#### **STRUCTURES**

and

The Urban Environment

#### STRUCTURAL STUDIES 1983

Beans - Mahazzini Gerbyrali Warehouse . . . . . .

David P. Billington

Robert Mark

in collaboration with

Julian Dumitrescu

Lisa Grebner Mulvey

Flying Buttress Systems - Bourges Cathedral

and Chartres Cathedral

Flying Buttress Systems - Amiens Cathedral . .

Copyright (c) David P. Billington and Robert Mark
Published by the Department of Civil Engineering,
Princeton University

ARCHES:

# SALGINATOBEL BRIDGE

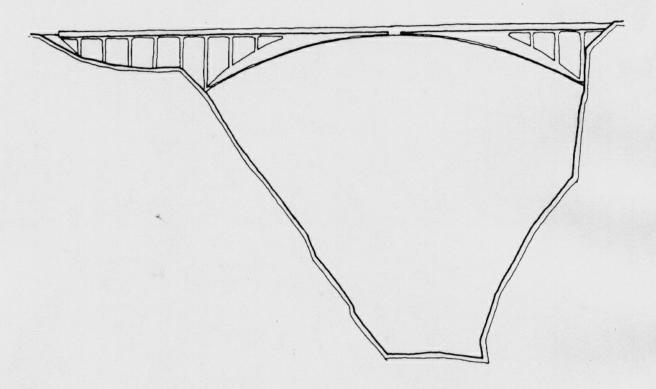


The Salginatobel Bridge in Shiers, Switzerland is perhaps the most famous and influential of Robert Maillart's bridges. The bridge, high up in the Swiss Alps, crosses a deep ravine to a small town, Shuders. The bridge is only about ten feet wide and was designed for pedest ians, animals, bicycles, and even cars and trucks. It springs from one very steep mountainside to a nearly shear one. The lightness and shallowness of its form make it a graceful crossing, as well as an efficient structure that was not exceptionally difficult nor costly to build. 1,2,3

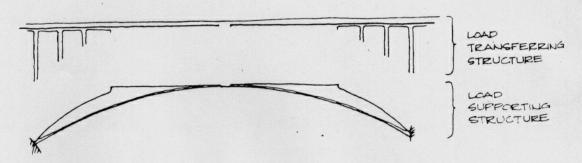
The design is the synthesis and abstraction of elements from Maillart's earlier bridges. The bridge represents thirty-four years worth of refinements to some very bold and beautiful ideas about both aesthetics and engineering. As Maillart evaluated the appearances and the structural responses of his earlier

bridges, he could modify his later designs accordingly. There is not a single new element in the Salginatobel bridge design, but the new form Maillart created through this synthesis is ideally suited to its purpose and place.

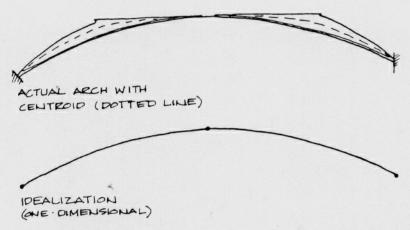
## Geometry



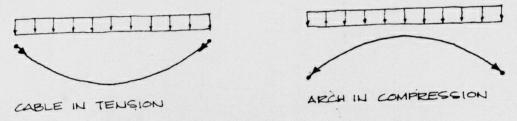
The basic structure of the Salginatobel Bridge is a three hinged arch. The road deck and the connecting verticals transfer the traffic and dead loads to the arch, which, in turn, transfers the loads to the supports on the mountainsides. The arch, however, is the stiff part of the structure, i.e., the part that makes spanning the ravine possible.



For analysis, the bridge must be simplified. First, the deck and the verticals are neglected, and only the supporting structure of the three-hinged arch is considered. This arch is further idealized into a one-dimensional model that is a parabola since the centroid of the U-shaped box section is close enough to a parabola to make the idealization valid.



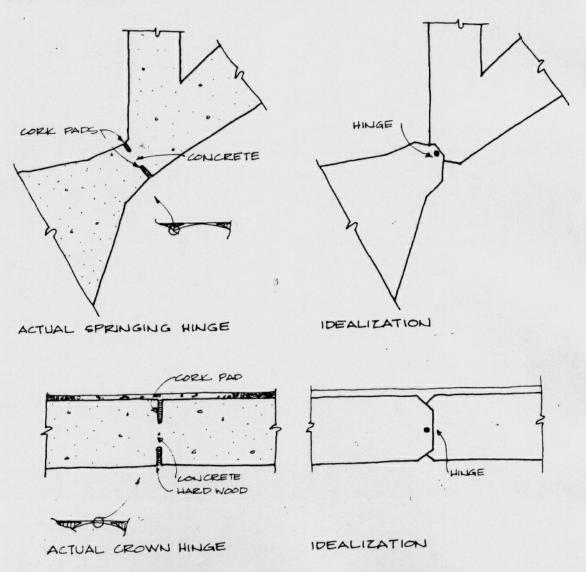
The basic shape of the arch - almost a parabola - was chosen because no bending forces are created in it from a uniform load. Recalling the analysis of the George Washington Bridge where the cables resisted only tension and hung in a parabolic shape under a uniform load, one can see the inverse of this principle in a parabolic arch. If a cable resists only tension from a uniform load, the arch will resist the opposite - only compression - from a uniform load.



Bending forces are not created in either structure from a uniform load. Non-uniform loads will cause the flexible cable to change shape to maintain pure tension forces, but the stiff arch, which cannot change its shape, must be strong enough to resist concentrated, varying loads and keep its shape. This will be further investigated in the analysis. At this point one must only realize the purpose of the parabolic shape.

The derivation of the shape can be found in the analysis of the Chiasso Warehouse shed.

The bridge was designed with the three hinges - one at each springing (end) and one at the crown (top). In the actual structure the hinges are made by decreasing the concrete section to the absolute minimum that is necessary to resist the compressive forces in the concrete. This analysis idealizes these hinges with the simpler, pinned hinges which are absolutely free to rotate and cannot transmit any bending moment forces.



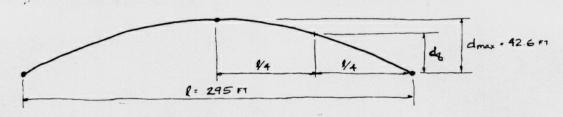
Incorporating three hinges into the structure allows the two supports to move relative to each other without creating any stresses in the arch itself. This is illustrated in the following diagrams where one of the supports moves while the other

remains fixed. (This would occur if the foundation should settle or even if the mountain should move.)



Support movement does not stress either half of the arch; the hinges merely adjust to the new support conditions. This flexibility is especially important to compensate for temperature changes in the bridge itself; the arch can either shrink or expand without creating internal stresses. The hinges also keep the overall arch from experiencing bending moments created by a large concentrated load on either half of the arch. If half of the arch is under a load, the hinges do not transmit any bending moments, so the other half does not experience any bending stresses. Also, with hinged supports, no bending moment reactions are created.

The dimensions of the idealization of the bridge are taken from the actual bridge:  $^{6}$ 

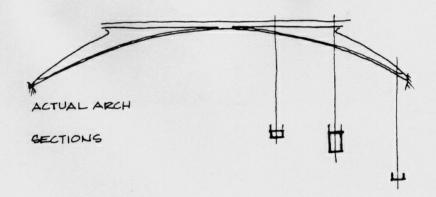


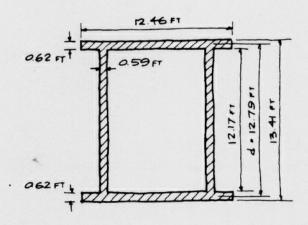
At the quarterpoint the depth,  $d_q$ , is three-fourths of the maximum,  $d_{max} = 42.6$  feet, because the arch is idealized as a parabola. This gives a depth,  $d_q$ , of

$$3d_{\text{max}}/4 = 3/4(42.6 \text{ feet}) = 31.95 \text{ feet}.$$

The actual arch has basically these same dimensions as well as section dimensions (the idealization is one-dimensional). (Refer to the diagram at the top of the next page.) The second diagram on the next page illustrates the quarterpoint section. This section has an area of

$$A = 2(0.62)(12.46)ft^{2} + 2(0.59)(12.17)ft^{2}$$
$$= 29.8ft^{2} = 4291in^{2}.$$



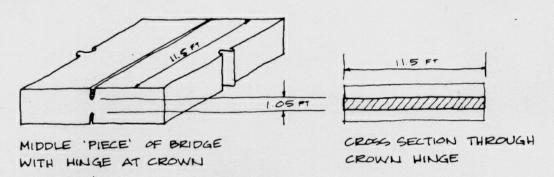


The flanges each have an area of

$$A_f = (0.62)(12.46)ft^2$$
  
= 7.73ft<sup>2</sup> = 1113in<sup>2</sup>.

The depth of the section is between the midpoints of the flanges: d = 12.79 feet.

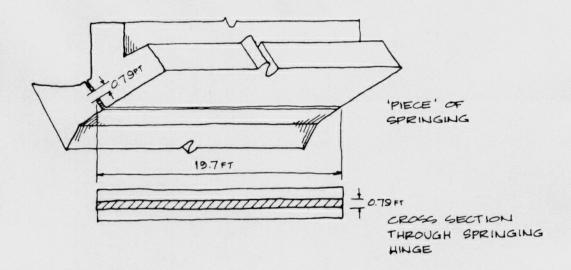
At the crown the section is the rectangle of concrete in the hinge.



The area of this concrete is

$$A_{cr} = (1.05)(11.48)ft^2$$
  
= 12.05ft<sup>2</sup> = 1735in<sup>2</sup>.

At the springing hinge the rectangular area has the following dimensions:

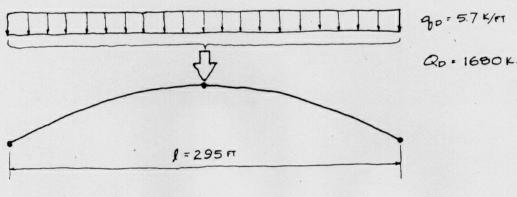


The area of concrete at the springing is then  $A_{sp} = (0.79)(19.68)ft^{2}$   $= 15.5 ft^{2} = 2240 in^{2}.$ 

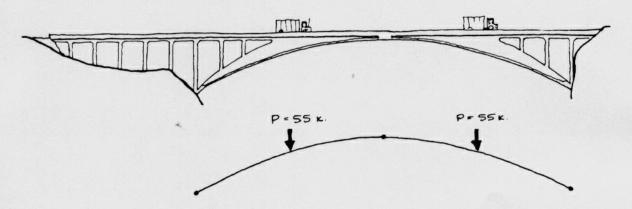
# Loads

Three loads act on the bridge: dead and live loads from the weight of the bridge and from the traffic and snow, respectively. The dead load is idealized as being distributed uniformly along the bridge span although it is actually greater where the arch is deeper and where the verticals are longer. Maillart found a total dead load of nearly 1680 kips. In his calculation this was not uniformly distributed, but for simplicity this analysis assumes that it is:

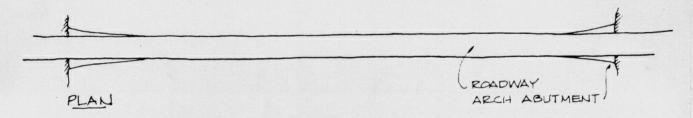
q = 1680 kips/29; feet = 5.7 kips/foot.



In Maillart's analysis a snow load was used; this analysis neglects the snow load for simplicity. The live traffic loads are not so easily defined as they constantly change with the traffic traveling over the bridge. However, large bending stresses are created in the arch when live loads are applied at the quarterpoints. This would happen on the actual bridge when a large load, such as a truck, has traveled either one or three quarters of the way acrosss. This analysis uses two point loads of 55 kips at each quarterpoint to create some of the larger stresses that could occur in the bridge.



Wind loads are not considered in this analysis. Although there is very little area of bridge facing the wind, there are still wind forces on it. These are resisted by increasing the width of the arch - it increases near the springings, as can be seen from the plan. (This is a horizontal example of the principle that was demonstrated by the Eiffel Tower.)



The live load considered has a total of 110 kips and the dead load has a total of 1680 kips. The total load is, therefore,

 $\rm Q_{tot}$  = 1680 kips + 110 kips = 1790 kips. Recalling the discussion of the geometry which said that the

arch takes all the dead load (and uniform loads, e.g., snow) in compression because of its parabolic shape, one can see that these create the largest internal force. This force is quite large enough to overcome any bending forces created by the non-uniform live loads and thus prevent any tension forces from being created in the concrete structure; this will be demonstrated in the analysis.

#### Reactions

A dead load of 1680 kips and a live load of 110 kips act vertically downward and must be resisted by vertical reactions in the supports. The reactions and forces due to dead and live loads will be kept separate for clarity since they are distributed differently and therefore have reactions that act at different angles. The reactions are found by taking moment balances about support A (none of the hinges can resist any moment). The dead load is idealized as a point load of Q = 1680 kips acting at the midpoint of the bridge.

due to dead load:

$$\sum M_A = 0 = Q(\ell/2) - V_{Bd}\ell$$

due to live load:

$$\sum M_A = 0 = P(\ell/4) + P(3\ell/4) - V_{B1}\ell$$

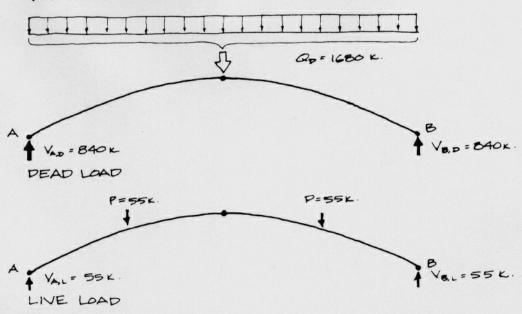
When these equations are solved with P = 55 kips, Q = 1680 kips and  $\ell$  = 295 feet,  $V_{Bd}$  is 840 kips and  $V_{Bl}$  is 55 kips. Vertical equilibrium finds  $V_{Ad}$  and  $V_{Al}$  equal to  $V_{Bd}$  and  $V_{Bl}$ , respectively.

$$zv = 0 = Q - V_{Ad} - V_{Bd}$$
  
 $zv = 0 = 2P - V_{A1} - V_{B1}$ 

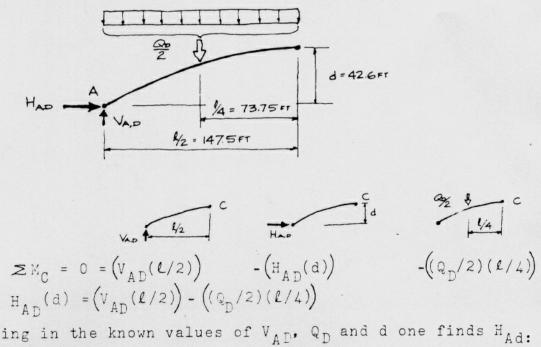
(The diagram at the top of the next page illustrates how the vertical reactions are found.)

To support the arch and its inclined axial force, the abutment must have horizontal reactions as well as vertical ones. These are found using the moment balance prinicple; one is taken about the crown hinge - point C - using one side of the arch. This is

#### VERTICAL REACTIONS:



first done with the dead load where  $Q_{\rm D}/2$  = 840 kips.



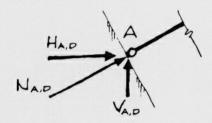
Substituting in the known values of  $V_{AD}$ ,  $Q_D$  and d one finds  $H_{Ad}$ :

$$H_{AD} = (840k)(147.5ft) - (840k)(73.75ft)$$
  
= 1455 kips.

To create horizontal equilibrium, the reaction at the other support, point B, must be equal since there are no other horizontal forces, loads or reactions.



The horizontal and vertical reactions just found can be thought of as components of an overall reaction.



The magnitude of the diagonal reaction is found from its components from the formula:

$$N_{Ad} = H_{Ad} \cos \alpha + V_{Ad} \sin \alpha$$

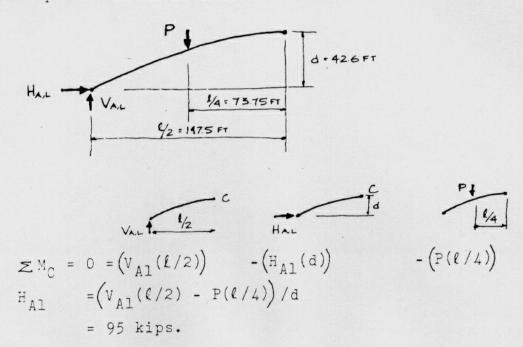
where  $\alpha$ , the angle that the arch meets the supports, is found by

$$\tan \alpha = V/H = 840/1455 = 0.577$$

$$\alpha = \tan^{-1}(0.577) = 30^{\circ}.$$

Then, 
$$N_{Ad} = (1455k)(\cos 30^{\circ}) + (840k)(\sin 30^{\circ})$$
  
= 1680 kips.

This procedure is repeated using the live loads; the moment balance is set up and solved in a similar manner:





The resulting diagonal force is

 $N_1 = H_1 \cos \alpha + V_1 \sin \alpha$ = (95k)(cos 30°) + (55k)(sin 30°) = 110 kips.

The diagonal reaction, N, created by a uniform load on a parabola acts along the axis of the arch. The diagonal reaction to concentrated loads, like the live loads, will not; it will have a slightly different angle. This is because a parabola is the ideal shape for supporting uniform loads, not concentrated ones.

# Internal Forces

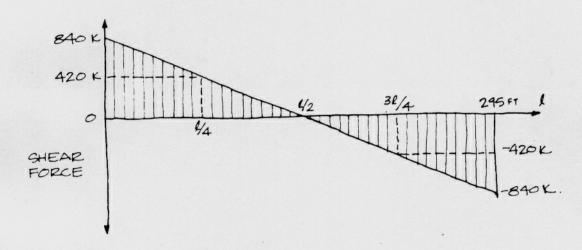
The loads and reactions can now be used to find the internal forces and to explain the shape of the arch more clearly. First the axial and shear forces are investigated, and then the bending moments. The dead and live loads will be considered separately in the investigation of the axial forces because the reactions and forces are somewhat different. The dead load is uniform, whereas the live load is concentrated; different types of loading create slightly different internal forces and reactions at different angles, so the loads are kept separate. The reactions are equal to the diagonal forces at the points where the arch meets its supports. The vertical force component decreases along the arch, whereas the horizontal component remains the same all along the arch. No horizontal loads are applied so the force is constant all along the entire span. At any point there is a compressive force of 1455 kips + 95 kips, the horizontal forces from dead and live loads respectively.

The vertical components vary as the shear force varies along the span. At the supports, the vertical reaction, the vertical component of the axial force, and the shear force all

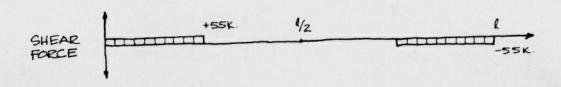
have the same values: 840 kips from the dead load and 55 kips from the live load. The dead load shear forces (or the vertical components of the diagonal force) can be found anywhere along the span from the formula for shear force in a uniformly loaded span:

 $S = q(\ell/2 - x).$ 

With a dead load of q = 5.7 kips/foot, the following shear diagram can be drawn:

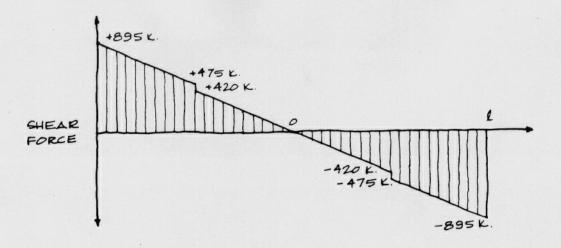


The concentrated live loads will create uniform shear forces between themselves and the supports. There is no shear in the middle of the arch between the point loads, and the resulting shear diagram is shown:



These shear diagrams are superimposed to find the actual shear from both loads. (The total shear force diagram is at the top of the next page.)

As mentioned before, the value of the shear is the value of the vertical component of the diagonal force. These vertical values are used with the constant horizontal components - 1455



and 95 kips - and with the angle of the slope of the arch, to find the diagonal force at any point from the formula:

$$N = H(\cos \alpha) + v(\sin \alpha).$$

The forces at the springings of the arch are equal to the diagonal reactions found earlier:  $N_{\rm sp,d} = -1680$  kips from the dead load and  $N_{\rm sp,l} = -110$  kips from the live load. At the quarterpoint, where the vertical components are 420 kips (dead) and 55 kips (live), the angle is

$$\tan \alpha = 2d/L$$
;  $\alpha = 16.1^{\circ}$ .

The axial forces are then

$$N_{qd} = (-1455k)(\cos 16.1^{\circ}) + (-420k)(\sin 16.1^{\circ})$$
  
= -1514 kips.

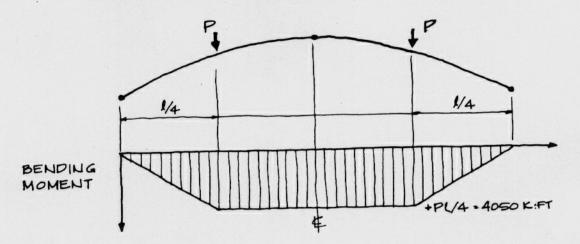
$$N_{ql} = (-95k)(\cos 16.1^{\circ}) + (-55k)(\sin 16.1^{\circ})$$
  
= -106 kips.

At the crown, where there is no vertical component, the axial forces are equal to the horizontal force:  $N_{\rm cr,d} = -1455 \, {\rm kips}$  (dead), and  $N_{\rm cr,l} = -95 \, {\rm kips}$  (live).

To find the total axial forces, the bending moment created by the live loads must be considered also. A uniform load on a parabolic arch does not create bending forces, only axial forces - those have been found. Therefore, only the 55 kip point loads must be analyzed to find bending forces. This is done in two steps: first the bending moments created by the live load alone are found, and then these are superimposed over the bending moments created by the horizontal reactions. Two

point loads at the quarterpoints create a maximum bending moment of P $\ell/4$  which is positive and constant between them.

$$P\ell/4 = (55k)(295ft)/4 = 4050 \text{ kips.}$$

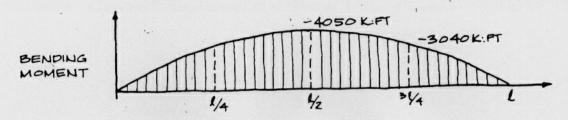


(This bending moment diagram is identical for either a flat or an arched span.) The bending moments from the horizontal reaction are described by

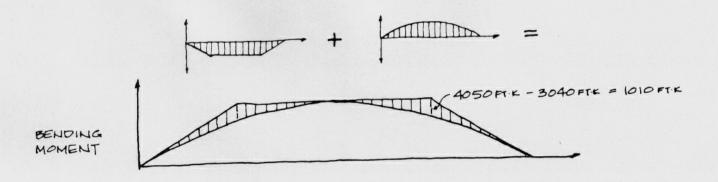
$$M = Hd_x$$

where d<sub>x</sub> is the depth of the arch at any point 'x' along the span. The bending moment is a maximum at the middle, and the diagram is the same parabolic shape as the arch. The bending moments are all negative because the horizontal reaction is acting counterclockwise whereas the load is acting clockwise (around the left-hand support).

$$M = -Hd_{x} = (-95k)d_{x}$$
where  $d_{x} = d_{max} = 42.6ft$ ,  $M = -4050ft$ -kips,
where  $d_{x} = 3d_{max}/4 = 31.95ft$ ,  $M = -3040ft$ -kips.



The overall bending moment from the live loads is found by adding these two bending moment diagram. (The superposition is shown at the top of the next page.)



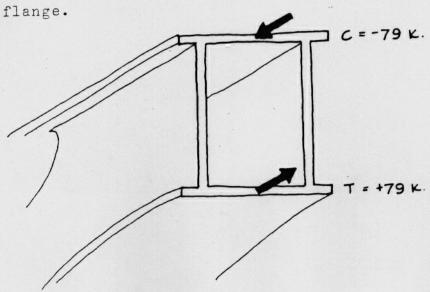
The remaining moment is positive. One can see how the actual shape of the arch follows this bending moment diagram for one of the most critical live load cases. This again illustrates the principle found in other analyses that an efficient structure imitates the shape of its bending force diagram. By proportioning a structure in this manner, there is enough material to resist forces where they are high, and conversely, where forces are low, there is not an excess of material.

Since the bending moment is greatest at the quarterpoints, these are the critical sections at which to analyze the axial force created by the bending moment. Using the formula from earlier analyses, +T = -C = M/d, with the M just found and the value of d at the quarterpoint section, 12.8 feet, the tension and compression forces resulting from the bending are found:

 $\pm M/d = 1010ft-kips/12.8 feet = \pm 79 kips,$ 

T = +79 kips; C = -79 kips.

The tension occurs in the bottom flange and the compression in the top flange.



Once the stresses are found for the axial force, they can be combined to find totals for the bridge.

## Internal Stresses

The axial stress at the springings is calculated first. Axial forces of  $N_{\rm sp,d}$  = -1680 kips and  $N_{\rm sp,l}$  = -110 kips were found from the dead and live loads. The area of concrete at the springing is  $A_{\rm sp}$  = 2240in<sup>2</sup>. Then the axial stresses from each load are:

$$f_{sp,d} = N_{sp,d}/A_{sp} = -1680k/2240in^2$$
  
= -0.75 ksi = -750 psi,  
 $f_{sp,l} = N_{sp,l}/A_{sp} = -110k/2240in^2$   
= -0.049 ksi = -49 psi

Then, at the springing, there is a total compressive stress of

 $f_{sp} = -750 \text{ psi} - 49 \text{ psi} = -799 \text{ psi}$ 

The same calculations are repeated to find the axial stresses at the crown, where  $A_{cr} = 1735in^2$ ,  $N_{cr,d} = -1455$  kips and  $N_{cr,l} = -95$  kips:

$$f_{cr,d} = N_{cr,d}/A_{cr} = -1455k/1735in^2$$
  
= -0.839 ksi = -839psi.  
 $f_{cr,l} = N_{cr,l}/A_{cr} = -95k/1735in^2$   
= -0.055 ksi = -55 psi.

The total compressive stress at the crown is  $f_{cr} = -839 \text{ psi} - 55 \text{ psi} = -894 \text{ psi}.$ 

The stresses at the quarterpoints are found from the axial compression forces from the dead and live loads ( $N_{qd} = -1514$  kips and  $N_{ql} = -106$  kips) and the area of that section found earlier ( $A_q = 4291$  in  $^2$ ). The axial forces found from the dead and live loads act across the entire section, therefore, the dead load stress is

$$f_{q,d,ax} = N_{qd}/A_q = -1514k/4291in^2$$
  
= -0.353 ksi = -353 psi,

and the live load stress is

$$f_{q,l,ax} = N_{ql}/A_q = -106k/4291in^2$$
  
= -0.025 ksi = -25 psi.

Together these create a total stress of -378 psi. The axial force created by the bending force from the live loads -  $\pm$ 79 kips - acts over the flange with an area of 1113in<sup>2</sup>. This yields a stress of

 $f_{\text{c,l,bend}} = (T \text{ or } C)/A_f = \pm 79k/1113in^2 = \pm 71 \text{ psi,}$ 

T = +71 psi in the bottom flange, and

C = -71 psi in the top flange.

These values must be added to the uniform stress of -378 psi across the section from the dead and live loads to find the total stress at the quarterpoints:

top flange: -378 psi - 71 psi = -449 psi (maximum)
bottom flange: -378 psi + 71 psi = -307 psi (minimum).

In this analysis many assumptions have been made for clarity and simplicity, and, while they are not incorrect, they do bring a certain amount of variance into the final answers. A more rigorous analysis with fewer assumptions has led to values very close to the ones Maillart determined while designing the bridge. 10 The more thorough analysis used the actual dead load from the shape of the arch. The analysis did not use the parabola assumed here, but the pressure line of the actual arch, which is the line of the bending moment diagram created by the actual loads. This line is slightly different than the line of the centroid of the section and would have resulted in no bending stresses from the uniform load. In the rigorous analysis as well as Maillart's analysis, some bending from the dead load was found. In addition greater bending forces were created by an assymetrical live load. This rigorous analysis used a single point load, whereas Maillart found a segment of a uniform live load that created the absolute maximum live load moment. Both analyses used more accurate section - the dimensions were not idealized - and this resulted in more precise bending moments. The rigorous analysis yielded results at the quarterpoints of

$$f_{max} = -675 \text{ psi and}$$

and Maillart found corresponding values of

 $f_{max} = -645 \text{ psi and}$  $f_{min} = +15.6 \text{ psi.}$ 

(The values of tension are small enough to be taken by the concrete.) These values are much closer than the ones from the analysis performed here. The greater accuracy can be attributed to the greater detail and thoroughness of the analyses.

# Summary of Formulas

Geometry: formula of parabola used to idealize the arch:  $d_x = d_m (4x(\ell-x)/\ell^2) = 42.6(4x(295-x)/295^2)$ = 0.002(295-x)xangle of the parabola at any point  $\tan \alpha = (4d_m/l)(1-2x)/l = 0.002(295-2x)$ quarterpoint:  $d_{q} = 3/4(d_{max})$ areas: Z(height)(width) Loads:  $Q_d = q_d(l)$  $Q_1 = 2P$  $Q_{tot} = Q_d + Q_1$ Reactions:  $\leq M_A = 0 = Q(\ell/2) - V_{Bd}(\ell)$  $z_{M_A} = 0 = P(\ell/4) + P(3\ell/4) - V_{B1}(\ell)$ ZV = 0 = Q - VAd - VBd  $= V_1 = 0 = 2P - V_{A1} - V_{B1}$  $H_{Ad}(d) = V_{Ad}(\ell/2) - Q_{d}(\ell/8)$  $\Sigma H = 0 = H_A - H_B$  for either dead or live loads  $H_{\Lambda \gamma}(d) = V_{\Lambda \gamma}(\ell/2) - P(\ell/4)$  $N = H(\cos \alpha) + V(\sin \alpha)$  where  $\alpha = \tan^{-1}(V/H)$ Internal Forces: shear force = vertical force =  $S = q(\ell/2 - x)$  $N = H(\cos \alpha) + V(\sin \alpha)$  where H is constant, V = shearforce and is found from the equation in "Geometry'.

 $M = P(\ell/4) - Hd_x$  for the live load moment

+T = -C = +M/d

Internal Stresses:

axial (for springing, crown or quarterpoint)

$$f_{dead} = N_d/A$$
 and  $f_{live} = N_l/A$ 

bending (live load at the quarterpoint)

axial and bending stresses superimposed to find totals

## Problems:

- 1. For the parabolic 3-hinged arch with a span of l = 524 feet and a midpoint rise of 187 feet, compute the midpoint, quarterpoint, and support compression axial forces for a load of 3.6 kips/foot uniformly distributed along a horizontal line.
- 2. A three-hinged arch spanning 200 feet is uniformly loaded vertically by 2 kips/foot. The rise at the midpoint is 20 feet, and the arch is constructed so that its form counteracts the bending moment due to the uniform loading.
  - a. At the quarterpoint, compute the horizontal and vertical forces and the resulting axial force in the arch.
  - b. This arch is now loaded by two concentrated 10 kip loads, one at each quarterpoint. At the quarterpoint, calculate the horizontal and vertical forces and the bending moment in the arch due only to the concentrated loads.
  - c. Draw the final moment diagram resulting from the two concentrated loads. How can the arch be designed to resist this moment?

### Notes

- Background on the bridge is provided from a biographical point of view by Max Bill in his book, Robert Maillart, 3rd edition, 1969.
- The bridge is discussed from an architectural point of view in Siegfried Giedion's Space, Time and Architecture, 5th edition, 1967, pp. 450 -476.
- The bridge is considered as a work of both art and enigneering in David P, Billington's Robert Maillart's Bridges:

  The Art of Engineering, Princeton, New Jersey: Princeton University Press, 1979.
- Ibid., This book references actual load tests performed by Mikro Roš on the Salginatobel Bridge: "Bericht zu den Ausfuhrungsplanen der Strassenbrucke uber das Salginatobel,' sent to J. Solen and dated July 20, 1929, 5 pages, Princeton Maillart Archives.
- 5 <u>Ibid.</u>, The drawings of these hinges were taken from this source, pp. 86 87.
- The dimensions were taken from: Billington, David P.,

  Structures and the Urban Environment: Structural Studies 1982,

  Princeton University Department of Civil Engineering.
- These dead loads were found from "Official Documents for the Arch Analysis", in <u>Background Papers</u> for the Second National Conference on Civil Engineering: History, Heritage, and the Humanities II, October 4, 5, 6, 1972, Princeton University, Editied by John F. Abel, Conference Co-directors: David P. Billington and Robert Mark, Conference sponsored by the National Endowment for the Humanities.
- A large uniform live load creates the worst axial stresses, but live loads over certain parts of the bridge create the worst bending stresses. Maillart found and used these in his analysis. To simplify the situation, the live load configuration from David P. Billington's Structural Studies 1980 is used in this analysis.
- The wind load considerations for a bridge like this are different than those for a bridge like the George Washington Bridge. Suspension bridges are flexible by nature (and usually have much longer spans) and are therefore more susceptible to dangerous wind influences. The Salginatobel Bridge is stiff, so wind forces will require an analysis similar to that used on the Eiffel Tower.
- This analysis is performed in: Billington, David P., Structures and the Urban Environment: Structural Studies 1982.