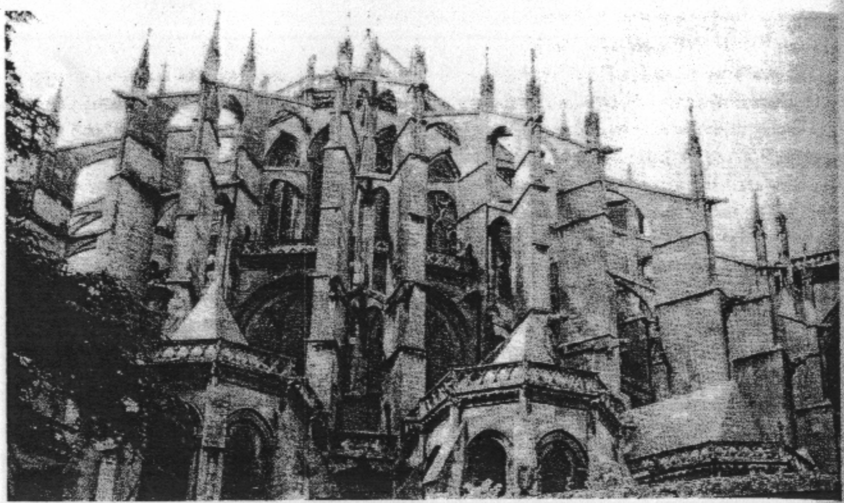
The end of the first phase of construction at Bourges is clearly marked on the exterior as one moves from east to west, first, by the enlargement of the flying buttresses and, second, by a change in the window motifs that permits larger areas of glass in the nave clerestory and side aisles. Construction of the nave in the later campaigns is characterized by increases in the height at which the upper flyers abut the pier extensions and by an increase in the size of three of the four flyers (figure 27).[19] The increased height of the abutment of the upper flyers might be explained on the grounds that it would result in much better resistance to roof and parapet wind loading. Yet the choir section had already proved stable without such a change.

Since the principal difference between the original design of the choir at Bourges and the later nave is the progressively more conservative construction of the flying buttress system, model testing of the nave under wind and dead loads would indicate what, if anything, was to be gained by this change. In a second photoelastic model test,[20] which compared the structural performance of the nave with that of the choir, two important observations resulted. The first demonstrated the inferior structural quality of the design of the nave's system of buttresses compared to that of the choir. The second demonstrated the ability of the modeling technique itself to identify in yet another case highly-localized, critical tensile regions that are verifiable from inspecting the buildings themselves.

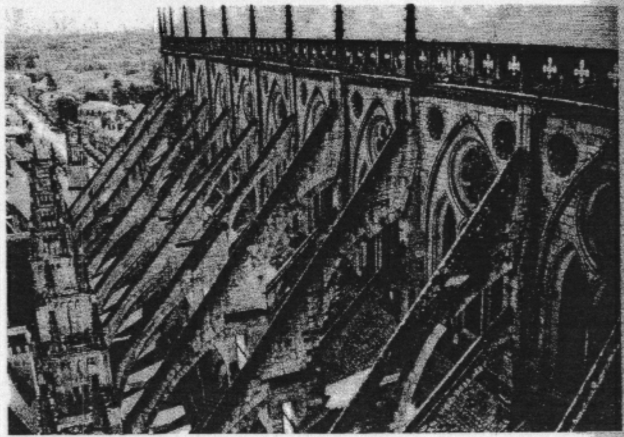
Whatever the impetus to increase the height at which the flying buttress abutted the clerestory, the increase was achieved by making both upper flyers deeper in section. The increased stiffness of these flyers is so great that under both dead and wind loading, only low levels of photoelastic activity were observed in the experiments (figure 28, point *a*). Interpretation of the patterns indicates that the material strength of the flyers is greater than is needed for their task.

The section depth of the lower outer flyer was also increased. This second enlargement may have been made solely for visual reasons. The two outer flyers, the visible pair, have been given similar proportions while the inner flyers, the lower of which is not readily visible from the ground, have not been similarly matched. There is some irony here, since it is the lower inner flyers that receive the thrust of the main vault.

At its heaviest, most conservative section, the flying buttresses of the nave employ approximately 60 percent more stone than the flyers of a typical choir section. In addition, the heavier nave section required passage openings for access to the roof through the intermediate pier buttresses that were unnecessary in the choir (plate 4).[21] The model analysis shows the intermediate pier buttresses to be free of tensile stresses under combined wind and dead loads below the first side aisle vault. Above the vault, though, the passages so weakened the fabric that a



26 Le Mans Cathedral. Choir buttressing is the most elaborate of the era.



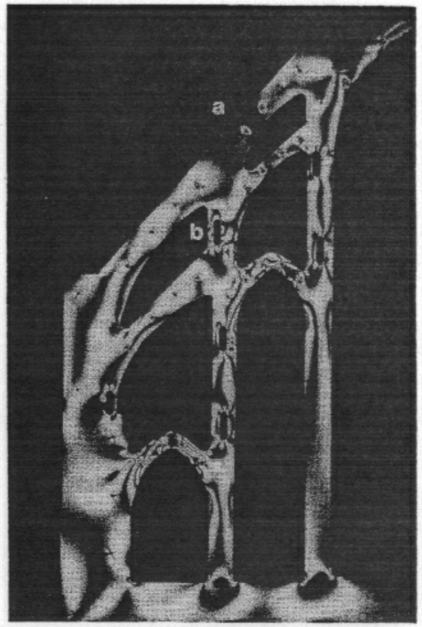
27 Bourges. Clerestory buttressing viewed from the northwest tower. Note the increase in size of the later, western flying buttresses (foreground).

combined wind- and dead-load tensile stress of approximately 4 kg/cm² (57 psi) developed on the outer segment of the intermediate pier buttresses next to the passageway (figure 28, point *b*).

The model's prediction of tensile stress was verified by examining the existing fabric of Bourges, which reveals the systematic replacement of the stone on the outside of these passages. The new stone is evident in plate 4 (this photograph was taken in 1971). It can be concluded, then, that even the enlargement of the flying buttresses in the nave, in comparison with those of the choir, could not ensure a more reliable structure. This change, in fact, produced problem areas that were not present in the original design.

One can speculate that a second Bourges architect, who clearly attempted to maintain the visual pattern of the choir buttresses when he designed the nave, was uneasy over his predecessor's daring because of his own familiarity with the structure of Chartres. He modified the original design by deepening the flying buttresses and raising the point of abutment of the higher buttress against the pier extension. Yet in doing so, he gave up some material economies of the original design for no significant gains in structural performance.

Thus, although Chartres made a major aesthetic contribution by becoming the model for the great High Gothic buildings that followed it, the model analyses have shown that, where technical matters are concerned, the cathedral's design was far less revolutionary than has been claimed. On the other hand, the analyses have also shown that the original structural solution at Bourges was unique. Even so, its full significance was not appreciated until late in the Gothic era when its steep, sloping buttresses reappeared in several large churches, including the choir of the parish church of Saint-Étienne at Beauvais and the nave of Bath Cathedral.



28 Bourges nave model. Photoelastic interference pattern produced by simulated wind loading.

NOTES

Material for this chapter was derived from Robert Mark, "The Structural Analysis of Gothic Cathedrals" (see note 13); Alan Borg and Robert Mark, "Chartres Cathedral: A Reinterpretation of its Structure" (see note 9); and from Maury I. Wolfe and Robert Mark, "Gothic Cathedral Buttressing: The Experiment at Bourges and its Influence" (see note 18). The most comprehensive writing on Bourges is Robert Branner's *La cathédrale de Bourges et sa place dans l'architecture Gothique* (see note 3). For the structure of Chartres, see John James, *Chartres les constructeurs* (see note 1).

1 The nave of Chartres, which embodies heavier flying buttresses than the choir, is generally (but by no means universally) assumed to be of earlier design than the choir and therefore more contemporaneous in design with Bourges. On the basis of recent archaeological research, John James infers that construction proceeded in horizontal layers across the entire building (*Chartres les constructeurs* [Chartres: Société Archéologique d'Eure-et-Loir, 1977], pp. 27ff.)

2 Jean Bony, "The Resistance to Chartres in Thirteenth-century Architecture," *Journal of the British Archaeological Association* XX–XXI (1957–1958):35–52.

3 Robert Branner, *La cathédrale de Bourges et sa place dans l'architecture Gothique* (Paris/Bourges: Éditions Tardy, 1962).

4 The names of the designers of both of these monuments have been lost to history, and there is the possibility that more than one master was responsible. James (*Chartres*) goes so far as to cite forty different building campaigns at Chartres under the direction of almost as many masters.

5 Paul Frankl, *Gothic Architecture* (Harmondsworth, England: Penguin Books, 1962), p. 79.

6 George Henderson, *Chartres* (Harmondsworth, England: Penguin Books, 1968), p. 111.

7 Otto von Simson, *The Gothic Cathedral* (New York: Harper, 1956), p. 204.

8 Eugène E. Viollet-le-Duc, *Dictionnaire raisonné de l'architecture française du XI^e au XVI^e siècle*, 10 vols. (Paris: Librairies-Imprimeries Réunies, 1854–1868), I, p. 65.

9 Alan Borg and Robert Mark, "Chartres Cathedral: A Reinterpretation of its Structure," *Art Bulletin* LV (September 1973):367–372.

10 The modeled section, at a scale of 1:180, was taken from a reproduction of a drawing by Robert Branner (*Chartres Cathedral* [New York: Norton, 1969], figure 14). Wind-loading information for Chartres was based on Paris data. A maximum velocity of 135 km/hr (84 mph) was assumed at the roof tip, 51 m (167 ft) above ground level. Model loadings were applied at a scale of 1:75,000. The ratio of the breadth of the upper flyers to the breadth of the lower flyers in the nave was maintained in the model.

11 A number of similar instances when prior modeling led to the disclosure of tensile distress or repairs necessitated by tension in the building fabric are reported throughout the text. These are summarized in chapter 9.

12 See section on sexpartite vaults in chapter 8.

13 The 1:107 scale modeled section is from Branner, *La cathédrale de Bourges.* Dead loads for the choir of Bourges were modeled at a scale of 1:148,000; wind loads at a scale of 1:100,000. The twin pinnacles on the pier buttresses, illustrated in figure 21, were not taken into account, as these were added for appearance in the nineteenth century. Photoelastic patterns for the choir under wind loads have been published in Robert Mark, "The Structural Analysis of Gothic Cathedrals," *Scientific American* 227 (November 1972):90–99.

14 The analysis also indicated that, without pinnacles, the lower pier buttress of Bourges would not be subjected to tension; the existing pinnacles have no structural role as they do, for example, on the Amiens pier buttress (see chapter 4).

15 Julien Guadet, *Éléments et théorie de l'architecture*, 3 vols. (Paris: Librairie de la Construction Moderne, 1909), III, pp. 340–349.

16 With the graphical-statical method, the interaction between the structural members cannot be taken into account. For example, in the actual structure, any deflection of the pier extension at its intersection with the arch must be accompanied by a corresponding deflection of the end of the arch. Considerable forces can be set up by these interactions.

17 Robert Mark, "The Church of St. Ouen, Rouen: A Reexamination of Gothic Structure," *American Scientist* 56 (1968):390–399.

18 Maury I. Wolfe and Robert Mark, "Gothic Cathedral Buttressing: The Experiment at Bourges and its Influence," *Journal of the Society of Architectural Historians* XXXIII (1974): 17–26.

19 These series of increases are not symmetric on the north and south sides of the cathedral. The historical import of this asymmetry is not clear, since some of the vaults at the west end of the nave were reconstructed after the north tower collapsed in 1506. See Branner, *La cathédrale de Bourges*, p. 71.

20 The second model approximates a section through the heaviest, westernmost bays. The model section was assumed to be through a primary pier, carrying the greater load, and scaled to 1:107. Dead loads were modeled at a scale of 1:72,000. The wind load, assumed to be the same as that on a choir bay, was modeled at a scale of 1:50,500. The model dimensions were established using a small section drawing from Amédée Boivet, *La cathédrale de Bourges*, Petites Monographies des Grands Édifices de la France (Paris: Henri Laurens, n.d.), p. 59, interpolations from Branner's drawing of the choir (*La cathédrale de Bourges*), and estimates from my own observations. The model ignores the existence of overvaulting (transverse walls above the vaults) at some of the nave sections.

21 The intermediate pier buttresses at Bourges, which support the center of the flying buttress system, maintain a relatively low profile both because of the steepness of the flyers and the great height of the first side aisles. At Le Mans and Beauvais the intermediate pier buttresses have the form of tall slender shafts (cf. figure 26, plate 4, and figure 43).

5

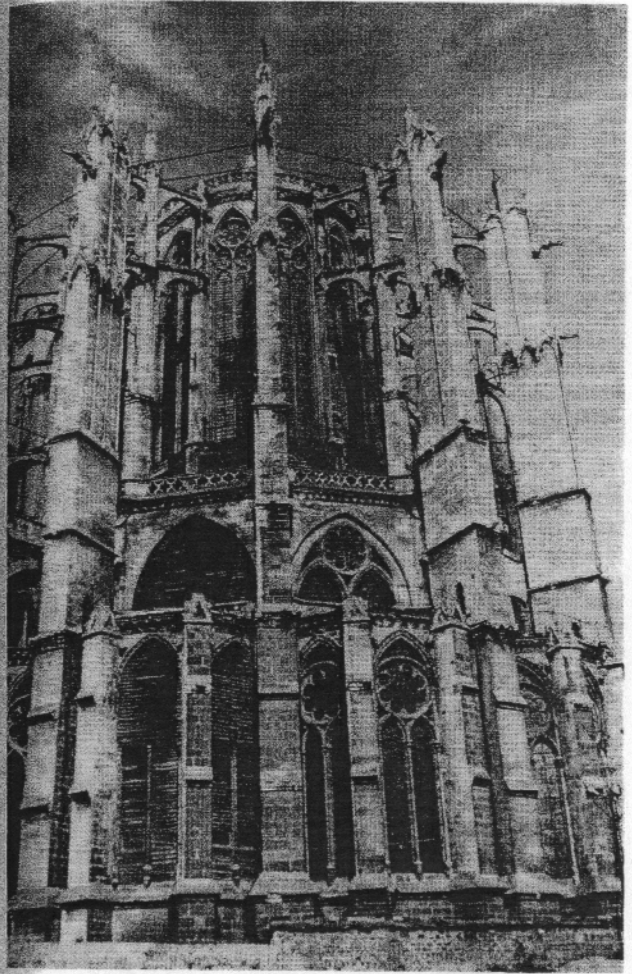
THE COLLAPSE OF BEAUVAIS CATHEDRAL

Major interest in Beauvais Cathedral (figure 33) has centered on the height of its vaults, 48 m (158 ft), and their collapse in 1284. Indeed, Robert Branner lamented that "the general admiration of Beauvais has been stimulated by a venture that was a tour-de-force, and an unsuccessful one at that, while the profoundly original stylistic part of the monument has been all but ignored."[1] Yet, in spite of the fascination with the collapse of the high vaults, the cause of the failure has never been adequately explained.

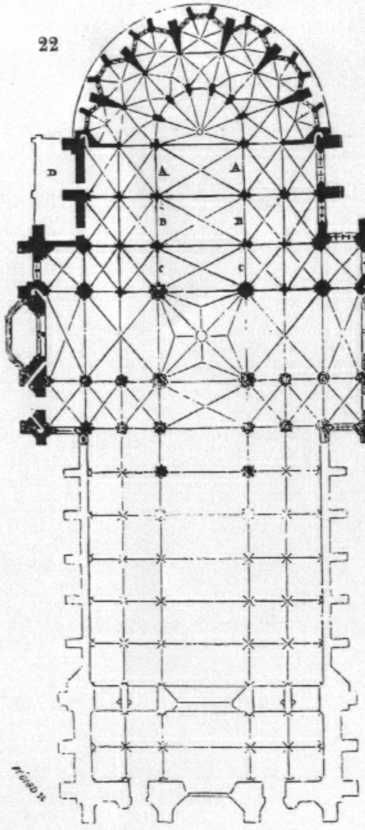
Beauvais was begun in 1225 by an unknown first master whose construction was concluded before 1245, by which time the choir below the main triforium was complete. A second master may have worked for a short period on the piers and vaults of the transept aisles, and from about 1250 or 1255, a third master took over the work and carried on until 1272, erecting the high choir vaults and buttressing. Figures 34, 35, and 36 show, respectively, reconstructions of the original plan for the whole building by Viollet-le-Duc, the interior of the original choir by Branner, and the cross section of the original northern half of the easternmost straight bay in the choir by the nineteenth-century architect, Benouville.

In 1284, all or part of the high vaults over the original three straight bays of the choir collapsed. Repairs, completed by about 1337, included replacement of all the original quadripartite vaults by sexpartite vaults and the erection of extra, unbuttressed piers in the three bays. Work ceased during the Hundred Years War, and the completion of the transept was not begun until about 1500.

Between 1564 and 1569, before work on the nave was far advanced, a gigantic stone tower of some 150 m (490 ft) in height was erected over the crossing. In 1573 the tower collapsed. Damage from this disaster was repaired by 1578, although the tower itself was not replaced. No further major construction was undertaken after that date, leaving the cathedral in its present truncated condition (see



33 Beauvais Cathedral. View of the hemicycle. Iron bars are reputed to be part of the fourteenth-century reconstruction.



34 Beauvais Cathedral plan. Reconstruction, showing unexecuted nave, by Viollet-le-Duc. The structure indicated by the darkened region was complete in 1272.

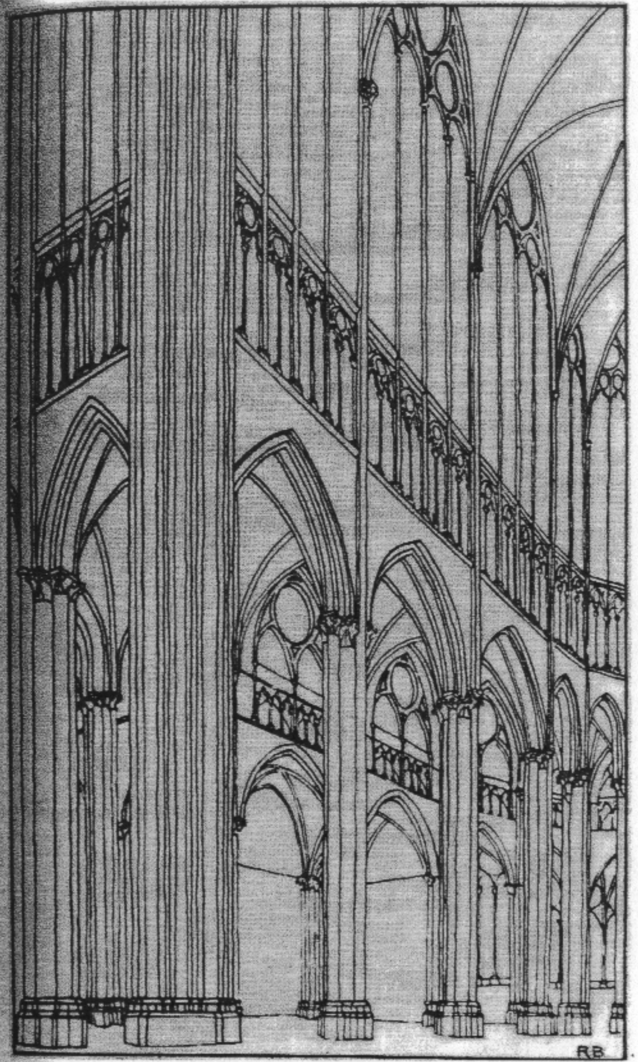
figure 34). A cross section of the existing southern half of the easternmost straight bay is shown in figure 37.[2]

Sixteenth-century accounts of the collapse of the tower and the recommendations of contemporary masons about how to prevent its fall make it clear that the two western piers in the crossing failed from progressive movement in a westerly direction because they received insufficient lateral support from the incomplete nave. In contrast to the tower debacle, which is well documented, there are no contemporary accounts of the collapse of the choir's high vaults three centuries earlier. Indeed, as Branner has noted, the entire bibliography on Beauvais is slight for a building of its importance.

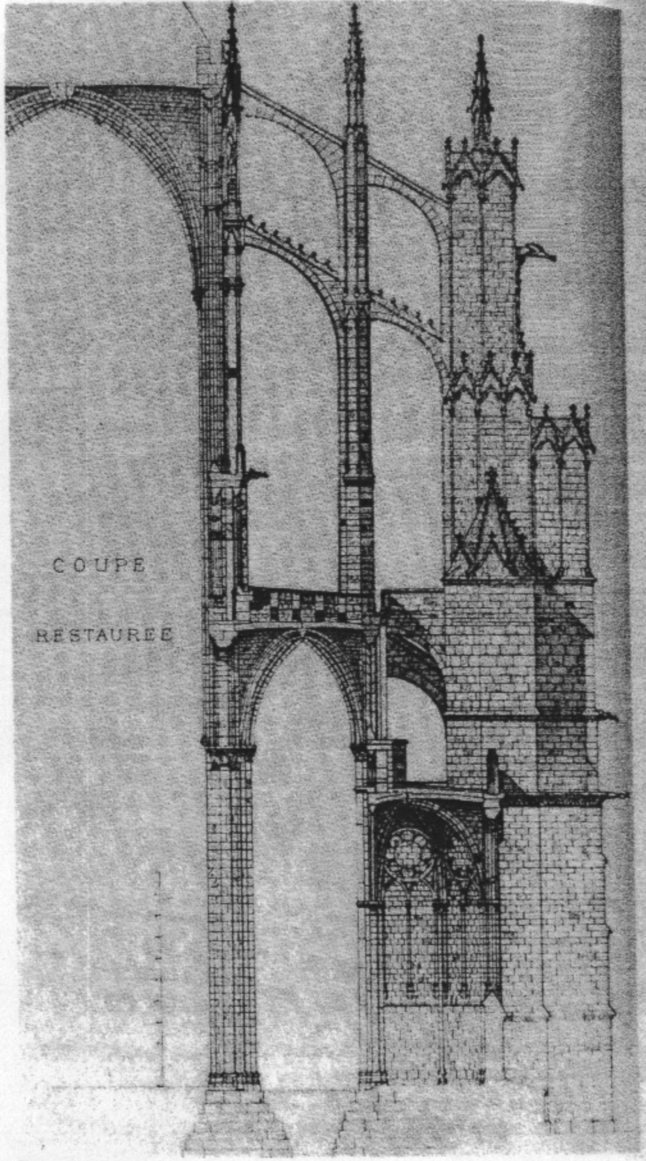
THEORIES CONCERNING THE FAILURE

In the absence of any evidence to suggest that the failure of the choir vaults in 1284 was due to some rare, excessive load on the structure such as an earthquake, there is a wide range of speculation about the cause of the collapse. One view is that the high vaults touched the limits of construction in stone. Viollet-le-Duc remarked that "the final limit at which construction of the great churches of the thirteenth century could arrive had been reached in the Beauvais structural system."[3] But, as will become apparent from our further investigations, there is no intrinsic structural principle that would hold Gothic bay construction to a height of 50 meters.

Paul Frankl's comment that the collapse was a failure "not of the architect as artist . . . but of the architect as engineer" is more plausible. Frankl suggests that the structural problem was due to inadequate foundations.[4] However, although differential foundation settlement is often the cause of severe structural damage to buildings,[5] there are no signs of any major deformity caused by settlement in the existing building fabric at Beauvais. Also, since the cathedral was founded on the site



35 Sketch of the original Beauvais choir, derived from archaeological evidence, by Branner.

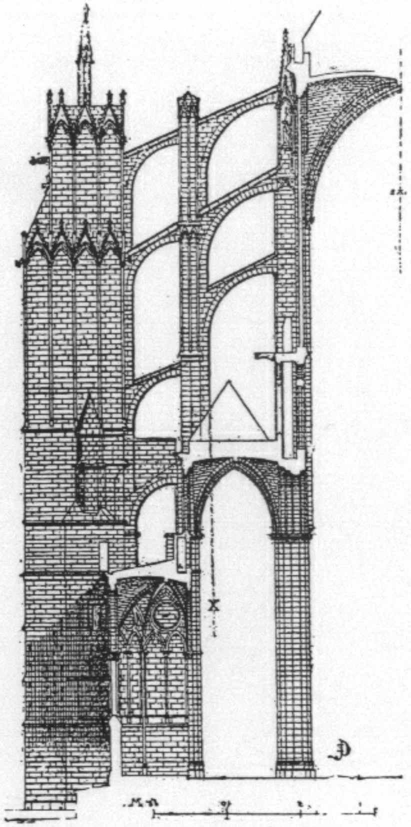


36 Beauvais Cathedral cross section. Reconstruction showing northern half of the eastermost straight bay, viewed from the east, by Benouville.

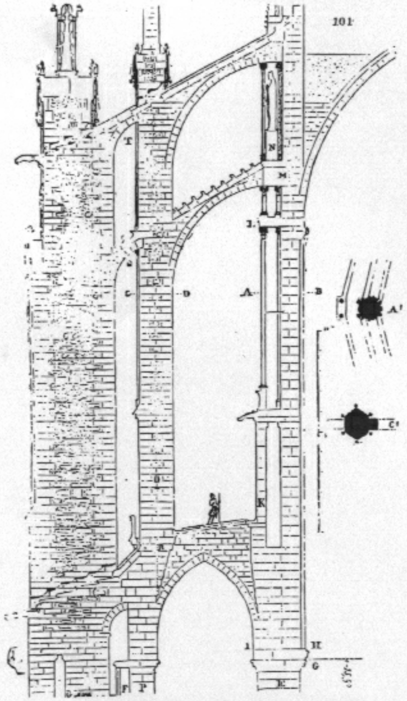
of an earlier building and within the walls of the old Roman precinct, it was less likely to have suffered from problems of settlement than major construction begun on fresh ground. The fact that the foundations were by the first Beauvais architect, whose masonry was evidently of higher quality than that of the master of the upper fabric, further weakens Frankl's suggestion.[6]

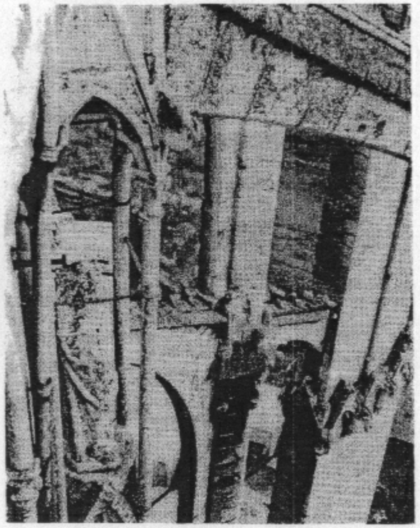
In the remaining theories, including the hypothesis presented here, the common idea is that there must have been some error in the design of the structure. These include the statements of nineteenth- and twentieth-century commentators who blame the failure on excessively large spans between piers that were too slender in relation to their height and the loads they had to carry.[7] According to this view, the change to sexpartite vaulting and the insertion of additional piers after the collapse of the vaults could have been an attempt to relieve the loads on the original piers. Branner's belief that the vaults were higher than intended by the first master may serve to give further support to this concept. No one, however, has produced a scientific analysis showing that the piers would have buckled under the loading from the original quadripartite vaults, and my own buckling calculation convinced me that the piers were in no such danger.[8]

Jacques Heyman, an engineer at Cambridge University, analyzed the Beauvais structural configuration for stability and concluded that "the fabric of Beauvais in 1272 seems to have been, in the large, almost perfectly designed to fulfill its function" and, hence, the explanation for the collapse had to be sought in some structural detail.[9] Heyman went on to endorse Viollet-le-Duc's theory of failure, which located the fault in the twin colonnettes below the lower tier of flying buttresses (letter A, figure 38). In Viollet-le-Duc's words,



37 Beauvais Cathedral cross section. Southern half of the existing eastermost straight bay, by Corroyer.



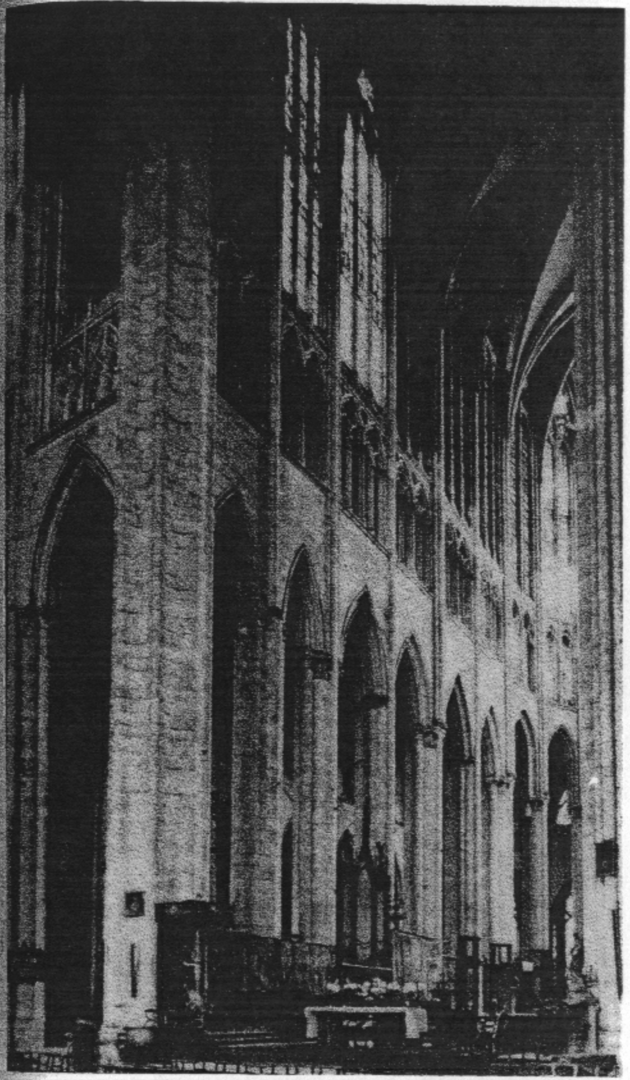


> [Beauvais] would have kept perfect stability if the architect had made the two pillars above the triforium stronger and more resistant; if he could have made them from cast iron, for instance. The disorders which have occurred in the structure have all come from this; these [monolithic] columns, too slender, have given way, for they could not resist the weight brought upon them when the inner piers began to settle in consequence of the drying of the mortar [of the coursed construction]. In the disorder, the lintels L [figure 38] were broken and the large blocks M, in swaying, rested too heavily upon the top of the first flying-buttress; this latter was deformed and, the vault following the movement, the pressure upon these flying-buttresses was such that they nearly all were forced outward and their action annulled. In consequence, the upper flying-buttresses were loosened somewhat, since the vault no longer pressed against them. The equilibrium was broken. . . .[10]

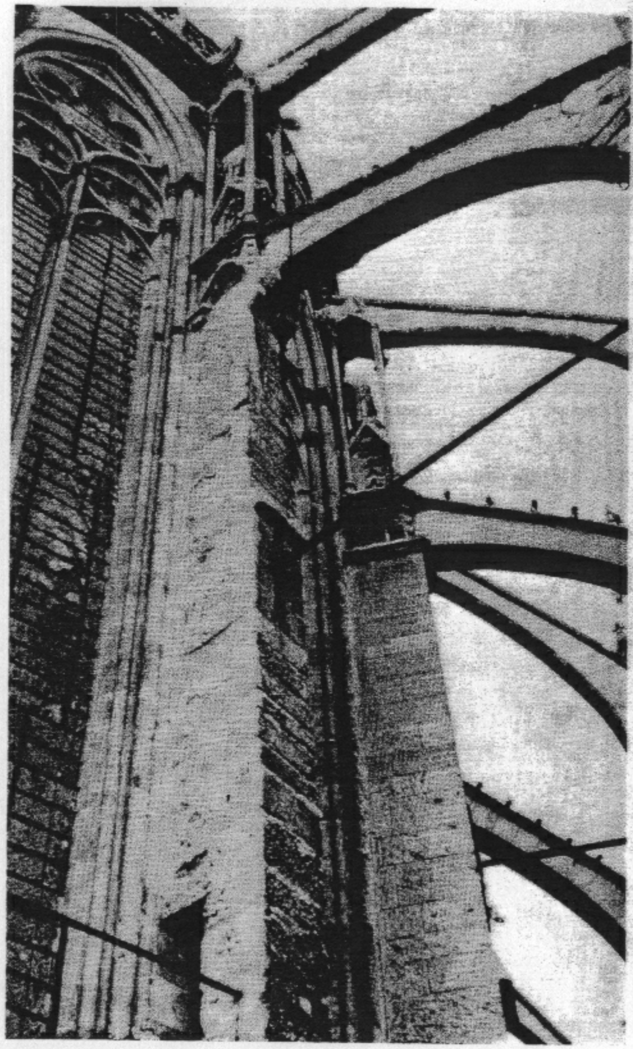
Even if it was true that the slender colonnettes buckled and failed by bearing the principal roof and vaulting dead loads, Viollet's explanation, endorsed by Heyman, is open to objection. First, it is crucial to this theory that the twin colonnettes supporting the statues were present in the original construction, prior to 1284. There is no doubt that the statues existed at that date, since they can still be seen on the easternmost choir bay and all around the hemicycle (figure 39). The absence of the statues and the presence of sexpartite vaulting, inserted piers (cf. figures 40 and 35), blocked lower clerestory window heads, and changed mullions and tracery details are the major archaeological evidence that the collapse was confined to the straight bays of the choir and that the easternmost bay is as a result substantially closer to the original state of the choir structure in 1272 than other portions of the straight bays. Evidence for the existence of the twin colonnettes in the thirteenth century is, however, far less convincing.

38 Cross section of the Beauvais hemicycle, by Viollet-le-Duc. In his text, Viollet-le-Duc implied that the wall details were similar for the original bays of the choir.

39 Beauvais Cathedral. Statue on hemicycle pier extension.



40 Beauvais Cathedral choir after 1337 showing sexpartite vaulting. Photo by R. Branner.

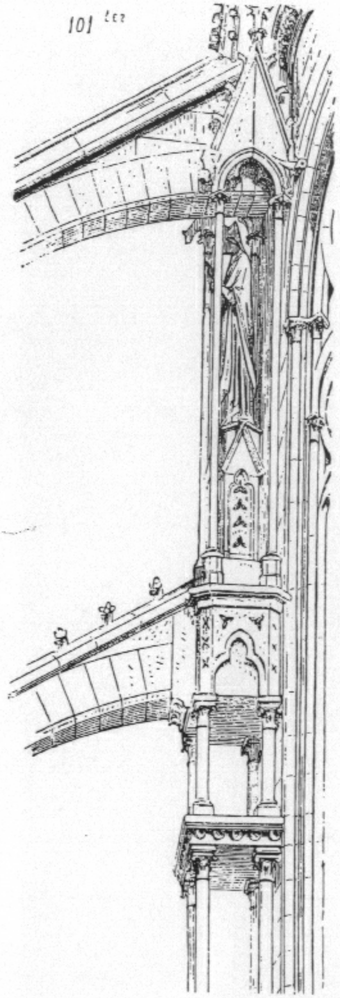


41 Beauvais Cathedral.
Hemicycle pier extensions
seen from the roof of the
ambulatory.

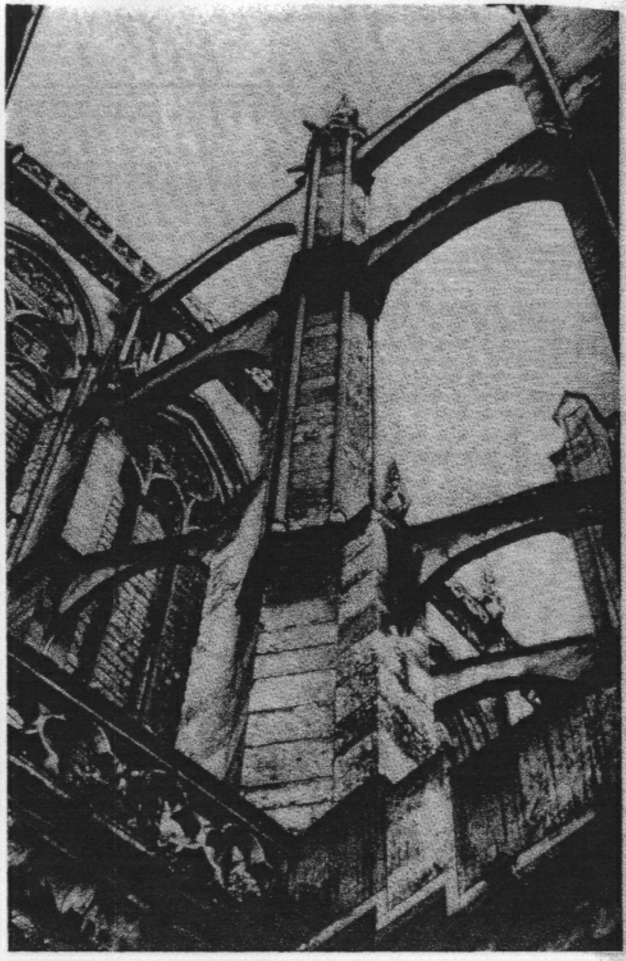
101

At present, all of the pier extensions, original and added, are solid masonry at clerestory level, except for the passage openings (figure 41). Viollet-le-Duc stated that one could still recognize the position and approximate the diameters of the twin colonnettes, but it does not seem possible that he could have seen the lower sockets or supports of the colonnettes, since those lower, exterior areas were covered by masonry.[11] If his inference was drawn from the carved blocks on which the statues stand and which receive the ends of the lower flyers, as seen in figure 41 and in his drawing (figure 42), then it is important to note the differences between these lower blocks and the ones above the statues. In the lower block, the trefoil is only a surface relief, and the block remains substantially solid; in the upper block the trefoil is fully pierced as part of a gabled canopy. Viollet-le-Duc's drawing is incorrect in that it shows the lower side of the upper flyer filling the canopy opening. In reality, the flyer abuts the canopy at a higher point (cf. figures 39 and 42). The motif on the surface-carved trefoil also is found on the intermediate pier buttresses that support the center of the flying buttresses and on the exterior pier buttresses (figure 43). On these buttresses, the surface-carved trefoils do receive monolithic colonnettes, but these colonnettes are not the sole support of the masonry above them. They are, rather, merely decorative elements.[12] Had these lower colonnettes been like the upper ones, then the trefoil surmounting them would have been more likely to have been carved through in the manner of the trefoil surmounting the upper colonnettes.

Benouville supports Viollet-le-Duc's theory by adducing as archaeological evidence for the existence of the colonnettes in the thirteenth century the fact that the solid exterior masonry of all the original wall buttresses is not bedded into the wall and is hence an obvious addition (figure 41). However, the only safe conclusion to be drawn from this observation



42 Beauvais Cathedral.
Detail of statue on
pier extension, by Viollet-le-Duc.



43 Beauvais Cathedral intermediate pier buttress on the north side of the easternmost straight bay. The upper portion of this buttress, with its attached shafts, appears to be part of the original construction. The lower portion is heavily reinforced, especially in the plane corresponding to the direction of thrusts from the flying buttresses.

today is that, while all the original wall buttresses in the choir were altered, their original form is still unknown.

Even if it were granted speculatively that the original section was accurately depicted by Viollet-le-Duc, it is still not clear that the failure he describes would have occurred. If the long colonnettes had broken, it is not obvious that the weight of the statue would have been sufficient to displace or significantly rotate the block below it. As shown in Viollet-le-Duc's drawing of a hemicycle section (figure 38), the block *M* was bedded into the wall. It was also the stone through which the horizontal vault thrust was transmitted to the lower flyer. This horizontal force on the block would have increased its frictional resistance to displacement by the weight of the statue.

The process that Viollet-le-Duc put forward as having caused the breaking of the colonnettes is also suspect. In his account the coursed section of the pier extension settled relative to the monolithic colonnettes because of shrinkage caused by drying of the mortar. From what is known of the behavior of medieval mortar and the nature of its construction, it is extremely unlikely that differential movement of the type described could have displaced sufficient load from the wall proper onto the colonnettes to break them after twelve years. Initial shrinkage of a lime mortar during setting could not have produced the dimensional change required by Viollet-le-Duc's account of the collapse because this initial dimensional change would have occurred in the course of construction and long before any vault loads were imposed.[13]

If we further allow that during the 30-year period from the beginning of the upper work at Beauvais to the collapse of 1284 all of the mortar within the pier extensions became carbonated, it still does not follow that the shrinkage associated with carbonation would have been sufficient to produce the dimensional change required to overload the colonnettes.[14] Using the very conservative assumption that the mortar joints make up 10 percent of the total height of the ashlar exterior of the pier extensions, a simple calculation shows that the carbonation shrinkage along the 11-m (36-ft) section of the pier extension adjacent to the colonnettes would be too small to endanger them as Viollet-le-Duc imagined: the largest ratio of dimensional change that could be expected is 0.0001, an amount approximately half that of the strain that would need to be produced before the colonnettes received a load sufficient to buckle them.

The one other mechanism by which dimensional change could occur is creep, the unrecoverable viscous flow of materials under load. Masonry construction may exhibit creep in the mortar whether or not the masonry blocks themselves are subject to creep. This deformation, although usually small, can be greater than that associated with shrinkage and could have been sufficient to produce the dimensional change required by Viollet-le-Duc's theory. However, it is characteristic of creep phenomena that the greater part of the total deformation occurs soon after the loadings are applied. Recent tests on brick and lime mortar piers under constant load indicated that the creep deformation had for all practical purposes nearly ceased within fifty to one hundred days from application of the load.[15] A period of two to three months after the centering was removed from the vaults would have been the time during which the structure was in greatest danger from the effects of creep deformation. The collapse at Beauvais did not come until twelve years after completion of the vaults, which indicates that the failure was not directly due to this sort of movement.

A further argument against Viollet-le-Duc's hypothesis is that the colonnettes are clearly so slender in relation to their height that it is unlikely that an experienced medieval builder would have planned to use them to support any great weight. Of course, the fact that the vaults fell at Beauvais indicates that their builder did not possess infallible judgment in construction; yet the general success of medieval builders with long monolithic shafts, such as bar tracery of windows, shows that they must have been in the habit of waiting until the mass of coursed construction had settled before fitting monolithic shafts. Even if the upper construction at Beauvais had been hasty, the twin colonnettes would not have been placed until the mortar's shrinkage from drying had almost certainly ceased and the principal part of the creep deformation had quite probably occurred. And, as indicated earlier, additional long-term change from carbonation-shrinkage of the mortar would have been too insignificant to overload the colonnettes.

ANALYSIS OF THE CHOIR STRUCTURE

In the absence of a convincing hypothesis as to why the choir vaults of Beauvais fell, I initiated a photoelastic analysis in order better to understand their structure and, possibly, to arrive at a more conclusive explanation for the collapse. The analysis, though, had to be speculative, first, because the full nature of the original structure is unknown. Assumptions had to be made about such dimensions as the thickness of the original quadripartite vaults, the height of the rubble and mortar surcharges over the vault, and the cross section of some of the original members. Second, modeling usually takes a typical structural section for analysis, but in the Beauvais choir there is no typical bay. The lengths of the three bays of the original construction diminish from east to west, the proportions from east to west being approximately 25:42, 24:42, and 23:42. Because each bay is a different size, the loads also change from bay to bay. Inspection of the existing fabric shows that sections of the various pier buttresses and flyers in the hemicycle and choir also vary considerably.[16] Third, analyses of this type are usually facilitated by the assumption that the whole structure can be adequately represented by a structural section; that is, that the resistance of the structure to dead weight and wind is largely in the plane of the section and that action outside the plane (in the third dimension) is negligible.[17] As discussed in chapter 2, the assumption of planar action is a reasonable one for nave or choir sections of High Gothic churches, but at Beauvais the choir is short, so there is likely to be some contribution to the support of the individual bays from the nearby more rigid structures of the hemicycle and transept. The varying lengths of the choir vaults indicate that the horizontal thrust components in the east-west direction from adjacent vaults will not balance each other, and this too contributes to action outside the north-south plane of a structural section. Even so, a three-dimensional model of the entire choir is not necessary as long as these limitations are kept in mind for the eventual interpretation of the model experiments.

The section that was drawn by Benouville (figure 36) was chosen for modeling because it is the only structural section of the choir that appears not to have been involved in the collapse and hence is most likely to resemble the original construction of the choir's other straight bays. The configuration of the high vaults was considered to be similar to that of Cologne (chapter 8); scaled colonnettes were incorporated in the model to test their behavior under expected loads.[18]

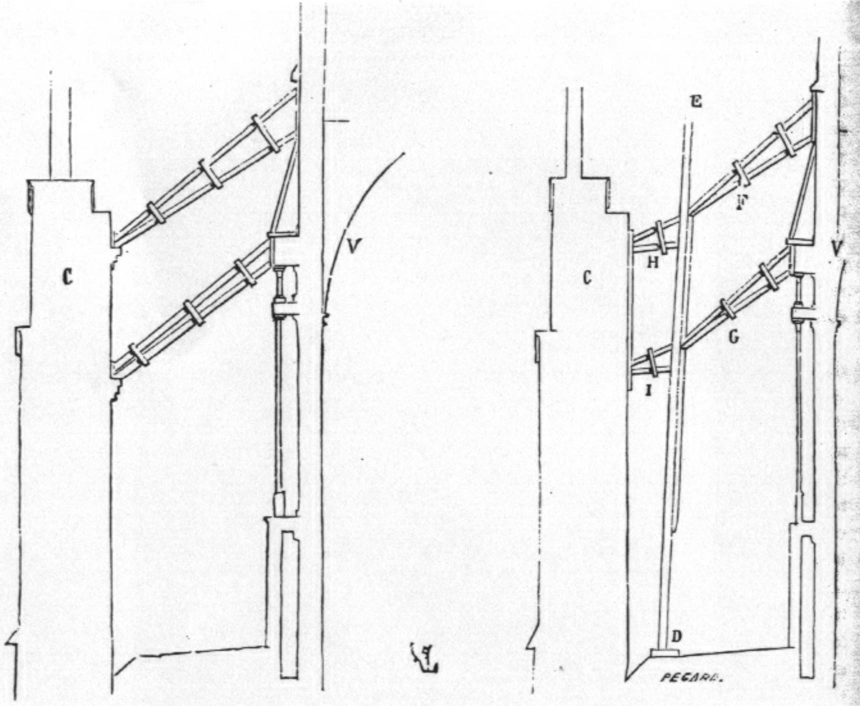
The results of the wind-load (plate 6) and dead-load experiments confirmed the reservations about many previous theories concerning the collapse of the vaults. They show that structural problems were not likely to be present in either the piers or the colonnettes. The highest compression stress level in the original choir piers under combined dead and wind loads was 28 kg/cm² (400 psi) at their bases; and the maximum compression in the clerestory wall was 21 kg/cm² (300 psi)

under combined loads. As already observed, these values are typical for High Gothic construction. Likewise, the colonnettes under dead and wind loads bore at most 7,000 kg (15,000 lbs) each, significantly less than the axial load required to buckle them. The analytical results of the experiments, therefore, discount theories of the failure that depend upon the slenderness of either the piers or the colonnettes.

However, the analysis produced a further result that suggested a new hypothesis. The dead-load experiment revealed that just above the side aisles and just below the junction with the lower flyers, the intermediate pier buttresses were bent by horizontal forces large enough to initiate cracking (see section C-D and region R in figure 38). When the tensile stresses on the exterior portion of the leeward intermediate pier buttress caused by wind loading were added to those already present from the dead loading, the indicated total tensile stress was 18 kg/cm² (260 psi) on the inside edge above the aisle and 15 kg/cm² (210 psi) on the outside edge below the flyers. On the windward intermediate pier buttress, the wind load combined with the dead load yielded tensile stresses of approximately 5 kg/cm² (70 psi) on the inside edge below the flyers. Cracking would have occurred in the mortar well below these levels of stress. Moreover, since the tensile stress on the buttresses would have alternated as the wind changed direction, cracks could have developed on both sides, and these cracks are so located that they probably would have avoided detection. The upper critical regions, near the flyers, would not have been readily accessible, and the lower cracks would have been concealed by a gabled, side aisle roof.

Though the experimental results were derived from an estimated "worst wind," winds of lower velocities could still have produced the predicted cracking. Since wind forces vary as the square of the wind velocity, two-thirds of the velocity of the worst wind would produce roughly one-half of the load simulated in the test. The resulting tensile stresses still exceeded the capacity of medieval masonry construction. Less violent storms that might have occurred relatively frequently, therefore, could have cracked the intermediate pier buttresses. Perhaps, even within the period of twelve years, cracking under alternating wind loads could have caused an intermediate pier buttress to deteriorate so badly that the horizontal forces acting on it might have caused a section to slide from its support. If an intermediate pier buttress had collapsed, the system of flying buttresses above it would have been unsupported at its center, causing the flyers to fall, allowing the horizontal thrust of the high vaults to push out a section of the clerestory wall and trigger the sequential fall of additional vaults.

There is some irony in the fact that the critical portion of the intermediate pier buttress most subject to cracking is cited by Viollet-le-Duc as an example of rational design.[19] Since the upper portion of the intermediate pier buttress is off center and only balanced on its lower support (figure 38), Viollet-le-Duc argued that it would tend to incline towards the clerestory to help to resist the thrust of the high vault. To illustrate his argument he drew an analogy with a system of inclined props (figure 44). Actually the buttress does not lean inward under its dead weight because, as Viollet-le-Duc himself stated, its center of gravity is still over the support. Further, for the buttress to act as he suggested, it must implicitly be assumed that hinges exist just above the side aisle and at the points of attachment to the flying buttresses. Since hinging in masonry construction is tantamount to its cracking, this explanation of the overhanging action might actually be taken as a demonstration of the structure's problems rather than its rationality.



44 Intermediate pier buttress as a prop. Sketch by Viollet-le-Duc.

Viollet-le-Duc's failure to understand the behavior of the pier buttresses is not surprising because the structural behavior of Beauvais is not intuitively obvious. He correctly sensed, however, that the relatively stiff exterior pier buttresses do not receive as much horizontal force from the vaults and wind as might be expected. In fact it is the extra share of these loadings that was received by the far more slender intermediate pier buttresses that caused them to bend and might have led to their deterioration.

THE RECONSTRUCTION

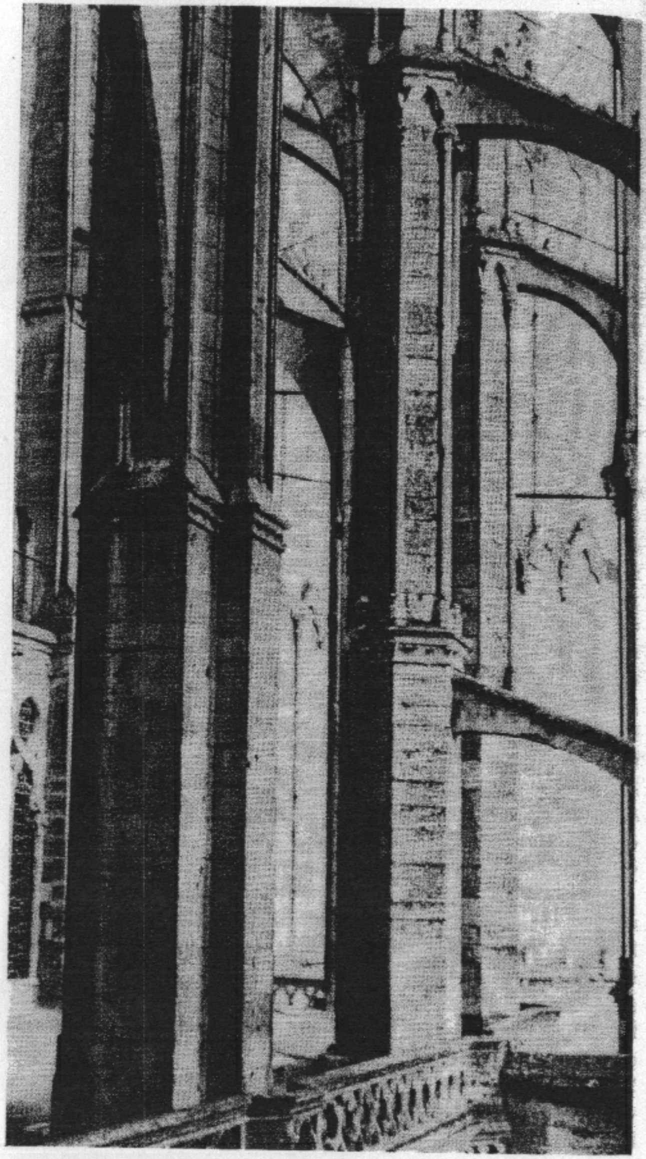
The experiments described here have not settled the question about whether the colonnettes were used as Viollet-le-Duc imagined, but they do show that even if they had been part of the original construction, they need not have played any role in the collapse of the vaults. Failure of one of the intermediate pier buttresses could have brought down all of the choir vaults, leaving, because of its greater stability, only the hemicycle.[20] If failure of an intermediate buttress was indeed the cause of collapse and the fact was known to the rebuilders, some aspects of the reconstruction and the existing fabric can more readily be explained.

It is likely that in the aftermath of the collapse the overhang of the intermediate buttress was noticed. If the rebuilders then understood that this buttress absorbed some of the transmitted thrust from the flying buttresses and that it was the locus of the failure that had caused the collapse, they would have tried to take steps to prevent a repetition. Such steps would have included measures to ensure that the horizontal shearing loads transmitted into the intermediate buttress were reduced, that the buttress support was made more stable, and that the buttress itself was increased in strength and was better supported laterally.

Examination of the existing fabric shows that all these steps were, in fact, taken. Replacement of the quadripartite by sexpartite vaults can be interpreted as an attempt to reduce the horizontal loads because the extra pier extensions needed to support the new vault would have absorbed some of the wind loads on the clerestory and hence reduced those transmitted to the intermediate pier buttresses through the flyers.

Moreover, the four western intermediate pier buttresses, two on the north and two on the south of the choir, that were rebuilt after the collapse, are of far greater section than those in the last straight bay and those in the hemicycle (the two types are illustrated in figure 45). Part of this increase is explained by their role in receiving the transept flyers (where those flyers are in place), but the buttresses are also much deeper in the north-south direction, that is, in the direction of the horizontal vault thrusts and wind loads, which they are therefore better able to carry than their predecessors. In addition, the intermediate pier buttresses are not only stronger but their centers of gravity also appear to have been moved outward from the overhanging position.[21]

At the easternmost straight bay on the north side, an intermediate pier buttress that is largely original has been reinforced substantially at its base in the region that we have indicated as being subject to severe cracking (figure 43). There is also a third and lowest set of flyers abutting this and other intermediate pier buttresses at mismatched elevations, and on those intermediate pier buttresses without the additional flyers, there are iron bars or traces of them in similar positions (figure 33). Benouville offered the suggestion that the lowest flyers had been added in the sixteenth century to prevent the separation of the solid masonry addition from the pier extensions. His dating of the addition is suspect because some of these flyers are not filleted at their



45 Intermediate pier buttresses on the south side of Beauvais Cathedral. Note the greatly increased section of the rebuilt buttress (foreground).

ends, as was sixteenth-century practice, and such flyers are used elsewhere at Beauvais. It is also implausible that if the flyers and iron bars were intended solely to hold the added masonry to the pier extensions, more than just an interior flyer or bar would have been constructed.

It is certainly more likely that the mismatched sets of flyers and bars were intended to support the intermediate pier buttresses, especially those around the hemicycle. In the hemicycle, where the vaults did not collapse, extensive rebuilding of the intermediate pier buttresses would have been uncalled for and difficult, since provision for support of the flyers and vaults would have had to be made during the work. In the choir proper, on the other hand, the intermediate buttresses could have been rebuilt completely and easily, whether one or all of them fell during the collapse, since reconstruction of the vaults would have followed the reconstruction of the buttress system. The alteration of the pier extensions all around the choir might have been considered necessary to receive the extra flyers and iron bars that were to be added to support the intermediate pier buttresses.

Although the mode of reconstruction of the choir appears to provide evidence to support this analysis, the new theory of the cause of the collapse of the high vaults in 1284 cannot be proved beyond a doubt because so much of the primary evidence disappeared so long ago. In 1975, however, when Stephen Murray, an architectural historian at Indiana University, examined the fabric of the cathedral with me, his archaeological observations further supported the basis of the model study on which this theory depends. In his view, the locus of the collapse was indeed the middle bay of the choir, which corroborated my assumption that this section must have been the weakest link, because the other bays were reinforced by the nearby rigid structures of the hemicycle and transept.[22]

This combination of archaeological observation and structural modeling can be immensely helpful in furthering our general understanding of how Gothic structures work. Particularly for Beauvais, the archaeological findings would seem to substantiate my hypothesis regarding a question of structural behavior that had baffled earlier investigators.

NOTES

Material for this chapter was derived from Maury I. Wolfe and Robert Mark, "The Collapse of the Vaults of Beauvais Cathedral in 1284," *Speculum* LI (July 1976):462–476. A recent, full account of the early history of Beauvais is given by Stephen Murray in "The Choir of the Church of St. Pierre, Cathedral of Beauvais: A Study of Gothic Architectural Planning and Constructional Chronology in its Historical Context" (see note 22).

1 Robert Branner, "Le Maître de la Cathédrale de Beauvais," *Art de France* II (Paris, 1962): 77–92.

2 Edward Corroyer's drawing in *L'Architecture Gothique* (New York: Macmillan, 1891) is actually a symmetric (right-left) inversion of the existing structure. The drawing shows the southern half-section as if it were viewed from the west; the small stairturret is, however, on the east face of the pier buttress.

3 Eugène E. Viollet-le-Duc, *Dictionnaire raisonné de l'architecture française du XI^e au XVI^e siècle*, 10 vols. (Paris: Librairies-Imprimeries Réunies, 1854–1868), IV, p. 175.

4 Paul Frankl, *Gothic Architecture* (Harmondsworth, England: Penguin Books, 1962), p. 101.

5 An example, particularly of differential settlement of towers, is found at Wells (chapter 6).

6 The higher quality of the masonry work of the first master compared to that of the builder of the upper fabric was pointed out by Léon Benouville, "Étude sur la Cathédrale de Beauvais," *Encyclopédie d'Architecture*, series 4, IV (1891–1892), pp. 52–54, 60–62, 68–70, and *tirage-à-part*.

7 A. P. M. Gilbert, *Notice historique et descriptive de l'église cathédrale de St. Pierre de Beauvais* (Beauvais, 1829), p. 10; Gustave Desgardins, *Histoire de la Cathédrale de Beauvais* (Beauvais, 1865), pp. 8ff., who cites as his source, Louvet, *Histoire et antiquitéz du diocèse de Beauvais* (Beauvais, 1635), 11, p. 474; L'Abbé L. Pihan, *Beauvais* (Beauvais, 1885), pp. 10ff.; Victor Leblond, *La Cathédrale de Beauvais*. Petites Monographies des Grands Édifices de la France (Paris: Henri Laurens, 1933), p. 15.

8 If the additional bracing of the attached shafts (see figure 35) is neglected, the pier core may be taken as a cylinder 14.6 m (48 ft) high and 1.5 m (4.9 ft) in diameter. Assuming an aggregate elastic modulus for the masonry of 200,000 kg/cm² (2,800,000 psi), the simple Euler equation (as described in any introductory text on strength of materials, for example, S. P. Timoshenko and D.H. Young, *Elements of Strength of Materials*, 5th ed. (Princeton: D. Van Nostrand, 1968)) yields a critical buckling stress that is two orders of magnitude greater than the nominal pier compression stress.

9 Jacques Heyman, "Beauvais Cathedral," *Transactions of the Newcomen Society* XL, 1967–1968 (London, 1971):20ff. Heyman's engineering analysis checks and elucidates Benouville's ("Étude sur la Cathédrale de Beauvais," plate 160) and confirms that the structure is stable under dead loadings. The method employed by both is a graphical-statical one which must assume the presence of "hinges" in a complex structure (see note 16, chapter 3). This method came into general use late in the nineteenth century and has been given new impetus by Heyman's work in limit analysis. See Jacques Heyman, "The Stone Skeleton," *International Journal of Solids and Structures* II (1966). The limit theorems extended by Heyman for use with masonry structures were developed originally and are used extensively for design of steel-framed structures. They provide a simple, reliable basis for determining safety factors against failure in structures of ductile materials, but they do not give information about their behavior under normal service loadings as do the elastic models described in this text.

10 Viollet-le-Duc, *Dictionnaire*, IV, pp. 180ff.; translation follows that of George M. Huss, *Rational Building* (New York: Macmillan, 1895), pp. 238ff. Heyman's account differs from Viollet-le-Duc's only on the specific nature of the movement of the block M (see Heyman, "Beauvais Cathedral," p. 30).

11 Viollet-le-Duc, *Dictionnaire*, IV, p. 179, fn. 1. Beauvais is one of the few major French High Gothic cathedrals that Viollet did not restore. Hence it may be assumed that he had no greater access than is available to present-day archaeologists.

12 Viollet-le-Duc's contention (*Dictionnaire*, IV, pp. 181ff.) that the use of monoliths, usually stones *en délit*, was a major principle and advance of Gothic construction is not consistent. *En délit* stones, like those of the colonnettes at Beauvais, are monoliths whose bedding plane is perpendicular to the natural sedimentary layering, or grain, of the rock from which it was cut. If a stone is bedded in this way and subjected to any substantial compressive stresses, it is likely to deteriorate more rapidly than stone bedded with its grain because the compressive strain will cause tensile strains perpendicular to the beds and split or exfoliate the stones (Robert J. Schaffer, *The Weathering of Natural Building Stones*, reprint (Watford: Garston, 1972), pp. 14ff.). The *en délit* colonnettes could not resist tensile bending stresses unless securely tied to their sockets, top and bottom. Such ties would be difficult to construct, and if iron ties were used in any place subject to moisture, they would be a hazard to the stonework due to rust expansion (Schaffer, *Weathering*, pp. 22ff.). And, for the idea that the colonnettes consolidate the adjoining stonework, the most that can be said is that if this stonework is subjected to high loadings, the frictional forces among its constituents are likely to be more than sufficient to hold the masonry together.

13 Tests show the shrinkage of a lime mortar to be roughly 0.35 percent of its initial dimensions after eight days of drying, after which it remains essentially constant. See Sven Shalin, *Structural Masonry* (Englewood Cliffs, N.J.: Prentice-Hall, 1971), p. 200, fig. J. 10.

14 Carbonation of lime mortar is discussed in chapter 2. From tests of concrete blocks and structural members, it appears that carbonation shrinkage is from 0.03 to 0.1 percent of the member dimensions. See Frederick M. Lea, *The Chemistry of Cement and Concrete*, 3rd ed. (London: E. Arnold and Company, 1970), pp. 544–546; and Shalin, *Structural Masonry*, p. 200, fig. J. 9.

15 Creep tests on brick and lime mortar piers under constant load indicated that creep deformation for the whole masonry pier is of the order 0.1 to 0.2 percent of the pier height. See Shalin, *Structural Masonry*, pp. 202–208, fig. J. 12. b., fig. J. 16. a.

16 Heyman ("Beauvais Cathedral," p. 20) states that his analysis is for a typical bay. He is careful to note that at Beauvais only the structural section free of the transept and the hemicycle could be considered as typical; hence his sense of "typical" is different from that employed here. He misidentifies the section he means as typical as the one chosen by Benouville also. All of Benouville's drawings are of the structural section of the northern half of the easternmost straight bay. This can be confirmed by noting that the Benouville drawings contain a turreted stair tower on the pier buttress which, as drawn, could only be seen on the last straight section just at the hemicycle on the north ("Étude sur la Cathédrale de Beauvais," Plate 159, Plate 161).

17 In cases where this assumption is untenable, as in the study of the structural behavior of Gothic ribbed vaulting (chapter 8), a full three-dimensional model analysis must be undertaken.

18 The model was machined from a 7.1-mm (0.28-in) thick sheet of epoxy at a model-to-prototype scale of 1:144. After preliminary calculations and experimentation, a dead-load experiment and a wind-load experiment were designed. The dead loads were applied at a model-load scale of 1:127,000 to the vaults, flyers, and intermediate buttresses (see figure 15). The wind-load experiment assumed a mean peak wind of 155 km/hr (96 mph) at the roof peak height of 67 m (220 ft). The total loading on the section, taking the gust factor into account (see chapter 2), was calculated to be approximately 188,000 kg (415,000 lbs): 104,000 kg (230,000 lbs) on the windward side and 84,000 kg (185,000 lbs) on the leeward side. The wind load was modeled at a scale of 1:100,000. These wind loads are considerable even in relation to an estimated total fabric weight for one bay of 4,000,000 kg (8,800,000 lbs).

19 Viollet-le-Duc, *Dictionnaire*, IV, pp. 177ff., where it is discussed as the overhanging *culée* or the pier *porte-à-faux*.

20 Whether all the choir vaults fell cannot be settled here. If only one intermediate pier buttress were to fall, its collapse would almost certainly bring down the vaulting of one whole bay, likely more. Heyman, in "Beauvais Cathedral," p. 30, also maintains this point.

21 As evidenced in Benouville's drawing ("Étude sur la Cathédrale de Beauvais," Plate 161), the center of gravity of the heavier intermediate pier buttress is moved outward toward the exterior pier buttress.

22 Stephen Murray, "The Collapse of 1284 at Beauvais Cathedral," *The Thirteenth Century, Acta* III (1976):17–44. Murray also reported on a later problem of deformation of an intermediate pier buttress at Beauvais that necessitated rebuilding in 1517 and which further bears out the hypothesis of a design weakness in this important structural element. Although he did not then concur in the new theory developed in this chapter, Murray has accepted it in his most recent account of the vault collapse. See Stephen Murray, "The Choir of the Church of St. Pierre, Cathedral of Beauvais: A Study of Gothic Architectural Planning and Constructional Chronology in its Historical Context," *Art Bulletin* LXII (December 1980):533–551.