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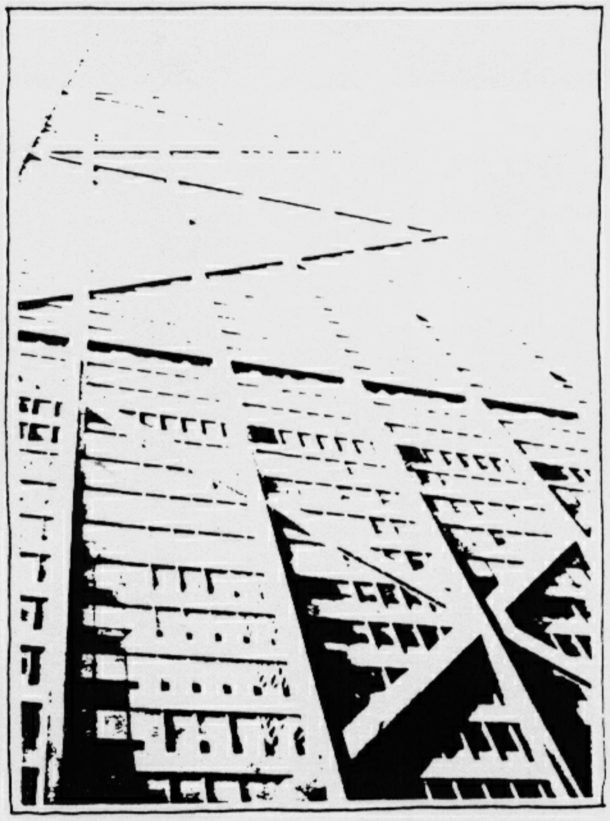
CANTILEVERED COLUMN:

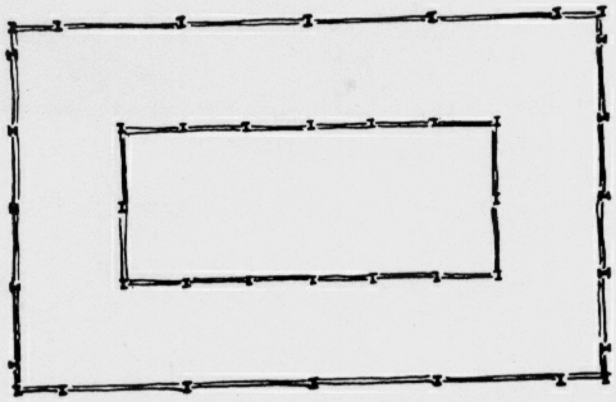
HANCOCK TOWER



Chicago's Hancock Tower was designed in 1966 by the architectural and engineering firm of Skidmore, Owings and Merrill. The building has one hundred stories and contains offices, apartments, commercial space and parking.[1,2]

Geometry and Materials

The unusual structural design of this skyscraper is of special interest because it is a hollow tube with a central core, i.e., the supporting structures are the outside walls and an inner core. This is shown in the plan at the top of the next page where the I shapes represent steel columns. These separate structural elements allow it to be identified here as a cantilevered column. The inner core acts as a column and is idealized as taking about fifty percent of the vertical load



and none of the horizontal load.[3] In reality the core will resist some shear, but no bending from the wind. The outside walls that make the tube act as a cantilever and are assumed to resist all the horizontal wind forces, as well as take the remaining half of the vertical load. The reactions of the tube against the horizontal forces must be idealized for simple analysis: the horizontal forces are carried in the two walls parallel to the wind and the bending moment is carried by the two walls perpendicular to the wind.[4]

The compound structure can be explained using the formula introduced in the earlier analyses:

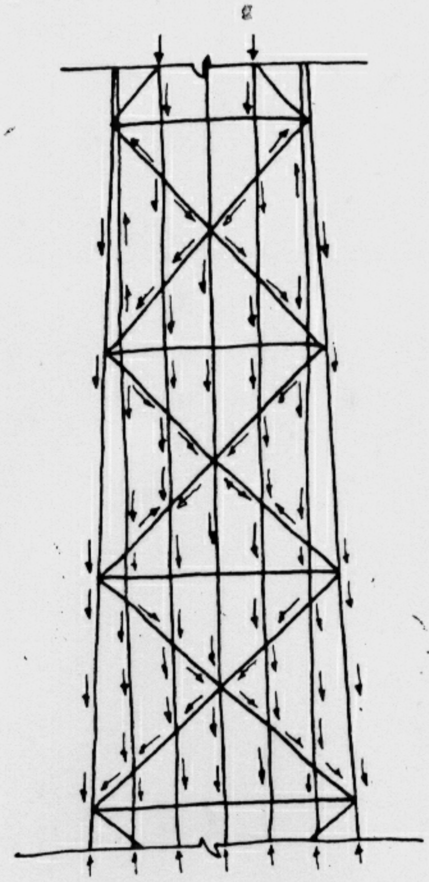
$$\frac{N_G + N_W}{N_G} \leqslant \frac{4}{3},$$

if the wind force is not a major factor in the design, i.e., it acts as a column. The subsequent analysis will find that for an individual column on the leeward side of the outer tube N_G = 14,300 kips and N_W = 5570 kips. Using these values in the equation given, a ratio of 1.39 is found:

$$\frac{14{,}300k + 5570k}{14{,}300k} = 1.39 > \frac{4}{3}$$

This means that the wind force is a factor in the design of the tube. The inner core, however, does not carry any horizontal loads and, therefore, it will have a ratio of 1.0, which means that it can be designed as a pure column. This building exemplifies a combination of the two types of structures introduced already, thus it is classified as a cantilevered column.

By using a tubular structure, the building's structural engineer has taken advantage of three characteristics of that form that make it superior to earlier skyscrapers.[5] First, by concentrating the structural elements on the outside and the inside, the interior is left column-free, making more flexible floor plans possible. Second, flexible floor plans use lighter, removable, non-load-bearing partitions which decrease the dead weight of the building. Third, the use of diagonals to tie the columns in the exterior walls creates a stiff wall with a minimum of structural material (and dead weight), leaving more room for windows. The diagonal braces are an important part of the structure because they make the columns act together as a tube. These braces also serve to distribute and transfer loads from one column to another, and to keep the loads on the columns essentially uniform.[6]



These braces connect the columns so effectively that they act together as a tube, rather than as individual columns around the exterior.

The columns are steel, and the floors, supported by steel beams, are concrete slabs six inches thick. Over the 100 stories of its height, there are 1,000,000 square feet each of office and of apartment space and 800,000 square feet of commercial and parking space, for a total of 2,800,000 square feet. The tower is tapered with base dimensions of 265 feet and 165 feet, and top dimensions of 160 feet and 100 feet.[7] Over the 1100 feet of its height this creates a maximum slope of 2.6^{o} which will be neglected in this analysis. Consequently, a simplification is made about the area of space on each floor: because of the taper there is less area on the top floors than on the bottom floors, but this analysis assumes that there is 28,000 sqaure feet per floor:

$$\frac{2,800,000 \text{ ft}^2}{100 \text{ floors}} = 28,000 \frac{\text{ft}^2}{\text{floor}}$$

The next page shows a dimensioned diagram of two different sides of the tower.

Loads

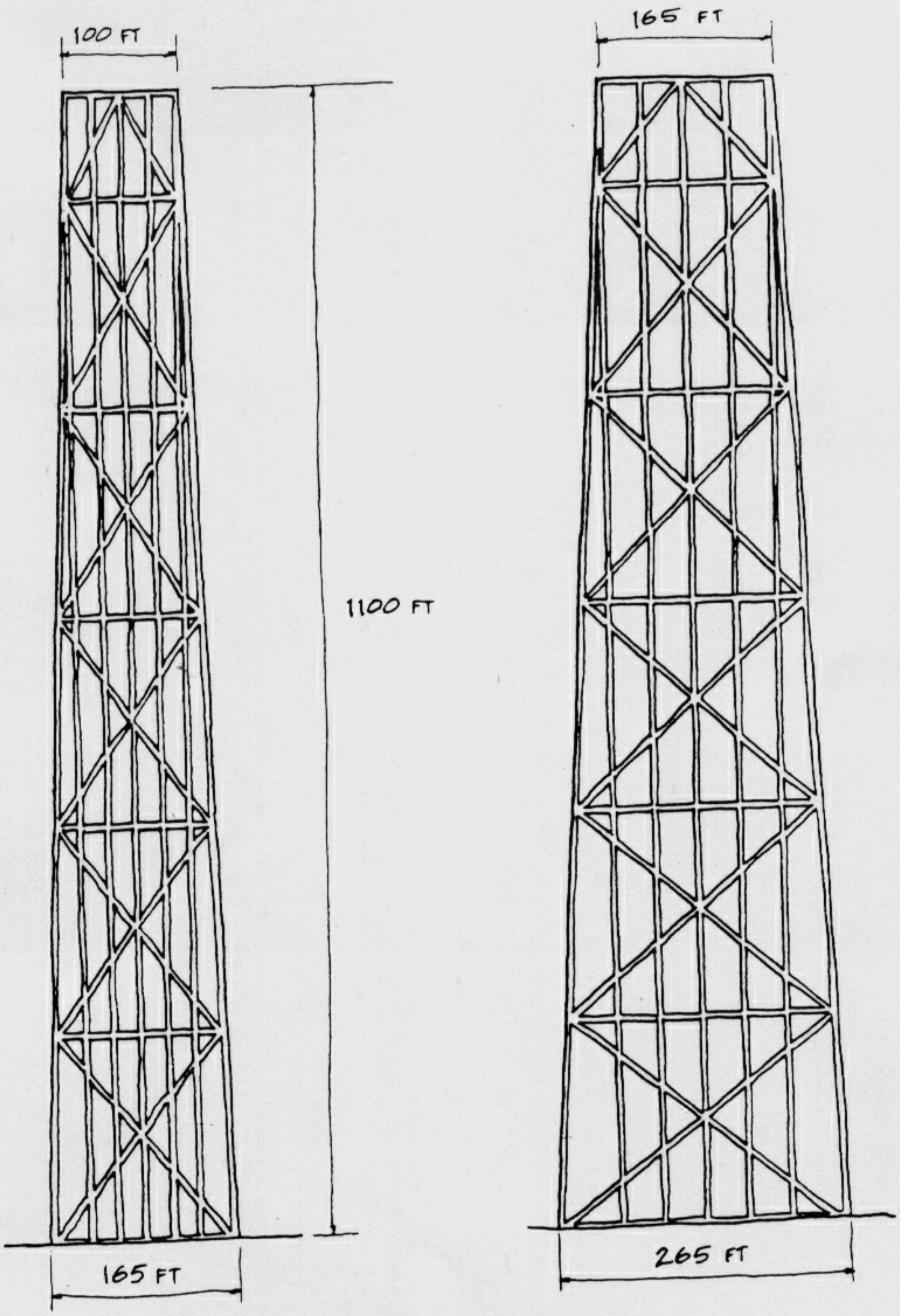
Three types of loads act on the Hancock Tower - dead, live, and wind loads. The dead and live loads act vertically and the wind load is idealized as acting horizontally. The dead and live loads are given in values of pounds per square foot and the total load is calculated from these values with the area of floor space in the tower, $A = 2,800,000 \text{ ft}^2$. The dead load is given in its components:

structural steel	30 psf
concrete floors: 6in @ 150pcf . . .	75 psf
partition, exterior walls and permanent equipment . . .	60 psf
total dead load	165 psf = 0.165 ksf

Over an area of $2,800,000 \text{ ft}^2$, the total dead load is

$$Q_d = 2,800,000 \text{ ft}^2(0.165\text{ksf}) = 462,000\text{k}.$$

The live load which includes people and furnishings is given as a standard value by building codes. This analysis uses a con-



servative estimate of about 80 psf (0.080 ksf). This results in a total live load over all the area of the tower of

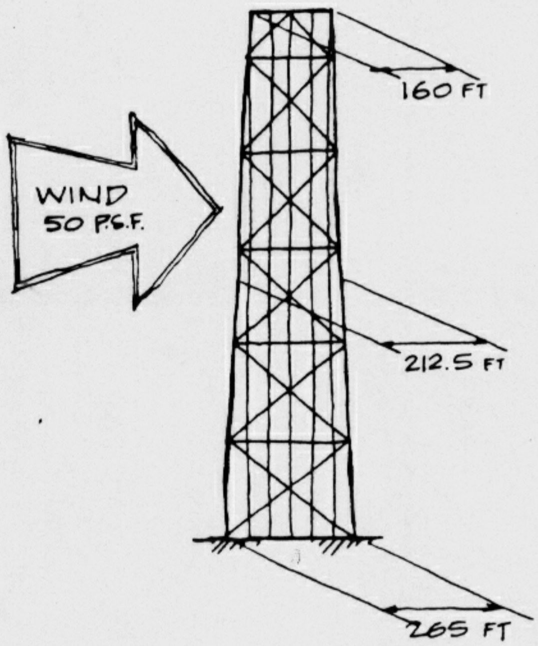
$$Q_l = 2{,}800{,}000 ft^2 (0.080 ksf)$$
$$= 224{,}000k.$$

A total vertical load is found from the sum of Q_d and Q_l:

$$Q_{tot} = Q_d + Q_l = 686{,}000k$$

The horizontal wind load is calculated over one of the larger sides, where the base dimension is 265 feet and the top dimension is 160 feet. The wind force increases with height and the building tapers, so the wind force is idealized as uniform all along the building's height. (This was done in the two previous analyses also.) An average width of the tower is

$$\frac{160ft + 265ft}{2} = 212.5ft.$$



The wind force is taken as 50 psf over a width of 212.5 feet, which gives a uniform load up the height of the tower of

$$p = 212.5ft(50psf) = 1062 5lbs/ft$$
$$= 10.6k/ft.$$

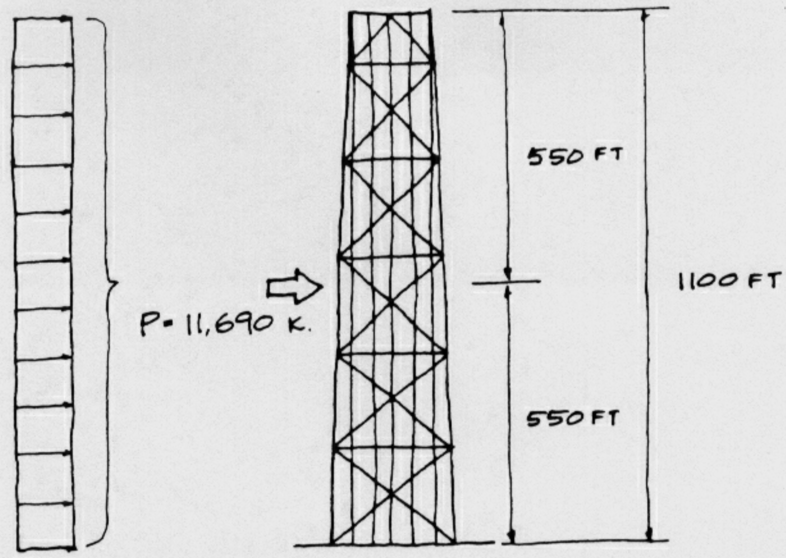
Over the height of ℓ = 1100 feet a total horizontal load is

$$P = p\ell = 10.6k/ft(1100ft)$$
$$= 11,690k.$$

This wind load acts at the midpoint of the tower, 550 feet, as shown in the diagram at the top of the next page.

Reactions

Because two different types of structure are incorporated in the Hancock Tower, the reactions are not quite as simple as



those of past analyses. The discussion of the geometry mentioned that the central core of structural steel columns takes about fifty percent of the vertical load. This overall vertical reaction, at the base of the core is

$$V_1 = 0.50(Q_{tot}) = 343,000k.$$

The remaining 343,000 kips is supported by the outside tube of columns:

$$V_2 = 343,000k.$$

The diagonals equally divide the force among the twenty-four columns of the tube, so each column takes 1/24th of this load and has its own reaction:

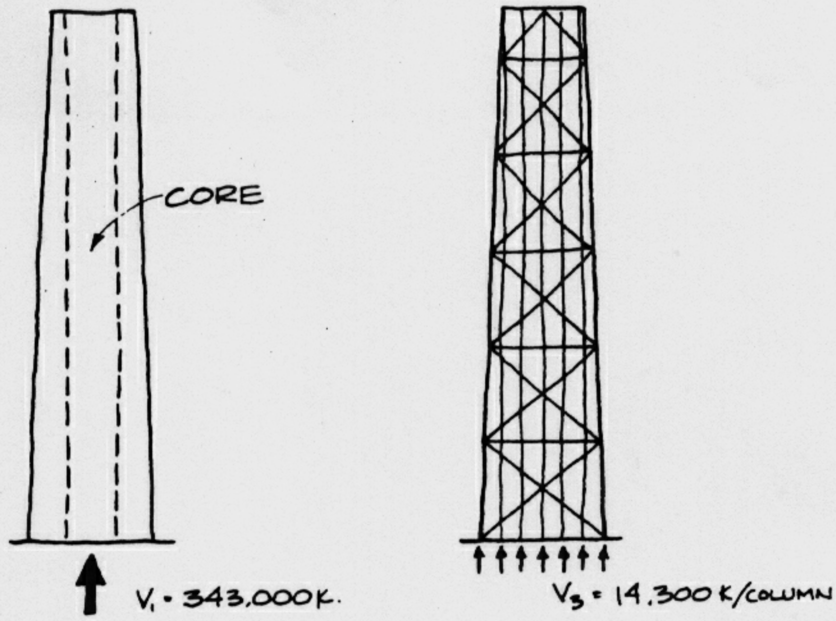
$$V_3 = 343,000k/24 \text{ columns}$$
$$= 14,300k/\text{column}.$$

The reactions V_1 and V_3 are shown on a diagram of the tower on the top of the next page.

Although both parts of the structure react to the vertical loads, only the tube reacts to the horizontal load. Therefore, it must develop both horizontal reactions and bending moment reactions. To resist the horizontal wind load, P = 11,690 kips, the tube must develop and overall horizontal reaction of

$$H_1 = P = 11,690k.$$

The horizontal reactions are idealized as developing in the two walls parallel to the direction of the wind. The diagonal braces help to transfer the force on the windward side to the two sides

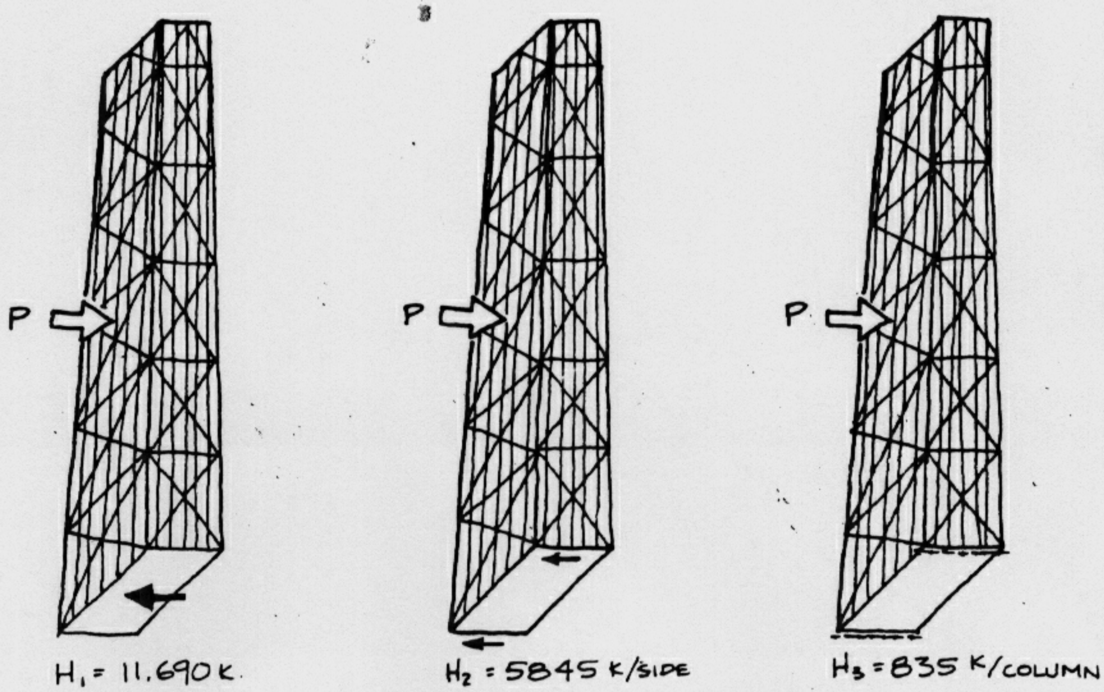


parallel to the wind. Each will develop a reaction:

$H_2 = H_1/2 = 5845k.$

On each side there are seven columns among which the horizontal reaction is divided:

$H_3 = H_2/7$ columns $= 835k.$

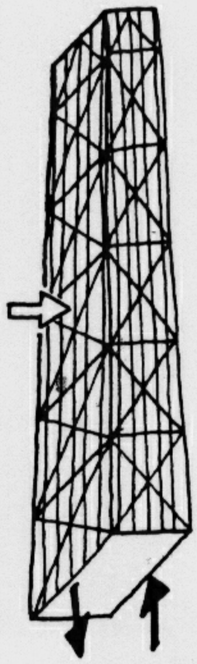


The horizontal load also creates a bending moment reaction.

The overall bending moment reaction can be found from P = 11,690 kips and $\ell/2$ = 550 feet:

$$M = P\ell/2 = 6,430,000 \text{ ftk.}$$

An idealization assumes that this moment reaction is resisted by the seven columns of each the windward side and the leeward side. The two walls act as a couple to resist the moment applied by the wind.



The total individual column reactions cannot be found yet, because the reaction force to bending has not been found. This will be found as an internal force since the reactions are equal to the internal forces at the base of the structure. Once this force is found at the base of the column, it can be added to the reaction from the dead load to find the total reaction at the base of each column.

Internal Forces

First, the forces from the vertical load are investigated at the base of the tube. The overall internal axial force is the same as the overall vertical reaction, V_2 = 343,000 kips. Similarly, the individual internal axial forces in the columns are equal to the vertical reactions found for each column:

V_3 = 14,300k.

Halfway up the tube one can find the axial forces from the vertical load, both overall and in the individual columns. The dead load is 0.165 ksf and the live load is 0.080 ksf, so the total load is q_{tot} = 0.245 ksf. The area of one floor, A = 28,000 ft^2, is used to find the load contributed by each floor:

$$Q_{floor} = q_{tot}(A_{floor}) = 6860k.$$

At the midpoint of the tower, fifty floors are exerting this vertical force, therefore, the internal vertical force at the midpoint is

$$V_4 = Q_{floor}(50 \text{ floors}) = 343,000k.$$

The actual value is less because this analysis has neglected the taper of the building. Again the core takes fifty percent of this load:

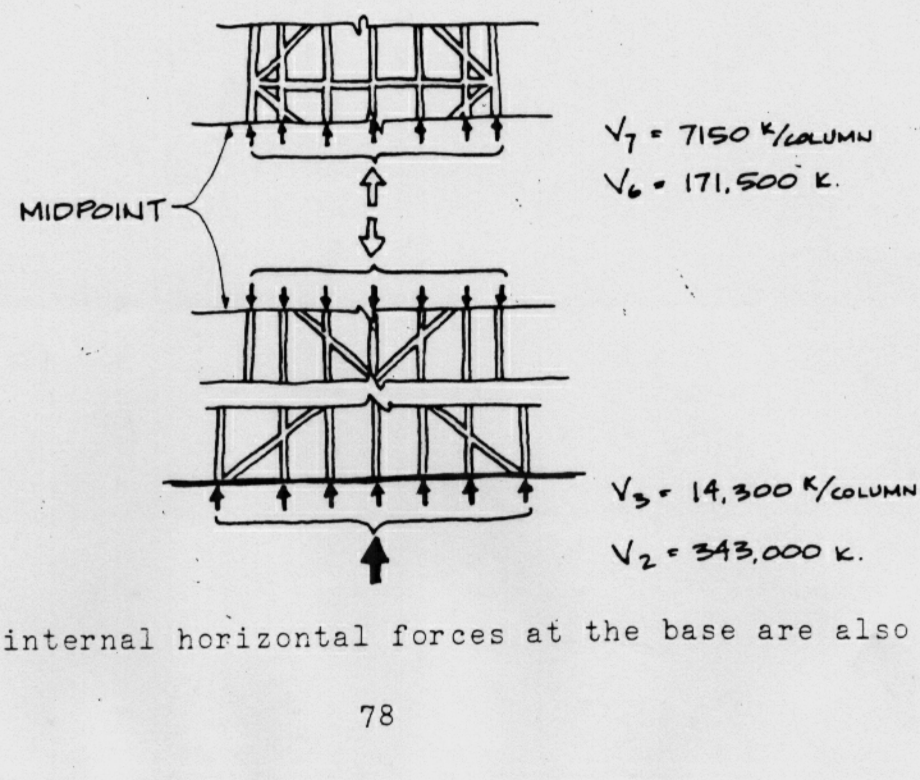
$$V_5 = V_4/2 = 171,500k,$$

and the exterior tube takes the other half:

$$V_6 = 171,500k.$$

This creates a vertical force in each exterior column at the midpoint of:

$$V_7 = V_6/24 \text{ columns} = 7150k/\text{column}.$$



The internal horizontal forces at the base are also equal to the reactions found earlier. The overall internal force at the base is equal to the overall horizontal reaction, and the internal forces in each of the column bases are equal to the reaction at the base of each of these columns. In the columns these internal forces are shear forces:

$$S_{overall} = H_1 = 11{,}690k,$$

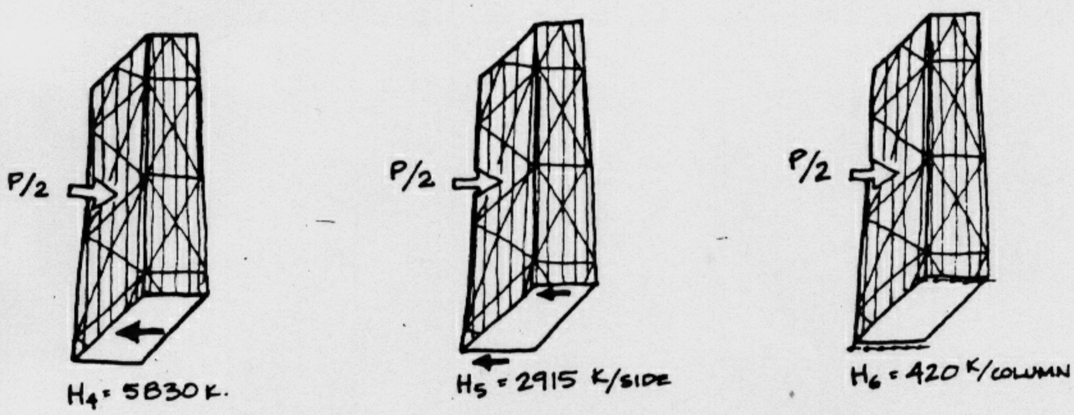
$$S_{wall} = H_2 = 5845k,$$

$$S_{column} = H_3 = 835k.$$

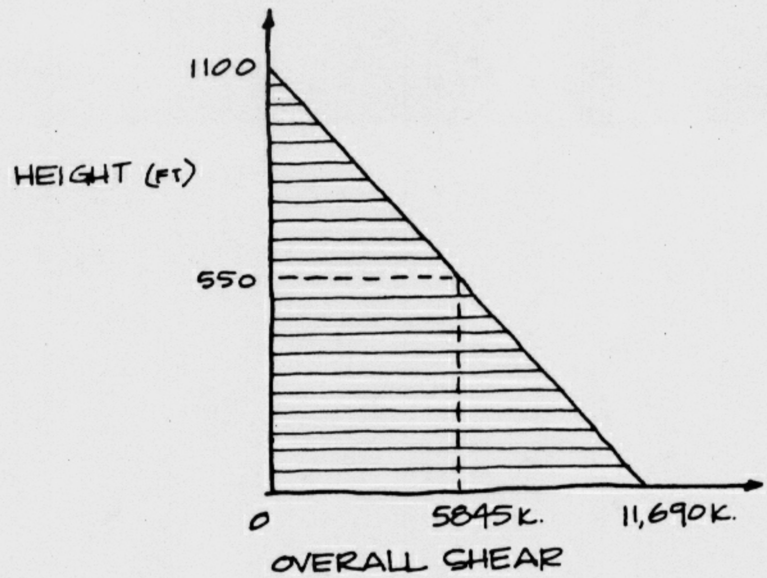
Earlier analyses have shown the shear diagram resulting from a uniform wind load. The maximum shear force value is at the base, and it decreases linearly to the top where there is no shear force. The shear force at the midpoint is found before the shear diagram is presented. Only half the wind force is acting at the midpoint:

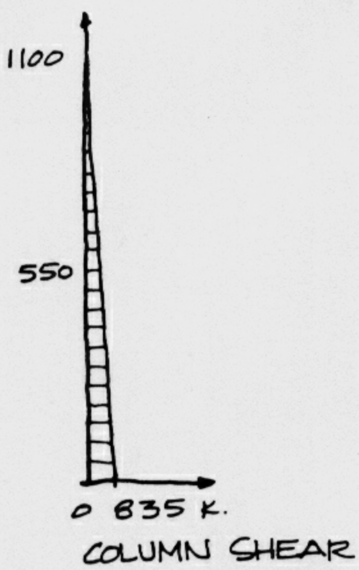
$$P_m = p\ell/2 = 10.6k/ft(550ft)$$
$$= 5830k.$$

The overall shear force developed equal the wind load, 5830 kips, and the individual shear forces in each of the fourteen columns resisting the horizontal load are as shown:



The overall force is used in the first shear diagram which is for the whole building. The three known values are called out on the diagram: those at the base, the midpoint, and the top. A shear diagram for each column can also be drawn using the corresponding values of shear force found in them. Both of these diagrams are found at the top of the next page.





To find the internal forces created by the bending forces of the wind, one uses the overall moment reaction already found: M = 6,430,000 ftkips. This reaction is created by the windward and leeward walls acting as a couple. The distance between these walls is about 165 feet, the value of d in the formula

$$T = C = \pm M/d.$$

Substituting in the values of M and d, one find the tension and compression forces acting on the entire wall. These values must be divided by seven, the number of columns in each wall, to find the values of the tension and compression forces in each of the columns.

$$T_1 = C_1 = \pm M/d = 6,430,000/165 = \pm 39,000k,$$

$$T_1 = +39,000k \ ; \quad C_1 = -39,000k.$$

In each of the seven columns resisting the moment on both sides,

$$T_2 = C_2 = \pm 39,000k/7 \text{ columns} = \pm 5570k,$$

$$T_2 = +5570k \ ; \quad C_2 = -5570k = N_W$$

These values are combined with the compressive axial forces from the dead load to find the total axial force at the base of the columns. A more accurate calculation of N_W must include the columns parallel to the wind and gives N_W = 4800 kips.[8]

Using the more approximate (and more conservative values for N_W, we find that the individual column forces are:

14,300k ± 5570k of axial compression.

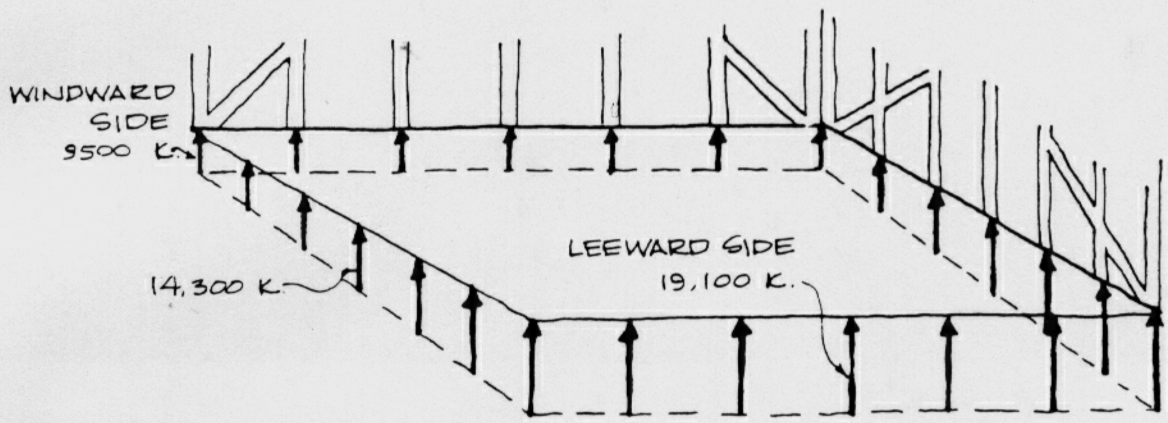
On the windward side

$$14{,}300k - 5570k = 8730k.$$

On the leeward side

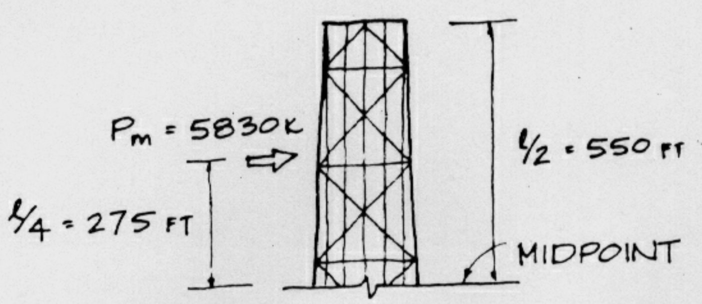
$$14.300k + 5570k = 19{,}870k.$$

On the sides parallel to the wind, the force in each column remains as 14,300 kips. These forces are shown on the following diagram. (In this diagram the forces on the sides parallel to the wind have been shown to vary linearly between the minimum and the maximum forces on the long sides. This represents the actual situation.)



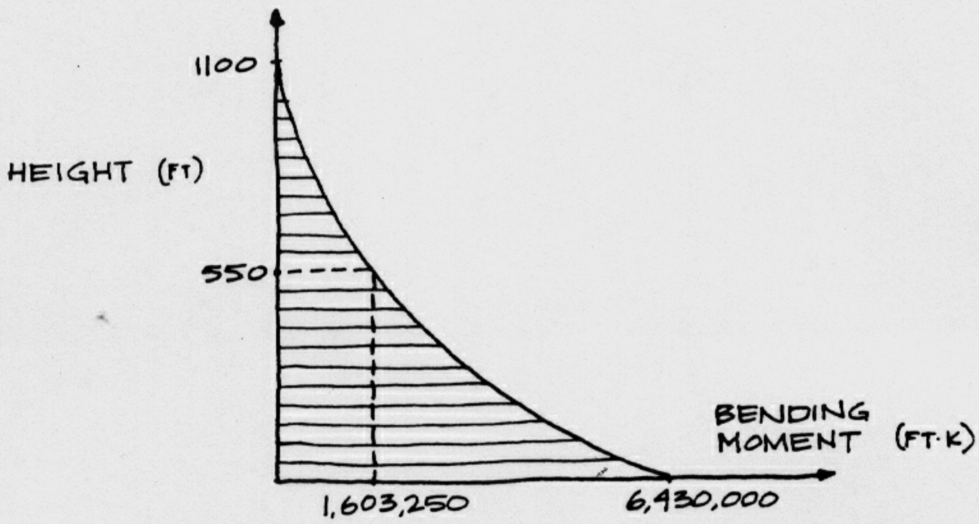
To find similar axial force values at the midpoint, the bending moment there must be found. This calculation uses the wind load acting at the top quarterpoint of the tower:

$$M_m = 5830k(275ft) = 1{,}603{,}250\ ftk.$$



Before the values of tension and compression at this point are found this bending moment can be used to help draw the overall bending moment diagram for the tower. The bending moment at the middle is one-fourth that at the base. This fits the formula for a parabola; the bending moment diagram is parabolic, just as inearlier analyses. This shape, standard for any uniformly loaded cantilever, has the formula

$$M = p(\ell - x)^2/2.$$



To find the tension and compression forces developed by the bending force at the midpoint, the width, d_m, must be found. This is the average of the width at the base, 165 feet, and the width at the top, 100 feet:

$$d_m = \frac{165ft + 100ft}{2} = 132.5ft.$$

Substituting the midpoint values of M_m and d_m into the equation for T and C one find

$$T_3 = C_3 = \pm M/d = 1,603,250/132.5 = \pm 12,100k,$$

$$T_3 = +12,100k \; ; \quad C_3 = -12,100k.$$

At each column this results in a force of

$$T_4 = +12,100k/7 \text{ columns} = +1730k/\text{colmun and}$$

$$C_4 = -12,100k/7 \text{ columns} = -1730k/\text{column}$$

at the midpoint. These values are combined with the axial force in each column from the dead load. This results in the following individual column axial forces at the midpoint:

windward side: 5420k/column

leeward side: 8880k/column

sides parallel to the wind: 7150k/column.

Stress and Efficiency

The only type of stress investigated here is the axial stress found from the total axial forces; these forces are a combination of the axial forces caused by the vertical loads and the horizontal loads. The axial stress is usually found from the formula

$$f_{axial} = N/A.$$

Rather than just repeat the calculation to find the stress, an illustration of the design procedure used by a structural engineer is given here. Once he has determined the general shape of the building and the resulting forces, the engineer uses allowable stresses to find the dimensions of his elements from the formula

$$f_{allowable} = N/A.$$

Usually a designer aims for an efficiency of 1.0, and, therefore, makes his actual stress equal to the allowable stress.

$$f_{actual} = f_{allowable} = N/A$$

In this example, prior calculations have found the axial forces, but the area of the columns are unknown. To find area, an allowable stress in steel of 20 ksi is used as the actual stress. The area is found from

$$f_{actual} = N/A$$

rearranged to

$$A = N/f_{actual} = N/f_{allowable}.$$

Substituting in the maximum value of N that a column in the building could experience at the base, and the value of f_{allow}, an area for the column is

$$A = 19{,}870k/20ksi = 994in^2.$$

The colums must therefore have an area of almost 1000 square inches to withstand the maximum loads the building could experience without any danger or drastic deformations. By using the allowable stress as a design value, the safety has been insured - safety factors are essentially built into the allowable stress values.

This design procedure is repeated for the midpoint of the outer columns to serve as a further illustration of this important method. This also illustrates how the area of the columns decreases as the loads decrease higher in the buildings. The maximum axial force found at the midpoint is 8880 kips. Using the same formula, the area is

$$A = 8880k/20ksi = 444in^2.$$

This area is less than half that necessary at the base because the bending force drops off quickly near the base. This can be seen in the moment diagram shown earlier.

The analysis has shown the basic actions of the combined cantilevered column form. The structures of these first three analyses are very similar and only small differences separate them. The same basic methods of analysis are used in each, with the difference coming from the proportions of gravity and wind forces found from the formula

$$\frac{N_G + N_W}{N_G} < \frac{4}{3} \quad \text{(for a column).}$$

For the calculations that assume all the wind force is resisted by the two rows of seven columns on the wide sides

$$\frac{N_G + N_W}{N_G} = \frac{14,300 + 5570}{14,300} = 1.39 > 4/3$$

For the more accurate N_W = 4800 kips (reference 8) then the ratio is found as precisely 1.33.

Individually, the Hancock Tower is important because of its innovative tubular structure with diagonal braces connecting all the columns so they act as a unit - a tube - rather than as a frame. A framed structure does not have the freedom in plan that the tube offers. The tube, made rigid by the diagonals, creates a light stiff structure that is an especially efficient way to resist the forces on the structure and conviently enclose the building spaces.

Summary of Formulas

Geometry:

$$\frac{N_G + N_W}{N_G} < \frac{4}{3}$$ for a columnar structure.

$$\text{average tower width} = \frac{\text{base width} + \text{top width}}{2}$$

Loads:

dead and live load

Q_d or Q_l = area(q_d or q_l)

$Q_{tot} = Q_d + Q_l$; $q_{tot} = q_d + q_l$

$Q_{floor} = q_{tot}(A_{floor})$

p = building width(pressure of wind)

$P = p\ell$

$P_m = p\ell/2$

Reactions:

$V_1 = 0.50(Q_{tot})$

$V_2 = V_1$

$V_3 = V_2/24$ columns

$H_1 = P$

$H_2 = H_1/2$

$H_3 = H_2/7$ columns

$M = P\ell/2$

Internal Forces:

$V_4 = Q_{floor}$(number of floors)

$V_5 = V_4/2$

$V_6 = V_5$

$V_7 = V_6/24$ columns

$S_{overall} = H_1$

$S_{wall} = H_2$

$S_{column} = H_3$

$H_4 = P_m$

$H_5 = P_m/2$

$H_6 = H_5/7$ columns

$M_m = P\ell/4$

$T = C = \pm M/d$

$M = p(\ell - x)^2/2$

total axial forces found by superposition (addition)

Stress and Efficiency:

$f_{axial} = N/A$

$f_{actual} = f_{allowable} = N/A$

$A = N/f_{actual} = N/f_{allowable}$

Problem:

1. Using the values of the dead, live, and wind loads given, calculate the internal axial forces in the windward and leeward columns at the lower quarterpoint, i.e.,at a height of 275 feet. Use the method and formulas from the analysis.

Notes

1 The principle structural engineer of the designing firm, Fazlur Khan, has written three different articles specifically about the Hancock Tower:

- a "Computer Design of 100-Story John Hancock Center", American Society of Civil Engineers, Proceedings, *Journal of the Structural Division*, volume 92, December 1966.
- b "John Hancock Center", *Civil Engineering*, volume 37, number 10, October 1967.
- c "100-Story John Hancock Center in Chicago - A Case Study of the Design Process", *International Association for Bridge and Structural Engineering Journal*, J-16/82, August 1982.

2 Further discussions of the building's scientific, social, and symbolic meanings can be found in: Billington, David P., *Structures and the Urban Environment: Lecture Notes and Structural Studies*, Princeton, New Jersey: Princeton University Department of Civil Engineering.

3 *Ibid*.

4 A discussion of the selection of the Tower's geometry can be found in Khan's article "100-Story John Hancock Center in Chicago - A Case Study of the Design Process." In this article he explains the rationale behind one tower (versus two), the tube structure, the diagonal braces, and the truncated pyramid shape.

5 Billington, David P., *Structures and the Urban Environment: Lecture Notes*.

6 Drawings from Billington, David P., *Structures and the Urban Environment: Structural Studies*.

7 In his article "100-Story John Hancock Center in Chicago - A Case Study of the Design Process", Khan explains the taper as creating proper floor areas for offices (lower and larger) and apartments (upper and smaller).

8 The moment is now resisted by two rows of: 7 columns separated by the distance d, 2 columns separated by 2d/3, and 2 columns separated by d/3. Therefore,

$$M = 7Cd + 2C_b 2d/3 + 2C_c d/3.$$

We assume that the forces in each column are linearly proportional to its distance from the centroidal axis, which means that $C_b = 2C/3$ and $C_c = C/3$ so that

$$M = 7Cd + 8Cd/9 + 2Cd/9 = 73Cd/9 \text{ and}$$

$$C = M/8.11 = 39{,}000\text{ft-k}/(8.11\text{ ft}) = 4800 \text{ kips.}$$